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PROBLEMS OF IDENTIFICATION OF MATERIAL PARAMETERS FOR DIFFERENT TYPES OF VISCOPLASTIC CONSTITUTIVE EQUATIONS

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In the paper the problem of identification of the material parameters of the elasto-viscoplastic constitutive equations is considered. A method based on the uniaxial tension tests for the Bodner – Partom law and two methods (from the tension and cyclic tests) for the Chaboche laws are presented. The parameters for both models for mild steel are directly calculated from the tension tests. Additionally, the numerical simulation of the cyclic loading tests is used for identification of the Chaboche model parameters. Results of the identification are verified in the viscoplastic analysis of vibrations of circular plates.

1. Introduction

Calculations of the viscoplastic behaviour of structures are one of the most challenging subjects for scientists. Andrade, Norton and Odquist have developed in 1910 – 1934 the mathematical model of this type of behaviour as the system of the first order differential equations. In the seventies, rapid progress of computer techniques and numerical methods gave new possibilities of verification of more complex models of the constitutive equations. The type of the viscoplastic law invented by Perzyna, has been extended by Chaboche, who used a more detailed description of the hardening effects. In this approach the elastic and inelastic parts of the deformation are separated. The viscous effects are assumed to appear after reaching the yield limit. Bodner and Partom have proposed a quite opposite description of the deformation process. In their type of constitutive equations, the viscous phenomena starts together with deformation of the structure. In order to use both types of equations, it is necessary to know at least 7 material parameters introduced in the system of the differential equations. The paper compares the methods of identification of them showing their advantages and drawbacks.

2. The Chaboche and Bodner-Partom models in uniaxial stress state

The dynamic response of structures subjected to the impact loading is a problem of great industrial importance. Application of one of many models of the viscoplastic constitutive equations depends on the possibilities of calculation of its parameters. Therefore, most often the simplest models, using small number of parameters, are used. For identification of these parameters, usually different kinds of uniaxial tests are applied. Preparing an identification procedure, the basic differential equations of the constitutive models must be transformed into

Table 1.

	Chaboche model [7]	Bodner-Partom model [8]
Inelastic strain rate	$\dot{\varepsilon}_I = \left\langle \frac{\lvert\sigma - X\rvert - R - k}{K} \right\rangle^n \operatorname{sgn}(\sigma - X)$	$\dot{\varepsilon}_I = \frac{2}{\sqrt{3}} D_0 \exp\left[-\frac{1}{2}\left(\frac{R+D}{\sigma}\right)^{2n} \times \frac{n+1}{n}\right] \operatorname{sgn}(\sigma)$
Additional definitions	$\langle x \rangle = \frac{1}{2}(x + \lvert x\rvert)$	$D = \sqrt{\frac{3}{2}} X \operatorname{sgn}(\sigma)$ $\dot{W}^I = \sigma \dot{\varepsilon}_I$
Kinematic hardening functions	$X = \operatorname{sgn}(\sigma)\frac{2}{3} \cdot \frac{a}{c} + \left(X_0 - \frac{2}{3} \cdot \frac{a}{c}\right) \exp(-c(\varepsilon_I - \varepsilon_{I0}))$ X_0, ε_{I0} – initial values of kinematic hardening and inelastic strains	$X = \sqrt{\frac{2}{3}} D_1 \operatorname{sgn}(\sigma)\left[1 - \exp(-m_2 W^I)\right]$ $D = \sqrt{\frac{3}{2}} X \operatorname{sgn}(\sigma)$
Isotropic hardening functions	$R = R_1(1 - \exp(-b\lvert\varepsilon_I\rvert))$	$R = R_1\left[1 - \exp(-m_1 W^I)\right] + R_0 \exp(-m_1 W^I)$
Model parameters	k, K, n, a, c, b, R_1	$n, D_0, D_1, R_0, R_1, m_1, m_2$

the uniaxial form. For the Chaboche and Bodner – Partom approaches, the necessary formulas are collected in Table 1.

The choice of the uniaxial tests, which can be carried out, often depends on the shape of specimens, which can be cut out. Plate and shell structures are usually made from thin sheets and the specimens taken from the same material are also thin. Therefore, the tests including compression in the inelastic range must be excluded. That is why identification of the viscous and the hardening properties from the full reversed cyclic tests, which is the most suitable, is not possible. In these conditions, it is important to find the procedure of calculation of the material coefficients if only the tension tests are available.

3. Determination of the Bodner – Partom model parameters

In the variant of the Bodner – Partom model, that is suitable in the dynamic analysis (without recovery effects) it is sufficient to identify 7 material parameters indicated in Table 1. The first of them D_0 (the limit of inelastic strain rate) is usually chosen arbitrarily. $D_0 = 10^4$ 1/s is most often used, but for rapid deformations $D_0 = 10^6$ 1/s or even $D_0 = 10^8$ 1/s can be applied [1, 2].

To identify parameters n and R_0, the inelastic strain rate function of the apparent yield limit has to be studied. When the material enters the plastic domain, the isotropic hardening is equal to its initial value $R = R_0$, and the kinematic hardening is negligible [3]. The value of the yield stress σ_0 for small values of the inelastic strain can be determined from equations for the accumulated inelastic strain rate given in Table 1 as follows [12]:

$$\sigma_0 = \frac{R_0}{\left[\dfrac{2n}{n+1} \cdot \ln\left(\dfrac{2D_0}{\sqrt{3} \cdot \dot{\varepsilon}_I}\right)\right]^{1/(2n)}}. \tag{3.1}$$

If the value of the apparent yield limit is known for several values of inelastic strain rates, the Marquardt-Levenberg regression [4] can be used and values of n and R_0 can be calculated.

To obtain the remaining four parameters, the hardening work rate function γ is build [5]:

$$\gamma = \frac{d\sigma}{dW^I} = \frac{d\sigma}{d\varepsilon^I} \cdot \frac{1}{\sigma}, \qquad dQ^I = \sigma \cdot d\varepsilon^I, \tag{3.2}$$

$$\gamma = f(\dot{\varepsilon}_I\left[m_1(R_1 - R) + m_2(D_1 - D)\right]. \tag{3.3}$$

A typical graph of this function is shown in Fig. 1.

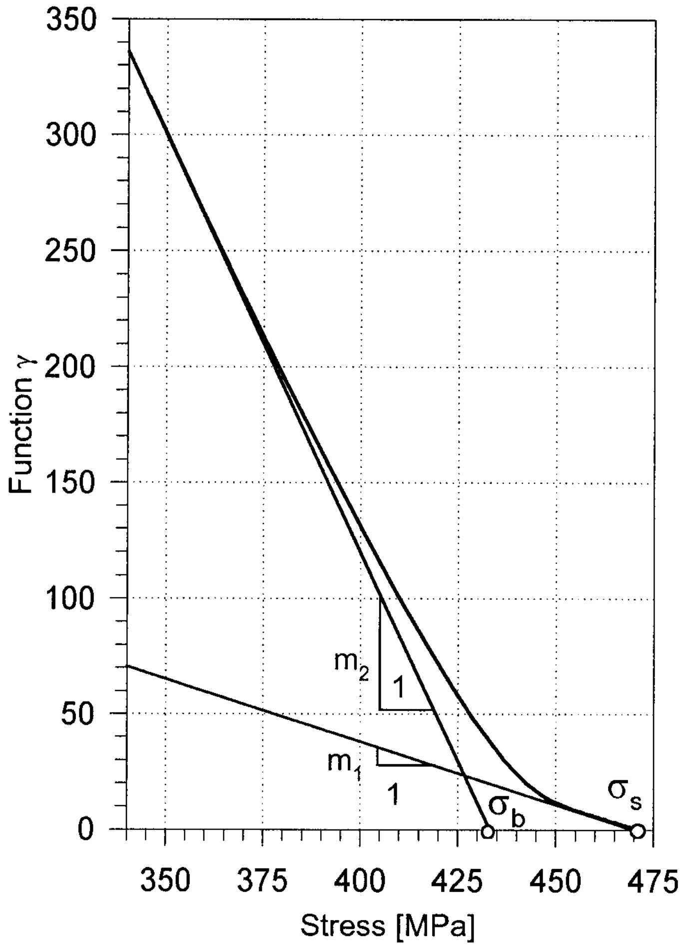


FIG. 1. Typical graph of γ – function.

The slope of the linear parts of the γ – function on both its ends gives values of the m_1 and m_2 parameters. The values of stresses σ_b and σ_s, indicated in Fig. 1, make it possible to build the system of two linear equations. The solution of this system defines D_1 and R_1 yielding [6]:

$$
\begin{aligned}
D_1 &= \frac{\sigma_b m_2}{f(\dot{\varepsilon}^I)(m_2 - m_1)} - \frac{\sigma_s m_1}{f(\dot{\varepsilon}^I)(m_2 - m_1)} - R_0, \\
R_1 &= \frac{\sigma_s m_2}{f(\dot{\varepsilon}^I)(m_2 - m_1)} - \frac{\sigma_b m_2}{f(\dot{\varepsilon}^I)(m_2 - m_1)} + R_0.
\end{aligned}
\tag{3.4}
$$

4. Determination of the Chaboche model parameters

4.1. Direct identification from the tension tests

For determination of the parameters K, k, n, a similar method as for the Bodner – Partom model can be used. The apparent yield limit for the Chaboche model is defined [12]:

$$\sigma_0 = K(\dot{\varepsilon} \cdot \mathrm{sgn}(\sigma))^{1/n} + k. \tag{4.1}$$

The approximation of experimental values of the yield limit for different strain rates by the function (4.1) leads to identification of K, k, and n. The hardening parameters are evaluated from the equation for stresses in the inelastic range of deformation:

$$\sigma = X(\varepsilon_I, X_0, \varepsilon_{I0}) + \mathrm{sgn}(\sigma)R(|\varepsilon_I|) + \mathrm{sgn}(\sigma)k + \mathrm{sgn}(\sigma)K|\dot{\varepsilon}_I|^{1/n}. \tag{4.2}$$

Also here the Marquardt-Levenberg regression [4] is applied. To guarantee correct results of the identification, it is necessary to choose proper starting values of the parameters. In the identification at least three different tests with different strain rates must be used. First single tests are used in each regression. The results of the previous calculation give the initial values for the next one. Finally, one more regression for all tests together is used. The details of this variant of identification are also discussed in [9].

4.2. Identification from the fully reversed cyclic tests

Identification of the Chaboche law parameters from the fully reversed cyclic tests is the most suitable method. Experiment series of such tests with the constant strain rate and different strain amplitudes $\Delta\varepsilon$ should be carried out (Fig. 2). In each test, stabilisation of the cycles should be reached.

Using the graph of Fig. 2b, separation of the stress into components related to the viscous effect (σ_ν), the kinematic hardening (X) and the isotropic hardening (R) is possible. The initial yield limit (σ_0) can be also indicated. According to the equations given in Table 1, the increment of stresses associated with viscous properties can be calculated from the expression:

$$\sigma_{\nu i} = K|\dot{\varepsilon}_{Ii}||^{1/n}. \tag{4.3}$$

From (4.3) the parameters K and n can be identified, and the function of the yield limit (4.1) gives the value of k parameter. For the stabilised cycles the maximum value of the total stress can be calculated from the relation given in [10]:

$$\sigma = \frac{a}{c}\,\mathrm{tgh}\,(c\varepsilon_{I\mathrm{stab}}) + R_1 + k + K(\dot{\varepsilon}_I)^{1/n}, \tag{4.4}$$

where: $\varepsilon_{I\mathrm{stab}} = \Delta\varepsilon_I/2$.

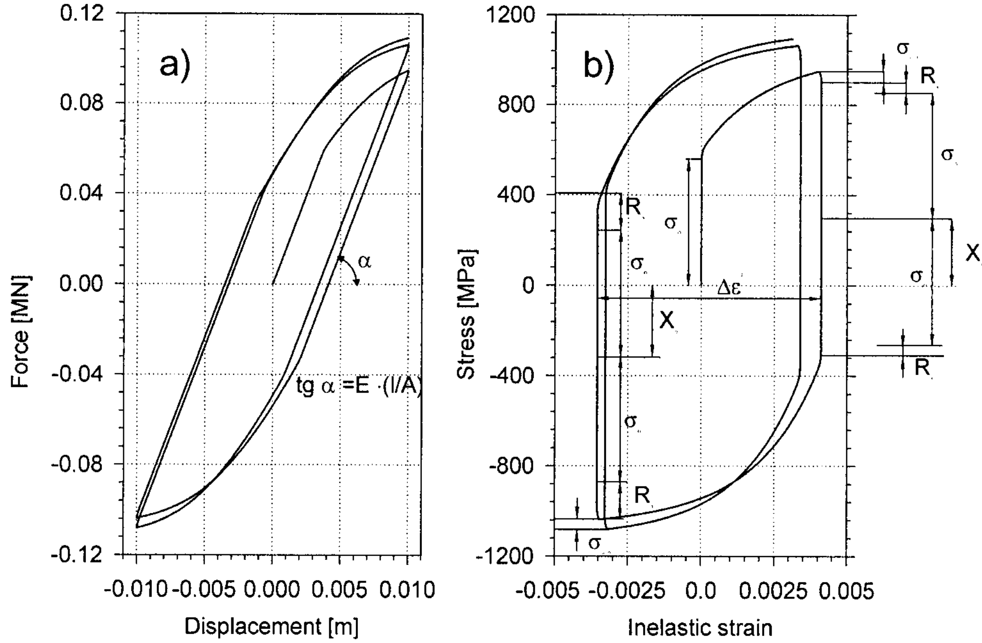


FIG. 2. Fully reversed cyclic tests. a) Force – displacement function; b) Inelastic strain – stress function.

Regression of the function (4.4) leads to the values of a, c, R_1 parameters. The current value of the isotropic hardening can be calculated from the following formula obtained from the first equation in Table 1:

$$R = \sigma - X - k - K(\dot{\varepsilon}_I)^{1/n}. \tag{4.5}$$

Using this equation, the parameter b can be calculated.

Identification of the Chaboche model parameters is more comfortable from the cyclic loading tests than from the tension tests only, but it can not be always used. Very often the examined structures are thin (e.g. plates and shells) and specimens cut out for uniaxial tests can not be compressed above the yield limit because of buckling. On the other hand, the direct identification is not always satisfying because of the difficulties with indication of the yield limit for current strain rate and because of large number of parameters, which have to be identified during a single regression. If the Bodner - Partom model parameters are determined (from the tension tests only), they can be used in the numerical simulation of the cyclic loading tests. For calculations only a simple computer program, in which only uniaxial relations are used, is necessary. The main advantage of this method is high accuracy of the results of such computer testing, as compared with real experiments. Unfortunately, inaccuracies of both identifications aggregate.

5. Verification of results of regressions

All three methods of identification were used for certain kind of mild steel 1mm thick. The cyclic tests for identification of the Chaboche law parameters were numerically simulated. The following values of coefficients were obtained. For the Bodner – Partom model:

$$\begin{aligned} E &= 215661 \text{ MPa}, \; D_0 = 10000 \text{ s}^{-1}, \; n = 9.61, \; D_1 = 21.35 \text{ MPa}, \\ m_1 &= 0.068 \text{ MPa}^{-1}, \; m_2 = 1.82 \text{ MPa}^{-1}, \; R_0 = 259.38 \text{ MPa}, \\ &\qquad\qquad R_1 = 422.90 \text{ MPa}. \end{aligned} \tag{5.1}$$

From direct identification of the Chaboche law parameters (variant I):

$$\begin{aligned} E &= 215661 \text{ MPa}, \; n = 1.0, \; k = 210 \text{ MPa}, \; K = 267 \text{ MPa}, \\ c &= 64, \; a = 535.5 \text{ MPa}, \; R_1 = 138.48 \text{ MPa}, \; b = 9.18. \end{aligned} \tag{5.2}$$

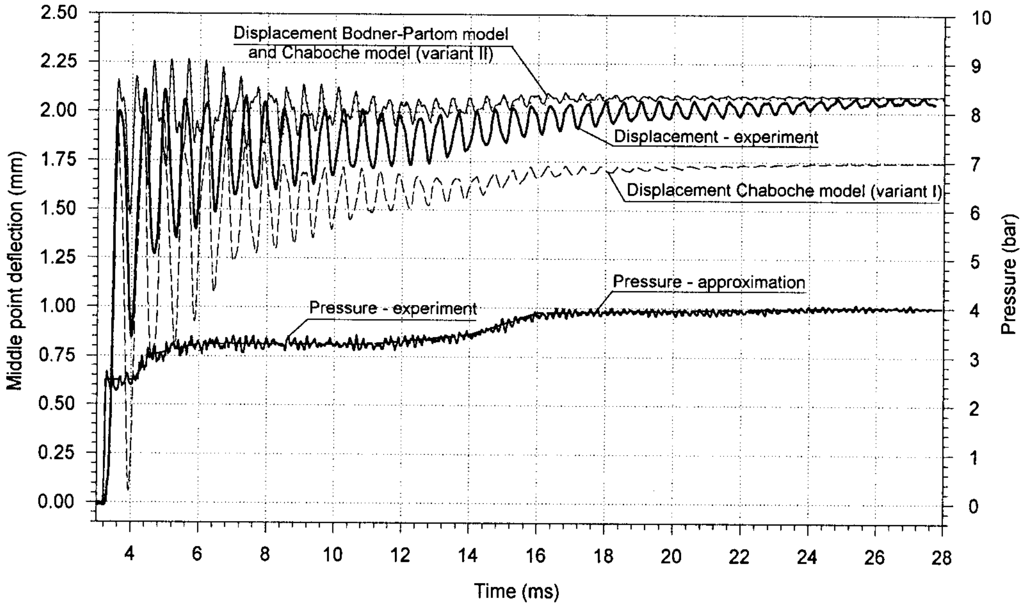
From numerical identification with the cyclic tests (variant II):

$$\begin{aligned} E &= 215661 \text{ MPa}, \; n = 9.51, \; k = 210.15 \text{ MPa}, \; K = 14.085 \text{ MPa}, \\ c &= 38840, \; a = 611700 \text{ MPa}, \; R_1 = 138.26 \text{ MPa}, \; b = 17.64. \end{aligned} \tag{5.3}$$

The value of the Young modulus E was found from separate typical elastic tests. To verify results of the identification, calculations of the circular plates subjected to the impulsive loading were compared with the experimental results. The details of description of the experimental equipment and the numerical aspects of calculations can be found in [11]. The time functions of the middle point displacements are compared for two levels of pressure in Fig. 3.

For lower pressure level, the Bodner – Partom model and the Chaboche model (variant II) give almost the same dynamic response during the whole time range examined. The results are also in a good agreement with the experimental result. Both the discussed calculations slightly overestimate the real values of displacements. The first variant of the Chaboche model has produced displacements smaller than those in tests. Vibrations have here the same characteristics, but the final deflection is about 17% smaller. During the first 5 ms, the amplitude of the vibrations is too large, later a similar decrease of the amplitude is observed in the calculations and in the experiment. The deflection level of plate vibrations calculated with the Bodner – Partom model and the second variant of the Chaboche model is higher than the experimental one. Finally, an error is smaller than 3%. The amplitude of vibrations is a little bit too small.

a)



b)

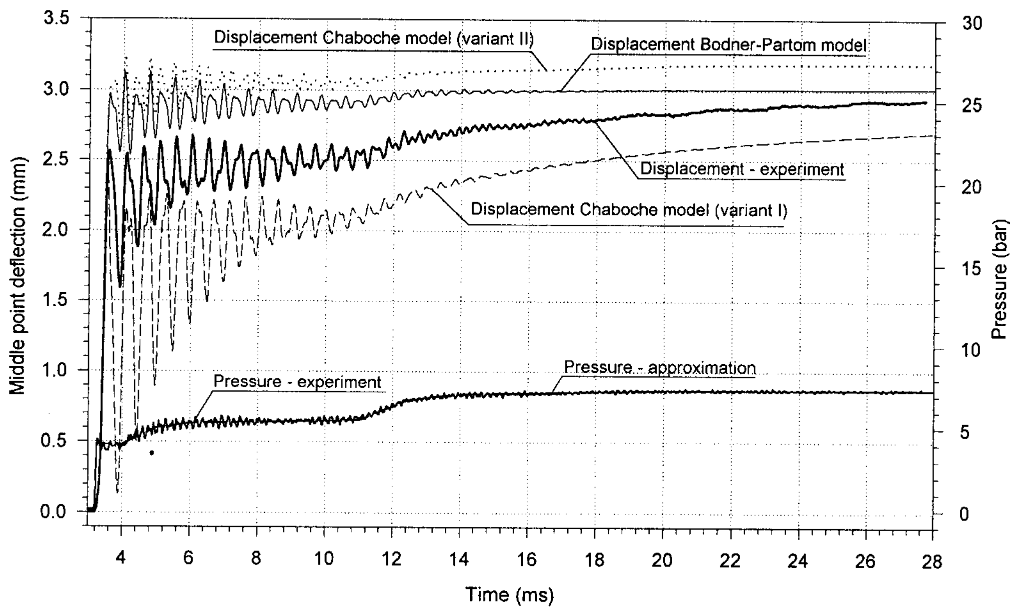


FIG. 3. Vibrations of circular plate – comparison of middle point displacement functions for different constitutive models and values of parameters.

Increase of the pressure (Fig. 3b) has more influence on the calculations with the variant I of the Chaboche model than on the Bodner – Partom model solution. Still a proper decline of the amplitude in all variants of calculations is visible. Calculations with the first variant of the Chaboche law give 12% error of the deflection level. Still a very good accuracy for the Bodner – Partom calculations (4% error) and an error less than 10% for the second variant of the Chaboche calculations is noticed. The shift of the graph for the Chaboche model (variant II) is rather not the result of improper identification of the material parameters, but shows the sensitivity of some material parameters to the change of the strain rate. This effect was also observed by CHAN *et al.* [5]. The Chaboche model is more sensitive to the change of the strain rate than the Bodner – Partom model.

6. CONCLUSIONS

1) The proposed methods of the material parameters identification can be effectively used for calculations of the material laws coefficients.

2) Only the constant strain rate tension tests are necessary for identification of the Bodner – Partom and the Chaboche models parameters.

3) The results of identification for such complex types of differential equations are not unique. Nevertheless, as it is shown on the example of the Chaboche law, they can produce similar answers in the structure dynamic analysis.

4) The different types of the viscoplastic models can also produce a similar dynamic answer of structures, which is in a good agreement with the experimental results. Very often in the literature can be found values of the parameters, which for the same material and in the same conditions, leads to completely different dynamic responses. Example of such disagreements and analysis of their reasons are discussed in [12].

5) If a set of values for one constitutive model is known, then the tests necessary for calculation of parameters for other law can be easily simulated numerically. Of course, is it better to identify parameters directly from the results of proper experiments. But very often the values of parameters for one constitutive law are taken from the literature and the results of experiments are not available. There is no way of direct recalculation of material parameters from the Bodner – Partom law into the parameters of the Chaboche law, because of different physical bases of both models. In this situation the method of numerical simulation of tests proposed in the paper can solve the problem. The results of plate calculations confirm the fact, that this procedure can be successful.

The main aim of this paper is not to show all details of the identification process, which can be different in particular cases. The authors wanted only to compare the methods of determination of parameters for the Chaboche and

Bodner – Partom models. The values of parameters for these laws taken from the literature for the same material in the same conditions usually lead to different results in calculation of the structures. Here even the numerical way of determination of parameters gives similar answers in the dynamic approach for both models.

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