ENGINEERING TRANSACTIONS • *Engng. Trans.* • **64**, 4, 449–456, 2016
Polish Academy of Sciences • Institute of Fundamental Technological Research (IPPT PAN)
National Engineering School of Metz (ENIM) • Poznan University of Technology

Multiobjective Optimization in Two-scale Thermoelastic Problems for Porous Solids

Adam DŁUGOSZ, Tomasz SCHLIETER

Institute of Computational Mechanics and Engineering
Faculty of Mechanical Engineering
Silesian University of Technology
Konarskiego 18a, 44-100 Gliwice, Poland
e-mail: {adam.dlugosz, tomasz.schlieter}@polsl.pl

The multiobjective optimization of a two-scale thermoelastic problem is considered in this paper. To compute the solutions, direct thermoelastic analysis with the representative volume element (RVE) and the finite element method (FEM) analysis are performed. Evolutionary algorithms (EAs) are used to find a set of Pareto-optimal solutions. The design variables of the optimization problem are defined so as to describe the microstructure of a porous solid, whereas the optimization criteria are defined on the basis of macro-scale thermal and mechanical quantities. A numerical example of optimization is included.

Key words: evolutionary algorithms, multiobjective optimization, multiscale modelling, porous solid, thermoelasticity.

1. Introduction

Recent trends in designing novel and smart materials require a combination of coupled field analysis, multiscale modelling and optimization methods. For structures under thermomechanical loading, optimization concerns both mechanical and thermal properties (e.g., strength, stiffness, low or high thermal conductivity). The proper functionals for considered criteria have to be defined in order to solve optimization tasks. Such functionals, which depend on quantities derived from different physical fields (e.g., mechanical, thermal), are very often contradictory. Moreover, for real engineering problemsoptimization functionals are strongly multimodal. Consequently, an efficient global optimization method has to be applied [1, 8]. The application of such methods in multiscale modeling, especially for the coupled problems, is an emerging area of research [2]. The current work is devoted to the optimization in two-scale thermoelastic problems by means of numerical homogenization and multiobjective evolutionary algorithm. The thermoelastic constants of the microstructure are calculated on the basis of the objective functionals, taking into account the quantities from the macro-scale level.

2. Multiobjective optimization algorithm

EAs, as a group of bioinspired methods, are resistant to getting stuck in local minima. Another advantage of such a method is avoiding the need to calculate the gradient of the fitness function. Multiobjective optimization problems are formulated, if more than one criterion is considered at the same time, and not one, but a set of optimal solutions is obtained for the contradictory criteria. Such solutions are optimal in the Pareto sense (Pareto-optimal solutions). The application of EAs is highly desirable in this case, because a population of solutions is processed at every iteration. The in-house implementation of the multiobjective evolutionary algorithm (MOOPTIM), based on Pareto's concept, is used. It is an improved version of the multiobjective evolutionary algorithm, inspired by the NSGA-II [6].

The pseudo-code of the algorithm is presented in Fig. 1. The algorithm uses two populations Q_i and P_i of the same size and uses non-dominated sorting procedure for classification of the individuals in population and a crowding coefficient to preserve diversity in the population [6]. The main difference between the proposed algorithm and the NSGA II is based on the changes in selection mechanism and the application of different evolutionary operators. The proposed implementation has more evolutionary operators in comparison to the NSGAII. Two types of mutation (uniform and Gaussian) and two types of crossover operators (simple and arithmetical) are used. The in-house implementation of the algorithm was tested on several benchmarks as well as in real optimization

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MOOPTIM algorithm
begin
     i←0
     randomly generate population Qi
     evaluate objective functions for Qi
     randomly generate population Pi
     while (not termination condition) do
     begin
       evaluate objective functions for Pi
           join population Qi and Pi (Ri = Qi + Pi)
           use selection (choose Pi+1 from Ri)
           copy Pi+1 to Qi
           apply evolutionary operators for Pi+1
       i←i+1
     end
end
```

Fig. 1. Pseudo-code of the MOOPTIM algorithm.

problems, showing its superiority to the NSGA-II [4]. The advantage of using one's own implementation instead of the NSGA-II algorithm is particularly remarkable in the case of multimodal functions and functions having non-convex Pareto's or discontinuous Pareto's front.

3. Formulation of the problem

A two-scale thermomechanical model of porous solids is modelled [9]. Linear uncoupled thermoelasticity is considered in [3]. Microstructures are locally periodical. The representative volume element (RVE) concept, coupled with FEM, is used [10]. The RVE is modeled with the periodical boundary conditions. The material parameters for the macro-scale are obtained on the basis of solving a few boundary-value problems for the RVE in the micro-scale [7, 11]. The elastic and thermal constants, such as: Young's modulus, Poisson's ratio and thermal conductivity, are homogenized. Six analyses for linear elasticity and three analyses of the heat conduction problem are performed to calculate tensor of elastic constants and tensor of heat conduction coefficients. The calculation of the values of the effective constants is done by means of volume averaging technique. The FEM software packages are adopted to solve boundary-value problems. Multiscale optimization tasks are formulated as a designing of a size and shape of the void in the microstructure. A linear thermoelasticity problem is described by differential equations of heat conduction and elasticity. These equations have to be supplemented by mechanical and thermal boundary conditions. An example of such a structure with thermal and mechanical boundary conditions is presented in Fig. 2, where $\overline{u_i}$, $\overline{t_i}$, $\overline{T_i}$, $\overline{q_i}$, α, T^∞ are known: displacements, tractions,

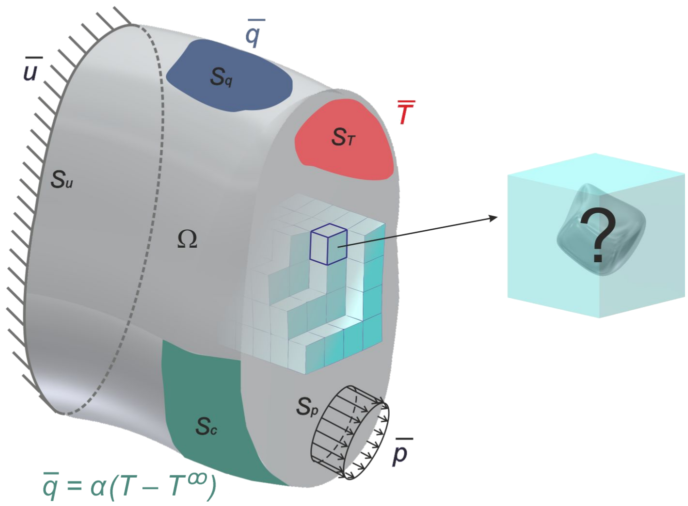

Fig. 2. Two-scale model of the 3D porous body under thermal and mechanical loads.

temperatures, heat fluxes, heat conduction coefficient and ambient temperature, respectively.

The implemented homogenization procedures are compared with analytical models and with non-homogenized numerical models, showing good accuracy of obtained results [5]. Optimization criteria are defined on the basis of thermal and mechanical quantities. The following criteria have been defined for: minimization of the displacement u on the selected part of the boundary Γ_u, maximization of the heat flux q on the selected part of the boundary Γ_q, maximization of the porosity defined as the ratio of pore volume to the volume of RVE (3.1)

$$\begin{aligned}
&\min_{\mathbf{x}} f_1 \overset{def}{=} \int_{\Gamma_u} u \mathrm{d}\Gamma_u, \\
&\min_{\mathbf{x}} f_2 \overset{def}{=} \int_{\Gamma_q} q \mathrm{d}\Gamma_q, \\
&\max_{\mathbf{x}} f_3 \overset{def}{=} \frac{\int_{\Omega_{por}} \mathrm{d}\Omega_{por}}{\int_{\Omega_{RVE}} \mathrm{d}\Omega_{RVE}}.
\end{aligned} \tag{3.1}$$

First functional is related to the stiffness of the structure, second to the ability to conduct the heat, whereas third to the total mass of the structure or amount of material needed for fabrication of porous material (cost).

4. Numerical example

As an example of two-scale micro-macro thermoelastic model, the cuboid solid of dimensions $100 \times 20 \times 20$ mm made of porous aluminum, is considered. Thermoelastic material constants for aluminum are as follows: Young's modulus $E = 70000$ MPa, Poisson's ratio $\nu = 0.35$, thermal conductivity $K = 200$ W/(m·K), thermal expansion coefficient $\alpha = 23 \cdot 10^{-6}$ K^{-1}. The geometry of the macromodel with boundary condition is presented in Fig. 3a. Solid is fixed in all degrees of freedom on the one surface, and loaded by a uniform distributed load $P = 360$ N on the opposite side. The temperature is equal to 0°C on the left side, whilst 100°C is applied on the right side of the block.

Multiobjective optimization tasks concern determining the size and orientation of the cylindrical void in the microstructure by minimization or maximization of the functionals calculated on the basis of results obtained from macromodel. The microstructure is modelled as the RVE with periodic boundary conditions. The effective elastic and thermal constants are obtained from

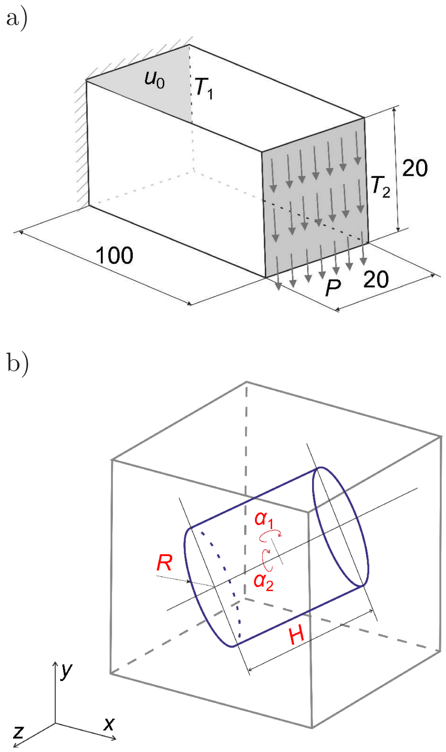

FIG. 3. a) The macromodel, b) parameterization of the void in the RVE.

the numerical homogenization. The void in the microstructure is modeled with cylindrical shape. Four design variables have been defined: radius and height of the cylinder as well as rotation angle along Y and Z axes. The geometry and parameterization of the RVE is presented in Fig. 3b. The multiobjective optimization is performed for different variants of optimization, taking into account different pairs of defined criteria. Moreover, multiobjective optimization that is taking into account simultaneously all three criteria is also performed.

The parameters of MOOPTIM are as follows: size of the population 20, number of generations 30, probability of uniform mutation 0.1, probability of Gaussian mutation 0.7, range of Gaussian mutation 0.5, and probability of simple and arithmetic crossover 0.1. Limitations of the design variables are as follows: radius of the void R $[0.1 \div 0.7]$, height of the void H $[0.1 \div 0.7]$, and rotations angles α_1 and α_2 $[0 \div 90]$.

The sets of Pareto-optimal solutions for different variants of multiobjective optimization are shown in Fig. 4, whereas the optimal parameters of the void in the microstructure for selected points on Pareto's front are collected in Table 1.

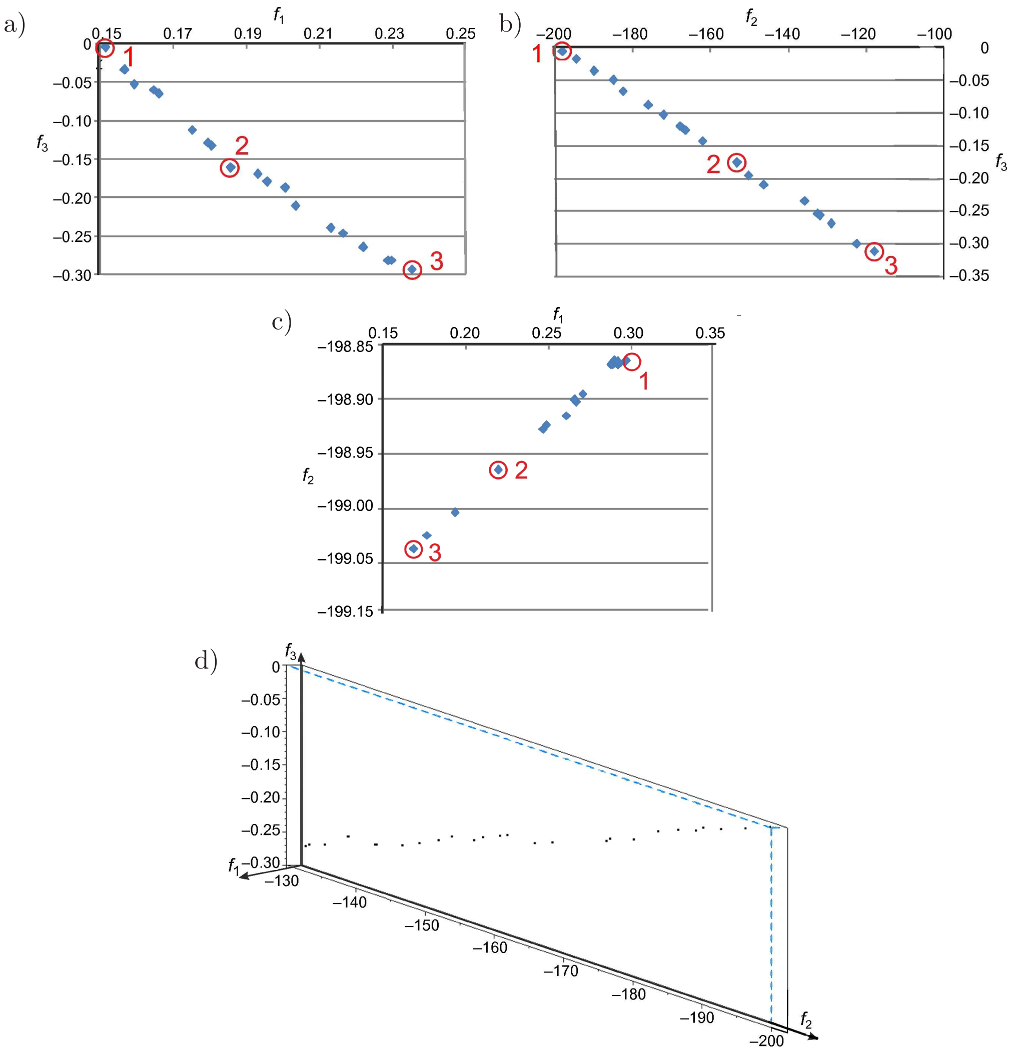

FIG. 4. The set of Pareto's optimal solutions for: a) variant 1 – functionals f_1 and f_3, b) variant 2 – f_2 and f_3, c) variant 3 – f_1 and f_2, d) variant 4 – functionals f_1, f_2 and f_2.

Table 1. Optimal parameters of void in the microstructure for selected points on Pareto's front (Fig. 4a, Fig. 4b, Fig. 4c).

Parameter	Variant 1 – f_1 and f_3			Variant 2 – f_2 and f_3			Variant 3 – f_1 and f_2		
	Point 1	Point 2	Point 3	Point 1	Point 2	Point 3	Point 1	Point 2	Point 3
R	0.130	0.321	0.587	0.213	0.483	0.587	0.106	0.116	0.106
H	0.102	0.400	0.400	0.100	0.341	0.392	0.107	0.100	0.100
α_1	44.9	53.5	53.5	36.8	82.9	44.1	85.2	86.2	86.2
α_2	47.8	74.9	73.4	69.6	36.8	77.1	4.4	0.7	0.7

5. Final remarks

The coupling of EAs with multiscale modeling of thermomechanical solid has been presented. Numerical homogenization with RVE concept and FEM has been used. The considered functionals are typically contradictory, thus the application of multiobjective evolutionary algorithm based on Pareto's concept is a good choice. The results of optimization have been presented in the form of the sets of Pareto-optimal solutions. It should be underlined that the sets of solutions are obtained by means of running a single optimization task. Moreover, such a representation can be very useful for a designer, because it carries information about the nature of the conflict between the criteria. Defining other criteria can be relatively easily implemented.

Acknowledgment

The research was financed from the Polish science budget resources as the project 10/040/BK_16/0032.

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Received October 18, 2016; accepted version November 3, 2016.