ENGINEERING TRANSACTIONS • Engng. Trans. • **47**, 2, 173–181, 1999
Polish Academy of Sciences • Institute of Fundamental Technological Research
10.24423/engtrans.627.1999

2-D AND 3-D ANALYSIS OF STOCHASTIC, ELASTIC SOIL MEDIUM

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The paper presents a stochastic description of a three-dimensional soil medium and its modelling under plane strain conditions. The main aim of the paper is to work out a computational model enabling incorporation of three-dimensional variability of soil properties into the plane strain state analysis. It is assumed that the soil medium is statistically homogeneous and its mechanical behaviour is governed by the linear elasticity theory. It is also assumed that elastic parameters can be modelled as the multidimensional random fields. The strip foundation on a soil layer in the 3-D and the 2-D strain states is analysed. Stochastic 2-D and 3-D finite element methods, based on the Monte Carlo technique, were used. The analysis performed enables determination of the standard deviations of components of the stress tensor and the displacement vector for the 3-D state, based on the solution for the 2-D plane strain state. Transfer functions between both states are determined.

1. Introduction

In many geotechnical engineering problems, concerning e.g. the retaining walls, strip foundations, or slopes and embankments (Fig. 1), the plane strain analysis is widely used. Such analysis is reasonable for elongated bodies of uniform cross-section subjected to a uniform loading along their longitudinal axes (x_1). It means that in the stochastic soil medium there is a full correlation in this direction, or in other words, a random variable model is assumed. Usually this is not so, and in some soils, depending on their origin, even a significant horizontal variability of the material elastic parameters may appear. Thus, the question of validity of the plane strain assumption for three-dimensional random fields of those parameters arises. It is fundamental to answer the following basic questions:

1. When is the plane strain schematization of the three-dimensional stochastic medium justified?
2. Is it possible to take into account 3-dimensional spatial variability of soil properties in the plane strain analysis, and how to do it ?

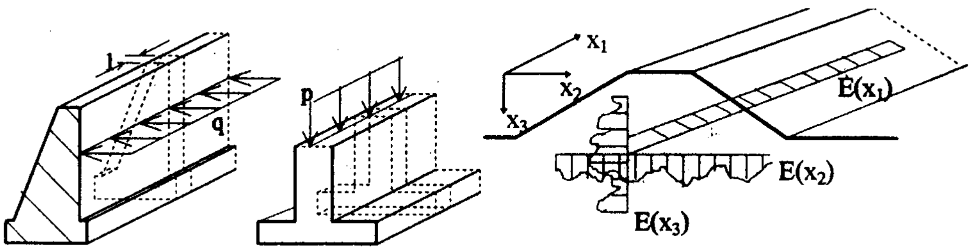

FIG. 1. Examples of elongated geotechnical structures.

In order to answer these questions, it seems to be necessary to compare similar boundary problems analysed both in the plane strain and 3-D state. It should allow us to determine a proper transfer function between particular states. The problem can be significantly simplified by comparing, for especially chosen boundary value problems, the solutions obtained in uni-axial and plane-strain states.

For an engineer, besides the expected values, most important are the standard deviations (variances) of stresses and displacements. So in the following, transfer functions for variances between different states of strains will be determined:

$$\begin{aligned} \mathrm{Var}\left[\tilde{u}_i^{\mathrm{III}}(x_2,x_3)\right] &= k_u \cdot \mathrm{Var}\left[\tilde{u}_i^{\mathrm{II}}(x_2,x_3)\right], \\ \mathrm{Var}\left[\tilde{\sigma}_{ij}^{\mathrm{III}}(x_2,x_3)\right] &= k_s \cdot \mathrm{Var}\left[\tilde{\sigma}_{ij}^{\mathrm{II}}(x_2,x_3)\right], \end{aligned} \tag{1.1}$$

where k_s and k_v are transfer functions for standard deviation of displacements and stresses, respectively.

2. Stochastic finite element method

The stochastic finite element method was used to obtain detailed results of the formulated problem. The computations were performed using a modified version of the NONSAP program for the stochastic finite element method (FEM), based on the Monte Carlo technique. The strip foundation of width B, laid on the elastic, random horizontal stratum, resting on a smooth, rigid base is considered. The load acting on the strip of intensity p is uniform and flexible. The finite element mesh applied in a plane strain analysis and in the 3-D state are shown in Fig. 2. It is assumed that the soil is weightless, the external loading $p = 10$ [kPa] and the average values of elastic parameters are $\overline{E} = 100$ MPa and $\overline{\nu} = 0.3$. The numerical calculations were performed both in the plane strain state and the 3-D state, for a $h = 20$ m thick soil stratum, $B = 2.5$ m. The coefficient of variation $\alpha_E = 0.1$ and three values of the decay coefficient $\lambda = 1$, 2 and 5 were taken into account.

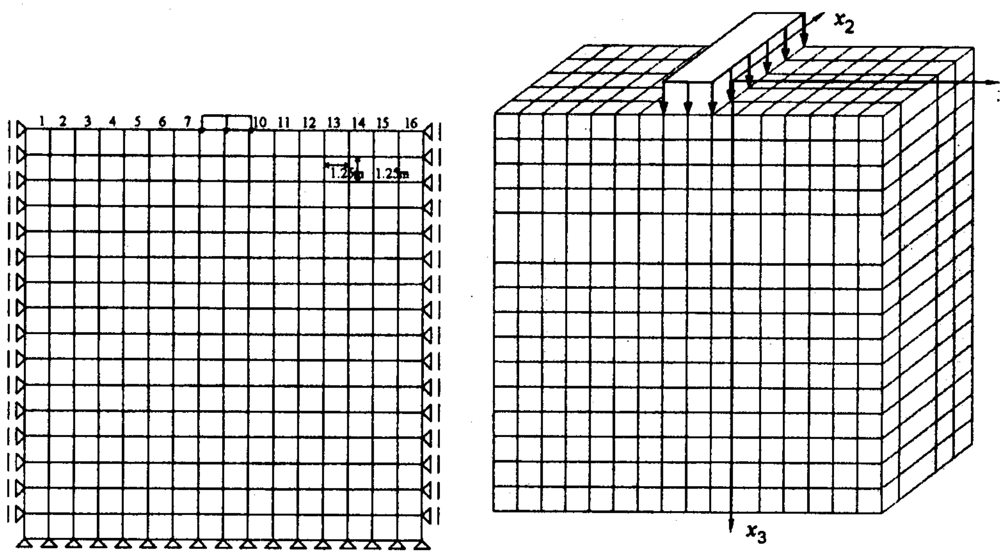

FIG. 2. Finite element mesh applied in 2-D and 3-D analysis.

3. STOCHASTIC SOIL DESCRIPTION

Randomness of the soil is described by the normal probability distribution function and the following covariance function:

$$R(x, y) = \sigma^2(1 + \lambda_x \cdot |x|)e^{-\lambda_x|x|}(1 + \lambda_y \cdot |y|)e^{-\lambda_y|y|}, \tag{3.1}$$

where: λ_x and λ_y are correlation decay coefficients in x and y directions, respectively, and σ is a standard deviation. The decay coefficients characterise spatial variability of soil properties while the standard deviation represents its scattering.

4. Two-dimensional schematization of three-dimensional problem

An attempt to solve the problem of the strip foundation on stochastic soil in the 2-D and 3-D states is presented. The comparison of solutions for both states enables us to find the transfer function. All results are presented in dimensionless co-ordinates.

4.1. *Displacements*

The change of the standard deviation of the vertical displacement with depth in the 2-D and 3-D states, for the symmetry axis ($x_1 = 0, x_2 = 0$) and for the decay coefficient $\lambda = 5$, is shown in the Fig. 3. In the plane strain analysis the standard deviations of vertical displacements are higher than in the 3-D state.

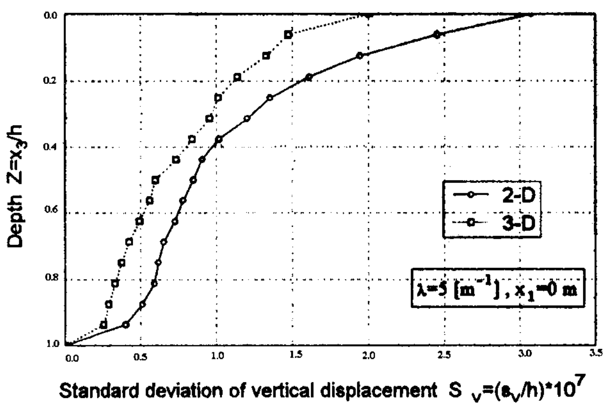

FIG. 3. Standard deviation of vertical displacement vs. depth.

The relation between the standard deviations of the vertical displacement for the 2-D and 3-D states, the symmetry axis and the decay coefficient $\lambda = 5$ is presented, in the Fig. 4.

Based on the obtained results (including 1-D schematization of a 2-D problem), the transfer function between the standard deviations of the vertical displacement for the 2-D and 3-D states, versus the decay coefficient, is of an exponential type and may be written as follows:

$$k_\nu = \exp(c \cdot \lambda^k), \tag{4.1}$$

The transfer function given by this expression is presented in the Fig. 5 as a continuous line. It is seen here that standard deviation of displacement for the 3-D case decreases in relation to the 2-D case, for increasing values of decay coefficients. The coefficients $c = -0.21$, $n = 0.24$ were determined for the average directional coefficients. The transfer function is presented in the figure as a continuous line.

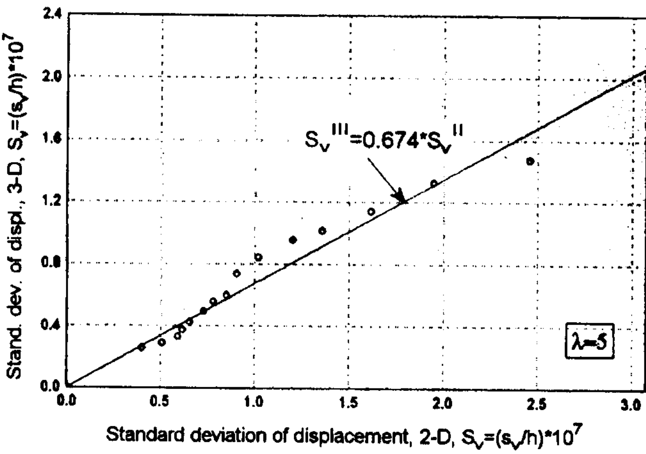

FIG. 4. Relation between standard deviations of vertical displacements for 2-D and 3-D analysis.

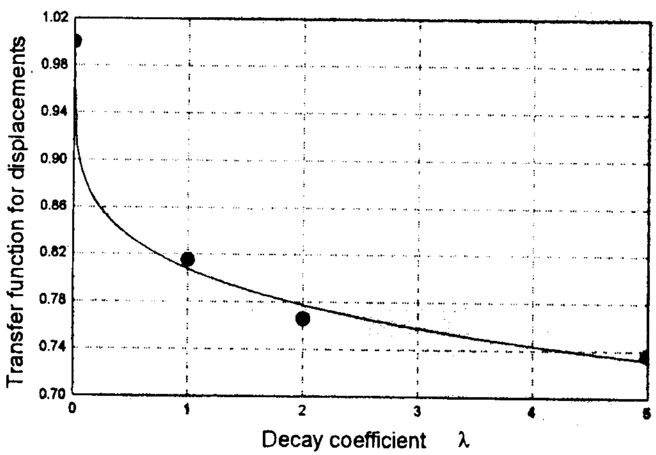

FIG. 5. Transfer function between standard deviations of vertical displacement in the 2-D and 3-D states.

4.2. Stresses

The change of the standard deviation of the normal vertical stress with depth, in the 2-D and 3-D states, for decay coefficient $\lambda = 5$ and four x_1 values, is shown in the Fig. 6. It can be seen there that the character of changes of the considered standard deviations in both states is similar. Respective curves, however, are shifted with respect to each other.

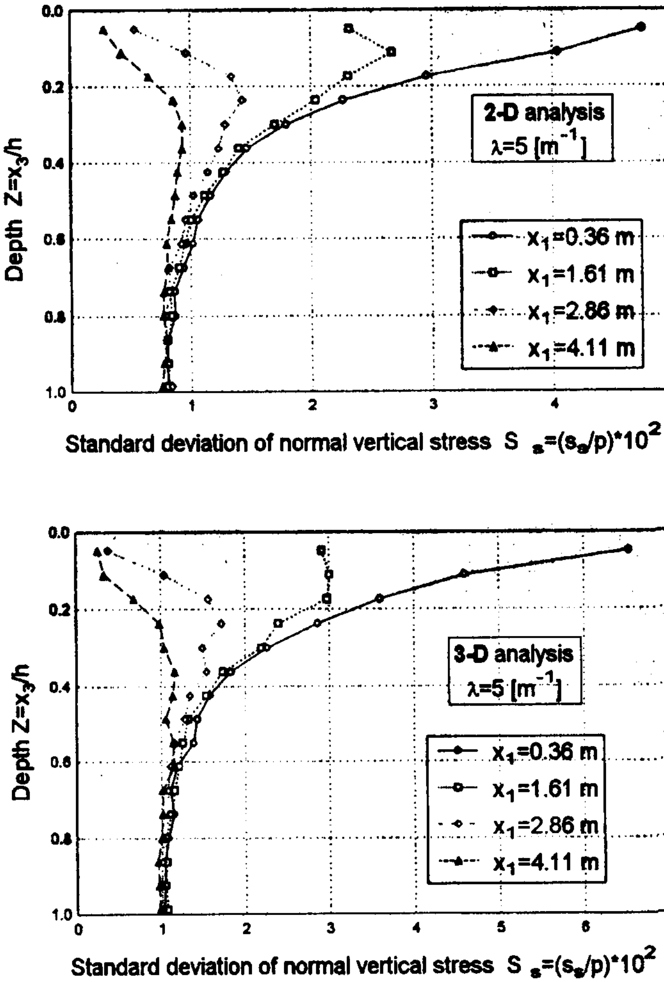

FIG. 6. Standard deviation of the normal vertical stress vs. depth.

The differences between the 2-D and 3-D standard deviations of normal vertical stress (for $x_1 = 0.36$ m) are distinctly visible in the Fig. 7.

The results obtained for the decay coefficients $\lambda = 1$, $\lambda = 2$, $\lambda = 5$ [m^{-1}] were approximated by linear functions. One of these relations for the decay coefficient $\lambda = 5$ is shown in the Fig. 8.

The transfer function between standard deviation of the normal vertical stress for the 2-D and the 3-D states is presented in the Fig. 9. It varies with the decay coefficient exponentially, and the same relations as that in the case of displacement (parameters $c = 0.16$, $n = 0.18$) can be assumed.

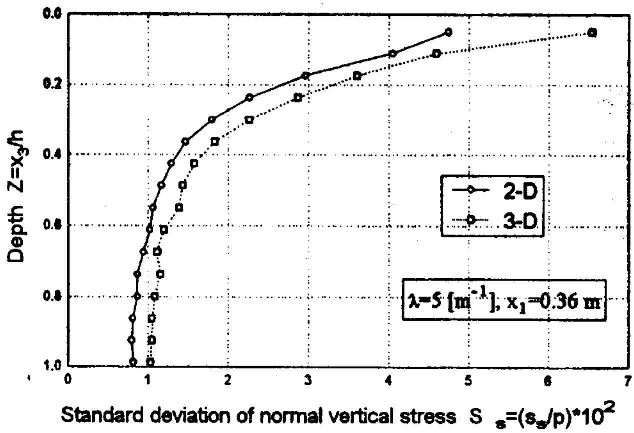

FIG. 7. Standard deviation of normal vertical stress vs. depth.

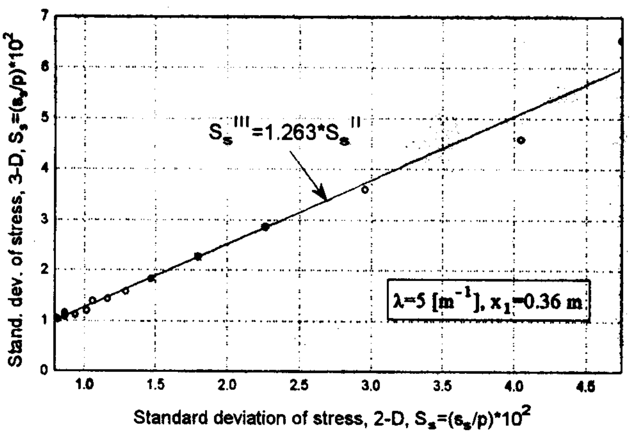

FIG. 8. Relations between standard deviations of normal vertical stress for 2-D and 3-D analyses.

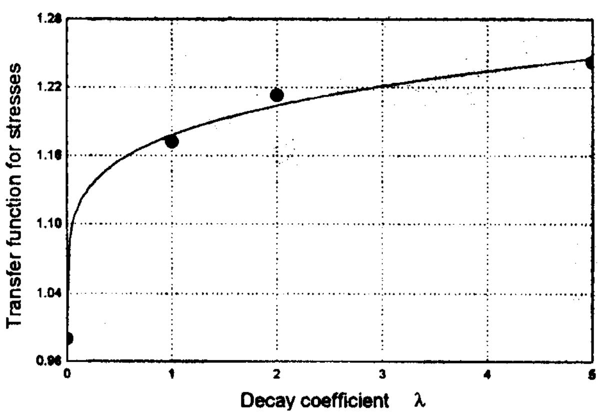

FIG. 9. Transfer function between standard deviations of normal vertical stress in the 2-D and the 3-D states.

5. Conclusions

The basic achievement of the paper is the proof of the thesis that it is possible to incorporate the spatial, three-dimensional variability into the plane strain analysis. The analysis of variances (standard deviations) as a measure of reduction of the space dimension is, from the engineering point of view, sufficient. The relations proposed in the paper enables us to express the standard deviations of displacements and stresses in the 3-D state in terms of respective standard deviations obtained in the 2-D analysis.

The considerable influence of both the displacements and stresses on the standard deviations have two parameters, namely the coefficient of variation of elasticity modulus α and the decay coefficient of the correlation function λ.

Standard deviation of the vertical displacement is the greatest for the uniaxial strain state. Taking into account the existence of the correlation in the third, longitudinal direction, leads to further decrease of the standard deviation of the vertical displacement. Contrary to this, the standard deviation of the normal vertical stress is the greatest for the 3-D state. For the full correlation it is equal to zero and increases to a constant value with the decrease of the correlation.

For the covariance function considered, the greatest changes of the standard deviations for both stresses and displacements take place for the relatively small decay coefficients, varying from $\lambda = 0$ to $\lambda = 2$.

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Received November 6, 1998.