ENGINEERING TRANSACTIONS • *Engng. Trans.* • **65**, 1, 201–208, 2017
Polish Academy of Sciences • Institute of Fundamental Technological Research (IPPT PAN)
National Engineering School of Metz (ENIM) • Poznan University of Technology

On Thermodynamically Consistent Form of Nonlinear Equations of the Cosserat Theory

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To describe motion in a micropolar medium a special measure of curvature is used that is a strain state characteristic independent of deformation process. The nonlinear constitutive equations of the couple stress theory are constructed using the method of internal thermodynamic parameters of state. The linearization of these equations in isotropic case yields the Cosserat continuum equations, where material resistance to the change in curvature is characterized by a single coefficient as against three independent coefficients of the classical theory. So, it turns out that the developed variant of the model gives an adequate description of generalized plane stress state in an isotropic micropolar medium, while the classical one describes this state only at a certain case. The complete system of nonlinear equations for the dynamics of a medium with couple stresses reduces to a thermodynamically consistent system of laws of conservation, which allows obtaining integral estimates that guarantee the correctness of the Cauchy problem and boundary-value problems with dissipative boundary conditions.

Key words: Cosserat continuum, curvature tensor, thermodynamically consistent system.

1. Plane stress

A key cause for undertaking this study became a property exhibited by the classical model of the elastic Cosserat continuum in the small strain approximation. It turns out that this model gives an adequate description of generalized plane stress state in an isotropic case exclusively at a certain ratio of elastic coefficients characterizing resistance of a material to the change in internal curvature. By way of illustration, Fig. 1 shows the stress state generated in an elastic plate made of a material with microstructure by induced rotation of particles, uniform with respect to the plate thickness, at a lateral boundary of the plate, or under influence of couple stresses at the boundary, causing no plate bending. It is natural in this state to expect nontrivial distribution of couple

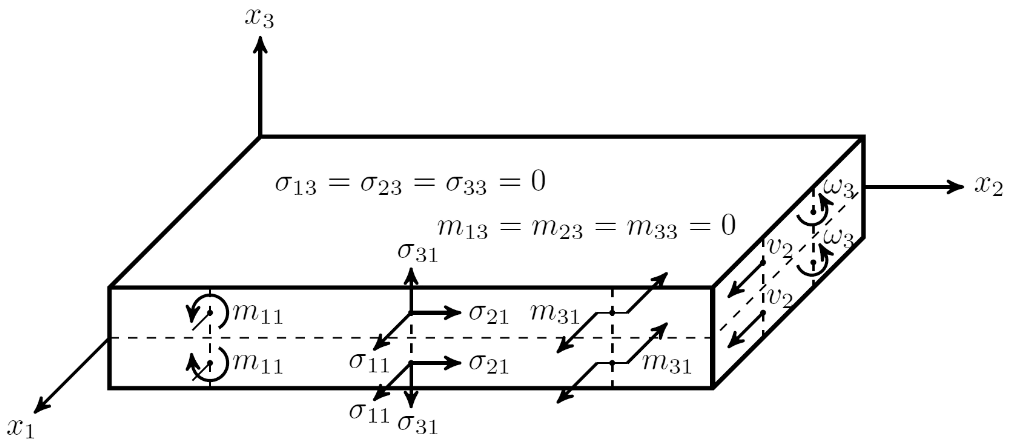

FIG. 1. Plane stress state.

stresses in the plate domain with the corresponding field of independent particle rotations.

The constitutive equations of the Cosserat continuum in tensor form

$$(1.1)\qquad \boldsymbol{\sigma} = \lambda\,(\mathbf{I} : \boldsymbol{\Lambda})\,\mathbf{I} + 2\mu\boldsymbol{\Lambda}^{\rm s} + 2\alpha\boldsymbol{\Lambda}^{\rm a}, \qquad \mathbf{m} = \beta\,(\mathbf{I} : \mathbf{M})\,\mathbf{I} + 2\gamma\mathbf{M}^{\rm s} + 2\varepsilon\mathbf{M}^{\rm a}$$

contain six independent elastic moduli of a material, denoted as in [1]. Here $\boldsymbol{\sigma}$ and $\mathbf{m}$ are the tensors of stresses and couple stresses, $\boldsymbol{\Lambda}$ and $\mathbf{M}$ are the tensors of strain and curvature, $\mathbf{I}$ is the unit tensor. The indices "s" and "a" denote symmetric and antisymmetric parts of tensors, respectively: $2\boldsymbol{\Lambda}^{\rm s} = \boldsymbol{\Lambda} + \boldsymbol{\Lambda}^*$, $2\boldsymbol{\Lambda}^{\rm a} = \boldsymbol{\Lambda} - \boldsymbol{\Lambda}^*$. The common operations of tensor analysis, conforming with multiplication of tensors by right-hand vectors, are used.

For the tensors of strain and curvature, the kinematic equations hold true: $\dot{\boldsymbol{\Lambda}} = \boldsymbol{\upsilon}_x - \boldsymbol{\Omega}$ and $\dot{\mathbf{M}} = \boldsymbol{\omega}_x$, where $\boldsymbol{\upsilon}_x$ is the tensor of velocity gradients, $\boldsymbol{\Omega}$ and $\boldsymbol{\omega}$ are the tensor and vector of angular velocity, a dot above a symbol means the time derivative. Turning to average values of the velocities and stresses over the plate thickness by averaging Eqs. (1.1), we obtain the constitutive equations for the generalized plane stress state:

$$(1.2)\qquad \begin{array}{ll} a_1\dot{\sigma}'_{11} + a_2\dot{\sigma}'_{22} = \upsilon'_{1,1}, & a_2\dot{\sigma}'_{11} + a_1\dot{\sigma}'_{22} = \upsilon'_{2,2}, \\ a_3\dot{\sigma}'_{21} + a_4\dot{\sigma}'_{12} = \upsilon'_{2,1} - \omega'_3, & a_4\dot{\sigma}'_{21} + a_3\dot{\sigma}'_{12} = \upsilon'_{1,2} + \omega'_3, \\ b_3 m'_{32} = \omega'_{3,2}, & b_4 m'_{32} = 0, \\ b_4 m'_{31} = 0, & b_3 m'_{31} = \omega'_{3,1}, \end{array}$$

written in terms of the moduli of elastic compliance of a material:

$$a_1 = \frac{\lambda + \mu}{\mu(3\lambda + 2\mu)}, \qquad a_2 = -\frac{\lambda}{2\mu(3\lambda + 2\mu)}, \qquad a_3 = \frac{\mu + \alpha}{4\mu\alpha}, \qquad a_4 = -\frac{\mu - \alpha}{4\mu\alpha},$$

$$b_1 = \frac{\beta + \gamma}{\mu(3\beta + 2\gamma)}, \qquad b_2 = -\frac{\beta}{2\gamma(3\beta + 2\gamma)}, \qquad b_3 = \frac{\gamma + \varepsilon}{4\gamma\varepsilon}, \qquad b_4 = -\frac{\gamma - \varepsilon}{4\gamma\varepsilon}.$$

If any other averaged projection of the vectors of translational and angular velocities, except of the used in Eqs. (1.2), or any component of the stress tensors is not equal to zero, then, by virtue of a mechanical sense, the considered state will not be the state of generalized plane stress because of the loss of symmetry with respect to the middle plane of a plate.

It follows from the general form of Eqs. (1.2) that couple stresses may only be no-zero if $b_4 = 0$, i.e. when $\gamma = \varepsilon$. Taking different coefficients γ and ε, it is mathematically incorrect to formulate boundary conditions in terms of the rotation angles or couple stress effects at the plate boundary, since couple stresses turn out to be zero everywhere in the plate domain. Thus, there is a problem of selecting a special curvature measure to be used as a parameter of state with intent to construct constitutive equations in the nonlinear case using the principles of equilibrium thermodynamics at smaller number of elastic coefficients of the linear approximation model.

2. Special tensor of curvature

The translational motion of a particle in a medium possessing microstructure is described by an equation $\mathbf{x} = \boldsymbol{\xi} + \mathbf{u}$, connecting the Lagrangian $\boldsymbol{\xi}$ and Eulerian $\mathbf{x}$ vectors of centers of masses with the displacement vector $\mathbf{u}(\boldsymbol{\xi}, t)$. The independent rotation of a particle is defined by an orthogonal rotation tensor $\mathbf{R}(\boldsymbol{\xi}, t)$. The antisymmetric tensor of angular velocity of a particle is calculated by the formula: $\boldsymbol{\Omega} = \dot{\mathbf{R}} \cdot \mathbf{R}^*$ (hereinafter star denotes the conjugate). As a measure of deformation of an infinitely small element, it is assumed to take the tensor $\boldsymbol{\Lambda} = \mathbf{R}^* \cdot \mathbf{x}_\xi - \mathbf{I}$. By differentiating with respect to time, it is found that this tensor satisfies the equation:

$$\mathbf{R} \cdot \dot{\boldsymbol{\Lambda}} = \boldsymbol{\upsilon}_\xi - \boldsymbol{\Omega} \cdot \mathbf{x}_\xi, \tag{2.1}$$

where $\boldsymbol{\upsilon} = \dot{\mathbf{x}}$ is the vector of velocity of translational motion. Aside from the tensor $\boldsymbol{\Lambda}$, a special curvature tensor $\mathbf{M}$ is used, calculated in terms of the rotation tensor $\mathbf{R}$ and its derivatives with respect to the Lagrangian variables in the Cartesian coordinate system: $\mathbf{R}_{,k} = \partial \mathbf{R}/\partial \xi_k$ $(k = 1, 2, 3)$. Let $\mathbf{M}^{(k)} = \mathbf{R}_{,k} \cdot \mathbf{R}^*$ be the antisymmetric curvature tensors along the coordinate lines. The Darboux vectors fitting with these tensors are assigned by the columns of $\mathbf{M}$. Differentiating $\mathbf{M}^{(k)}$ with respect to time and $\boldsymbol{\Omega}$ with respect to the space variables ξ_k yields kinematic equations that admit the tensor representation:

$$\dot{\mathbf{M}} = \boldsymbol{\omega}_\xi + \boldsymbol{\Omega} \cdot \mathbf{M}. \tag{2.2}$$

Note, that it differs from the equation for commonly used curvature measures [2, 3]. It follows from (2.2) that $\mathbf{M}$ is neither an invariant nor an indifferent tensor, i.e. it changes both under rotation of the actual configuration and under rotation of the original configuration. It can be shown that the invariance is the property of the product $\mathbf{M}^* \cdot \mathbf{M}$, that must be used as an independent parameter of state to construct constitutive equations accounting for the couple properties of a medium, and that leads to a thermodynamically consistent system of conservation laws.

The expanded form of the curvature measure $\mathbf{M}$ in the Cartesian coordinate system is

$$\mathbf{M} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}, \qquad \mathbf{M}^{(k)} = \begin{pmatrix} 0 & -M_{3k} & M_{2k} \\ M_{3k} & 0 & -M_{1k} \\ -M_{2k} & M_{1k} & 0 \end{pmatrix}. \tag{2.3}$$

When solving static problems, the equation (2.3) for $\mathbf{M}$ can be used instead of (2.2) together with the general representation of orthogonal tensor

$$\mathbf{R} = \mathbf{I} + \sin\phi\, \mathbf{Q} + (1 - \cos\phi)\, \mathbf{Q}^2, \qquad \mathbf{Q} = \begin{pmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{pmatrix},$$

via the angle of rotation ϕ and the skew-symmetric tensor $\mathbf{Q}$, associated with the unit vector $\mathbf{q} = (q_1, q_2, q_3)$ of rotational axis. In the case of the rotation around the x_3 axis

$$\mathbf{M} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \phi_{,1} & \phi_{,2} & \phi_{,3} \end{pmatrix}.$$

Note that, in accordance with Eqs. (2.2), the curvature measure coincides with the classical measure in geometrically linear case.

3. Thermodynamically consistent form

System of equations of the dynamics of a medium with couple stresses is constructed based on the integral laws of impulse, momentum and energy conservation. For the case of continuous motions, the integral conservation laws are equivalent to the differential equations of translational and rotational motion and the equation for internal energy:

$$\rho_0 \dot{\boldsymbol{\upsilon}} = \mathrm{div}_\xi \boldsymbol{\sigma} + \mathbf{f}, \qquad \frac{\partial}{\partial t}(\mathbf{J}\cdot\boldsymbol{\omega}) = \mathrm{div}_\xi\, \mathbf{m} + 2\left(\boldsymbol{\sigma}\cdot\mathbf{x}_\xi^*\right)^{\mathrm{a}} + \mathbf{g},$$
$$\dot{\Phi} = \boldsymbol{\sigma}^* : (\boldsymbol{\upsilon}_\xi - \boldsymbol{\Omega}\cdot\mathbf{x}_\xi) + \mathbf{m}^* : \boldsymbol{\omega}_\xi - \mathrm{div}_\xi \mathbf{h} + H. \tag{3.1}$$

Here ρ_0 is the initial density, $\mathbf{J} = \mathbf{R}\cdot\mathbf{J}^0\cdot\mathbf{R}^*$ ($\mathbf{J}^0$ is a given symmetric and positively definite inertia tensor), $\boldsymbol{\sigma}$ is the Piola-Kirchhoff stress tensor, $\mathbf{m}$ is the couple stress tensor, Φ is the internal energy of a medium per unit volume, $\mathbf{f}$ and $\mathbf{g}$ are the bulk densities of mass forces and couple forces, $\mathbf{h}$ is the heat flux vector, H is the intensity of internal heat sources, div_ξ is the operator of divergence with respect to Lagrangian variables, the colon means a double convolution of tensors, the superscript "a" denotes a vector corresponding to the antisymmetric part of a tensor.

For the reversible processes, the state of which is characterized with the thermodynamic parameters represented by the strain measure $\boldsymbol{\Lambda}$, curvature measure $\mathbf{M}$ and entropy s, the latter equation in the system (3.1) can be rewritten with regard to (2.1) and (2.2) as

$$\frac{\partial\Phi}{\partial\boldsymbol{\Lambda}^*} : \dot{\boldsymbol{\Lambda}} + \frac{\partial\Phi}{\partial\mathbf{M}^*} : \dot{\mathbf{M}} + T\dot{s} = \boldsymbol{\sigma}^* : \left(\mathbf{R}\cdot\dot{\boldsymbol{\Lambda}}\right) + \mathbf{m}^* : \left(\dot{\mathbf{M}} - \boldsymbol{\Omega}\cdot\mathbf{M}\right) - \mathrm{div}_\xi\mathbf{q} + \mathbf{Q},$$

where $T = \partial\Phi/\partial s$ is the absolute temperature, decomposes due to linear independence of the values $\dot{\boldsymbol{\Lambda}}$, $\dot{\mathbf{M}}$ into the constitutive equations:

$$\mathbf{R}^*\cdot\boldsymbol{\sigma} = \frac{\partial\Phi}{\partial\boldsymbol{\Lambda}}, \qquad \mathbf{m} = \frac{\partial\Phi}{\partial\mathbf{M}}, \tag{3.2}$$

heat influx equation:

$$T\dot{s} = -\mathrm{div}_\xi\mathbf{h} + H,$$

and a complementary equation: $\mathbf{m}^* : (\boldsymbol{\Omega}\cdot\mathbf{M}) = 0$. In view of the linear independence of the projections of the angular velocity vector, the complementary equation reduces to the symmetry condition for the tensor $\mathbf{m}\cdot\mathbf{M}^*$, confining general relationship between the elastic potential Φ and the curvature tensor $\mathbf{M}$. With a pass to the coordinate representation, it is possible to prove that this condition holds true only when Φ is a function of the symmetric tensor $\mathbf{M}^*\cdot\mathbf{M}$.

In the case of adiabatic approximation of the model, when $\mathbf{h} = 0$, $H = 0$, a closed system consists of equations of translational and rotational motion from (3.1), constitutive equations (3.2), equation $\dot{\mathbf{R}} = \boldsymbol{\Omega}\cdot\mathbf{R}$ for the tensor of rotation and equation $\dot{s} = 0$ for the entropy.

Adiabatic internal energy takes the simplest form in the case of physically linear approximation, when

$$\Phi(\boldsymbol{\Lambda},\mathbf{M}) = \frac{\lambda}{2}(\mathbf{I}:\boldsymbol{\Lambda})^2 + \mu\boldsymbol{\Lambda}^{\mathrm{s}}:\boldsymbol{\Lambda}^{\mathrm{s}} - \alpha\boldsymbol{\Lambda}^{\mathrm{a}}:\boldsymbol{\Lambda}^{\mathrm{a}} + \gamma\mathbf{M}^*:\mathbf{M}.$$

Constitutive equations (3.2) in this case are as follows

$$\boldsymbol{\sigma} = \lambda(\mathbf{I}:\boldsymbol{\Lambda})\,\mathbf{R} + 2\mu\mathbf{R}\cdot\boldsymbol{\Lambda}^{\mathrm{s}} + 2\alpha\mathbf{R}\cdot\boldsymbol{\Lambda}^{\mathrm{a}}, \qquad \mathbf{m} = 2\gamma\mathbf{M}.$$

Let $\boldsymbol{\tau} = \mathbf{R}^*\cdot\boldsymbol{\sigma}$ be a stress tensor making a dual couple with $\boldsymbol{\Lambda}$,

$$\Psi(\boldsymbol{\tau},\mathbf{m},s) = \boldsymbol{\tau}^*:\boldsymbol{\Lambda} + \mathbf{m}^*:\mathbf{M} - \Phi(\boldsymbol{\Lambda},\mathbf{M},s)$$

be a dual potential equal to the Legendre transform from internal energy. Written in terms of the dual potential, the constitutive equations (3.2) are given in inverted form:

$$\mathbf{R}\cdot\boldsymbol{\Lambda} = \mathbf{R}\cdot\frac{\partial\Psi(\boldsymbol{\tau},\mathbf{m},s)}{\partial\boldsymbol{\tau}} = \frac{\partial\Psi(\mathbf{R}^*\cdot\boldsymbol{\sigma},\mathbf{m},s)}{\partial\boldsymbol{\sigma}}, \qquad \mathbf{M} = \frac{\partial\Psi(\mathbf{R}^*\cdot\boldsymbol{\sigma},\mathbf{m},s)}{\partial\mathbf{m}}.$$

Using (2.1) and (2.2), these equations can be written as

$$\frac{\partial}{\partial t}\frac{\partial\Psi(\mathbf{R}^*\cdot\boldsymbol{\sigma},\mathbf{m},s)}{\partial\boldsymbol{\sigma}} = \boldsymbol{\upsilon}_\xi, \qquad \frac{\partial}{\partial t}\frac{\partial\Psi(\mathbf{R}^*\cdot\boldsymbol{\sigma},\mathbf{m},s)}{\partial\mathbf{m}} = \boldsymbol{\omega}_\xi + \boldsymbol{\Omega}\cdot\frac{\partial\Psi(\mathbf{R}^*\cdot\boldsymbol{\sigma},\mathbf{m},s)}{\partial\mathbf{m}}.$$

This allows representing the model by a thermodynamically consistent system of the laws of conservation in the following sense [4]: it is possible to indicate generating potentials L^0 and L^j, the use of which modifies the complete system of equations in the Cartesian coordinates as follows

$$\begin{gathered}
\rho_0\dot{\upsilon}_i = \sigma_{ij,j} + f_i, \qquad \frac{\partial}{\partial t}(J_{ij}\omega_j) = m_{ij,j} + \varepsilon_{ijk}\sigma_{kl}\frac{\partial\Psi(\mathbf{R}^*\cdot\boldsymbol{\sigma},\mathbf{m},s)}{\partial\sigma_{jl}} + g_i,\\
\frac{\partial}{\partial t}\frac{\partial\Psi(\mathbf{R}^*\cdot\boldsymbol{\sigma},\mathbf{m},s)}{\partial\sigma_{ij}} = \upsilon_{i,j},\\
\frac{\partial}{\partial t}\frac{\partial\Psi(\mathbf{R}^*\cdot\boldsymbol{\sigma},\mathbf{m},s)}{\partial m_{ij}} = \omega_{i,j} + \varepsilon_{ikl}\omega_k\frac{\partial\Psi(\mathbf{R}^*\cdot\boldsymbol{\sigma},\mathbf{m},s)}{\partial m_{lj}},\\
\dot{R}_{ij} = \varepsilon_{ikl}\omega_k R_{lj}, \qquad \dot{s} = 0, \qquad J_{ij} = J^0_{kl}R_{ik}R_{jl}
\end{gathered} \tag{3.3}$$

(ε_{ijk} is the discriminant tensor) and makes it uniform:

$$\frac{\partial}{\partial t}\frac{\partial L^0(\mathbf{DU})}{\partial\mathbf{U}} = \frac{\partial}{\partial\xi_j}\frac{\partial L^j(\mathbf{U})}{\partial\mathbf{U}} + \mathbf{F}(\mathbf{D},\mathbf{U}), \qquad \frac{\partial\mathbf{D}}{\partial t} = \mathbf{G}(\mathbf{D},\mathbf{U}). \tag{3.4}$$

Here $\mathbf{U}$ is the column-vector composed of unknown functions, except for the entropy, namely, projections of vectors of velocity of translational motion and angular velocity, components of tensors of stresses and couple stresses; $\mathbf{D}$ is the nonsingular matrix, which non-zero and non-unit elements are given by the values R_{ij}; $\mathbf{F}$ and $\mathbf{G}$ are the preset vector and matrix functions readily determinable from the form of the equation. The generating potentials are equal to:

$$L^0(\mathbf{DU}) = \rho_0 \frac{\upsilon_i \upsilon_i}{2} + \frac{1}{2}(\mathbf{R}^* \cdot \boldsymbol{\omega})_i J^0_{ij} (\mathbf{R}^* \cdot \boldsymbol{\omega})_j + \Psi(\mathbf{R}^* \cdot \boldsymbol{\sigma}, \mathbf{m}, s),$$

$$L^j(\mathbf{U}) = \upsilon_i \sigma_{ij} + \omega_i m_{ij}.$$

The equation for the entropy from (3.3) is not included in the system (3.4), as it automatically yields an equivalent equation in the form of auxiliary law of conservation:

$$\frac{\partial}{\partial t}\left(\mathbf{U} \cdot \frac{\partial L^0(\mathbf{DU})}{\partial \mathbf{U}} - L^0(\mathbf{DU})\right) = \frac{\partial}{\partial \xi_j}\left(\mathbf{U} \cdot \frac{\partial L^j(\mathbf{U})}{\partial \mathbf{U}} - L^j(\mathbf{U})\right) + \mathbf{U} \cdot \mathbf{F} - \frac{\partial L^0(\mathbf{DU})}{\partial \mathbf{U}} \cdot \mathbf{D}^{-1}\mathbf{GU}.$$

The system of equations (3.4) possesses some essential properties reflective of mathematical correctness of the model. It has a divergent form and can serve to describe generalized solutions with discontinuous velocities and stresses – shock waves and contact discontinuities at interfaces of media having different mechanical properties. Solving such systems involves effective computational algorithms adapted to calculation of discontinuities [5].

Differentiation brings the system (3.4) into symmetric form:

$$\begin{pmatrix} \mathbf{I} & 0 \\ 0 & \mathbf{A} \end{pmatrix} \frac{\partial}{\partial t}\begin{pmatrix} \mathbf{D} \\ \mathbf{U} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{B}^j \end{pmatrix} \frac{\partial}{\partial \xi_j}\begin{pmatrix} \mathbf{D} \\ \mathbf{U} \end{pmatrix} + \begin{pmatrix} \mathbf{G} \\ \mathbf{H} \end{pmatrix}. \tag{3.5}$$

Here the matrices $\mathbf{A}$ and $\mathbf{B}^j$ are symmetric, and, moreover, when the potential $L^0(\mathbf{DU})$ is strongly convex, the matrix $\mathbf{A}$ is positively definite. Therefore, the system of equations (3.4) is of hyperbolic type. The strong convexity condition L^0 is fulfilled, when the dual potential $\Psi(\boldsymbol{\tau}, \mathbf{m}, s)$ is a strongly convex function with respect to the set of variables $\boldsymbol{\tau}$ and $\mathbf{m}$. In this case a priori integral estimates in characteristic cones for the system (3.5) can be obtained, which guarantee the uniqness and continuous dependence on the initial data of solutions of the Cauchy problem and the boundary-value problems with dissipative boundary conditions.

In [6, 7] the questions about symmetry and hyperbolicity of the governing system of equations of the nonlinear Cosserat theory were studied using an unnatural curvature measure, that could not be represented in terms of actual displacements and rotations.

Acknowledgment

This work was supported by the Complex Fundamental Research Program no. II.2P “Integration and Development” of SB RAS (project no. 0356-2016-0728).

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Received October 11, 2016; accepted version February 14, 2017.