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ESTIMATION OF THE DEFLECTION ANGLE OF PLATE DRIVEN BY THE PRODUCTS OF GRAZING DETONATION

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The deflection angle of a metal plate launched by the products of detonation grazing along the surface of the plate has been estimated theoretically. The influence of parameters characterising the plate and an explosive charge on quantitative characteristics of the launching process has been analysed. Results of the numerical calculations presented in the form of diagrams enable simple estimation of a dynamic deflection angle of the plate for given parameters of the explosive system. Estimated values of the angle can be applied in construction of the explosive schemes destined for cladding and strengthening purposes.

1. Introduction

In the explosive methods destined for cladding and strengthening purposes the systems are widely used, in which the surface of a metal sample to be loaded is struck by an oblique impact of a metal plate. The oblique collision of metal sheets can be accomplished in the explosion system, in which the sample and the plate with touched explosive layer are placed in configuration with an angle between them (an angle system). The plate is being launched by the gaseous products of detonation moving in a direction perpendicular to the plate surface. Similar effect of an oblique collision can be also achieved in a parallel system, in which the surface of the metal sample to be loaded is parallel to the metal plate. In this case the detonation wave moves along the plate surface (a grazing detonation) and the plate is deflected due to the action of the detonation product. The dynamic deflection angle of the plate, defined as an angle of tangent to the plate, changes during the motion of the tube. Intensity and configuration of shock waves inside the sample depend in an appropriate manner on the value of deflection angle of the plate at the moment of the collision, thus the design of the explosive system requires determination of the relation between the deflection angle and parameters of the system. These parameters are the thickness of plate and the density of the plate material, the thickness of the explosive layer, the density and detonation velocity of the explosive. In case of the angle system, the deflection angle is constant and depends on the configuration of system elements, and a normal velocity of the plate can be easily estimated from the Gurnay formula, for example. In case of the parallel system, the estimation of the dynamic deflection angle and the plate velocity is much more difficult. The complex process of the detonation moving along the plate must be considered.

In Ref. [1], a problem of driving a plate by the products of grazing detonation was solved by making use of some simplifications. Material of the plate was treated as ideal incompressible liquid, detonation products were described by a model of polytropic gas, the lateral expansion of the products was neglected and finite dimensions of the plate were not taken into considerations. Solving of the problem was divided into two stages. It is assumed in the first stage that the expansion of the detonation products does not influence the initial phase of plate motion, that is, the acceleration of the plate by the explosive layer of finite thickness is the same as that for the explosive of infinite thickness. This assumption is justified up to the time when a rarefaction wave from the free surface of explosive reaches the plate. It is assumed that after that moment, the velocity of plate increases insignificantly, that is, the plate moves with constant velocity. In the second stage of the solution, the time is determined, at which the rarefaction disturbances propagating from the free surface attain the plate.

Complete solution of the stationary two-dimensional problem of driving a metal plate is given in Refs. [2, 3]. In the model describing the process of launching the plate by the products of detonation grazing along its surface, the solution of the problem of lateral expansion of the detonation products of a plane charge proposed in Ref. [4] was made use of. This solution is composed of the solution in the region adjacent to the detonation wave, obtained by expanding the parameters of state and motion in power series of the axial and radial coordinate, and numerical solution of the problem of supersonic flow of detonation products obtained by the method of characteristics. Refs. [2, 3] contain only some examples of computation results, without detailed analysis of the influence of particular parameters of the system on the deflection angle of the plate. The monograph [5] is also limited to the quotation of some selected results obtained by the method described in Ref. [3].

The present paper contains a detailed discussion of the influence of the parameters of the explosive system on the dynamic deflection angle of the plate on the grounds of the model proposed in Ref. [3]. The problem was solved by the method based on the idea of S.K. GODUNOV'S difference scheme [6], but instead of complex solution in the region adjacent to the detonation wave obtained by expanding the parameters of state and motion in power series, the solutions of the Prandtl-Mayer problem of flow were employed. Having in view practical use of the results, the relations between the dynamic angle of the plate and the dimensionless displacement are presented in the form of diagrams. The diagrams can be used to estimate the deflection angle for concrete explosive systems without performing complicated computations.

The physical model of the driving process of the plate and its mathematical formulation will be presented in Sec. 2. Sec. 3 is devoted to presentation of the results of numerical calculations. This section includes also some examples, in which the results of calculations are used.

2. Description of the model

Let us consider the process of driving a plate by the products of detonation grazing on the surface of the plate (Fig. 1). The detonation wave moves at a velocity D in a layer of explosive of thickness δ_0. The gaseous detonation products expand, thus generating a shock wave in the ambient air. The contact boundary between the detonation products and air follows the shock wave. At the same time, the pressure of the detonation products acting on the surface of the plate makes it move.

It is assumed that the time of propagation of the detonation wave is long enough to neglect the influence of initial conditions. Moreover, the width of the plate is assumed to be sufficient to miss the influence of the lateral rarefaction waves on the driving process. Then the motion of the detonation products and air may be treated as stationary in a reference frame connected with the detonation wave. To describe this motion, use is made of the integral form of the equation of stationary gasdynamic flow

$$\frac{d}{dx}\int\limits_{y-}^{y+} \mathbf{A}\,dy = \left(\mathbf{A}\frac{dy}{dx} - \mathbf{B}\right)\Big|_{y-}^{y+} \tag{2.1}$$

where

$$\mathbf{A} = \begin{bmatrix} \rho u \\ p + \rho u^2 \\ \rho u \nu \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} \rho \nu \\ \rho u \nu \\ p + \rho \nu^2 \end{bmatrix}.$$

The symbols u, ν, p, and ρ in (2.1) denote the velocity component in the x- and y-axial directions (in a reference frame attached to the detonation wave),

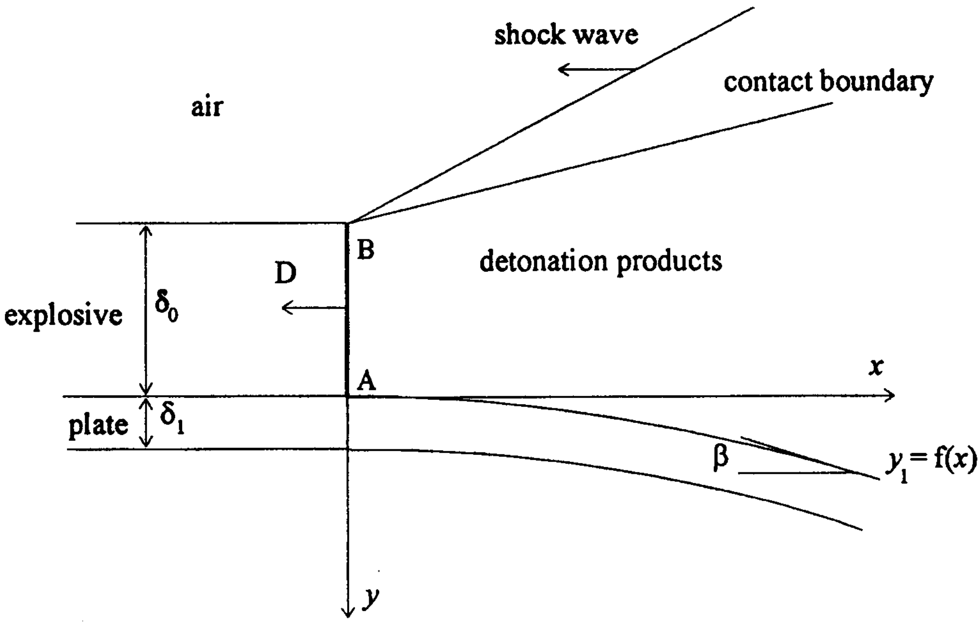

FIG. 1. Diagram of the system investigated.

the pressure and the density, respectively. The co-ordinates y- and y+ are the boundaries of any infinite layer in the x = const plane.

This set of equations is completed by the Bernoulli equation

$$u^2 + \nu^2 + 2i(p, \rho) = \text{const} \tag{2.2}$$

Here i denotes the enthalpy of the medium.

The thermodynamic properties of the detonation products will be described by using as a model of polytropic gas. Then

$$i(p, \rho) = \frac{k}{k-1}\frac{p}{\rho}, \tag{2.3}$$

where k is the polytropic exponent of gas. As regards the outer medium (air), the polytropic model is also used with the exponent γ.

To describe the motion of the plate use will be made of the fact that, during the driving process of the plate, the variations in density of the material of which it is made in relation to the initial density are, by at least one order of magnitude, lower than the relative variations in density of the detonation products. Moreover, during the detonation of a condensed explosive there occur pressures much above the dynamic plasticity limit of the plate material. In the description of the driving process of the plate, the strength characteristics of the plate material can, therefore, be disregarded. Thus, the material of the plate can be treated as an incompressible fluid. In monograph [5] the following equation is given describing the motion of an incompressible layer launched by the pressure pulse moving with a constant velocity D along its surface

$$\frac{d^2y_1}{dx^2} = \frac{p(x, y_1(x))}{\rho_1\delta_1 D^2}\left[1+\left(\frac{dy_1}{dx}\right)^2\right]^{3/2}, \qquad y_1(0)=0. \tag{2.4}$$

where y_1 denotes the current value of the co-ordinate of the internal surface of the plate (Fig. 1). The symbols ρ_1 and δ_1 denote the density of the material of the plate and its thickness, respectively.

The boundary conditions for the problem considered are as follows.

The line $x = 0$.

For $y \in [-\delta_0^+, 0]$ (detonation wave front) we have

$$u(0,y) = \frac{k}{k+1}D, \tag{2.5}$$

$$\nu(0,y) = 0, \tag{2.6}$$

$$p(0,y) = \frac{\rho_0 D^2}{k+1}, \tag{2.7}$$

$$\rho(0,y) = \frac{k+1}{k}\rho_0. \tag{2.8}$$

For $y \in [-\infty, -\delta_0^-]$ (undisturbed outer medium – air) we have

$$u(0,y) = D, \tag{2.9}$$

$$\nu(0,y) = 0, \tag{2.10}$$

$$p(0,y) = p_0, \tag{2.11}$$

$$\rho(0,y) = \rho_p, \tag{2.12}$$

where ρ_0, ρ_p denote the initial densities of explosive and air, respectively, p_0 denotes the initial pressure in air.

The boundary condition for the set (2.1) at the line $y_1 = f(x)$ (Fig. 1), constituting a solution of Eq. (2.4), is a slip condition and has the form

$$\frac{\nu(x,y_1)}{u(x,y_1)} = \frac{dy_1}{dx}. \tag{2.13}$$

To reduce the number of parameters on which the solution depends, let us introduce the dimensionless variables

$$X = \frac{x}{\delta_0}, \quad Y = \frac{y}{\delta_0}, \quad U = \frac{u}{c_H}, \quad V = \frac{\nu}{c_H}, P = \frac{p}{kp_H}, \quad R = \frac{\rho}{\rho_H}, \tag{2.14}$$

where p_H, ρ_H denote the pressure and density of the detonation products at the Chapman-Jouguet plane, expressed by Eqs. (2.7) – (2.8). The quantity

$$c_H = \frac{kp_H}{\rho_H},$$

denotes the sound velocity at that plane.

On introducing the dimensionless variables, the mathematical formulation of the problem has the form of the following equations.

Equations of motions of the detonation products and air

$$\frac{d}{dX}\int\limits_{Y-}^{Y+} \mathbf{A} dY = \left(\mathbf{A}\frac{dY}{dX} - \mathbf{B}\right)\Big|_{Y-}^{Y+}, \tag{2.15}$$

the form of the vectors **A** and **B** being analogous to that of the corresponding vectors in Eqs. (2.1).

Equation of conservation of total enthalpy for the detonation products

$$\frac{2k}{k-1}\frac{P}{R} + U^2 + V^2 = \frac{k+1}{k-1}. \tag{2.16}$$

Equation of conservation of total enthalpy for air

$$\frac{2\gamma}{\gamma-1}\frac{P}{R} + U^2 + V^2 = \frac{2\gamma}{\gamma+1}\frac{P_0}{R_p} + \frac{(k+1)^2}{k^2}. \tag{2.17}$$

Conditions at the detonation wave front for $Y \in [-1^+, 0]$

$$U(0,Y) = 1, \tag{2.18}$$

$$V(0,Y) = 0, \tag{2.19}$$

$$P(0,Y) = \frac{1}{k}, \tag{2.20}$$

$$R(0,Y) = 1. \tag{2.21}$$

Initial conditions in air for $Y \in [-\infty, -1^-]$

$$U(0,Y) = \frac{k+1}{k}, \tag{2.22}$$

$$V(0,Y) = 0, \tag{2.23}$$

$$P(0,Y) = P_0, \tag{2.24}$$

$$R(),Y) = R_p. \tag{2.25}$$

Conditions at the surface of the plate

$$\frac{V(X,Y_1)}{U(X,Y_1)} = \frac{dY_1}{dX}. \tag{2.26}$$

Equation of motion of the plate

$$\frac{d^2Y_1}{dX^2} = \frac{rkP}{k+1}\left[1+\left(\frac{dY_1}{dX}\right)^2\right]^{3/2}, \tag{2.27}$$

$$Y_1(0) = 0,$$

where $r = \dfrac{\rho_0\delta_0}{\rho_1\delta_1}$ denotes the ratio of the mass of explosive to the mass of plate material per unit surface of the plate.

The solution of the problem in dimensionless variables depends on the parameters r, k, P_0, R_p, γ. Their influence on the solution will be analysed in Sec. 3.

The numerical solution of the equations of motion of the detonation products and gaseous medium surrounding the explosive charge was constructed by the method described in Ref. [7], based on the idea of S.K. GODUNOV difference scheme [6]. This scheme may be used only in a region of supersonic flow, therefore no initial condition can be imposed with the difference method at the front of the detonation wave (line AB in Fig. 1), because, in agreement with the Chapman-Jouguet postulate, the flow at the wave front is, in a system of co-ordinates attached to the wave, equal to the local speed of sound. To circumvent this difficulty, the solution in a certain neighbourhood of the line AB was constructed by making use of the solutions of Prandtl-Mayer flows centred at the points A and B. Moreover, to limit a region of calculations, the move of shock wave in air was being traced.

3. Results of calculations

To verify the theoretical model and the method of solution, experimental data given in Ref. [8] were used. In that work the process of driving a copper plate of the thickness of 1 mm by the detonation products of a layer of explosive was investigated by the method of X-ray photography. Charges used were elaborated from ammonale (AN/Al 95/5), the mixture of crystalline RDX and zinc oxide (RDX/ZnO 80/20), and RDX-based plastic explosive (PE). The thickness of explosive layer was 10 mm. In calculations, the values of polytropic exponents for the detonation products of applied explosive were taken from Refs. [9 – 11].

In that works, so-called effective exponents of isentrope of detonation products were determined from a cylinder test by the method described in Ref. [12]. The experimental and theoretical profiles of the surface of copper plates driven by the detonation products of explosives tested are shown in Fig. 2. The values of parameters r and k used in calculations are also given in Fig. 2.

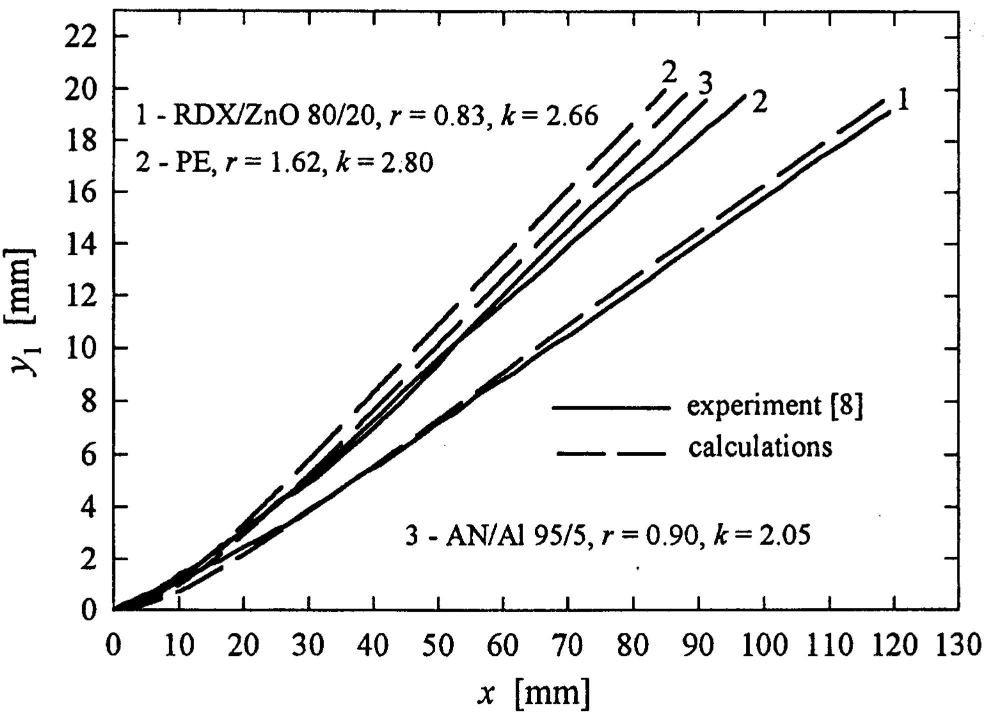

FIG. 2. Experimental and theoretical profiles of a surface of copper plates driven by the products of grazing detonation.

Comparison of the plate profiles shown in Fig. 2 reveals some discrepancies between experimental and theoretical data, especially in the second stage of plate motion. This means that there are some factors influencing the driving process, which are neglected in the model applied. In particular, the finite dimension of the plate is not taken into considerations, thus the influence of lateral rarefaction waves on the state of detonation products is ignored. However, as it follows from experiments, the waves weaken the action of the detonation products on the edges of plate causing its deformation, that is, displacements of the edges are smaller than the displacement of the central part of plate. The assumption of incompressibility of the plate material is the next factor causing discrepancies between experimental and calculated profiles of the plate. Theoretical analysis of driving a cylindrical tube performed in Ref. [13] shows that the deflection angles of a compressible tube are smaller as compared with those of an incompressible tube as a result of taking into account the wave processes proceeding in the material of tube. Moreover, constant value of the exponent k is assumed for each explosive tested, while it is changing during expansion of the detonation products.

The difference between the experimental profile of the plate and the calculated one is most remarkable for the scheme with PE. This explosive is characterised by relatively great values of exponent k and parameter r. The latter denotes the charge-to-plate mass ratio. Higher value of the exponents indicates higher sound velocity in the detonation products. High velocity of the rarefaction waves facilitates the process of dispersal of an energy due to scattering the detonation products in directions of lateral edges of the charge. This process is also reinforced by higher acceleration of the plate in the system with greater value of the ratio r. Hence, the transport of energy from more remote regions of the detonation products to the plate surface is made more difficult.

However, in the explosive schemes met with in practice, a distance of movement of the plate is relatively small. As a rule, the displacement of the metal plate is not longer than the thickness of the explosive layer. Due to this fact, the influence of the lateral rarefaction waves is depressed and plane symmetry of the driven plate is preserved. From Fig. 2 it follows that the discrepancies between the experimental and calculated displacements of copper plates at the distance of 10 mm (the thickness of explosive layer used) do not exceed 12%. Thus, the results of simulation of launching metal plates by the detonation products can be used in the first stage of construction of the explosive systems for cladding and strengthening purposes.

From the analysis made in Sec. 2 it follows that the solution of the problem of driving an incompressible plate by grazing detonation depends on the exponent k characterising the detonation products, the charge-to-plate mass ratio r and the parameters P_0, R_p and γ describing an external gaseous medium. The medium surrounding the explosive system is usually air under normal conditions. The influence of the initial pressure of air on the results of simulation of driving plates in the systems discussed was determined. The results of computation of the dynamic deflection angle β for the pressure varying within the limit from 0,1 to 1 MPa differ with each other by no more than 0,1% for the range of y_1 shown in Fig. 2. Such a small difference allows us to assume that, within the values of the pressure met with in practice, values of β do not depend on parameters P_0 and R_p. The polytropic exponent γ is generally assumed to be constant for air shocked by the lateral scatter of the detonation products of condensed explosives. Thus, the solution of the problem depends on only two parameters r and k.

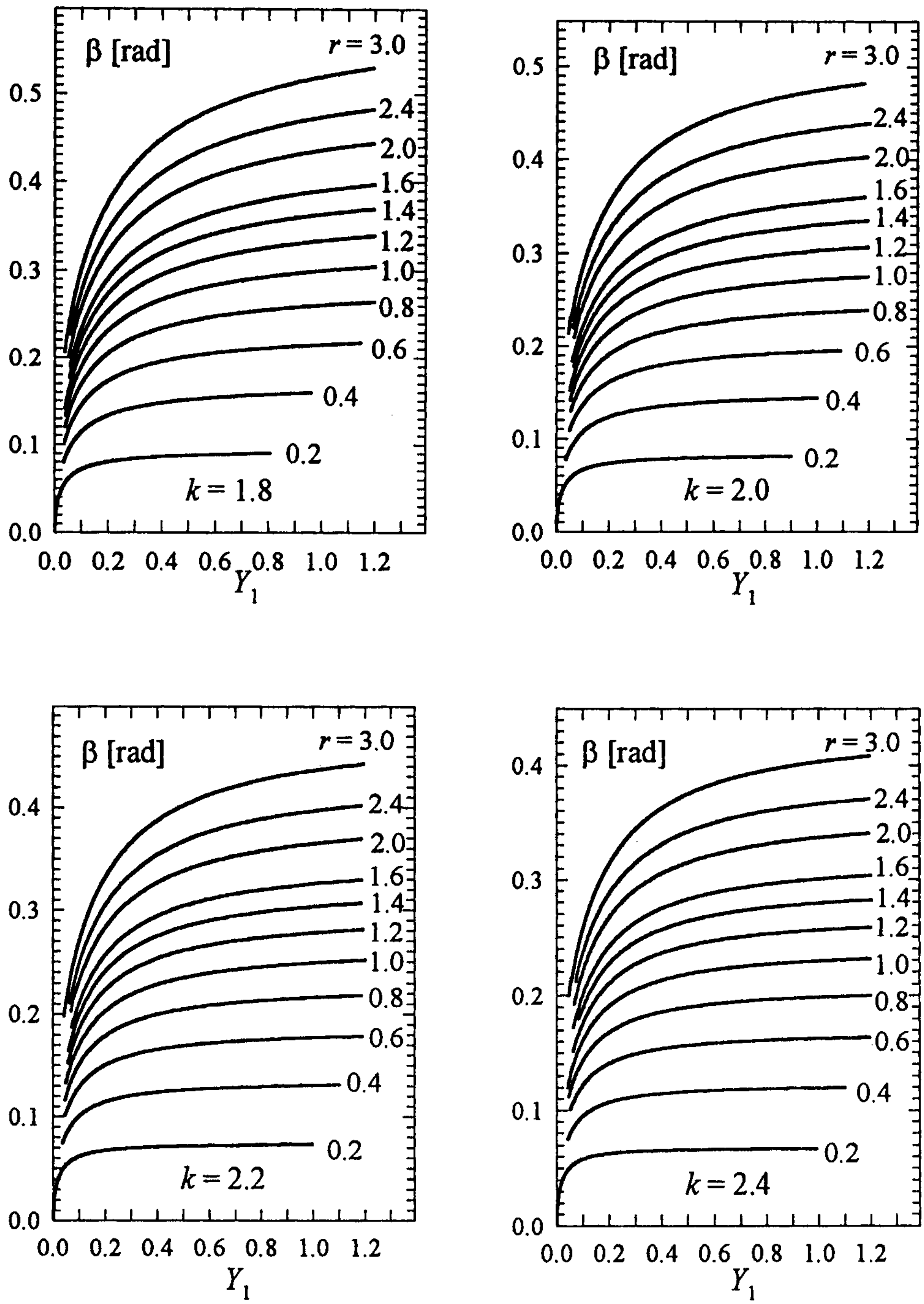

FIG. 3. a) Deflection angle β of a plate as a function of displacement Y_1 for different k and r.

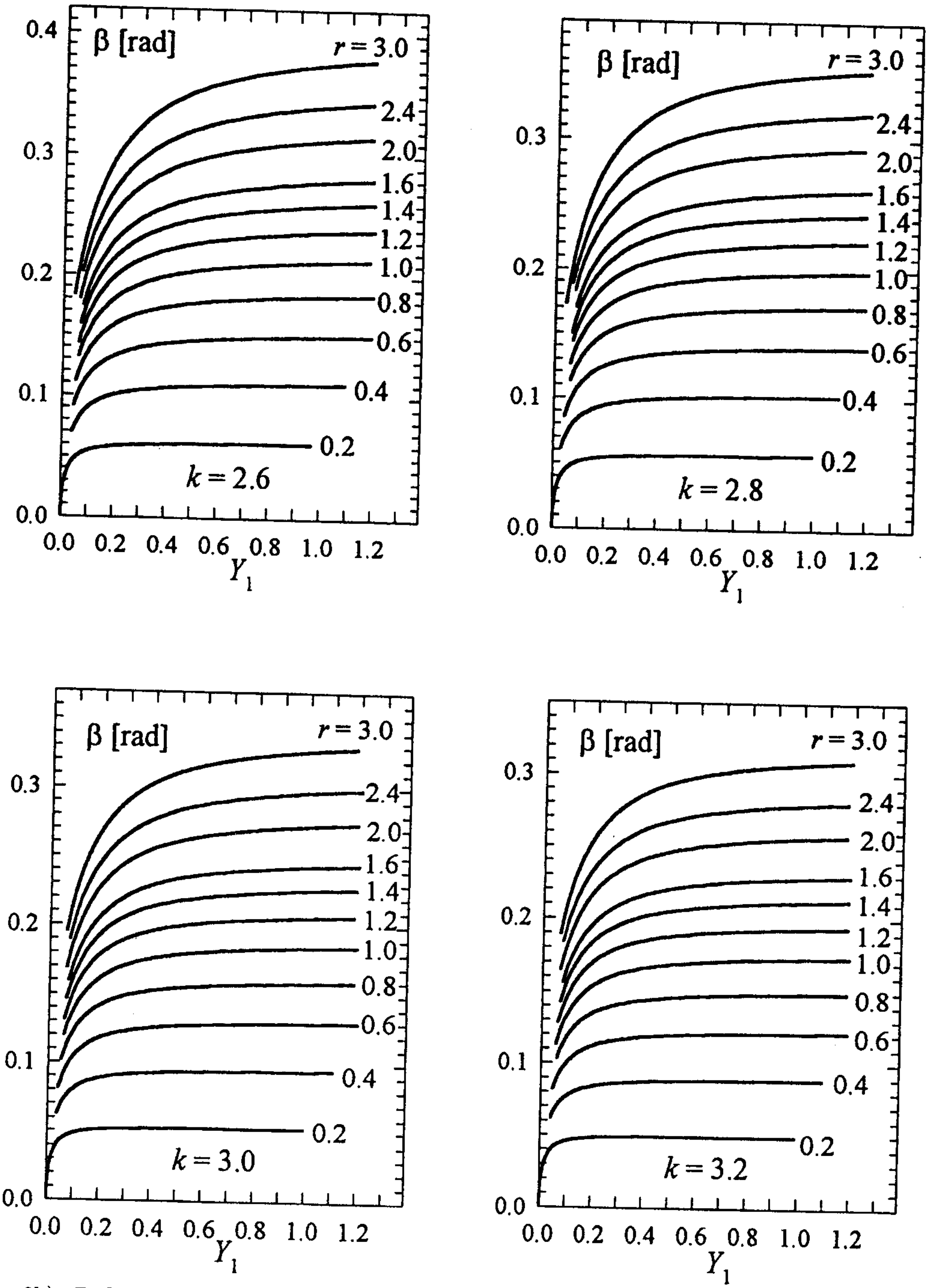

FIG.3b). Deflection angle β of a plate as a function of displacement Y_1 for different k and r.

In Figs. 3a and 3b, theoretical dependencies of the dynamic deflection angle β of a plate on the dimensionless displacement Y_1 for different parameters r and k are presented. The range of variation of these parameters contains almost all values met in the systems applied in the explosive methods of cladding and strengthening of metals. Calculated diagrams can be also used to estimate the deflection angle for the schemes with the values of r and k excluded on the Figs. 3a and 3b. In this case the deflection angle β can be calculated by interpolation between the values found on diagrams. As an illustration of such procedure let us consider the following examples.

Examples

Let us estimate the deflection angle of a steel plate of the thickness of 0.5 mm driven at a distance of 2 mm by grazing detonation of plastic explosive (PE). The following values for the characteristic parameters of the explosion system are assumed: $\delta_1 = 0.5$ mm, $\rho_1 = 7850$ kg/m^3, $y_1 = 2$ mm, $\rho_0 = 1450$ kg/m^3, $k = 2.85$, $\delta_0 = 2$ mm. First, we determine $r = 0.739$ and $Y_1 = 1$. From Fig. 3b the values of the angle β for $Y_1 = 1$ are found for four sets of parameters k and r: $\beta(k = 2.8, r = 0.8) = 0.17$; $\beta(k = 2.8, r = 0.6) = 0.14$; $\beta(k = 3.0, r = 0.8) = 0.16$; $\beta(k = 3.0, r = 0.6) = 0.13$. Making use of interpolation between these values we calculate $\beta = 0.16$ for $r = 0.739$ and $k = 2.85$. From numerical simulation the value $\beta = 0.159$ was obtained.

If the same plate is driven by the products of the charge of PE, thickness of which is the twice previous one, then $r = 1.478$ and $Y_1 = 0.5$. From diagrams we find: $\beta(k = 2.8, r = 1.6) = 0.25$; $\beta(k = 2.8, r = 1.4) = 0.23$; $\beta(k = 3.0, r = 1.6) = 0.24$; $\beta(k = 3.0, r = 1.4) = 0.22$, and after interpolation we achieve $\beta = 0.24$ for $r = 1.478$ and $k = 2.85$. In this case the numerical calculations gave $\beta = 0.235$. Therefore, the use of the interpolation method produces an error less than 1%.

The above examples show that the results of numerical simulation presented in the form of diagrams enable us to estimate with satisfactory accuracy the dynamic deflection angle of a metal plate for any explosive system. The results can be also used in construction of practical schemes giving the expected deflection angles at the assumed displacements of the plate.

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