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TRANSIENT VIBRATION OF UNSTEADY LINEAR CLOSED LOOP SYSTEMS

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A problem of unsteady linear closed loop system vibration is formulated and solved in this paper. Proportional, ideal derivative and delayed feedback interaction is considered. The mixed method of rigid and deformable finite elements is extended by including coupling elements, and applied to modelling the system. An unconditionally stable numerical integration variant of the Newmark method is used for solving the problem. Special computer software is implemented and illustrative calculations of the traverse shaper's model are performed. Time and frequency plots for chosen generalised coordinates of the model and for several cases of the feedback interaction, are obtained as results.

Key Words: Structure vibration. Closed loop systems. Finite element method. Numerical methods. Computation.

1. INTRODUCTION

An analysis of dynamic phenomena of real mechanical structures has recently become a basis of knowledge for the development and improvement of engineering instrumentation. It usually depended on investigations of the structural behaviour due to the presence of external exciting forces, called "INPUT SIGNALS". Referred generalised displacements of chosen structure's parts have been defined as the "OUTPUT SIGNALS". However, a special behaviour of many kinds of machine structures (e.g. machine tools, manipulators, driving systems) is observed. These dynamic systems, which show a feasibility of interacting between the desired input and output signals quite naturally, are called closed-loop systems [1]. One approach, based on simultaneous investigation of the structure with an accompanying process of the feedback interaction, was worked out a few years ago [2].

If we define, that:

• the $\mathbf{x}(t)_{q\times 1}$ vector with q components is an input to the chosen part of the system called: FEEDBACK UNIT;

• the $\mathbf{y}(t)_{p\times 1}$ vector with p components is a response of the FEEDBACK UNIT to the $\mathbf{x}(t)_{q\times 1}$ vector;

the following relationship will be derived [3]:

$$\mathbf{y}(s)_{p\times 1} = \mathbf{T}(s)_{p\times q}\mathbf{x}(s)_{q\times 1}, \tag{1.1}$$

where $\mathbf{T}(s)_{p\times q} = [t_{ij}(s),\ j = 1,2,...,q,\ i = 1,2,...,p]_{p\times q}$ – complex matrix of one-dimensional closed loop transfer functions, $\mathbf{y}(s)_{p\times 1}$ – Laplace transform of the $\mathbf{y}(t)_{p\times 1}$ vector at zero initial conditions, $\mathbf{x}(s)_{q\times 1}$ – Laplace transform of the $\mathbf{x}(t)_{q\times 1}$ vector at zero initial conditions, t – time, s – complex variable.

Moreover, the presence of large relative motions of several substructures is observed during the work of real mechanical devices (e.g. machine tools, lifting machines and manipulators). These motions are usually implied by the desired functions of the machine. Considering the following aspects of the machine structure behaviour, that is to say: the large desired motion of several machine's substructures and small vibration of elastic parts of the structure, we will get a parametric (unsteady) model of the system, whose configuration changes with time [4]. The transient vibration analysis is to be performed in this case.

A more difficult mathematical description of the parametric closed-loop system's transient vibration, implies the effective solution of the problem only for particular types of feedback interaction. Therefore, the following types of interaction are included in this paper: proportional, ideal derivative and delayed [1, 3]. Basic equations, which describe transfer functions of one-dimensional closed loop interactions, are shown in Table 1.

Table 1. Transfer functions of some one-dimensional feedback interactions.

Type of feedback interaction	Transfer function
Proportional	$t_{ij}(s) = k_{ij}$
Ideal derivative	$t_{ij}(s) = k_{ij}s$
Delayed	$t_{ij}(s) = k_{ij}e^{-T_{ij}s}$
NOTATION: $t_{ij}(s)$ – transfer function of one-dimensional feedback interaction k_{ij} – gain coefficient T_{ij} – time-constant	

2. The calculation model description

The calculation model of the parametric linear closed-loop system contains (Fig. 1):

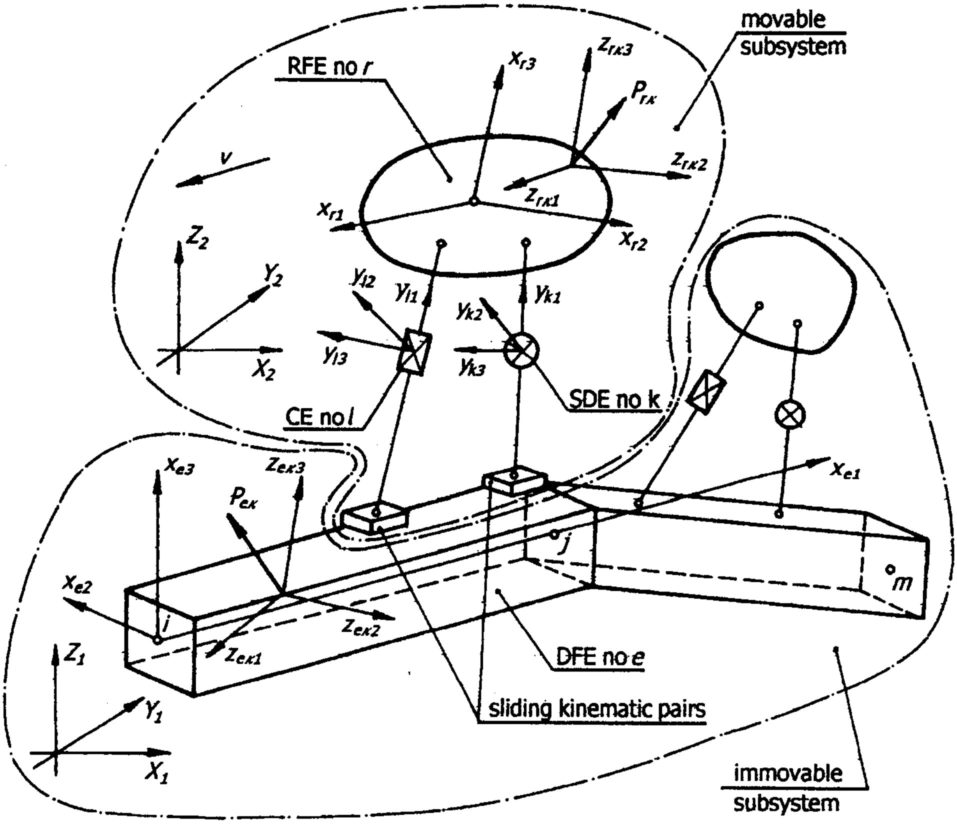

FIG. 1. Discrete model of the closed loop system whose configuration changes with time.

• rigid bodies, which have mass properties only, and are called rigid finite elements (RFEs). A motion of RFE no. r is described in the local $x_{r1}x_{r2}x_{r3}$ coordinate system. These axes are principal axes of inertia for RFE no. r;

• flexible bodies, which have mass, damping and elastic properties. They are called deformable finite elements (DFE). Two nodes, i and j, are defined for DFE no. e whose motion is described using the local coordinate system $x_{e1}x_{e2}x_{e3}$;

• massless points which have elastic properties and an ability of dissipating energy. They are called spring-damping elements (SDEs). The deformability of SDE no. k is described using the local coordinate system $y_{k1}y_{k2}y_{k3}$;

• massless coupling elements (CEs), which are introduced additionally in order to idealise the process of feedback interaction. Their behaviour is described using the $y_{l1}y_{l2}y_{l3}$ local coordinate system.

The approach described above is called the hybrid method of rigid and deformable finite elements (RDFEM) [3, 5]. The discrete model of a closed loop system is obtained as a result.

The following basic assumptions are taken into consideration:

• each SDE or CE connects either one DFE with one RFE, two DFEs, or two RFEs together;

• two DFEs may also be connected to each other in their nodes;

• an immovable subsystem, which may experience small vibration only, is separated. The immovable coordinate system $X_1Y_1Z_1$ is introduced for this purpose;

• there is also a separate movable subsystem, which displaces with constant linear velocity ν with respect to an immovable one. Its small vibration is described using a local (movable) co-ordinate system $X_2Y_2Z_2$;

• sliding kinematic pairs are considered in the model.

Moreover, an external force $\mathbf{P}_{r\kappa}$ is acting on RFE no. r, while an external force $\mathbf{P}_{e\kappa}$ – on DFE no. e. Coordinate axes systems $z_{r\kappa1}z_{r\kappa2}z_{r\kappa3}$ and $z_{e\kappa1}z_{e\kappa2}z_{e\kappa3}$ correspond with RFE no. r and DFE no. e, respectively.

The first step of consideration depends on analysing the discrete system as a steady system. It was proved that the dynamic equation of this system with n degrees of freedom, which is composed of i_u RFE, i_ν SDE, i_s CE and i_o DFE, has the form [1]:

$$(\mathbf{M}s^2 + \mathbf{L}s + \mathbf{K} - \mathbf{D}(s))\mathbf{q}(s) = \mathbf{p}(s), \tag{2.1}$$

where: $\mathbf{M}$ – inertia matrix of the whole discrete system, $\mathbf{L}$ - damping matrix of the whole discrete system, $\mathbf{K}$ - stiffness matrix of the whole discrete system, $\mathbf{D}(s)$ – complex matrix of feedback interactions of the whole discrete system, $\mathbf{q}(s)$ – Laplace transform of the generalised coordinate vector, $\mathbf{p}(s)$ – Laplace transform of the generalised force vector, s – complex variable.

From the $\mathbf{q}$ vector we separate the following subvectors: one of the independent coordinates $\mathbf{q}_i$, one of the dependent coordinates $\mathbf{q}_d$ and one of the given coordinates $\mathbf{q}_g$ [4]. By analogy, from the $\mathbf{p}$ vector we get one which corresponds to independent coordinates $\mathbf{p}_i$, one which corresponds to dependent coordinates $\mathbf{p}_d$ and one which corresponds to given coordinates $\mathbf{p}_g$. It is expressed by the notation:

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_i \\ \mathbf{q}_d \\ \mathbf{q}_g \end{bmatrix}, \tag{2.2}$$

$$\mathbf{p} = \begin{bmatrix} \mathbf{p}_i \\ \mathbf{p}_d \\ \mathbf{p}_g \end{bmatrix}. \tag{2.3}$$

A relationship between the coordinates $\mathbf{q}_i$ and $\mathbf{q}_d$ is determined by the equation of constraints, which may be written as [1, 4]:

$$[\mathbf{W}_i \vdots \mathbf{W}_d] \begin{bmatrix} \mathbf{q}_i \\ \mathbf{q}_d \end{bmatrix} = \mathbf{0}, \tag{2.4}$$

where: $\mathbf{W}_i$, $\mathbf{W}_d$ – constraints' matrices of independent and dependent subsystems, respectively, or after small rearrangements:

$$\mathbf{q}_d = \mathbf{W}\mathbf{q}_i, \tag{2.5}$$

where

$$\mathbf{W} = -(\mathbf{W}_d^T \mathbf{W}_d)^{-1} \mathbf{W}_d^T \mathbf{W}_i. \tag{2.6}$$

If the dynamics of each subsystem (i.e. movable or immovable) is described in its local reference system, matrix $\mathbf{W}$ will become one of the given time functions:

$$\mathbf{W} = \mathbf{W}(t). \tag{2.7}$$

After differentiation with respect to time, we get:

$$\dot{\mathbf{q}}_d = \dot{\mathbf{W}}\mathbf{q}_i + \mathbf{W}\dot{\mathbf{q}}_i, \tag{2.8}$$

$$\ddot{\mathbf{q}}_d = \ddot{\mathbf{W}}\mathbf{q}_i + 2\dot{\mathbf{W}}\dot{\mathbf{q}}_i + \mathbf{W}\ddot{\mathbf{q}}_i. \tag{2.9}$$

A complex dynamic equation of the closed loop system is to be presented using the time-dependent form. If we use the notation

$$\mathbf{E}(s) = \mathbf{D}(s)\mathbf{q}(s), \tag{2.10}$$

and suppose that there exists an inverse Laplace transformation, i.e.:

$$\mathcal{L}^{-1}(\mathbf{E}(s)) = \mathbf{E}(t), \tag{2.11}$$

the following matrix differential equation will result:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{L}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{p}(t) + \mathbf{E}(t). \tag{2.12}$$

Considering Eqs. (2.2) and (2.3), and separating from the $\mathbf{E}(t)$ vector subvectors: $\mathbf{E}_i$ whose components are acting along the independent coordinates, $\mathbf{E}_d$ whose components are acting along the dependent coordinates, and $\mathbf{E}_g$ whose components are acting along the given coordinates, Eq. (2.12) may be written in the form:

$$\begin{bmatrix} \mathbf{M}_{ii} & \mathbf{M}_{id} & \mathbf{M}_{ig} \\ \mathbf{M}_{dI} & \mathbf{M}_{dd} & \mathbf{M}_{dg} \\ \mathbf{M}_{gi} & \mathbf{M}_{gd} & \mathbf{M}_{gg} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_i \\ \ddot{\mathbf{q}}_d \\ \ddot{\mathbf{q}}_g \end{bmatrix} + \begin{bmatrix} \mathbf{L}_{ii} & \mathbf{L}_{id} & \mathbf{L}_{ig} \\ \mathbf{L}_{di} & \mathbf{L}_{dd} & \mathbf{L}_{dg} \\ \mathbf{L}_{gi} & \mathbf{L}_{gd} & \mathbf{L}_{gg} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_i \\ \dot{\mathbf{q}}_d \\ \dot{\mathbf{q}}_g \end{bmatrix}$$
$$+ \begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{id} & \mathbf{K}_{ig} \\ \mathbf{K}_{di} & \mathbf{K}_{dd} & \mathbf{K}_{dg} \\ \mathbf{K}_{gi} & \mathbf{K}_{gd} & \mathbf{K}_{gg} \end{bmatrix} \begin{bmatrix} \mathbf{q}_i \\ \mathbf{q}_d \\ \mathbf{q}_g \end{bmatrix} = \begin{bmatrix} \mathbf{p}_i \\ \mathbf{p}_d \\ \mathbf{p}_g \end{bmatrix} + \begin{bmatrix} \mathbf{E}_i \\ \mathbf{E}_d \\ \mathbf{E}_g \end{bmatrix}. \tag{2.13}$$

These are ordinary differential equations of a de-coupled mechanical system, because the movable subsystem and immovable subsystem at this stage are separated from each other. In order to "connect" them, the constraints' Eqs. (2.5), (2.8) and (2.9) with the respective constraint reactions' Eq. [4]:

$$\mathbf{R}_i + \mathbf{W}^T\mathbf{R}_d = \mathbf{0}, \tag{2.14}$$

where $\mathbf{R}_i$, $\mathbf{R}_d$ – vectors of constraints' reactions, which are acting along independent and dependent coordinates, respectively, must be included into the description of the model. We then obtain an equation of motion, referring to independent coordinates:

$$\mathbf{A}'^*\ddot{\mathbf{q}}_i + \mathbf{B}^*\dot{\mathbf{q}}_i + \mathbf{C}^*\mathbf{q}_i = \mathbf{p}^* + \mathbf{E}^*, \tag{2.15}$$

where:

$$\mathbf{A}^* = \mathbf{M}_{ii} + \mathbf{M}_{id}\mathbf{W} + \mathbf{W}^T\mathbf{M}_{di} + \mathbf{W}^T\mathbf{M}_{dd}\mathbf{W}, \tag{2.16}$$
$$\mathbf{B}^* = \mathbf{L}_{ii} + \mathbf{L}_{id}\mathbf{W} + \mathbf{W}^T\mathbf{L}_{di} + \mathbf{W}^T\mathbf{L}_{dd}\mathbf{W} + 2\mathbf{M}_{id}\dot{\mathbf{W}} + 2\mathbf{W}^T\mathbf{M}_{dd}\dot{\mathbf{W}}, \tag{2.17}$$
$$\mathbf{C}^* = \mathbf{K}_{ii} + \mathbf{K}_{id}\mathbf{W} + \mathbf{W}^T\mathbf{K}_{di}\mathbf{W} + \mathbf{W}^T\mathbf{K}_{di} + \mathbf{L}_{id}\dot{\mathbf{W}} + \mathbf{W}^T\mathbf{L}_{dd}\dot{\mathbf{W}} + \mathbf{M}_{id}\ddot{\mathbf{W}} + \mathbf{W}^T\mathbf{M}_{dd}\ddot{\mathbf{W}}, \tag{2.18}$$
$$\mathbf{p}^* = \mathbf{p}_i - (\mathbf{M}_{ig}\ddot{\mathbf{q}}_g + \mathbf{L}_{ig}\dot{\mathbf{q}}_g + \mathbf{K}_{ig}\mathbf{q}_g) + \mathbf{W}^T[\mathbf{p}_d - (\mathbf{M}_{dg}\ddot{\mathbf{q}}_g + \mathbf{L}_{dg}\dot{\mathbf{q}}_g + \mathbf{K}_{dg}\mathbf{q}_g)], \tag{2.19}$$
$$\mathbf{E}^* = \mathbf{E}_i + \mathbf{W}^T\mathbf{E}_d. \tag{2.20}$$

Matrices $\mathbf{A}^*$, $\mathbf{B}^*$, $\mathbf{C}^*$ and $\mathbf{p}^*$ are time-dependent functions, but the vector $\mathbf{E}^*$ is a function of time, generalised coordinates and their derivatives. Thus, we can write:

$$\mathbf{E}^* = \mathbf{E}^*(t, \mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i, \ldots). \tag{2.21}$$

In a particular case, when only proportional, ideal derivative and delayed feedback interaction are considered, the matrix of feedback interaction $\mathbf{D}(s)$ is able to be expressed, as

$$\mathbf{D}(s) = \left[d_{Pij} + s d_{Vij} + d_{Oij} e^{-Ts}\right]_{n\times n}, \tag{2.22}$$

or

$$\mathbf{D}(s) = \mathbf{D}_P + s\mathbf{D}_V + \mathbf{D}_O e^{-Ts}, \tag{2.23}$$

where: $\mathbf{D}_P = [d_{Pij}]$ – matrix of displacement feedback interaction coefficients, $\mathbf{D}_V = [d_{Vij}]$ – matrix of velocity feedback interaction coefficients, $\mathbf{D}_O = [d_{Oij}]$ – matrix of delayed feedback interaction coefficients. Here it was supposed that the delayed feedback interaction had been common for the whole dynamic system, i.e. one value of time-delay T is needed to describe the interaction only. Then, the $\mathbf{E}(s)$ vector can be written as

$$\mathbf{E}(s) = (\mathbf{D}_P + s\mathbf{D}_V + \mathbf{D}_O e^{-Ts})\mathbf{q}(s), \tag{2.24}$$

or after the inverse Laplace transformation

$$\mathbf{E}(t) = \mathbf{D}_V\dot{\mathbf{q}}(t) + \mathbf{D}_P\mathbf{q}(t) + \mathbf{D}_O\mathbf{q}(t-T). \tag{2.25}$$

Following this we can write, that

$$\begin{bmatrix} \mathbf{E}_i(t) \\ \mathbf{E}_d(t) \\ \mathbf{E}_g(t) \end{bmatrix} = \begin{bmatrix} \mathbf{D}_{Vii} & \mathbf{D}_{Vid} & \mathbf{D}_{Vig} \\ \mathbf{D}_{Vdi} & \mathbf{D}_{Vdd} & \mathbf{D}_{Vdg} \\ \mathbf{D}_{Vgi} & \mathbf{D}_{Vgd} & \mathbf{D}_{Vgg} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_i(t) \\ \dot{\mathbf{q}}_d(t) \\ \dot{\mathbf{q}}_g(t) \end{bmatrix} \tag{2.26}$$
$$+ \begin{bmatrix} \mathbf{D}_{Pii} & \mathbf{D}_{Pid} & \mathbf{D}_{Pig} \\ \mathbf{D}_{Pdi} & \mathbf{D}_{Pdd} & \mathbf{D}_{Pdg} \\ \mathbf{D}_{Pgi} & \mathbf{D}_{Pgd} & \mathbf{D}_{Pgg} \end{bmatrix} \begin{bmatrix} \mathbf{q}_i(t) \\ \mathbf{q}_d(t) \\ \mathbf{q}_g(t) \end{bmatrix}$$
$$+ \begin{bmatrix} \mathbf{D}_{Oii} & \mathbf{D}_{Oid} & \mathbf{D}_{Oig} \\ \mathbf{D}_{Odi} & \mathbf{D}_{Odd} & \mathbf{D}_{Odg} \\ \mathbf{D}_{Ogi} & \mathbf{D}_{Ogd} & \mathbf{D}_{Ogg} \end{bmatrix} \begin{bmatrix} \mathbf{q}_i(t-T) \\ \mathbf{q}_d(t-T) \\ \mathbf{q}_g(t-T) \end{bmatrix}.$$

Then, the $\mathbf{E}^*$ vector (see (2.20)) will take the form:

$$
\begin{aligned}
(2.27)\qquad \mathbf{E}^* = {}& (\mathbf{D}_{Vii} + \mathbf{D}_{Vid}\mathbf{W} + \mathbf{W}^T\mathbf{D}_{Vdi} + \mathbf{W}^T\mathbf{D}_{Vdd}\mathbf{W})\dot{\mathbf{q}}_i \\
& +(\mathbf{D}_{Vig} + \mathbf{W}^T\mathbf{D}_{Vdg})\dot{\mathbf{q}}_g + (\mathbf{D}_{Pii} + \mathbf{D}_{Vid}\dot{\mathbf{W}} + \mathbf{D}_{Pid}\mathbf{W} \\
& + \mathbf{W}^T\mathbf{D}_{Pdi} + \mathbf{W}^T\mathbf{D}_{Vdd}\dot{\mathbf{W}} + \mathbf{W}^T\mathbf{D}_{Pdd}\mathbf{W})\mathbf{q}_i \\
& + (\mathbf{D}_{Pig} + \mathbf{W}^T\mathbf{D}_{Pdg})\mathbf{q}_g + (\mathbf{D}_{Oii} + \mathbf{D}_{Oid}\mathbf{W} \\
& +\mathbf{W}^T\mathbf{D}_{Odi} + \mathbf{W}^T\mathbf{D}_{Odd}\mathbf{W})\mathbf{q}_i(t-T) \\
& + (\mathbf{D}_{Oig} + \mathbf{W}^T\mathbf{D}_{Odg})\mathbf{q}_g(t-T).
\end{aligned}
$$

If we consider Eq. (2.27) in the (2.15), the final form of the dynamic equation will result. This is an equation of excited vibration of the parametric system, considering proportional, ideal derivative and delayed feedback interactions. It has the form:

$$
(2.28)\qquad \hat{\mathbf{A}}\ddot{\mathbf{q}}_i + \hat{\mathbf{B}}\dot{\mathbf{q}}_i + \hat{\mathbf{C}}\mathbf{q}_i = \hat{\mathbf{p}}(t, t-T),
$$

where:

$$
\begin{aligned}
\hat{\mathbf{A}} &= \mathbf{A}^*, \\
\hat{\mathbf{B}} &= \mathbf{B}^* - (\mathbf{D}_{Vii} + \mathbf{D}_{Vid}\mathbf{W} + \mathbf{W}^T\mathbf{D}_{Vdi} + \mathbf{W}^T\mathbf{D}_{Vdd}\mathbf{W}), \\
\hat{\mathbf{C}} &= \mathbf{C}^* - (\mathbf{D}_{Pii} + \mathbf{D}_{Vid}\dot{\mathbf{W}} + \mathbf{D}_{Pid}\mathbf{W} + \mathbf{W}^T\mathbf{D}_{Pdi} \\
& \quad +\mathbf{W}^T\mathbf{D}_{Vdd}\dot{\mathbf{W}} + \mathbf{W}^T\mathbf{D}_{Pdd}\mathbf{W}), \\
\hat{\mathbf{p}}(t, t-T) &= \mathbf{p}^* + (\mathbf{D}_{Vig} + \mathbf{W}^T\mathbf{D}_{Vdg})\dot{\mathbf{q}}_g + (\mathbf{D}_{Pig} + \mathbf{W}^T\mathbf{D}_{Pdg})\mathbf{q}_g \\
& \quad +(\mathbf{D}_{Oii} + \mathbf{D}_{Oid}\mathbf{W} + \mathbf{W}^T\mathbf{D}_{Odi} + \mathbf{W}^T\mathbf{D}_{Odd}\mathbf{W})\mathbf{q}_i(t-T) \\
& \quad + (\mathbf{D}_{Oig} + \mathbf{W}^T\mathbf{D}_{Odg})\mathbf{q}_g(t-T).
\end{aligned}
$$

Equation (2.28) is a linear differential equation whose coefficients change with time. Its solution depends on determining the time response of the system to the explicitly known function $\hat{\mathbf{p}}(t, t-T)$. In order to find a numerical solution of Eq. (2.28) we have applied an unconditionally stable numerical integration variant of the Newmark method [6, 7, 8].

3. Numerical example

A process of machining long pieces of material by the traverse shaper (Fig. 2a) is analysed. The slide with the tool is displaced along the shear direction with constant velocity of the work motion ν_w and lost motion ν_l (along run distance s_r, cutting distance s and coasting distance s_c). The parameters of fixed and movable joints are steady and their values were determined previously [1, 5].

Using the RDFEM, a discrete plane model of the system (Fig. 2b) is created. Some RFEs are used to idealise the frame (RFE 1), two legs (RFE 3 and 4) and the slide with the tool (RFE 2). Some 2-nodal DFEs, which are classical Timoshenko beams [1, 5], are applied for idealising the shear, the workpiece, the table and the bed. Their nodes are shown in the figure. RFEs and DFEs are connected by SDEs, which are also presented. In order to include the cutting dynamics interaction, one CE is introduced. Suitable parameters of the discrete model are given in Table 2.

It is easy to separate two subsystems: the immovable subsystem, which experiences small vibration only, and the movable one. Both these subsystems are accompanied by reference coordinate systems X_1Y_1 and X_2Y_2, respectively.

In order to "connect" the subsystems mentioned above, two massless RFEs 5 and 6 are introduced into the model. The first one idealises the conventional contact point of tool and workpiece, while the second idealises the conventional contact point of slide and shear. All their degrees of freedom are dependent coordinates, while the degrees of freedom of other elements (as well of RFEs as of DFEs) are independent. The parts of the system with given motion (i.e.. solid rest bodies, which idealise the machine foundation), are shaded in the picture. Those degrees of freedom refer to the given coordinates.

Let us analyse the geometry of the shaping process (Fig. 3). An ideal situation occurs, when a desired constant value of the cutting zone thickness h_0 is maintained during the whole process. Also the desired longitudinal velocity ν_w of the tool with respect to the workpiece (called cutting velocity) is maintained all the time. In this case we would observe constant values of interaction forces F_y and F_z (called desired forces) between the tool and the workpiece, which are strictly accompanying steady conditions of the process.

Due to dynamic interaction between the tool and the workpiece, the change Δh in the cutting zone thickness occurs, while additional relative displacement of the edge q_z along the main direction of motion is observed. It implies a real cutting zone thickness h during the process.

Table 2. Parameters of the traverse shaper model

INERTIA COEFFICIENTS OF RFEs		
r^*	m_{r1} [kg]	m_{r6} [kgm^2]
1	150.00	15.6250
2	30.00	0.4062
3	100.00	7.4167
4	100.00	7.4167
5	0.00	0.0000
6	0.00	0.0000

PARAMETERS OF DFEs								
i	j	l[m]	A[m^2]	I_3[m^4]	E[N/m^2]	ρ[kg/m^3]	η[Ns/m^2]	ν
1	2	0.5	0.03	2.2500×10^{-4}	2.1×10^{11}	7.9×10^3	1.0×10^7	0.3
2	3	0.5	0.03	2.2500×10^{-4}	2.1×10^{11}	7.9×10^3	1.0×10^7	0.3
4	5	0.5	0.02	4.1667×10^{-6}	2.1×10^{11}	7.9×10^3	1.0×10^7	0.3
6	7	0.8	0.05	4.1667×10^{-6}	2.1×10^{11}	7.9×10^3	1.0×10^7	0.3
8	9	0.6	0.12	4.0000×10^{-4}	2.1×10^{11}	7.9×10^3	1.0×10^7	0.3
9	10	0.6	0.12	4.0000×10^{-4}	2.1×10^{11}	7.9×10^3	1.0×10^7	0.3

Table 2. [cont.]

STIFFNESS COEFFICIENTS AND CO-ORDINATES OF THESE SDEs WHICH CONNECT ONLY RFEs

k	r	p	c_{k1} [N/m]	c_{k2} [N/m]	c_{k6} [Nm]	s_{rk1} [m]	s_{rk2} [m]	s_{rk3} [m]	s_{pk1} [m]	s_{pk2} [m]	s_{pk3} [m]
1	3	3	7.0×10^9	7.0×10^8	7.0×10^8	0.0	−0.400	0.0			
2	4	4	7.0×10^9	7.0×10^9	7.0×10^8	0.0	−0.400	0.0			
3	1	3	7.0×10^8	7.0×10^8	7.0×10^7	0.0	−0.500	0.0	0.0	0.400	0.0
4	2	6	5.0×10^8	7.0×10^8	3.0×10^7	0.0	0.175	0.0	0.0	0.000	0.0

STIFFNESS COEFFICIENTS AND CO-ORDINATES OF THE OTHER SDEs

k	r	i_1	j_1	i_2	j_2	c_{k1}	c_{k2}	c_{k6}	s_{e1k1}	s_{e1k2}	s_{e1k3}	s_{rk1}	s_{rk2}	s_{rk3}
													or	
												s_{e2k1}	s_{e2k2}	s_{e2k3}
						[N/m]	[N/m]	[Nm]	[m]	[m]	[m]	[m]	[m]	[m]
1	4	9	10			7.0×10^8	7.0×10^8	7.0×10^7	0.35	−0.100	0.0	0.0	0.40	0.0
2		6	7	9	10	7.0×10^7	7.0×10^7	7.0×10^6	0.70	−0.050	0.0	0.3	0.10	0.0
3		6	7	8	9	7.0×10^7	7.0×10^7	7.0×10^6	0.10	−0.050	0.0	0.3	0.10	0.0
4		4	5	6	7	7.0×10^7	7.0×10^7	7.0×10^6	0.05	−0.025	0.0	0.2	0.05	0.0
5		4	5	6	7	7.0×10^7	7.0×10^7	7.0×10^6	0.45	−0.025	0.0	0.6	0.05	0.0

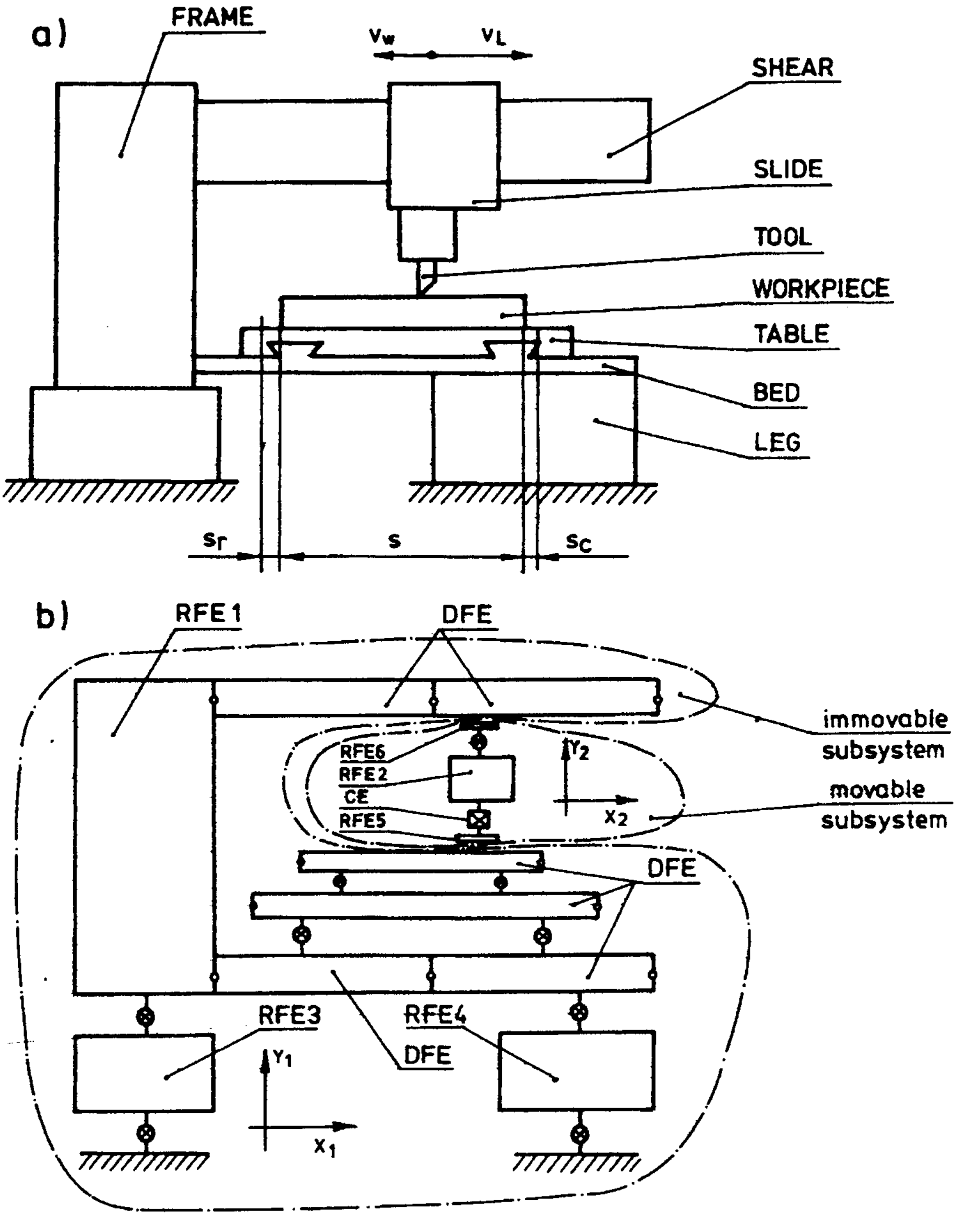

FIG. 2. The traverse shaper: a) overall design, b) discrete model of the finite element method.

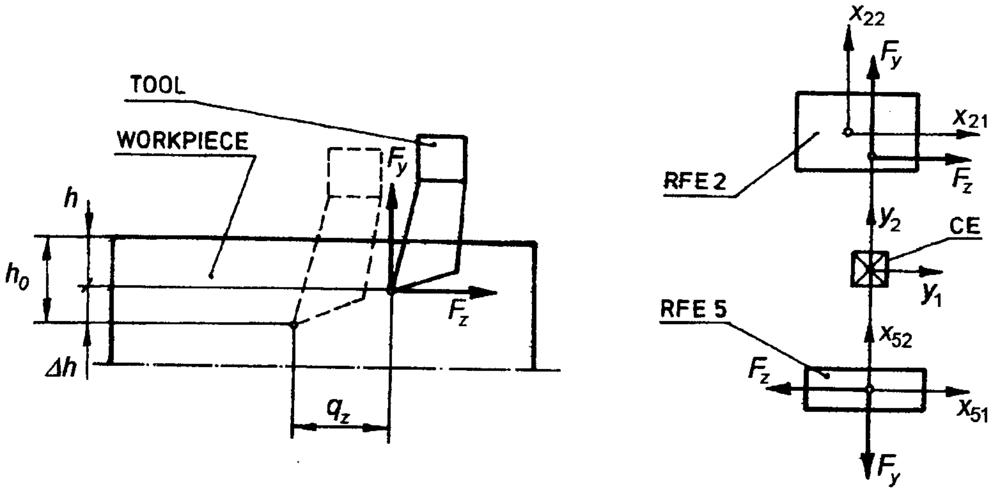

FIG. 3. Modelling the cutting process interaction using the coupling element (CE).

As a result, the real cutting force components are as follows:

$$F_z^{(r)} = F_z - \Delta F_z, \tag{3.1}$$

$$F_y^{(r)} = F_y - \Delta F_y, \tag{3.2}$$

where ΔF_z, ΔF_y – dynamic changes in cutting force components, which are described by a modified theory of GRASSO - NOTTO LA DIEGA - PASSANNANTI [9], that is to say:

$$\Delta F_z = K_1 \dot{q}_z(t) + K_4 \Delta\dot{h}(t) + K_3 \Delta h(t) - K_7 \Delta h(t-T), \tag{3.3}$$

$$\Delta F_y = K_2 \dot{q}_z(t) + K_6 \Delta\dot{h}(t) + K_5 \Delta h(t) - K_8 \Delta h(t-T). \tag{3.4}$$

Here K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8 – constant coefficients, which are usually determined experimentally [9], T is the time-delay between two subsequent passes of the tool.

It should be noticed that this dynamic interaction of the cutting process may be idealised by CE. The results of the above considerations are the following:

One CE, which is involved in order to present the dynamics of the cutting process;

Outer desired cutting forces F_z and F_y of the CE, which also act on RFE 2 and on RFE 5.

Using the matrix form for the plane case of dynamics, we can write

$$\mathbf{P}_l(t) = \mathbf{P}_l^{(z)} - \mathbf{D}_{lV} \Delta\dot{\mathbf{w}}_l(t) - \mathbf{D}_{lP} \Delta\mathbf{w}_l(t) - \mathbf{D}_{lO} \Delta\mathbf{w}_l(t-T), \tag{3.5}$$

where:

$$\mathbf{P}_l(t) = \mathrm{col}(F_z^{(r)}, F_y^{(r)}, 0), \tag{3.6}$$

• vector of real cutting forces during the shaping process for CE no. l,

$$\mathbf{D}_{lV} = \begin{bmatrix} K_1 & K_4 & 0 \\ K_2 & K_6 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{3.7}$$

• matrix of derivative feedback interactions for CE no. l,

$$\mathbf{D}_{lP} = \begin{bmatrix} 0 & K_3 & 0 \\ 0 & K_5 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{3.8}$$

• matrix of proportional feedback interactions for CE no. l,

$$\mathbf{D}_{lO} = \begin{bmatrix} 0 & -K_7 & 0 \\ 0 & -K_8 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{3.9}$$

• matrix of delayed feedback interactions for CE no. l,

$$\mathbf{P}_l^{(z)} = \mathrm{col}(F_z, F_y, 0), \tag{3.10}$$

• vector of desired cutting forces during the shaping process for CE no. l,

$$\Delta\mathbf{w}_l(t) = \mathrm{col}(q_z(t), \Delta h(t), 0), \tag{3.11}$$

• vector of connecting points displacement differences for CE no. l, for an instant of time t,

$$\Delta\mathbf{w}_l(t-T) = \mathrm{col}(q_z(t-T), \Delta h(t-T), 0), \tag{3.12}$$

• vector of connecting points displacement differences for CE no. l, for an instant of time $t - T$.

It is evident that no. l refers to only one CE in this case. Suitable values of feedback interaction coefficients for the shaping process (Table 3) are the basis of the numerical computation. The desired cutting forces have the values $F_z = 3260$ N and $F_y = 1240$ N.

Table 3. Coefficients of the feedback interaction for the shaping process.

Workpiece: steel 40						Tool: sintered carbide $\gamma_0 = 5°$, $\lambda_s = 0°$				
b	ν_w	ν_l	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8
mm	m/s		Ns/m		N/m	Ns/m	N/m	Ns/m	N/m	
2	0.5	2	625.4	2097.0	3.8×10^6	−703.0	1.6×10^6	1355.0	3.8×10^6	1.6×10^6

In order to determine the correct step of numerical integration, preliminary computation of natural frequencies of the traverse shaper's model is performed. The calculation is made for a chosen time-instant position of the movable subsystem (i.e. at a central position of the slide with respect to the workpiece) so that the whole system of the machine tool is considered as temporarily steady for this purpose. Some lowest values of these frequencies (Table 4) should influence the dynamic response of the system. The reason is that a contribution of eigenmodes accompanying them is significant for those parts of the system which model the tool and the workpiece [1]. Thus, here it is supposed that the step of integration is $h = 1.4 \times 10^{-3}$ sec. It means that four of the first eigenmodes influence the solution of the differential equations system [7].

Table 4. Some lowest natural frequencies and time-periods of the corresponding free vibrations for the traverse shaper discrete model.

Number of natural frequency	Natural frequency f[Hz]	Corresponding time-period T [s]
1	76.20	0.0130
2	84.70	0.0118
3	115.56	0.0086
4	140.52	0.0071
5	176.96	0.0057
6	217.41	0.0046
7	237.30	0.0042

Computer calculation (IBM-PC) of the conventional tool and workpiece contact point vibration q_1 along horizontal co-ordinate x_{51}, and similarly, of the vibration q_2 along vertical co-ordinate x_{52}, is performed for the following cases:

a) the model without feedback interaction (Fig. 4 and 7). All components of matrices (3.7), (3.8) and (3.9) are equal to zero all the time;

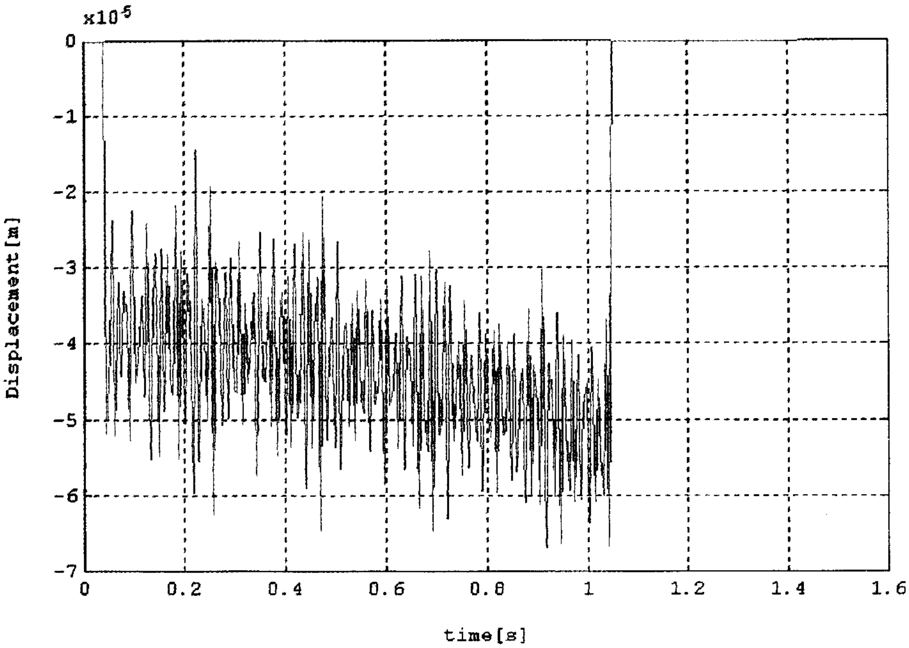

FIG. 4. The tool and workpiece conventional contact point vibration along the x_{51} coordinate for the system without feedback interaction.

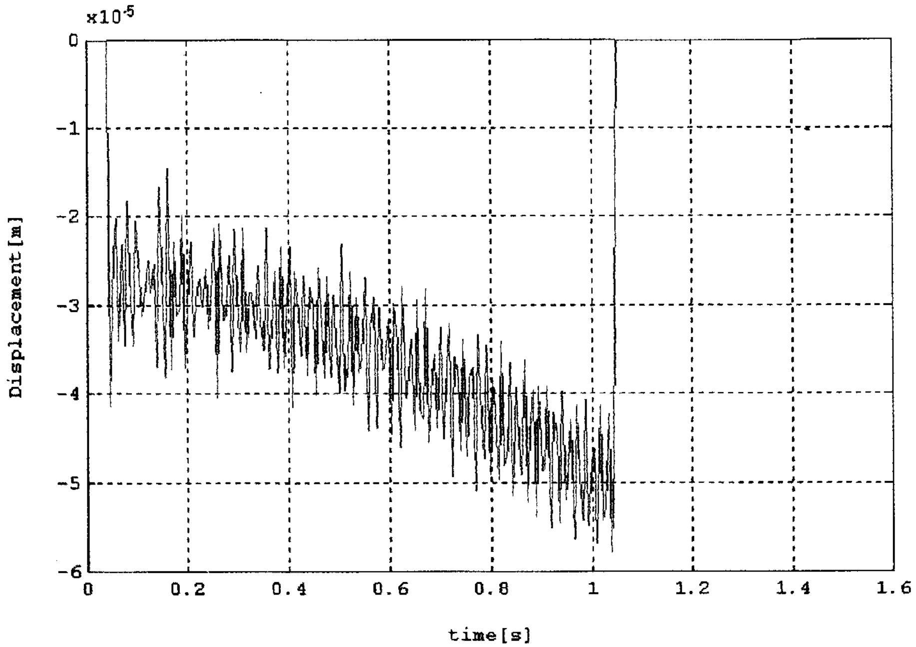

FIG. 5. The tool and workpiece conventional contact point vibration along the x_{51} coordinate for the system with proportional and ideal derivative feedback interaction.

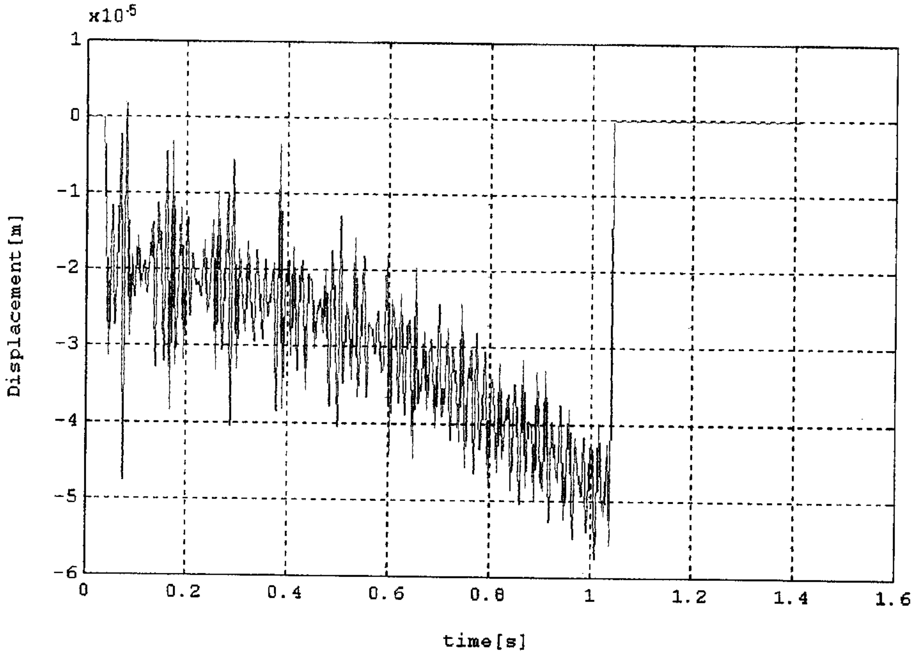

FIG. 6. The tool and workpiece conventional contact point vibration along the x_{51} coordinate for the system with proportional, ideal derivative and delayed feedback interaction.

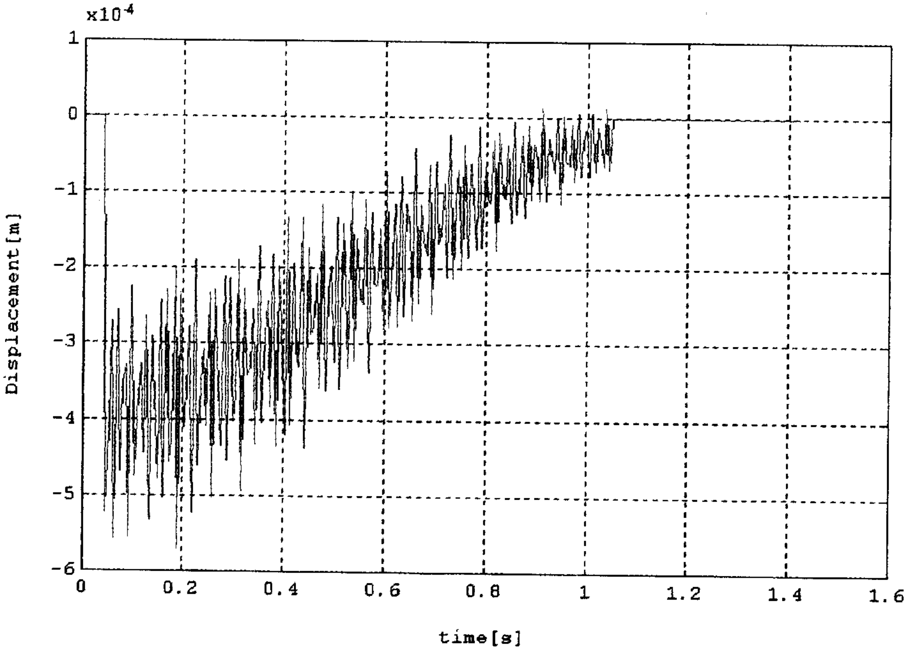

FIG. 7. The tool and workpiece conventional contact point vibration along the x_{52} coordinate for the system without feedback interaction.

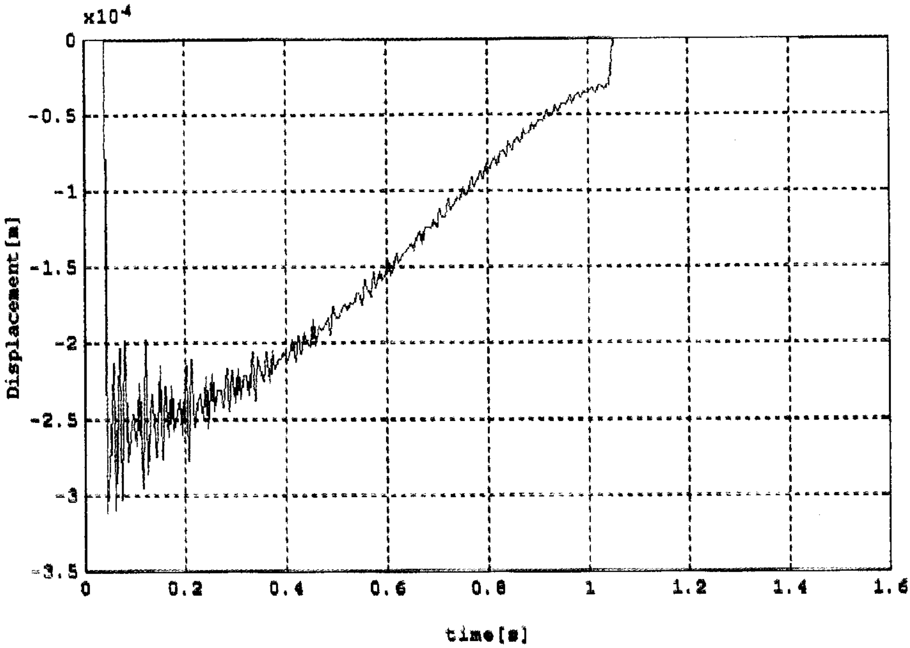

FIG. 8. The tool and workpiece conventional contact point vibration along the x_{52} coordinate for the system with proportional and ideal derivative feedback interaction.

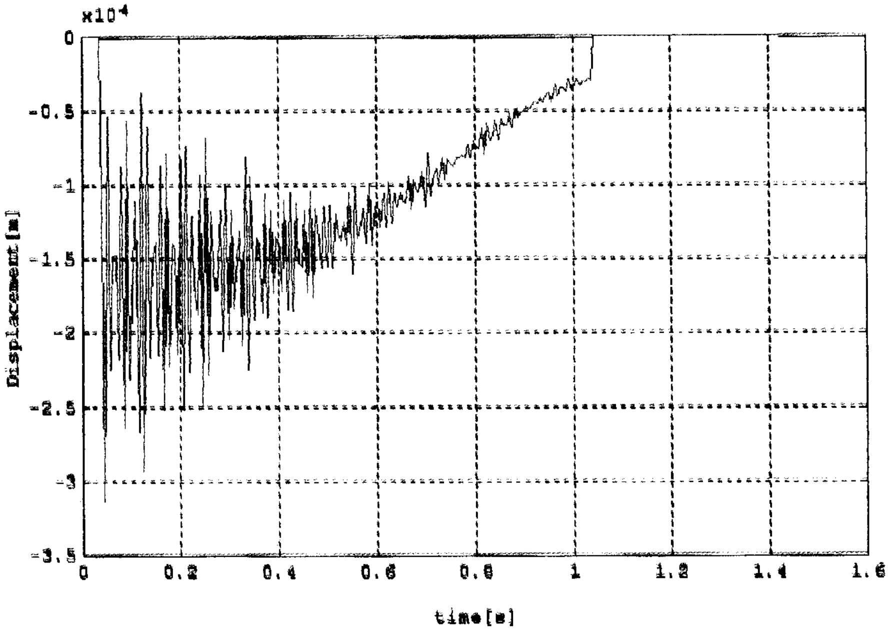

FIG. 9. The tool and workpiece conventional contact point vibration along the x_{52} coordinate for the system with proportional, ideal derivative and delayed feedback interaction.

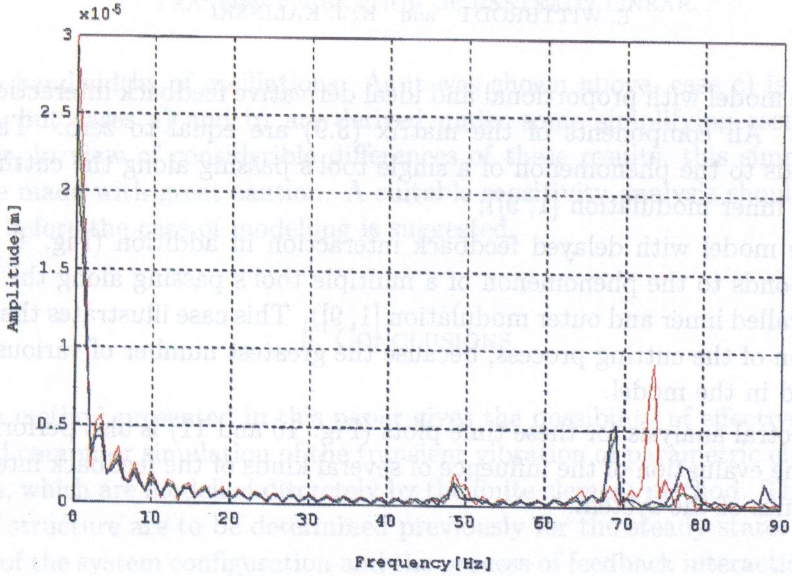

FIG. 10. The spectrum of the tool and workpiece conventional contact point vibration along the x_{51} coordinate: red line –the system without feedback interaction; green line – the system with proportional and ideal derivative feedback interaction; blue line – the system with proportional, ideal derivative and delayed feedback interaction.

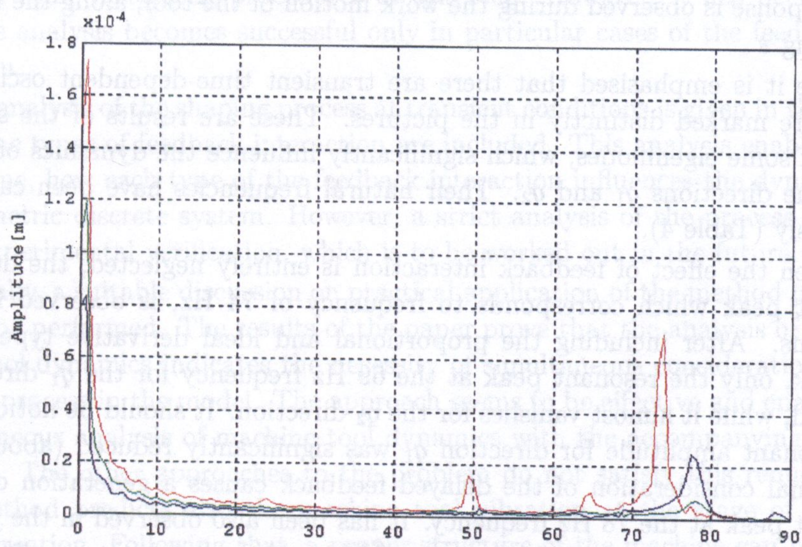

FIG. 11. The spectrum of the tool and workpiece conventional contact point vibration along the x_{52} coordinate: red line – the system without feedback interaction; green line – the system with proportional and ideal derivative feedback interaction; blue line – the system with proportional, ideal derivative and delayed feedback interaction.

b) the model with proportional and ideal derivative feedback interaction (Fig. 5 and 8). All components of the matrix (3.9) are equal to zero. This case corresponds to the phenomenon of a single tool's passing along the cutting zone (so-called inner modulation [1, 9]);

c) the model with delayed feedback interaction in addition (Fig. 6 and 9). It corresponds to the phenomenon of a multiple tool's passing along the cutting zone (so-called inner and outer modulation [1, 9]). This case illustrates the perfect description of the cutting process, because the greatest number of various effects is involved in the model.

A spectral analysis for these time plots (Fig. 10 and 11) is also performed. It enables the evaluation of the influence of several kinds of the feedback interaction on dynamics of the system.

4. Discussion of results

The plots present dynamic responses of the discrete parametric model along horizontal (i.e. q_1) and vertical (i.e. q_2) axes, due to the desired cutting force. This response is observed during the work motion of the tool, along the distance of cutting s.

Here it is emphasised that there are transient time-dependent oscillations, which are marked distinctly in the pictures. These are results of the superposition of some eigenmodes, which significantly influence the dynamics of motion along the directions q_1 and q_2. Their natural frequencies have been calculated previously (Table 4).

When the effect of feedback interaction is entirely neglected, the dominant resonant peak which corresponds to frequency of 74 Hz, is observed for both directions. After including the proportional and ideal derivative types of the feedback, only the resonant peak at the 69 Hz frequency for the q_1 direction is observed, while it almost vanishes for the q_2 direction. It should be noticed, that the resonant amplitude for direction q_1 was significantly reduced (about 40%). Additional consideration of the delayed feedback causes a generation of a new resonant peak at the 78 Hz frequency. It has been also observed in the previous case, but its value was rather meagre. Although the resultant amplitudes are much less than those in the first case, an appearance of new resonance yields a danger of losing the system stability, when the b width of cutting will be increased. It means that including the effect of feedback interaction in the model improves the quality of the discrete system modelling.

Results of the calculation show that several models of the cutting interaction influence the dynamic response. It concerns both the averaged values of the plots, and the bandwidths of oscillations. As it was shown above, case c) is the most general, but cases a) and b) are derived under some simplifying assumptions. However, in view of considerible differences of these results, this simplification must be made with great caution. A suitable sensitivity analysis should be performed before the case of modelling is suggested.

5. Conclusions

The method presented in this paper gives the possibility of effective calculation and computer simulation of the transient vibration of parametric closed-loop systems, which are modelled discretely by the finite element method. The parameters of structure are to be determined previously for the steady state. Hence, a change of the system configuration and the process of feedback interaction should be included. This concept allows to include various types of feedback into the same structure, and a variety of large, desired motions of several substructures. Then, the structure is analysed as a whole multidimensional parametric closed-loop system. A convenience of this approach is obvious, because it allows the user to avoid some measuring procedures in this step. However, it has been shown that the analysis becomes successful only in particular cases of the feedback interaction.

An analysis of the shaping process at transient conditions is given in the paper and some types of feedback interaction are included. This analysis enables us to determine, how each type of the feedback interaction influences the dynamics of a parametric discrete system. However, a strict analysis of the process requires some experimental verification, which is to be worked out in the future.

Finally, a suitable discussion on practical application of the method proposed should be performed. The results of the paper prove that the analysis of the machine tool dynamics indicates the necessity of simultaneous consideration of the cutting process in the model. The approach seems to be effective and enables the simultaneous analysis of machine tool dynamics with the accompanying cutting process. The other approaches to the problem do not satisfy this requirement. The method predicts also the machine tool vibration at the stage of discrete model creation. Following that, a proper structure of the machine can be established and correct values of cutting parameters (e.g. cutting speed, feed, depth of cutting) can be determined.

Although the computation has been worked out at stable conditions of the manufacturing process, the presence of the closed loop interaction during the cutting process may lead to a loss of stability and generate self-excited vibration, which is called *chatter*. In particular, delayed feedback (that is to say: regenerative feedback) is the main reason for it. This phenomenon is very dangerous, because it causes tool damages during the cutting process. Also the accuracy of the process decreases and efficiency of machining is extremely reduced in this case. The method proposed enables computer simulation of the model in order to preview and assess the cutting process behaviour before its real performance. Thus, it may secure the machine tool vibrostability conditions and eliminate the unstable chatter vibration in real structures.

References

1. K. Kaliński, *The finite element method in the closed loop mechanical systems vibration's calculation. Ph.D. Thesis* [in Polish], Technical University of Gdańsk, Gdańsk 1988.
2. K. Kaliński and E. Wittbrodt, *An influence of feedback on dynamics of systems with changing in time configuration modelled by the finite element method*, Proc.: XV Int. Conference on Dynamics of Machines, Frankfurt/Oder, 265–274,1985.
3. K. Kaliński, *The finite element method application to linear closed loop steady system vibration analysis*, Int. J. Mech. Science, **39**, 3, 315–330, 1997.
4. E. Wittbrodt, *Dynamics of systems whose configuration changes with time, using finite element method* [in Polish], Zeszyty Naukowe Politechniki Gdańskiej No. 369, Mechanika XLVI, 1983.
5. W. Gawroński, J. Kruszewski, W. Ostachowicz, J. Tarnowski and E. Wittbrodt, *The finite element method in the dynamics of structures* [in Polish], Arkady, Warszawa 1984.
6. K.J. Bathe and E.L. Wilson, *Numerical methods in finite element analysis*, Prentice Hall, Englewood Cliffs *1976.*
7. K. Kaliński and W. Przybylski, *Modelling and analysis of the shaft burnishing process*, Machine Vibration, **2**, 150–156, 1993.
8. M. Kleiber, *The finite element method in nonlinear mechanics of continuum* [in Polish], PWN, Warszawa-Poznań 1985.
9. V. Grasso, S. Notto La Diega and A. Passannanti, *Dynamic cutting coefficients in three-dimensional cutting*, Int. J. Mach. Tool Des., **20**, 235–249, 1980.

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