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ANALYSIS OF THIN-WALLED BARS WITH OPEN AND CLOSED-OPEN CROSS-SECTIONS

A. GARSTECKI, W. KĄKOL, K. RZESZUT

POZNAŃ UNIVERSITY OF TECHNOLOGY, INSTITUTE OF STRUCTURAL ENGINEERING
ul. Piotrowo 5, 60–965 Poznań, Poland

The paper presents the numerical analysis of global and local buckling of columns made of steel cold-rolled, very thin-walled cross-sections of sigma (Σ) and double sigma (2Σ) type. Variation of the buckling stress for a wide range of slenderness ratio is presented. The deformation of the contour associated with different buckling modes and warping of open and closed-open sections is discussed, too. The exactness and numerical efficiency of different methods are studied on several examples. Finite Element Method incorporating the Vlasov beam element and shell element is compared with the Finite Strip Method.

1. INTRODUCTION

In practical engineering approach to the design of metal structures and in the respective design codes, the local and global stability problems are considered separately. The first reason for distinguishing one from the other is that rather different physical phenomena are observed in these two classes of problems. The second reason is that for the analysis of global buckling of many structures, namely columns, beams, frames or grids, simplified theories can be used. For example, when these structures are made of thin-walled elements of open cross-sections, the theory proposed by TIMOSHENKO [1] and developed by VLASOV [2] is widely used. When implemented to Finite Element Method (FEM), it results in addition of only one degree of freedom per node, namely warping. A warping function that extends Vlasov's theory to closed-open sections was proposed in [19] and also implemented in FEM [20]. However, there are only few commercial codes that employ the Vlasov type finite elements, where warping is incorporated in the formulation as a nodal variable.

The local buckling analysis requires implementation of the shell theory. In case of FEM, shell elements and fine mesh must be used. This approach provides correct results for both, the local and global structural stability response as well as in the case of their interaction. Different boundary conditions and a wide class of cross-sections, i.e. open, closed and closed-open, can be considered in a unified way. The application of FEM to the solution of buckling problems of thin-walled structures has a wide survey in the literature. The method became a routine approach in many construction industry applications. However, the number of degrees of freedom is very large, the analysis is expensive and therefore this approach cannot be used for built up structures nor for structures subjected to multiple load combinations. Hence, beam elements are often used for the analysis of global stability, whereas shell elements are used for the analysis of local stability effects limited only to small parts of a structure. Unfortunately, in these mutually independent approaches the interaction of local and global instability taking the form of multi-mode buckling cannot be properly studied. It is a serious disadvantage, because the structures subject to multi-mode buckling are very sensitive to imperfections what is important in design.

The shell theory can be implemented numerically, among others, by means of the Finite Strip Method (FSM). The FSM has remarkable advantages relative to the FEM in the number of equations required to obtain a solution of a given accuracy and a variety of cross-sectional shapes can be treated within the same computational scheme. So the box columns, I-columns, channels, tee sections, angles, stiffened plates of various shape and position of stiffeners, can be studied. This method proved to be very useful and reliable in the stability analysis and optimal design [18], where numerical efficiency plays an important role.

Theoretical background of FSM and its practical application to plates and shells was presented by CHEUNG [3, 4]. At this stage, only simple harmonic and polynomial approximations were used in the longitudinal and transverse directions, respectively. Extension to physically nonlinear problems was presented in [5] and [6]. A general version of FSM applicable to geometrically nonlinear problems has been presented in [7], whereas in [8] and [9] the interactive local and global buckling problems were formulated and solved, proving once again the efficiency of FSM. Further extension of FSM to postbuckling was discussed in [10] and [11]. Implementation of spline approximation to FSM ([12, 13]) allowed to overcome some former limitations of the method to structures of regular geometry and boundary conditions. The application of FSM to structural optimisation was demonstrated for prismatic structures in [14], for arbitrary platform structure in [15] and for cold-rolled beams in [18]. In the field of structural sensitivity analysis, where numerical efficiency is also vital, FSM was used in [16] and [17].

In the present paper the attention will be focused on cold-rolled sections. The stability analysis will be carried out for a wide range of lengths of columns, so that global and local buckling will appear. The numerical examples will be solved using different methods and therefore the precision and computational effectiveness of different approaches will be compared. The general purpose FEM code ABAQUS [21] and the FSM program developed in [10] are used.

2. Problem formulation

The buckling loads are calculated by solving the eigenvalue problem which can be written in the matrix form as

$$(\mathbf{K}^{\mathrm{O}} + \lambda \mathbf{K}^{\mathrm{G}} \mathbf{U}) = 0, \tag{2.1}$$

where $\mathbf{K}$ denotes the linear elastic stiffness matrix, $\mathbf{K}^{\mathrm{G}}$ is the initial stress (geometric) matrix (being a linear function of a base stress state S_{ij}), λ is the load multiplier and eigenvectors $\mathbf{U}$ represent the buckling mode shapes. In Eq. (2.1) we assumed the proportional loading and linearization of the pre-buckling state. The critical buckling loads are $\lambda_i^* \mathbf{P}$, where $\mathbf{P}$ is the reference load (the base state). Usually the lowest value of λ_i is of designer's interest.

Simultaneous calculation of both global and local buckling stress using FEM requires very fine mesh. The smallest element must be equal at least to half-wavelength of the expected buckling mode. On the contrary, in FSM the calculation of the buckling stress for both local and global modes is very efficient due to the application of trigonometric series as shape functions in longitudinal direction. The orthogonality of harmonic functions results in decoupling of Eq. (2.1) for each harmonic term of the expansion. Therefore in FSM the Eq. (2.1) can be written in the form

$$(\mathbf{K}_m^{\mathrm{O}} + \mathbf{K}_m^{\mathrm{G}}(S_{ij}) \mathbf{U}_m) = 0, \tag{2.2}$$

where m denotes the consecutive number of a harmonic term. The lowest critical load factor λ_{cr} for a given length of a bar is calculated by changing the number of harmonic terms. The assumed displacement interpolation functions have the form

$$u = f_{1m}(\eta) g_{1m}(\xi), \qquad v = f_{2m}(\eta) g_{2m}(\xi), \qquad w = f_{3m}(\eta) g_{3m}(\xi), \tag{2.3}$$

where $\eta = \dfrac{y}{b}$ and $\xi = \dfrac{x}{l}$ (Fig. 1).

In Eq. (2.3) summation with respect to $m = 1, 2..., M$ is carried out. Usually, the satisfactory results are obtained when the number of harmonics $M = 15$. In the examples presented in Sec. 3 we assumed $M = 30$ to be sure that all local effects are obtained. We use the classical approximation functions, namely the 3-rd order polynomial functions f_{im} in transverse direction, and harmonics in longitudinal direction

$$g_{1m} = \cos(m\pi\xi), \qquad g_{2m} = \sin(m\pi\xi), \qquad g_{3m} = \sin(m\pi\xi). \tag{2.4}$$

Functions g_{1m}, g_{2m} and g_{3m} satisfy the boundary conditions of a strip produced by diaphragms.

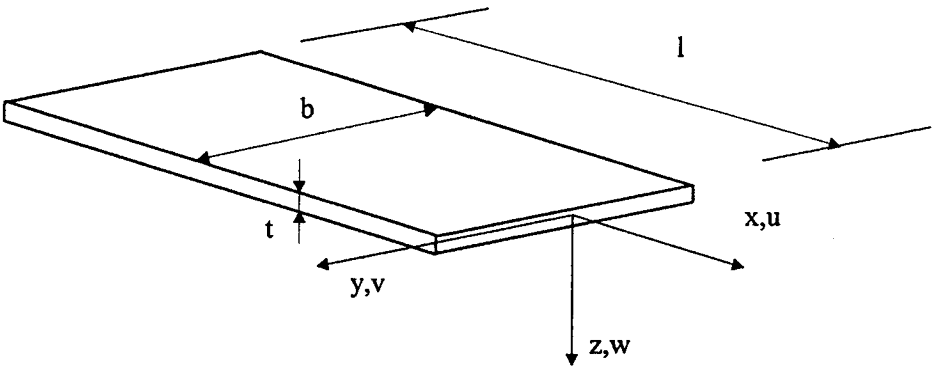

FIG. 1. Finite strip element.

3. Numerical examples

Consider a simply supported column loaded by a concentrated tip load. The problem of global and local buckling will be analysed for variable lengths of columns. For this simple structure the critical stress for global buckling, allowing for combined flexural-torsional modes, can be evaluated analytically. Therefore, using these examples, coupling of local and global buckling and the range of applicability of Vlasov's theory to thin-walled cold-rolled columns can be studied, and effectiveness of FSM versus FEM can be discussed. Two types of cross-sections, shown in Fig. 5, are considered.

Note that the open cross-section (Fig. 2) has different widths of flanges, therefore there is no symmetry and hence the governing equations for Vlasov buckling are mutually coupled. This column buckles in combined flexural-torsional modes. The second type of cross-section has a closed-open form and is composed of two sections shown in Fig. 2. It is a symmetric section and the equations are decoupled, hence purely bending mode can appear. The elastic behaviour of the material was assumed with Young's modulus $E = 210$ GPa. For small slenderness ratio and particularly for the second and third buckling modes, the critical stress exceeds the yield limit. In the Figs. 3 and 4 this is illustrated by two horizontal lines corresponding to the yield stresses for two types of steel.

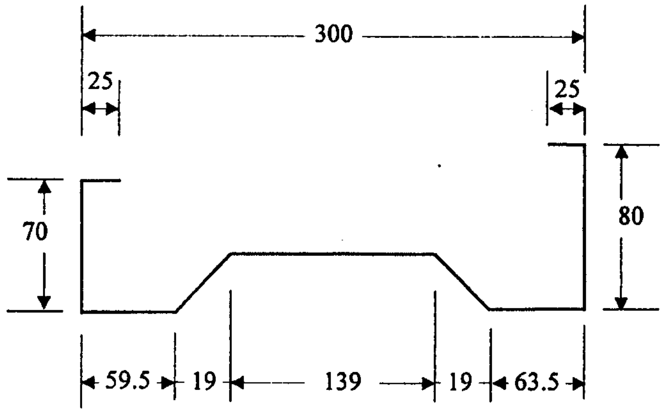

FIG. 2. Sigma cross-section.

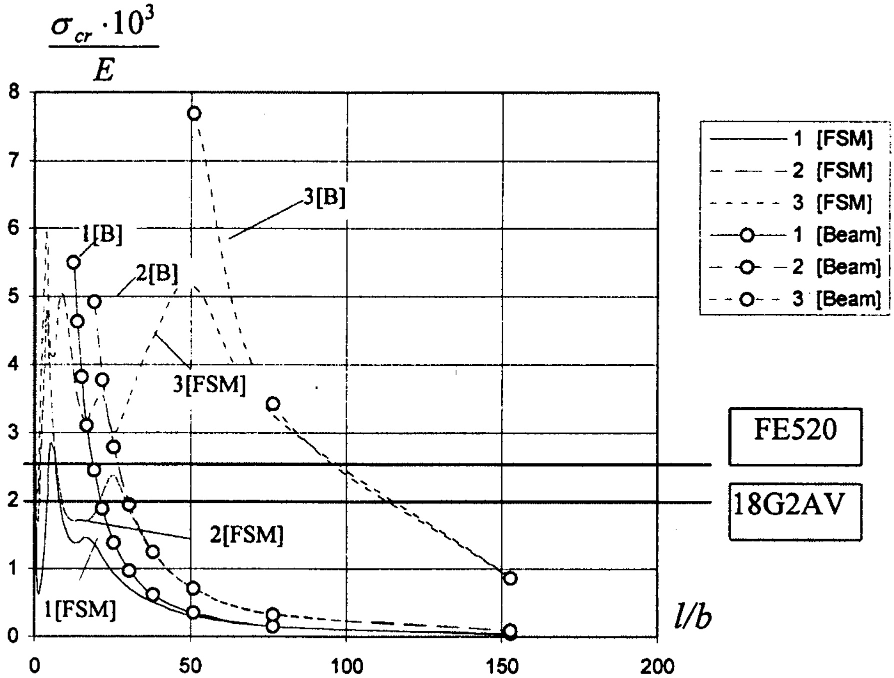

FIG. 3. Buckling stresses calculated by FSM and the Vlasov beam theory for open cross-section: Σ profile.

The buckling stress was computed using different methods. The global stability problem for columns with open cross-sections was solved using the Vlasov theory of thin-walled beams. The buckling stresses were computed employing FEM with Vlasov-type element that had 7 degrees of freedom at each node (the seventh DoF is the warping amplitude). These results were in perfect agreement with analytical results for the whole range of column lengths, therefore in the next figures and discussions, these results will be simply referred to as results of the beam theory. Next, the thin shell theory was used, employing both the FEM and FSM.

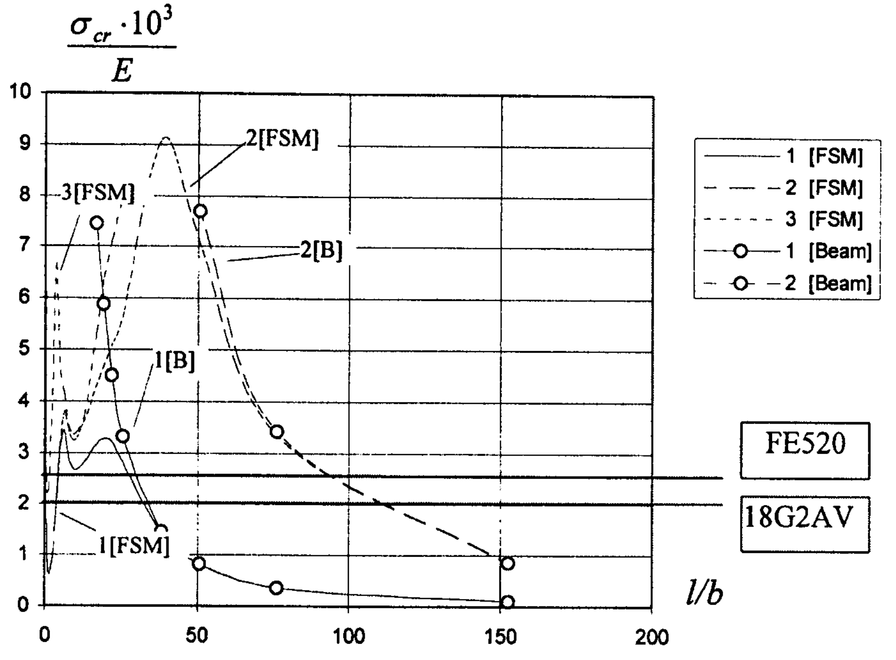

FIG. 4. Buckling stresses calculated by FMS and the Vlasov beam theory for closed-open cross-section: 2 × Σ profile.

In the FEM approach the code ABAQUS was used with 4-node doubly curved general-purpose shell element. In the FSM approach the program [10] was used, with shape functions of (2.3) and (2.4) form.

The non-dimensional critical stress for variable non-dimensional length l of columns is plotted in Figs. 3 and 4 for open and closed-open sections, respectively. Width of flange b was used in the non-dimensional abscissa, because it plays an essential role in the local buckling. Note that for sufficiently high slenderness ratio we arrive at the same results using the Vlasov beam theory and the shell theory implemented in FEM and FSM. In this slenderness range only global buckling appears.

The lower limit of slenderness for the region of validity of the beam theory depends on the mode of buckling. Below this limit the divergence of plots representing the beam theory and shell theory is observed, what is the result of interaction of local and global buckling. Hence, for small slenderness ratio, only the shell theory approach provides a correct $\sigma_{\rm cr}$. Similar stability response is observed in the case of columns with closed-open cross-section shown in Fig. 4. In this case, as mentioned above, the first two eigenvalues in the beam approach are associated with bending modes.

The characteristic features of local stability response of thin-walled structures are displayed in Figs. 5 and 6, where the close-ups of the regions of low slenderness ratio from Figs. 3 and 4 are shown. We observe local minima and maxima associated with the respective wavelengths. The typical feature is that the global minima of all three eigenvalues appear in the vicinity of $l/b = 1$.

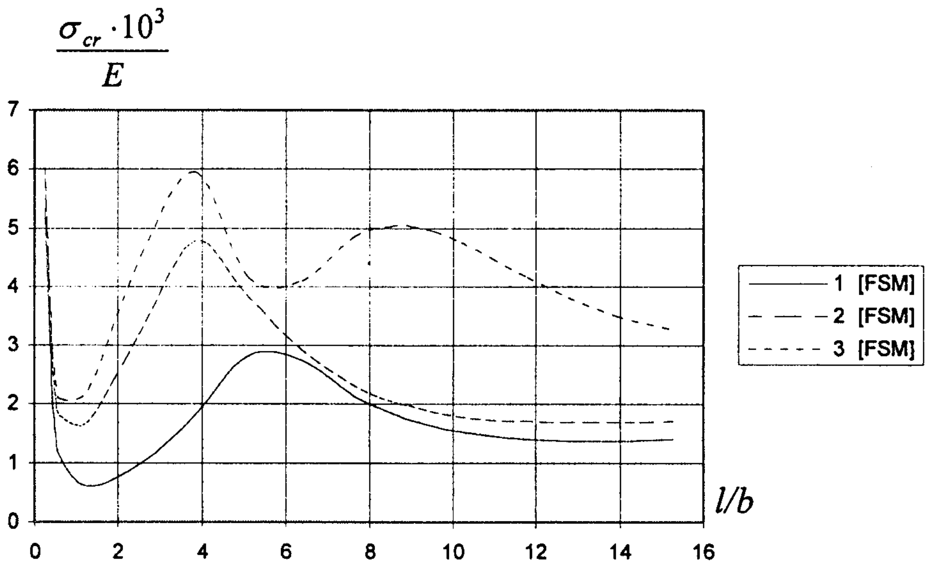

FIG. 5. Local buckling stresses calculated by FSM for open cross-section: Σ profile.

The interaction of local and global stability and its influence on the resulting critical stress is illustrated in Fig. 7. This interaction plays an important role in structural design, because it results not only in the decrease of $\sigma_{\rm cr}$, but also in sensitivity to imperfections. Note the excellent agreement between the results obtained by FSM and FEM using shell elements.

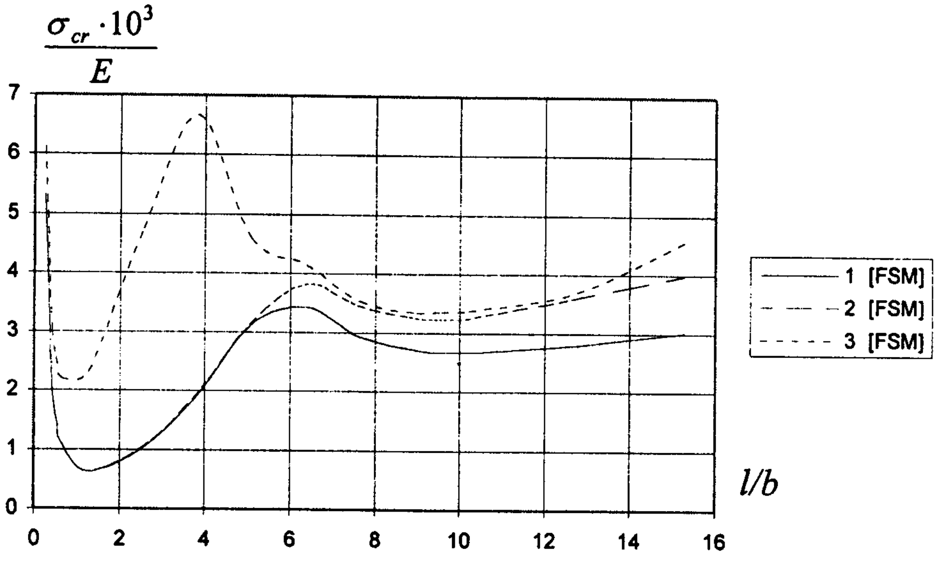

FIG. 6. Local buckling stresses calculated by FSM for closed-open cross-section: $2 \times \Sigma$ profile.

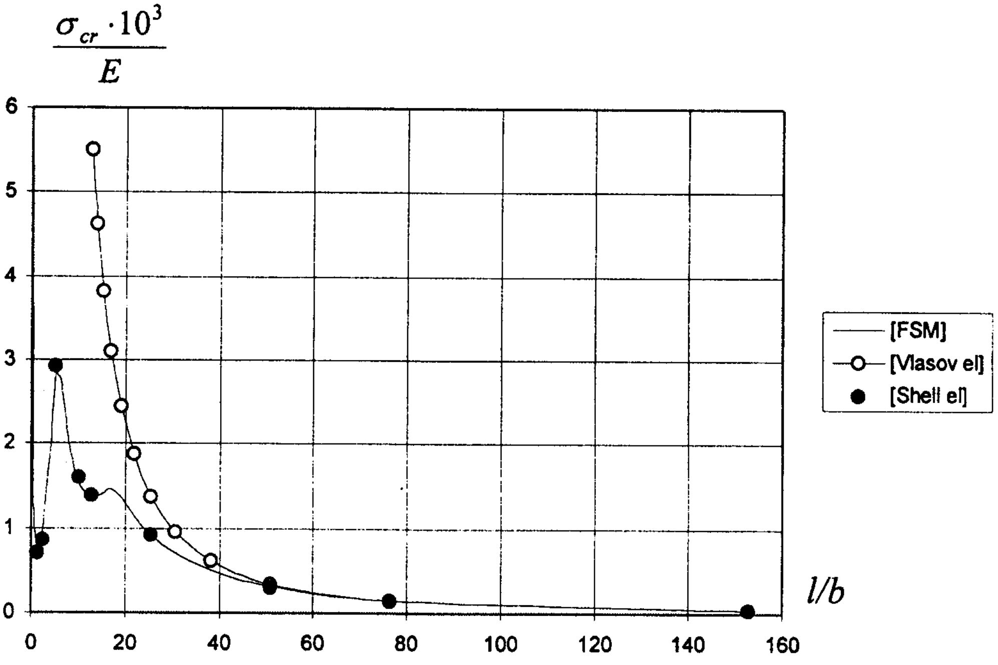

FIG. 7. Buckling stresses calculated by FEM for open cross-section: Σ profile.

Interesting information concerning variation of the shape of contour and form of warping was provided by FSM. The modes of variation of the contour observed in the examples under discussion are shown in Fig. 8. Note that the deformation of the contour is not allowed for in the Vlasov beam theory.

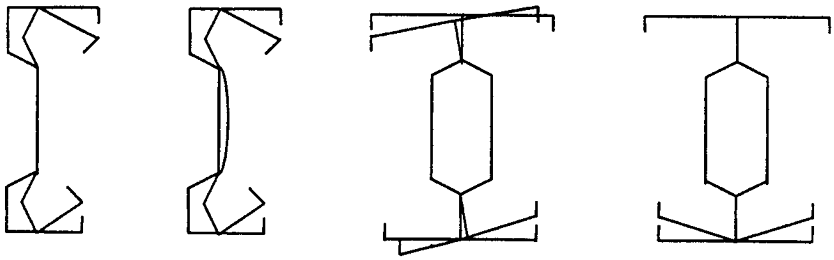

FIG. 8. Modes of deformation of the contour.

The form of warping is presented in Fig. 9. The plots presented in Figs. 8 and 9 contain only qualitative data because they were computed for normalized eigenmodes. The warping of open cross-section (Fig. 9a) is in good agreement with the sectorial coordinate of the Vlasov theory. Small warping of the closed portion is observed. The forms of warping presented in Fig. 8 provide information on the distribution of normal stress in torsional modes of buckling.

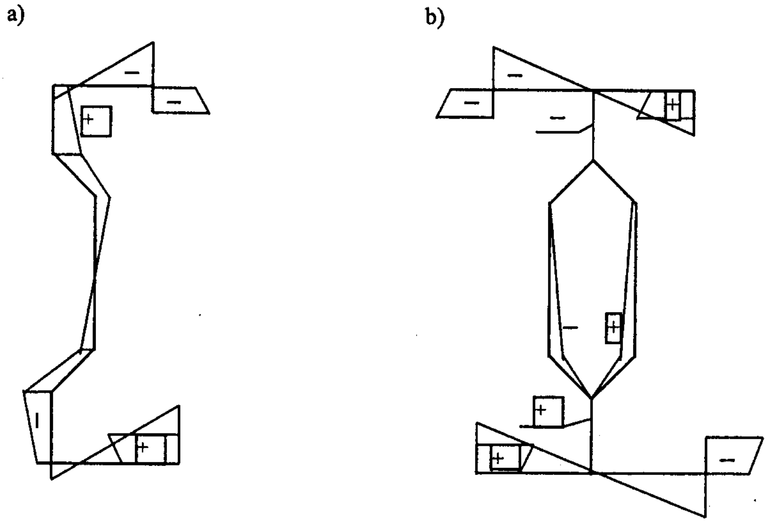

FIG. 9. Distribution of warping.

The numerical examples demonstrated excellent efficiency of the FSM. The results were in very good agreement with the results obtained by FEM but at a much lower computational cost.

4. Concluding remarks

Global and local stability of columns made of steel thin-walled cold-rolled Σ profiles was studied for a wide range of lengths. Open and closed-open cross-sections were considered. Buckling stresses were computed using different methods. Vlasov's beam elements incorporated in FEM were used in a study of global buckling of columns of open cross-section. Shell elements were used in both FEM and in FSM in a study of global and local buckling of columns of open and closed-open cross-sections. Numerical examples demonstrated that the Vlasov beam theory precisely estimated the critical load and the distribution of warping for slender Σ columns of open cross-section. All examples proved that there was excellent agreement between the results obtained by FEM (shell elements) and FSM for both the open and closed-open sections in a wide range of slenderness ratio, though FSM provided the results at a much lower computer time.

Different modes of deformation of cross-sectional contour were observed depending on the mode of buckling. Multimodal buckling of short columns produced the local maxima and minima of the critical stress $\sigma^{cr}(\lambda)$. Warping of the core (closed part) of $2 \times \Sigma$ is negligible.

Acknowledgment

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References

1. S. P. Timoshenko, J. M. Gere, *Theory of elastic stability*, McGraw-Hill Book Co., Inc., New York 1961.
2. V. Z. Vlasov, *Thin-walled elastic rods* [in Russian], Izd. Akad. Nauk SSSR, Moscow 1963.
3. M. S. Cheung, Y. K. Cheung, *Static and dynamic behaviour of rectangular plates using higher order finite strips*, Build. Sci., **7**, 151–158, 1972.
4. Y. K. Cheung, *Finite strip method in structural analysis*, Pergamon Press, Oxford 1976.
5. S. C. W. Lau, G. J. Hancock, *Buckling of thin-walled structures by a spline finite strip method*, Thin-Walled Structures, **4**, 4, 269–294, 1986.
6. M. S. Cheung, Li. Wenchang, *Finite strip method for materially nonlinear analysis of reinforced concrete slabs*, Computers and Structures, **35**, 603–607, 1990.
7. J. T. Gierliński, T. R. Graves-Smith, *The geometric nonlinear analysis of thin-walled structures by finite strips*, Thin-Walled Structures, **2**, 27–35, 1984.
8. S. Sridharan, *Doubly symmetric interactive buckling of plate structures*, Int. J. Solids and Structures, **19**, 7, 625–641, 1983.

9. S. SRIDHARAN, ALI H. ASHRAF, *An improved interactive buckling analysis of thin-walled columns having doubly symmetric sections*, Int. J. Solids and Structures, **22**, 429–434, 1986.

10. W. KĄKOL, *Non-linear stability analysis of rectangular stiffened plates* [in Polish], Ph. D. Thesis, Techn. Univ. Poznań 1987.

11. W. KĄKOL, *Stability analysis of stiffened plates by finite strips*, Thin-Walled Structures, **10**, 277–297, 1990.

12. S. C. W. LAU, G. J. HANCOCK, *Inelastic buckling analyses of beams, columns and plates using the spline finite strip method*, Thin-Walled Structures, **7**, 213–238, 1989.

13. A. H. SHEIKH, M. MUKHOPADHYAY, *Analysis of stiffened plate with arbitrary platform by general spline finite strip method*, Computers and Structures, **42**, 1, 53–67, 1992.

14. E. HINTON, N. V. R. RAO, *Analysis and shape optimization of variable thickness prismatic folded plates and curved shells, Part 1: Finite strip formulation*, Thin-Walled Structures, **17**, 2, 1993.

15. E. HINTON, N. V. R. RAO, *Analysis and shape optimization of variable thickness prismatic folded plates and curved shells, Part 2: Shape optimization*, Thin-Walled Structures, **17**, 3, 1993.

16. A. GARSTECKI, W. KĄKOL, *Structural sensitivity analysis in eigenvalue problems using Finite Strip Method*, ASME, 393–398, 1994.

17. A. GARSTECKI, W. KĄKOL, *Accuracy of sensitivity analysis in eigenvalue problems using FEM*, ZAMM, **75**, 385–486, 1995.

18. G. THIERAUF, *Thin-walled structures and related optimization problems*, Thin-Walled Structures, **9**, 241–256, 1990.

19. A. PROKIC, *New warping function for thin-walled beams, I: Theory*, J. Struct. Engng., ASCE, **122**, 12, 1437–1442, 1996.

20. A. PROKIC, *New warping function for thin-walled beams, II: Finite Element Method and applications*, J. Struct. Engng., ASCE, **122**, 12, 1443–1452, 1996.

21. ABAQUS-Standard, Hibbitt, Karlsson & Sorensen, Inc, 1995.

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