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Multiscale Identification of Parameters of Inhomogeneous Materials by Means of Global Optimization Methods

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This paper deals with the identification of material parameters at a microscale on the basis of measurements at a macroscale. Inhomogeneous materials such as composites and porous media are considered. Numerical homogenization with the use of a representative volume element is performed to obtain a macroscopically homogenized equivalent material. The evolutionary algorithm is applied as the global optimization method to solve the identification task. Modal analysis is performed to collect data necessary for the identification. Different ranges of measurement errors are considered. A finite element method is employed to solve a boundary-value problem for both scales.

Key words: identification, numerical homogenization, evolutionary algorithm, porous material, composite, measurement error.

1. Introduction

Microscopically inhomogeneous materials like composites or porous materials are an important group of structural materials. Their macroscopic material properties depend on such parameters as the properties of constituent materials, the volume fraction of constituents and the shape and the location of reinforcement or voids. The determination of the properties of constituents of inhomogeneous materials is an essential problem in many engineering applications.

It is assumed that the considered material is macroscopically homogeneous and microscopically heterogeneous. To determine the effect of microstructure of heterogeneous materials on their behaviour at the macro level, different homogenization methods may be applied [1]. A numerical homogenization method with the representative volume element (RVE) concept is employed to obtain the connection between the two considered scales [2].

The aim of this paper is to perform the identification of the micro-properties of the constituents of composite and porous media on the basis of the measurements performed at a macroscale. The identification is performed by means of global optimization methods in the form of the evolutionary algorithm (EA) to avoid problems with multimodal objective functions and the calculation of the objective function gradient [3]. The finite element method software ANSYS Workbench is applied to both scales to solve the boundary-value problem. Modal analysis of the components made of inhomogeneous materials is carried out to obtain necessary measurement data [4].

2. Formulation of the problem

The aim of the identification is to estimate the properties of the constituents of an inhomogeneous material at the microscale level by means of the measurements performed at the macrolevel [5]. The following properties are identified: (i) the volume ratio and the elastic constants (Young's modulus E and Poisson's ratio v) for both isotropic constituents of composite materials, and (ii) the porosity p and the elastic constants of the material without pores in the case of the porous material.

The identification is performed as the minimization of an objective function J_0:

$$\min\colon J_0(\mathbf{x}) = \sum_{i=1}^{N} (\widehat{q}_i - q_i)^2, \tag{2.1}$$

where $\mathbf{x} = (x_i)$ is a vector of identified parameters, $\widehat{q}_i$ are measured values of state fields, q_i are values of the same state fields calculated for the numerical model, N is a number of measurement data.

To solve the identification problem, multi-objective genetic algorithm (MOGA), included in the ANSYS Workbench software, is employed. The application of the FEM software and an optimization tool available in the same software package reduces the time necessary for data exchange between different algorithms. MOGA is based on a well-known Non-dominated Sorting Genetic Algorithm II (NSGA-II) [6]. MOGA takes advantage of controlled elitism concepts and supports binary and floating-point input parameters. It can be applied to single-objective and multi-objective optimization problems. The application of the evolutionary algorithm to the multi-objective and multi-scale optimization of inhomogeneous materials is presented in [7].

3. Numerical homogenization

The aim of the homogenization is to obtain a medium macroscopically equivalent to a non-homogeneous medium in a microscale [8]. The microscale behaviour of the considered materials is described by differential equations with discontinuous coefficients. To obtain the equivalent properties of macroscopic material, a different method of homogenization can be used (e.g., mean-field approach, variational methods). In this paper, the numerical homogenization has been performed due to its ability of modelling the complex geometry of microstructure [9].

To perform the numerical homogenization a proper RVE has to be created. A RVE describes the structure of the whole material (global periodicity) or its part (local periodicity) [9]. The analysis of the RVE allows to determine the constitutive relation between averaged field variables, like stresses or strains, of the microscopic model. To apply the RVE homogenization concept, some conditions have to be satisfied: the Hill-Mandel condition, appropriate boundary conditions and separation of scales rule [10].

As the FEM method is applied to solve the boundary-value problem in both considered scales, the RVE has to be assigned to each integration point at the micro-scale.

4. Numerical examples

Two different structures: a porous cantilever beam and a particle-reinforced composite are considered. The aim of the identification is to find some parameters of the structure at the microscale with some quantities measured at the macroscale. Modal analysis data are considered as the macroscale measurements. The uniform displacement boundary conditions are imposed on RVEs in both cases. As the unit cell (a RVE with one inclusion) is not sufficient for this type of boundary conditions due to inadequate homogenization precision [11], the more complicated RVEs have been proposed.

The identification is performed for both structures with the following parameters of the evolutionary algorithm: the number of individuals $pop\_size = 100$, the arithmetic crossover probability $p_{ac} = 0.98$, the uniform mutation probability $p_{um} = 0.01$, and the maximum number of iterations $n_{it} = 12$.

4.1. Numerical example 1

The porous steel cantilever beam of dimensions $b \times h \times l = 20 \times 30 \times 100$ mm is considered (Fig. 1a). Two elastic constants (E, ν) and the porosity of the beam p are the design variables. Homogenized material properties are obtained by the numerical homogenization of a RVE containing 27 uniformly distributed spherical voids (Fig. 1b). First five eigenfrequencies of the beam are taken into account as macroscopic measurement data for the objective function calculation. Three cases of measurements data were considered: the ideal one and the data with two levels of maximum measurement error of normal distribution: 1% and 5%. The design variables ranges are 70–400 GPa for E, 0.25–0.35 for v, and 12.5–17.5%

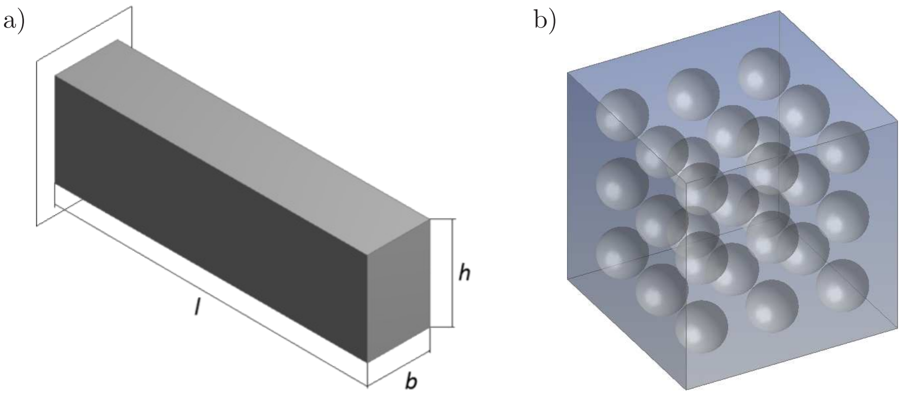

FIG. 1. a) The cantilever beam and b) the RVE geometry.

for p with the actual values $E = 200$ GPa, $v = 0.3$ and $p = 15\%$. The porosity range is the result of the assumption that the porosity can be estimated with the accuracy of 5% [12]. The density is determined by the measurement of mass of the beam and the assumed value of porosity.

The values of the design variables and identification errors for different measurement errors are collected in Table 1. In this case, the imprecise measurements decrease the identification precision proportionally to the measurement error.

Table 1. The cantilever beam – the identification results.

Measurement error [%]	Found values			Error [%]		
	p [%]	E [GPa]	v [ ]	p	E	v
0	14.590	198.803	0.299	2.73	0.59	0.25
1	14.585	199.570	0.303	2.77	0.22	0.96
5	15.598	200.827	0.298	3.99	0.41	0.74

4.2. Numerical example 2

The particle-reinforced composite connecting rod is considered (Fig. 2a). It is assumed that the material of the matrix of the composite is known (aluminium) and as a result two elastic constants of the reinforcement material (E, ν) and the reinforcement volume fraction V are the design variables. Homogenized material properties are obtained by the numerical homogenization of a RVE containing 27 uniformly distributed spherical reinforcements. First six eigenfrequencies of the structure are the measurement data. As in the first example, three cases of macroscopic measurements precision are considered. The design variables ranges are 300–700 GPa for E, 0.12–0.25 for v and 5–40% for V. The actual values of the identified constants are: $E = 410$ GPa, $v = 0.14$ and $V = 20\%$.

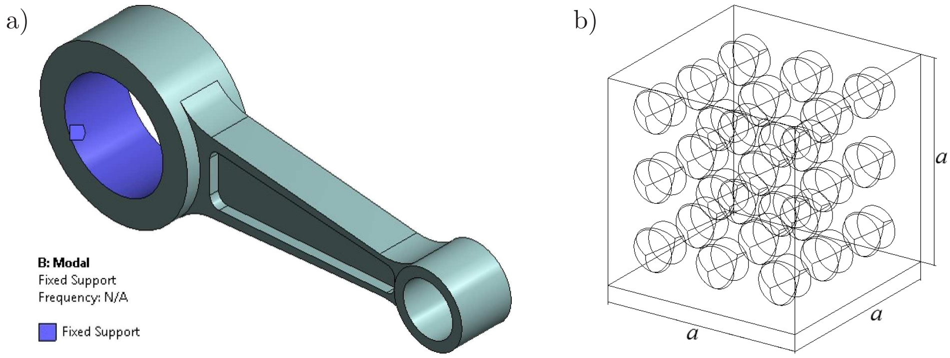

FIG. 2. a) The connecting rod geometry and constraints, b) the RVE geometry.

The identified values of the design variables and the identification errors are collected in Table 2. It can be observed that the precise measurements do not guarantee the best identification results, which may be an important issue in real problems.

Table 2. The connecting rod – the identification results.

Measurement error [%]	Found values			Error [%]		
	V [%]	E [GPa]	v [ ]	V	E	v
0	21.339	366.36	0.154	6.70	10.64	10.16
1	19.933	403.74	0.124	0.34	1.53	11.41
5	19.853	435.718	0.156	0.73	6.27	11.42

5. Final conclusions

The multiscale identification of the properties of microscopically inhomogeneous material has been performed. To solve the identification problem, the numerical homogenization methods, evolutionary algorithms and finite element method software have been simultaneously employed. RVEs with 27 uniformly distributed voids/reinforcements have been used instead of RVEs with single inclusion in order to obtain the required homogenization accuracy. The identification of the selected parameters at a microscale has been performed on the basis of macroscale measurements. Numerical experiments have been performed to collect necessary data. As the real measurements are always imprecise, different levels of the measurement error have been introduced. Positive evolutionary identification results have been obtained in all cases, including a relatively high measurement error. The proposed method may be applied to different non-homogeneous structures and different mechanical, thermal and other problems, including coupled ones.

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References

1. Zohdi T.I., Wriggers P., *An introduction to computational micromechanics*, Springer, Berlin – Heidelberg, 2005.
2. Kouznetsova V., *Computational homogenization for the multi-scale analysis of multi-phase materials*, PhD thesis, Technische Universiteit Eindhoven, 2002.
3. Michalewicz Z., Fogel D., *How to solve it: modern heuristics*, Springer Science & Business Media, Berlin – Heidelberg, 2004.
4. He J., Fu Z-F., *Modal analysis*, Butterworth-Heinemann, Oxford (UK) – Woburn (United States), 2001.
5. Beluch W., Burczyński T., *Two-scale identification of composites' material constants by means of computational intelligence methods*, Archives of Civil and Mechanical Engineering, **14**(4): 636–646, 2014.
6. Deb K., *Multi-objective optimization using evolutionary algorithms*, Wiley, New York, 2001.
7. Beluch W., Długosz A., *Multiobjective and multiscale optimization of composite materials by means of evolutionary computations*, Journal of Theoretical and Applied Mechanics, **54**(2): 397–409, 2016, doi: 10.15632/jtam-pl.54.2.397.
8. Ptaszny J., Fedeliński P., *Numerical homogenization by using the fast multipole boundary element method*, Archives of Civil and Mechanical Engineering, **11**(1): 181–193, 2011, doi: 10.1016/S1644-9665(12)60182-4.
9. Hill R., *Elastic properties of reinforced solids: Some theoretical principles*, Journal of the Mechanics and Physics of Solids, **11**(5): 357–372, 1963, doi: 10.1016/0022-5096(63)90036-X.
10. Zohdi T.I., Wriggers P., *An introduction to computational micromechanics*, Lecture Notes in Applied and Computational Mechanics, Springer, Berlin – Heidelberg, 2005.
11. Miehe C., *Strain-driven homogenization of inelastic microstructures and composites based on an incremental variational formulation*, International Journal for Numerical Methods in Engineering, **55**(11): 1285–1322, 2002, doi: 10.1002/nme.515.
12. Mokhles Gerami F., Kakuee O., Mohammadi S., *Porosity estimation of alumina samples based on resonant backscattering spectrometry*, Nuclear Instruments and Methods in Physics Research Section B: Beam Interactions with Materials and Atoms, **373**: 80–84, 2016, doi: 10.1016/j.nimb.2016.03.016.

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