



## DELAYED-DAMAGE MODELLING FOR FRACTURE PREDICTION OF LAMINATED COMPOSITES UNDER DYNAMIC LOADING

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The basic aspects of a damage meso-model with delay effects for laminated composites are presented. The applications concern fracture prediction under severe and multiaxial dynamic loading. In order to illustrate the possibilities and the mechanical significance of the proposed model for the prediction of rupture, especially the role of the delay effect, various simulations of one-dimensional wave propagation are performed. Then, a more complex finite element calculation is presented in order to show the ability of the model to predict the response of a composite structure until complete fracture.

### 1. INTRODUCTION

The aim of this paper is to present the basic aspects of a delay-damage meso-model of laminated composites, such as SiC-SiC or carbon-epoxy laminates. The application concerns the fracture prediction of this kind of structure under severe and multiaxial dynamic loading. For such loading, the notion of homogenised material is meaningless. The first key point is thus to define the scale at which the material may be properly described. A pragmatic approach consists in determining a characteristic length of the main damage mechanisms which are the same in statics and in dynamics. For example, in the case of carbon-epoxy materials, these mechanisms are (i) delamination (ii) inside the layer: matrix microcracking, fibre/matrix debonding and fibre fracture. Moreover, at least in statics, they appear to be nearly homogeneous throughout the thickness of each layer.

Our proposal is thus to test an extension to dynamics of a damage meso-model previously defined [1] and developed in statics [2, 3]. The main feature of this damage meso-model is to introduce the damage mechanisms previously described by means of internal damage variables which are constant throughout the thickness of each ply. Furthermore, an interface damage model is introduced to deal with delamination [4]. Since the damage evolution due to force variations cannot be instantaneous, a delayed-damage model is introduced [1].

It is well-known that the use of classical (i.e. local and time-independent) damage models is inconsistent with the prediction of fracture. This leads, when used in F.E. codes, to a spurious mesh-dependency [5]. The paper focuses on the delayed-damage model formulation and its use. In order to illustrate the improvement obtained in the prediction of rupture, various simulations of one-dimensional wave propagation are performed. The results are shown to be mesh-insensitive in terms of fracture area and dissipated energy. Then, an example of a three-dimensional finite element calculation involving strongly damaged areas is presented. It shows the ability of the model to predict the response of a composite structure subjected to dynamic loading [6].

## 2. MESO-MODELLING OF LAMINATED COMPOSITES [1]

When dealing with composites, the first key point is the scale at which the model is constructed. This is also the scale at which the computation will have to be performed. On the one hand, the use of the micro-scale, independent of other and numerous difficulties, would not enable us to keep the cost of computation within sensible limits. On the other hand, the use of the macro-scale would not enable us to respect the main and basic feature of the laminate and its deterioration. Moreover, for severe dynamic loading, the notion of homogenised material is meaningless. The first key point is thus to define the scale at which the material may be properly described without going into too much detail. A pragmatic approach is to determine a characteristic length of the main damage mechanisms. For 2D composites, between the macro-scale of the structure and the micro-scale of the single fibre, an intermediate and preferred modelling scale exists which is called the meso-scale. It is the scale associated with the thickness of the layer and of the different interlaminar interfaces. At this scale, the main damage mechanisms (delamination, matrix microcracking, fibre/matrix debonding and fibre fracture) appear to be nearly uniform in each meso-constituent, at least under quasi-static loading, and thus may be described in a rather simple way. Our idea is that, due to the smallness of the meso-scale (one tenth of a mm), the previous description of the damage mechanisms should be valid even for high-rate loading (Fig. 1).

Thus, our proposal is to adapt a meso-model, previously defined for static loadings [1–4], to the dynamic case. This meso-model is initially defined by means of two meso-constituents:

- a single layer,
- an interface which is a mechanical surface connecting two adjacent layers and depending on the relative orientation of their fibres.

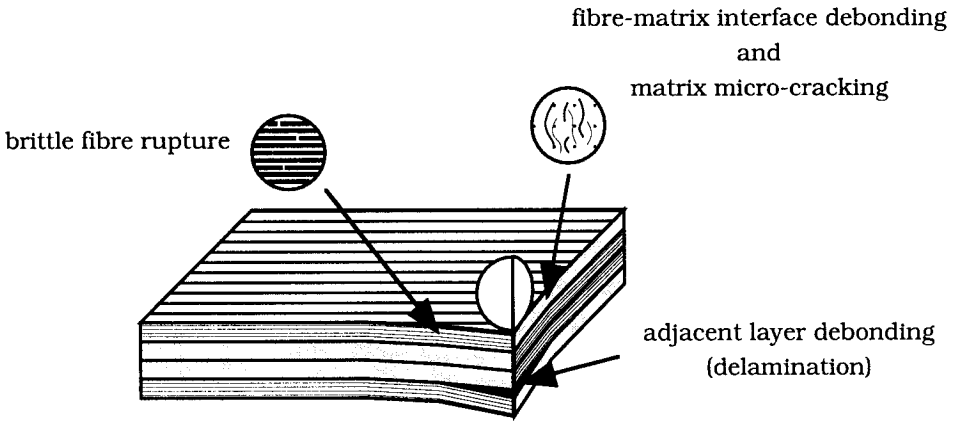


FIG. 1. Damage and failure mechanisms.

The damage mechanisms are taken into account by means of internal damage variables. A meso-model is then defined by adding another property: a uniform damage state is prescribed throughout the thickness of the elementary ply. This point plays a major role when trying to simulate a crack with a damage model. As a complement, delayed-damage models are introduced (Fig. 2).

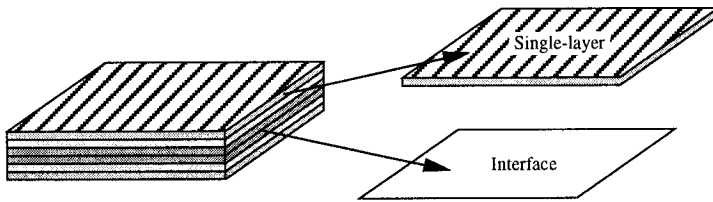


FIG. 2. Laminate modelling.

One limitation of the proposed meso-model is that the fracture of the material is described by means of only two types of macrocracks:

- delamination cracks within the interfaces,
- cracks, orthogonal to the laminate mid-plane, with each cracked layer being completely cracked throughout its thickness.

This paper is focused on inner-layer damage mechanisms, and therefore the interface model is not discussed.

### 3. DAMAGE MODEL WITH DELAY EFFECT

It is well known that classical damage models are inadequate for a proper description of the fracture. Consequently, the numerical simulation of failure,

initiated by strain softening, exhibits an excessive mesh dependency [5]. One way to avoid such numerical difficulties is to use localisation limiters [7]. It is a regularisation procedure based on the introduction of additional terms in the continuum formulation. A large class of these limiters has been studied by de Borst and SLUYS [8]. For example, non-local theory [9] or the second-gradient approach [7, 10] include higher-order gradient terms. Alternatively, the use of material rate dependence in the constitutive model implicitly introduces a length scale into the governing equations of the problem and then eliminates the pathological mesh sensitivity [11, 12]. Let us note that a rate-dependent damage model has been proposed to deal with the fracture of concrete [13].

At present, the question is to propose and identify a physically sensitive damage model that provides a consistent prediction of fracture. Of course, such models strongly depend on the type of material under consideration. In particular, the length scale which has to be introduced is connected to the material's internal length scales (i. e. heterogeneity). This is the idea governing the damage meso-model previously described. This paper is focused on the formulation and use of a delayed-damage model in dynamic situations. First, unidirectional examples are considered; then, application to the fracture prediction of the single layer under two-dimensional loads is presented.

### 3.1. Main properties

In order to investigate the performance of the damage model with delay effect, we consider the classical example, previously proposed by BAZANT [5], of a bar submitted to two tension pulses (Fig. 3). The magnitude of the initial force,  $F_{\max}$ , is chosen such that no damage occurs during the propagation of waves until the middle of the bar. Then, the doubling in stress due to the superposition of the tension wave causes the initiation of fracture. The material enters the softening regime and a localisation zone emerges.

The analysis is based on a simple one-dimensional damage model with only one scalar damage variable. The model is defined by its strain energy,  $E_D$ , which is split into two parts according to the fact that the cracks can be closed or open.

$$\begin{aligned}
 \sigma &= E^0(1-d)\langle\varepsilon\rangle_+ + E^0\langle-\varepsilon\rangle_+, \\
 (3.1) \quad E_D &= \frac{1}{2} \left[ \frac{\langle\sigma\rangle_+^2}{E^0(1-d)} + \frac{\langle-\sigma\rangle_+^2}{E^0} \right], \\
 Y &= \left. \frac{\partial E_D}{\partial d} \right|_{\sigma:\text{cst}} = \frac{\langle\sigma\rangle_+^2}{2E^0(1-d)^2} = \frac{E^0\langle\varepsilon\rangle_+^2}{2},
 \end{aligned}$$

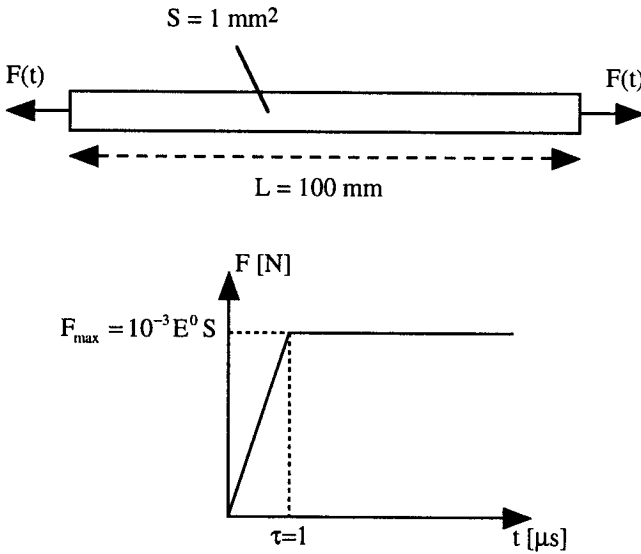


FIG. 3. One-dimensional bar problem.

where  $Y$  is the damage energy release rate.  $Y$  is supposed to drive the damage evolution. In fact, for many long fibre-composites [2, 3] and for a progressive damage mode, a typical quasi-static damage evolution law is:

$$(3.2) \quad \begin{cases} d = \langle f(Y) \rangle_+ & \text{if } d < 1 \\ d = 1 & \text{otherwise} \end{cases} \quad \text{with} \quad \begin{cases} Y|_t = \sup_{\tau \leq t} Y|_\tau, \\ f(Y) = \frac{\sqrt{Y} - \sqrt{Y_0}}{\sqrt{Y_c}}. \end{cases}$$

For the numerical simulation, a central difference scheme with a lumped mass matrix is used. The numerical consequence of the poorly-posed problem is demonstrated by a mesh sensitivity analysis. The bar is divided into 100, 200 and 400 elements, respectively. Figure 4 gives the dissipated energy in the bar versus time. It is clear that the numerical result is inconsistent. Indeed, the size of the fracture area is the same as the size of one element of the mesh. So, the dissipation value directly depends on the spatial discretisation and tends to zero with the size of the element.

In order to obtain a consistent and “physical” computational damage approach, a damage model with delay effect is introduced. The physics in such a model is that the damage evolution is not instantaneous but rather must be governed by some internal characteristic time. The idea is that, in combination with a dynamic analysis, the characteristic time will introduce a characteristic length by means of a characteristic speed. The type of model under study herein respects the following properties:



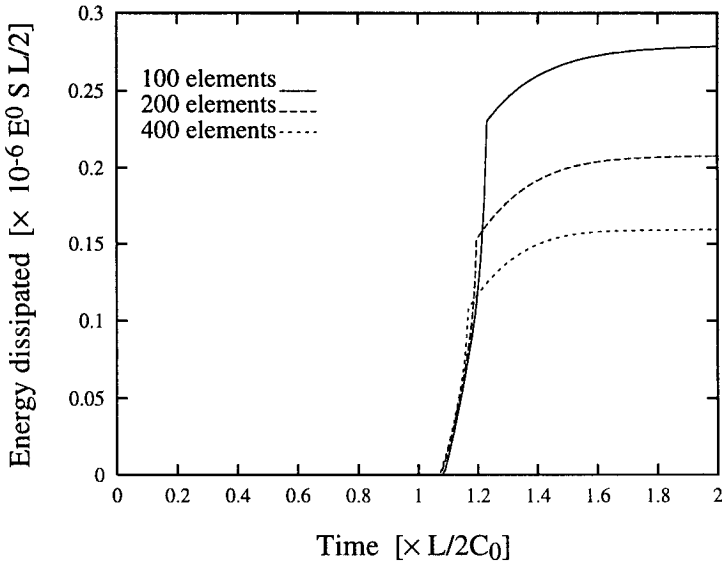


FIG. 4. Energy dissipated versus time for a classical damage model.

- agreement with the static analysis,
- the size of the fracture process zone is comparable to the thickness of the ply.

This last requirement leads to a clear distinction between delayed-damage models and visco-elastic or visco-anelastic models: the characteristic time introduced in the delayed-damage model is of several orders of magnitude weaker than in the viscous case. Certain composites, like glass-epoxy laminates, display significant viscosities which obviously influence their dynamic response [14]. However, the characteristic times introduced in such cases are not connected with the fracture process.

In order to ensure compliance with the first condition along with bounding the damage rate, the following damage evolution law is introduced:

$$(3.3) \quad \begin{cases} \dot{d} = \frac{k}{a} [1 - \exp(-a\langle f(Y) - d \rangle_+)] & \text{if } d < 1, \\ d = 1 & \text{otherwise.} \end{cases}$$

Therefore, for this model, a variation of the force  $Y$  does not lead to an instantaneous variation of the damage variable  $d$ . There is a certain delay defined by the characteristic time  $a/k$ . Moreover, a maximum damage rate, which is  $k/a$ , does exist. The consequence of the delayed-damage can be seen, from the stress-strain curves, in terms of rate effects (Fig. 5). For this problem, the parameters of the model are:  $E_0 = 57 \cdot 10^3$  MPa,  $\rho = 2.28 \cdot 10^3$  kg/m<sup>3</sup>,  $C_0 = 5 \cdot 10^3$  m/s,  $Y_0 = 0.05$  MPa,  $Y_c = 0.23$  MPa,  $a = 10$  and  $k = 5 \cdot 10^6$  s<sup>-1</sup>.

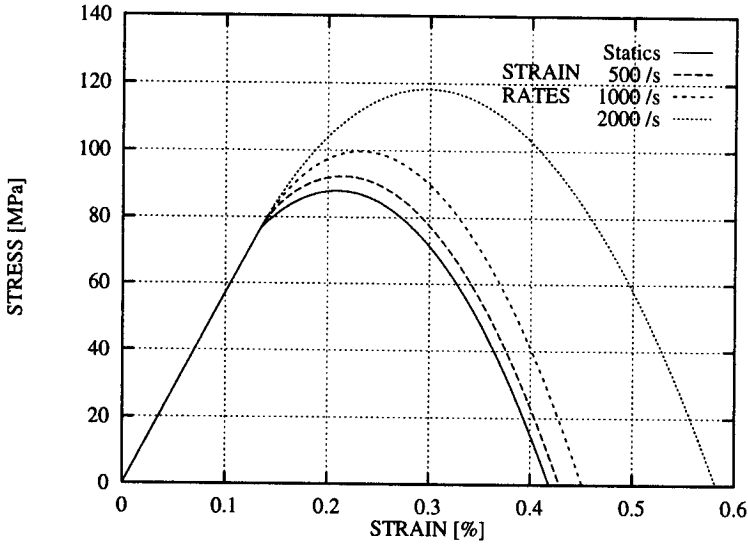


FIG. 5. Stress-strain curves for different imposed strain rates.

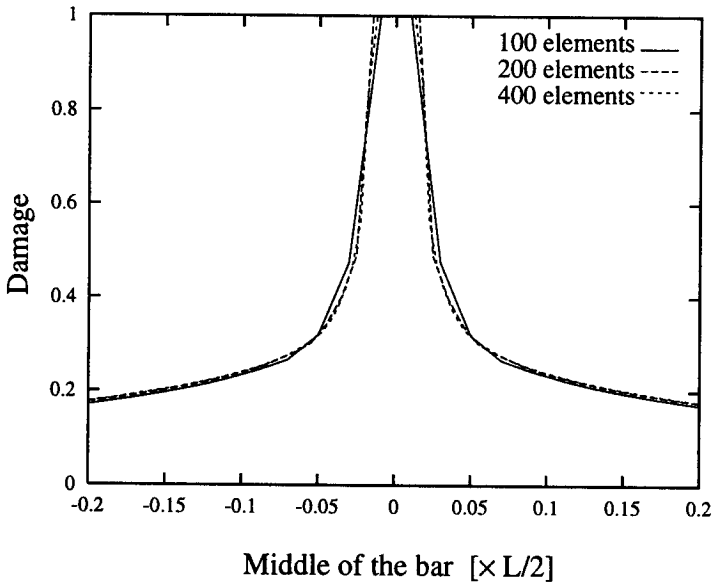


FIG. 6. Damage along the middle part of the bar at  $t = L/C_0$ .

Figures 6 and 7 show what happens for the model with delay effect. It can be seen that the numerical results do not depend on the mesh size. For example, at time  $t = L/C_0 = 20 \mu s$ , Fig. 6 displays the damage value in terms of the spatial variable for the middle part of the bar. The fracture region is about 2 mm long.

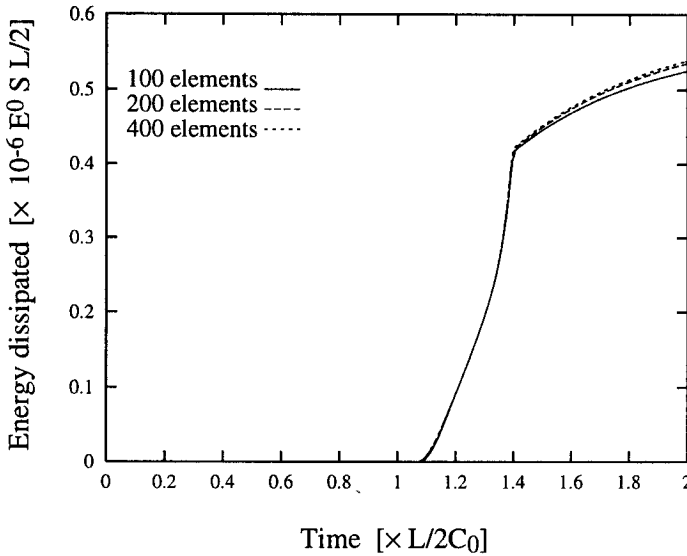


FIG. 7. Energy dissipated versus time for a damage model with delay effect.

### 3.2. Influence of the delayed-damage parameters $k$ and $a$

The study of the influence of the delayed-damage parameters on fracture prediction is conducted using the one-dimensional bar example previously defined (Fig. 8). For various stress values, we seek the minimum time,  $\Delta t$ , after which fracture occurs in the middle of the bar.

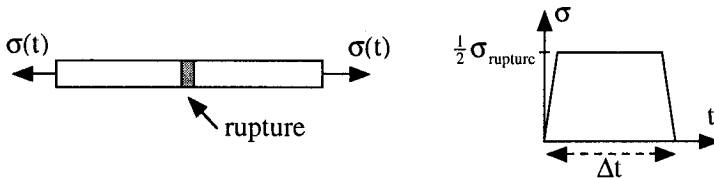


FIG. 8. Fracture prediction.

Examining these graphs (Fig. 9), we find that all curves have a similar shape:

- a vertical asymptote corresponding to the characteristic time,
- a horizontal asymptote which seems to coincide with the static instability stress value.

This type of results is qualitatively representative of plate-plate impact experiments with spallation in which the stress at the fracture threshold varies with the width of the pulse [15]. Thus, one can expect to identify  $k$  and  $a$  by means of experimental dynamic testing.



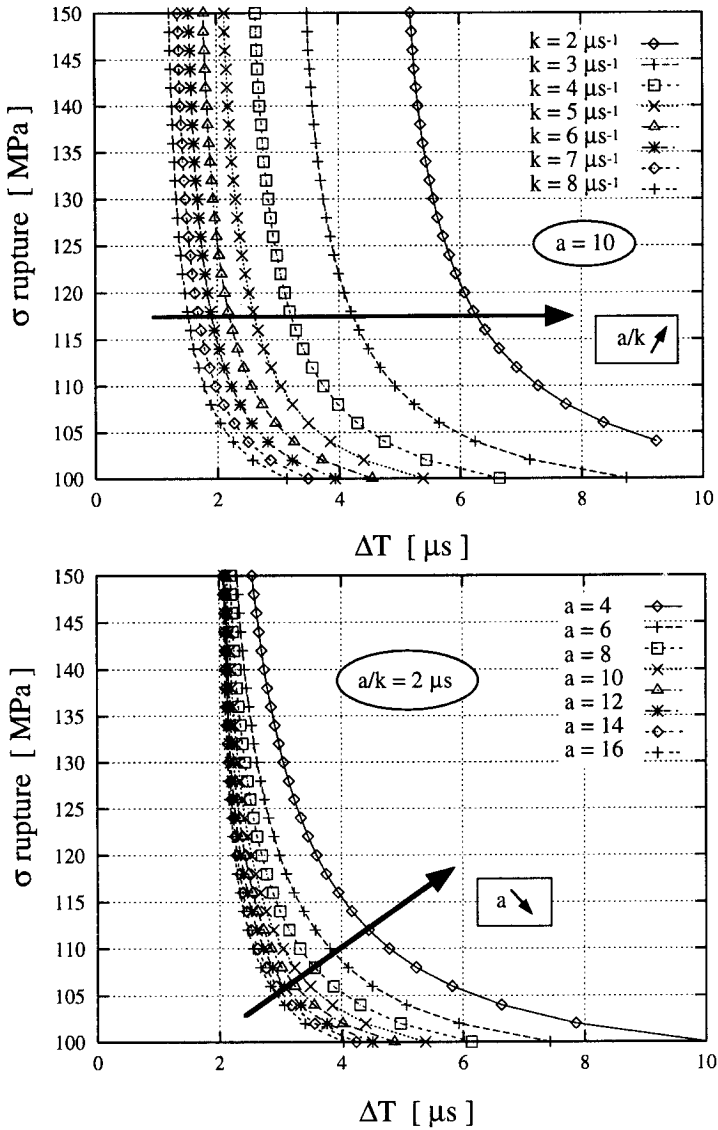


FIG. 9. Influence of  $k$  and  $a$  on the fracture prediction.

#### 4. MODELLING OF THE ELEMENTARY PLY

In order to deal with perhaps more realistic situations, let us consider for example the case of a laminated SiC/MAS-L composite, with Silicon Carbide fibres and a glass matrix, submitted to in-plane loading. This material is made by the French Aerospace Society. The fibre stiffness (200 GPa) is higher than the matrix stiffness (75 GPa), and cracks appear first in the matrix. In the case

of static loading, the material has been modelled and characterised in [16] on the basis of previous studies on carbon-epoxy laminates. Each layer is reinforced in only one direction. In what follows, subscripts 1, 2 and 3 designate the fibre direction, the transverse direction inside the layer and the normal direction, respectively.

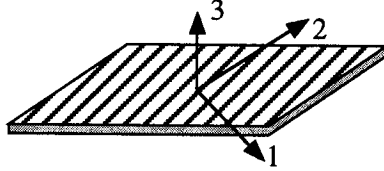


FIG. 10. Elementary ply.

Three scalar damage variables, assumed to be constant throughout the thickness of the ply, are used:  $d_1$  associated with cracks orthogonal to the fibre direction,  $d_2$  and  $d_{12}$  associated with cracks parallel to the fibre direction.

#### 4.1. Damage kinematics

The model is built in order to respect two experimental observations:

- a different behaviour between tension and compression,
- a constant ratio between  $\nu_{12}$  and  $E_1$  constants.

Therefore, in the case of plane stress, the following expression of the strain energy density of the damaged elementary layer is introduced:

$$(4.1) \quad E_D = \frac{1}{2} \left[ \frac{\langle \sigma_{11} \rangle_+^2}{E_1^0(1-d_1)} + \frac{\langle -\sigma_{11} \rangle_+^2}{E_1^0} - 2 \frac{\nu_{12}^0}{E_1^0} \sigma_{11} \sigma_{22} \right. \\ \left. + \frac{\langle \sigma_{22} \rangle_+^2}{E_2^0(1-d_2)} + \frac{\langle -\sigma_{22} \rangle_+^2}{E_2^0} + \frac{\sigma_{12}^2}{G_{12}^0(1-d_{12})} \right],$$

where  $\langle \cdot \rangle_+$  denotes the positive part. This allows for distinguishing the tension and compression behaviour according to whether the cracks are closed or open. The damage energy release rate associated with  $d_1$ ,  $d_2$  and  $d_{12}$  have the following expressions:

$$(4.2) \quad Y_1 = \frac{\partial [E_d]}{\partial d_1} \Big|_{\sigma:\text{cst}} = \frac{[\langle \sigma_{11} \rangle_+^2]}{2E_1^0(1-d_1)^2}, \\ Y_2 = \frac{\partial [E_d]}{\partial d_2} \Big|_{\sigma:\text{cst}} = \frac{[\langle \sigma_{22} \rangle_+^2]}{2E_2^0(1-d_2)^2}, \\ Y_{12} = \frac{\partial [E_d]}{\partial d_{12}} \Big|_{\sigma:\text{cst}} = \frac{[\sigma_{12}^2]}{2G_{12}^0(1-d_{12})^2},$$

where  $[\cdot]$  denotes the mean value in the thickness of one ply.

#### 4.2. Static damage evolution law

For the sake of simplicity, the behaviour in the fibre direction is assumed to be independent of the transversal and shear behaviours. Moreover, the model introduces a coupling, by means of the material parameter  $b$ , between the evolution of  $d_2$  and  $d_{12}$  which, on average, are both associated with the same type of cracks.

$$(4.3) \quad \begin{aligned} d_1 &= f(\mathbf{Y}_1), \\ d_2 &= g(\mathbf{Y}_{12} + b\mathbf{Y}_2), \\ d_{12} &= f(\mathbf{Y}_{12} + b\mathbf{Y}_2), \quad \text{with } \mathbf{Y}|_t = \sup_{\tau \leq t} \mathbf{Y}|_\tau. \end{aligned}$$

The expression of  $\mathbf{Y}$  ensures the property that for quasi-monotonic loading, the damage variables depend on the maximum of the damage forces over the time interval of their action. The identification of the ply, performed by GASSER [16], is carried out by macro-tests (in tension-compression) on different laminate stacking sequences. The classical laminate theory is then used to obtain information at the scale of the elementary ply. For example, we plot the tension-compression test on the unidirectional specimen (Fig. 11) and the evolution law of  $d_1$  versus  $Y_1$  (Fig. 12).

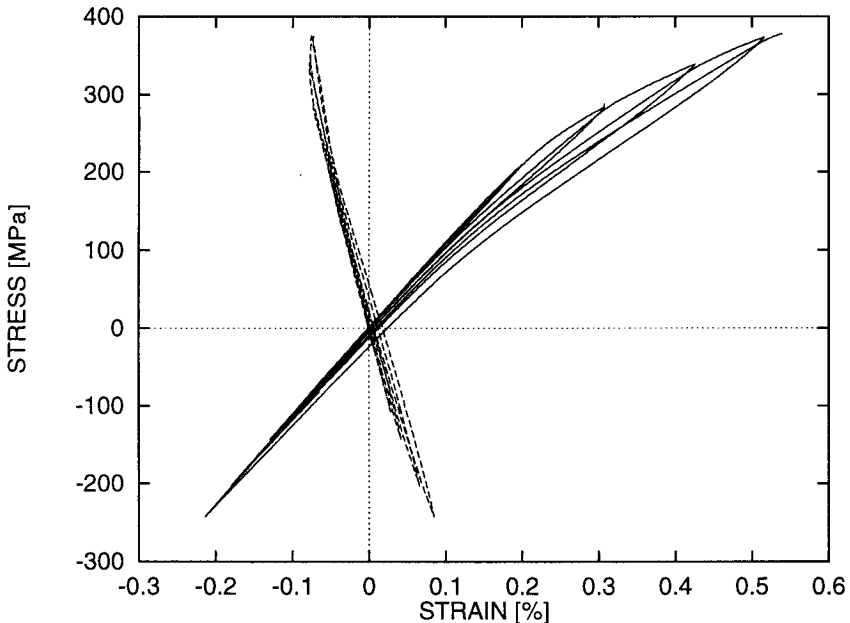
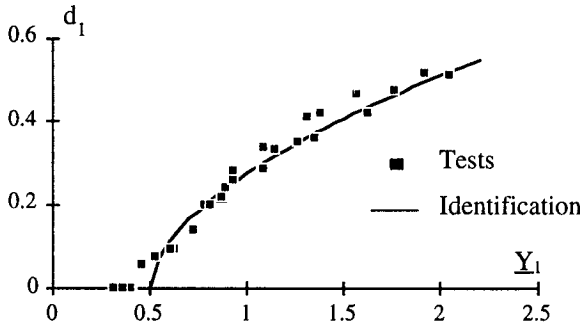


FIG. 11. Tension-compression test on unidirectional SiC/MAS-L.

FIG. 12. Evolution law  $d_1 = f(Y_1)$ .

#### 4.3. Delayed-damage model of the single layer

In accordance with the main lines of Sec. 2, the delayed-damage model of the single layer is defined as follows:

$$(4.4) \quad \begin{aligned} \dot{d}_1 &= \frac{k}{a} \left\{ 1 - \exp \left[ -a \langle f(Y_1) - d_1 \rangle_+ \right] \right\}, \\ \dot{d}_2 &= \frac{k}{a} \left\{ 1 - \exp \left[ -a \langle g(Y_{12} + bY_2) - d_2 \rangle_+ \right] \right\}, \\ \dot{d}_{12} &= \frac{k}{a} \left\{ 1 - \exp \left[ -a \langle h(Y_{12} + bY_2) - d_{12} \rangle_+ \right] \right\}. \end{aligned}$$

Initially, the same delayed-damage parameters ( $k$  and  $a$ ) are introduced for the various damage mechanisms, which are in fact all mechanisms related to a matrix deterioration. Based on our experiment, the values of  $k$  and  $a$  which have been used are the following:

$$k = 0.5 \mu\text{s}^{-1} \quad \text{and} \quad a = 1$$

corresponding to a characteristic time

$$t_c = 2 \mu\text{s}$$

such that the size of the fracture zone is of the order of magnitude of ply's thickness.

## 5. RUPTURE COMPUTATION

An example of three-dimensional finite element computation is presented in order to show the ability of the model with delay effect to predict the response of a composite structure until the ultimate fracture. This example, computed

with the explicit dynamic code LS-DYNA3D [17], involves a unidirectional plate with a hole subjected to compression (Fig. 13). Here, the direction of loading is the same as the direction of fibres, and the impact velocity 5 m/s is set. The symmetries of the problem mean that we only need to simulate one-quarter of the structure. All the results presented below have been plotted for the three different finite element discretisations:  $10 \times 35$ ,  $20 \times 75$  and  $40 \times 140$  hexahedron elements with one integration point.

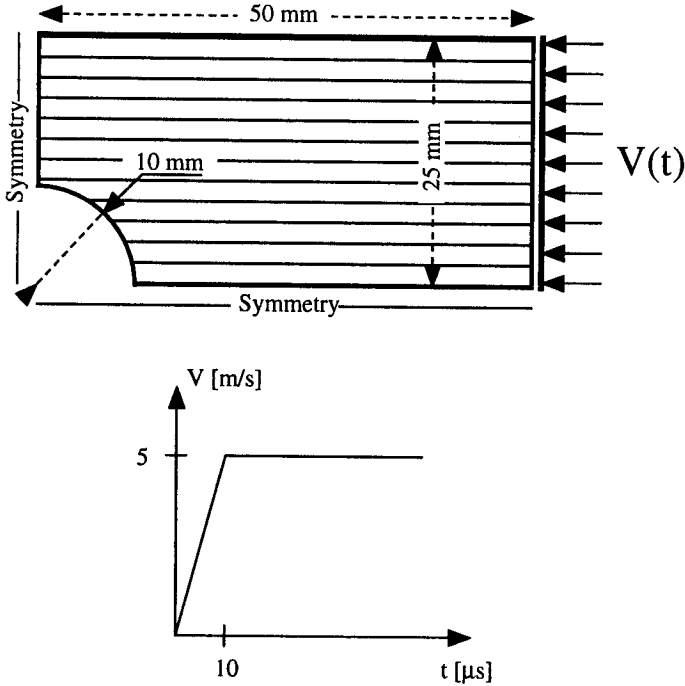


FIG. 13. Unidirectional plate with a hole subjected to compression.

In this example, the initiation of fracture appears around the hole and then propagates in the longitudinal direction. It is the damage variable  $d_2$ , associated with microcracks parallel to the fibre direction, that first reaches an ultimate value of one.

### 5.1. Classical damage model

Prior to investigating the damage model with delay effect, we present the difficulties concerning a classical damage model. Like for the one-dimensional case, the width of the fracture band is determined by element's size. Indeed, as regards the contours of the fracture area at time  $t = 35 \mu\text{s}$  (Fig. 14), the total rupture is localised along a line in just one element.

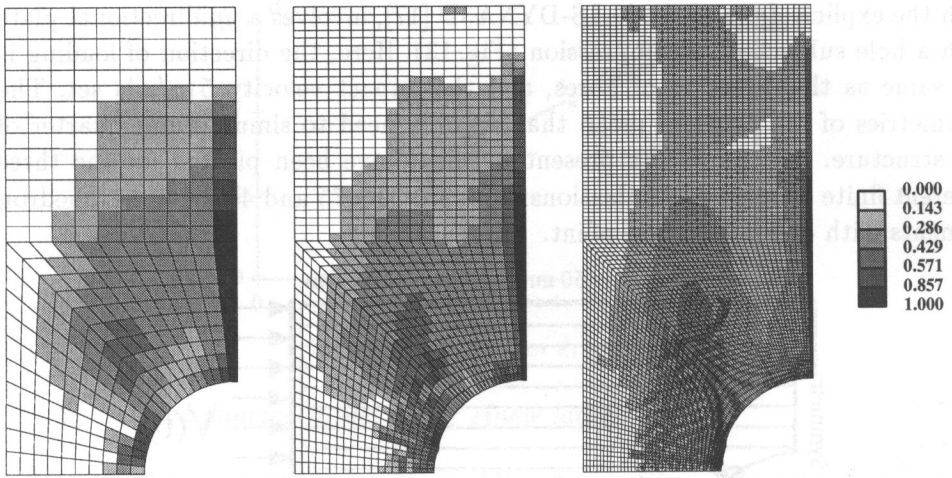


FIG. 14. Damage variable  $d_2$  at  $t = 35 \mu\text{s}$  for a classical damage model.

Mesh dependency is also obvious from Figs. 15 and 16 in which load-displacement curves and energy consumption in the structure are plotted for the various meshes. In the force-displacement curves, it can be observed that, up to the global instability, the solutions coincide. Nevertheless, during the second stage of the rupture process (after the critical point), the three curves are very different. Moreover, the effect of mesh refinement leads to different predictions for the global instability and to slightly steeper unstable branches. These figures thus suggest that the behaviour becomes more brittle upon mesh refinement.

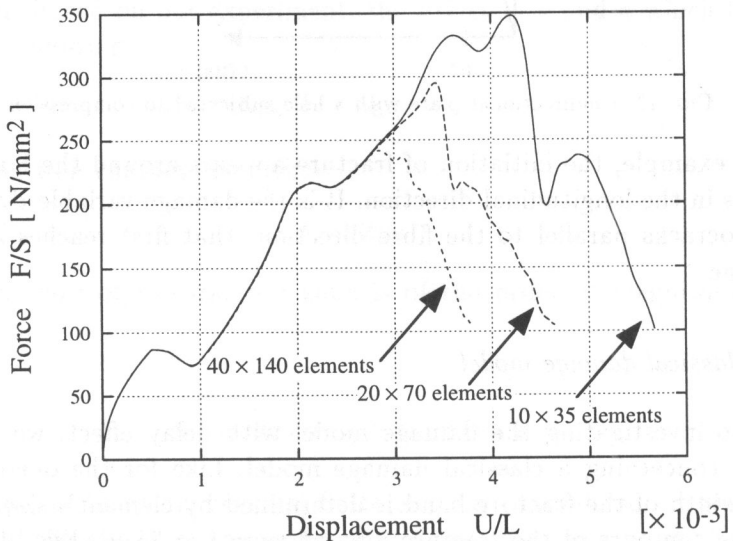


FIG. 15. Load-displacement curves for a classical damage model.

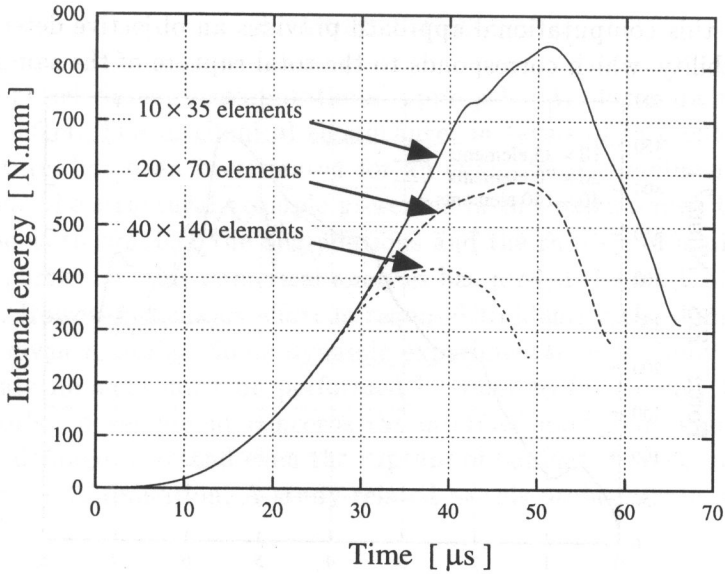


FIG. 16. Energy consumption in the structure for a classical damage model.

5.2. Damage model with delay effect

This computation now makes use of the delayed damage of the single layer. The totally damaged areas are as such reasonably similar for the various meshes (Fig. 17).

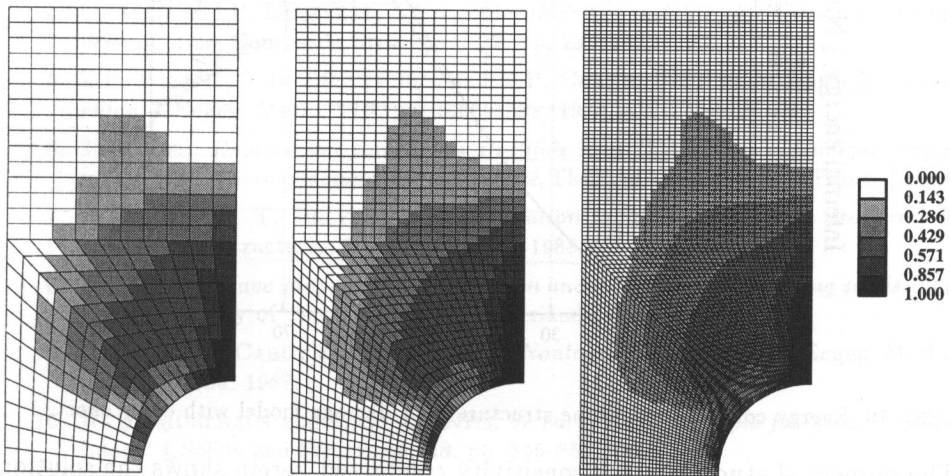


FIG. 17. Damage variable  $d_2$  at  $t = 35 \mu s$  for a damage model with delay effect.

Moreover, the load-displacement curves (Fig. 18) and the energy consumption in the composite structure (Fig. 19) are approximately the same for the three

meshes. So, this computational approach provides an objective detection of the global instability, which corresponds to the total rupture of the sample.

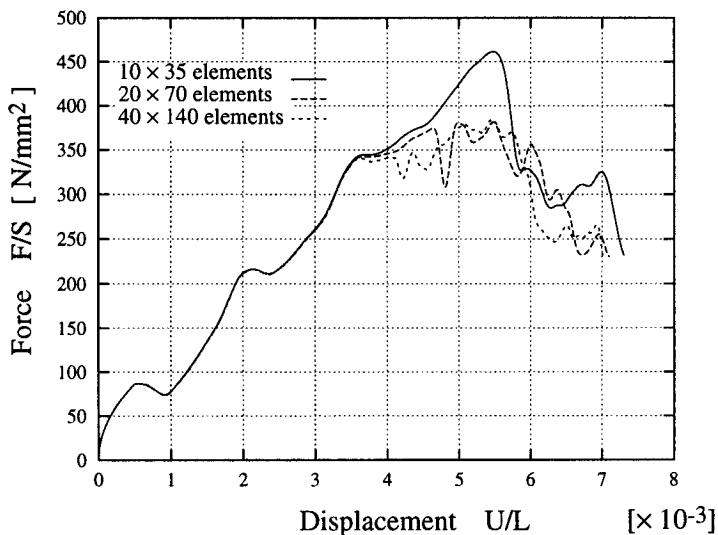


FIG. 18. Load-displacement curves for a damage model with delay effect.

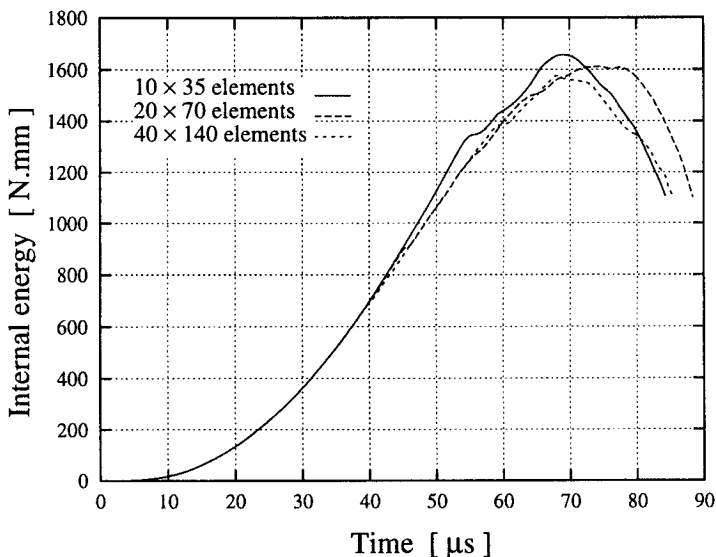


FIG. 19. Energy consumption in the structure for a damage model with delay effect.

The numerical study of mesh sensitivity conducted herein shows the solution to be mesh-independent for the delayed damage modelling. Obviously, the numerical results must be compared with experimental data. In this case, it will be certainly necessary to update the new damage parameters and to use very fine spatial discretisations.



## 6. CONCLUSIONS

The proposed damage computational approach introduces damage models with delay effect. The mechanical significance, in terms of rate effects and dynamic fracture, has been investigated for an example of one-dimensional wave propagation. The structural example presented in this paper shows the aptitude of such models to simulate the degradations and the rupture of laminated composites. In particular, this numerical analysis has proved the mesh objectivity of the results. However, a major effort is required to identify the damage parameters for a given material. Some dynamic experimental tests (Split-Hopkinson or plate-plate impact) must be performed in order to fully define the damage model. Another development concerns the interface modelling which allows to predict the damage state and even the rupture of laminated structures for loadings leading to delamination. A study related to this problem is in progress [6].

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