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Optimal Parameters of a Dynamically Loaded Patch/Layer Structure Against the Elastic-Brittle Interface Debonding

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A one-dimensional shear-lag model was developed to study the high frequency, dynamic, time-harmonic mechanical behavior of the overlap zone of a piezoelastic patch attached to an elastic host layer and subjected to electric, temperature and moisture excitation. It was interesting to see that the change of the geometry of the overlap zone leads to different solutions after some frequency, which is responsible for different dynamic behavior of the considered structure. Furthermore, the model was involved in an optimization framework (genetic algorithm-GA) in order to find the optimal values of the model parameters of the patch/layer configuration.

Key words: PZT-4/glass-fiber epoxy, shear lag model, genetic algorithm.

1. Introduction

During the past decades the piezoelectric materials with high electromechanical coupling properties have been used as sensors or actuators in structural health-monitoring systems. The use of piezoelectric patches for the damage detection of structures based on elastic wave propagation is due to the fact that elastic waves are very sensitive to changes of the structure geometry and its material parameters. It is important that these patches are convenient for the fast analysis when a detailed inspection of structures is performed. However, the presence of material discontinuity between the patches and the host structure generates complicated local electromechanical fields near the edges of patches.

2. Problem formulation

The shear-lag method which was first introduced in the papers of Volkersen [1] and Cox [2] for layered and fiber-reinforced composites is now a common analytical tool known in the engineering society. The shear-lag hypothesis involves a simplification of in-plane shear stress and decouples the 2D problem into two 1D problems.

The following system of ordinary differential equations for the overlap zone of the structure patch/layer (Fig. 1) is obtained reducing the respective 2D case to a 1D case and shear-lag hypothesis:

$$\begin{gathered}\frac{d\sigma_A}{dx} - \frac{\tau_I}{h_A} = \rho_A \frac{\partial^2 u_A}{\partial t^2}, \qquad \frac{d\sigma_B}{dx} + \frac{\tau_I}{h_B} = \rho_B \frac{\partial^2 u_B}{\partial t^2}, \\ \frac{d^2 T}{dx^2} = 0, \qquad \frac{d^2 H}{dx^2} = 0, \qquad \frac{dD_{zA}}{dz} = 0,\end{gathered} \tag{2.1}$$

where T, H, D_{zA} are the temperature, moisture concentration and electrical displacement, respectively, σ_A, σ_B, τ_I are the axial stresses of the patch and host layer and interface shear stress, respectively, and h_A, h_B, $h_I = h_A + h_B$ are the thicknesses of the adherends. The system of reduced Eqs. (2.1) is obtained and used in [3].

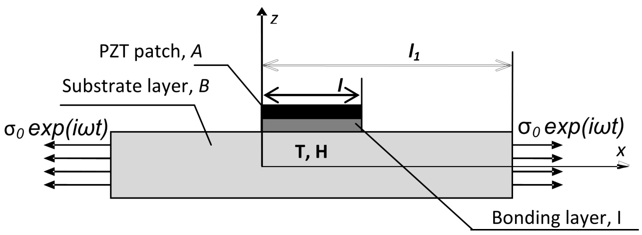

Fig. 1. The model of smart patch/layer structure.

It is assumed that the considered structure consists of the patch A that is piezoelectric and transversaly elastically isotropic, influenced by the temperature and mechanical and electrical time harmonic load and the hosting layer B that is isotropic elastic. As shown in Fig. 1 the hosting layer is loaded by tensile time harmonic stress at the temperature and moisture excitation. Both the patch and the layer work in axial tension, while the interface (the bonding or the adhesive layer) works only in shear.

It is assumed that the kinematic behavior within the structure is given by $\varepsilon_k^{\mathrm{tot}} = du_k/dx$, and then the mechanical strain is expressed by $\varepsilon_k^{\mathrm{mech}} = \varepsilon^{\mathrm{tot}} - \varepsilon_k^{\mathrm{pzel}} - \varepsilon_k^T - \varepsilon_k^H$, $k = A, B, I$. According to the mechanical, electrical and environmental conditions assumed above for the interface I we have $\varepsilon_I^{\mathrm{pzel}} = 0$, $\varepsilon_I^T = 0$, $\varepsilon_I^H = 0$, for the patch A $\varepsilon_A^H = 0$, and for the layer B $\varepsilon_B^{\mathrm{pzel}} = 0$.

The problem will be solved in the selected overlap zone for the structure of the smart patch and hosting layer. The constitutive equations for the patch A, the layer B and for the interface I are as follows:

$$
\begin{aligned}
\sigma_A &= \left(c_{11}^{A*} + \frac{e_{13}^{A*} e_{31}^{A*}}{\varepsilon_{33}^{A*}}\right)\varepsilon_A - \frac{e_{13}^{A*}}{\varepsilon_{33}^{A*}} D_{zA} - \left(\alpha_{11}^{A*} - \frac{e_{13}^{A*}}{\varepsilon_{33}^{A*}} p_3^{A*}\right) T,\\
\sigma_B &= c_{11}^{B*}\varepsilon_B - \alpha_{11}^{B*} T - \beta_{11}^{B*} H,\\
\tau_I &= G_I \frac{(u_A - u_B)}{h_I},\\
E_{zA} &= \frac{D_{zA}}{\varepsilon_{33}^{A*}} - \frac{e_{31}^{A*}}{\varepsilon_{33}^{A*}}\varepsilon_A - \frac{p_3^{A*}}{\varepsilon_{33}^{A*}} T.
\end{aligned}
\tag{2.2}
$$

Here $c_{ij}^{A/B*}$, $e_{ij}^{A/B*}$, $\varepsilon_{ij}^{A/B*}$ are the elastic constants transformed from a 3D case to a 1D case (measured at constant electric field), piezoelectric and dielectric constants (measured at constant strain), α_{11}^{A*}, β_{11}^{A*}, α_{11}^{B*}, β_{11}^{B*} are thermal stress coefficients for the respective adherends, p_3^{A*} is the pyroelectric coefficient for the patch A, and G_I is the shear modulus of the interface (for details see [3]). E_{zA} is the electric gradient of the patch. The poling axis is along the axis $0z$, the origin of Oxz is set at the left end of the overlap zone. It is assumed that all functions in (2.1) obey the Fourier presentation, for example: $u_i(x,t) = \exp(-i\omega t)u_i(x)$, $i = A, B$.

For the solutions of the temperature, moisture and electric equations we take the simplest case when $T_A(x) = T$, $T_B(x) = T$, $H_B(x) = H$, $D_{zA} = D_0$, where T, H, D_0 are constant for $0 < x < l$.

$$
\begin{aligned}
&\frac{d^2u_A}{dx^2} - \frac{G_I}{h_I E_A h_A}(u_A - u_B) = -\frac{\rho_A}{E_A}\omega^2 u_A,\\
&\frac{d^2u_B}{dx^2} + \frac{G_I}{h_I E_B h_B}(u_A - u_B) = -\frac{\rho_B}{E_B}\omega^2 u_B.
\end{aligned}
\tag{2.3}
$$

For the sake of simplicity we introduce the following parameters:

$$
\xi = \frac{\rho_A}{E_A}\omega^2, \qquad \eta = \frac{\rho_B}{E_B}\omega^2, \qquad \kappa = \frac{1}{h_I h_A}\frac{G_I}{E_A}, \qquad \lambda = \frac{1}{h_B h_I}\frac{G_I}{E_B},
$$

$$E_A = \left(c_{11}^{A*} + \frac{e_{13}^{A*} e_{31}^{A*}}{\varepsilon_{33}^{A*}} \right), \qquad E_B = c_{11}^{B*}, \qquad D_A = \frac{e_{13}^{A*}}{\varepsilon_{33}^{A*}} D_0,$$

$$\alpha_A = \left(\alpha_{11}^{A*} - \frac{e_{13}^{A*}}{\varepsilon_{33}^{A*}} p_3^{A*} \right), \qquad \alpha_B = \alpha_{11}^{B*}, \qquad \beta_B = \beta_{11}^{B*}.$$

Then the equilibrium equations (2.3) take the form:

$$u_A'' - \kappa(u_A - u_B) = -\xi u_A, \qquad u_B'' + \lambda(u_A - u_B) = -\eta u_B. \tag{2.4}$$

After some transformation of the system of Eqs. (2.4) we come to the following equation (2.5) defining the patch displacement and the boundary conditions (2.6):

$$u_A^{IV} - A u_A'' - B u_A = 0, \qquad A = k - \xi + \lambda - \eta, \qquad B = \kappa\eta - \xi\eta + \lambda\xi, \tag{2.5}$$

$$\sigma_A(0) = 0, \qquad \sigma_A(l) = 0, \qquad \sigma_B(0) = \sigma_0, \qquad \sigma_B(l) = \sigma_0. \tag{2.6}$$

The characteristic equation is $r^4 - Ar^2 - B = 0$, $r^2 = z$, $z^2 - Az - B = 0$ with the roots $z_{1,2} = \frac{1}{2}\left(A \pm \sqrt{A^2 + 4B}\right)$. For the structure considered in Secs. 3 and 4, the following analytical solutions for u_A, u_B, τ_I at $z_{1,2} < 0$ are obtained:

$$\begin{aligned} u_A &= A_1 \cos(r_1 x) + A_2 \sin(r_1 x) + A_3 \cos(r_2 x) + A_4 \sin(r_2 x), \\ u_B &= \frac{1}{k} \left\{ \left(k - \xi + r_1^2\right) \left[A_1 \cos(r_1 x) + A_2 \sin(r_1 x)\right] \right. \\ &\qquad \left. + \left(k - \xi + r_2^2\right) \left[A_3 \cos(r_2 x) + A_4 \sin(r_2 x)\right] \right\}, \\ \tau_I &= \frac{G_I}{h_I k} \left\{ \left(\xi - r_1^2\right) \left[A_1 \cos(r_1 x) + A_2 \sin(r_1 x)\right] \right. \\ &\qquad \left. + \left(\xi - r_2^2\right) \left[A_3 \cos(r_2 x) + A_4 \sin(r_2 x)\right] \right\}. \end{aligned} \tag{2.7}$$

The integration constants A_i are calculated from (2.6). To find the elastic-brittle debond length of the possible delamination along the interface of the patch/layer structure the following criterion is used: $\tau_I(x) = \tau^{cr}$.

3. Illustrative numerical example

The first layer A of the bi-material structure is made from piezoelectric PZT-4 material, and the second layer B is made from glass-fiber epoxy material. The adhesive bonding layer I is taken to be polyacrylate thermoplastic glue with $\tau^{cr} = 18$ MPa, $G = 800$ MPa. For both cases and in all the results shown below, the magnitude of the applied moisture concentration, temperature and electric displacement are given respectively as $H = 0.25\%$, $T = 300$ K, $D_0 = 2.55$ C/m^2, unless stated otherwise. In Fig. 2, the interface shear stresses at/for different overlap zones are presented for $\omega = 30\,000$ Hz, $D_0 = 2.55$ C/m^2, $h_A = 0.01$ m, $h_B = 0.1$ m.

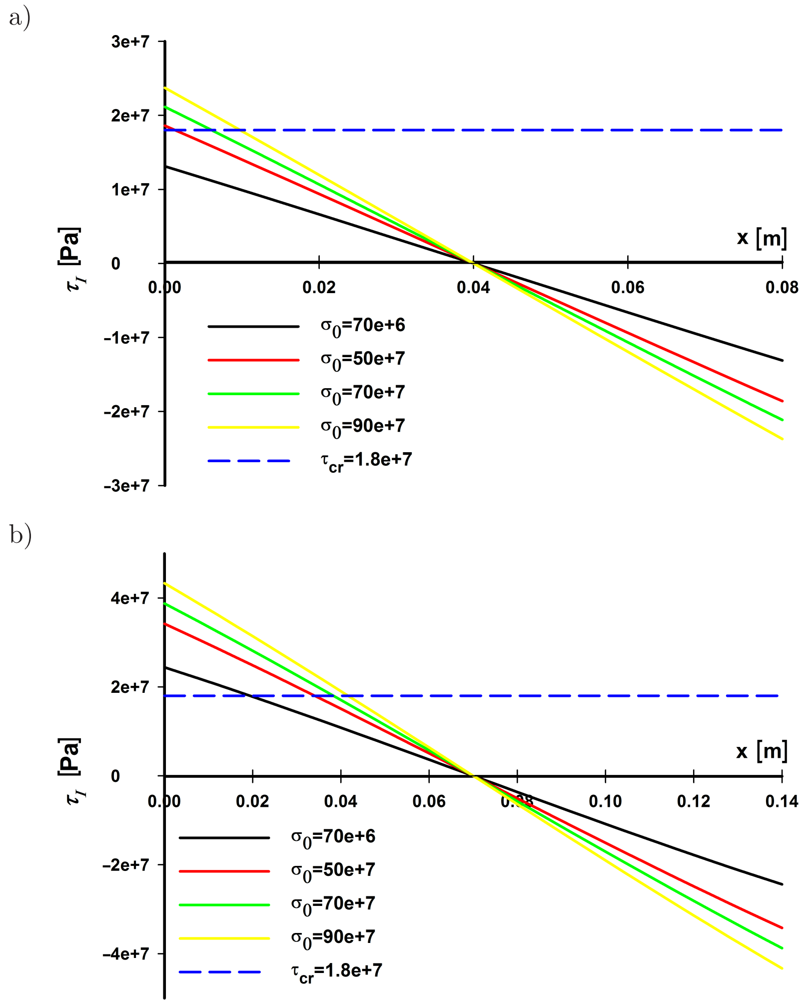

FIG. 2. Interface shear stress vs length of the structure x for different axial loadings: a) $l = 0.08$ m and b) $l = 0.14$ m.

4. Objective function

The aim of the optimization is to find the values of the model parameters σ_0, ω, h_A, h_B, D_0, H and T, at which the debond length (a nonlinear function of model parameters) is minimal, e.g., as close as possible to 0. The following optimization problem is formulated:

Objective function:

$$MIN_{\sigma_0,\omega,D_0,h_A,h_B,H,T}(l_e), \tag{4.1}$$

subjected to $\tau_I(l_e) = \tau^{cr}$, which pushes τ_I to approach τ^{cr}.

$$0.1 \leqslant h_A/h_B \leqslant 0.2, \qquad 0 \leqslant x \leqslant l, \qquad z_{1,2} = \frac{\left(A \pm \sqrt{A^2 + 4B}\right)}{2} < 0$$

and all control variables σ_0, ω, h_A, h_B, D_0, H and T are constrained between their lower and upper boundaries.

Two optimization problems for the structures with two different lengths of the overlap zones have been formulated. The formulated optimization problems belong to the constrained non-linear programming (NLP) because of the introduced non-linear constraints and are solved by a genetic algorithm – BASIC GA [4].

Two cases of the structure with PZT-4/glass-fiber epoxy with short l = 0.08 m and long l = 0.14 m overlap zones are under consideration. For both cases the obtained optimal value of the objective function is $1 \cdot 10^{-5}$ m, which corresponds to the structure where no interface delamination appears. The values of all control variables in (4.1) at which the optimal value of the objective function is obtained by GA for the short and long structures are given in Table 1. The optimal solution for the first case is obtained at 25 000 generations and for the second case is obtained at 37 000 generations.

Table 1. Results of GA optimization for short and long structure, for PZT-4/glass-fiber epoxy.

l [m]	h_A [m]	h_B [m]	D_0 [C/m^2]	ω [kHz]	σ_0 [MPa]	H [%]	T [K]
0.08	0.0099	0.0873	1.6648	31.65	590.31	0.1228	288.31
0.14	0.0069	0.0562	0.5033	49.18	125.60	0.1927	302.89

5. Conclusions

The applicability of the proposed approach is demonstrated on one structure (PZT-4/glass-fiber epoxy) with different lengths of the overlap zone. The conclusion is that the short length overlap zone is a preferable structure at the given combined load. The provided optimal analysis shows that for both considered cases of the structure configurations a vanishing (minimal) debond length is ensured.

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