ENGINEERING TRANSACTIONS • Engng. Trans. • 49, 1, 47–63, 2001
Polish Academy of Sciences • Institute of Fundamental Technological Research
10.24423/engtrans.573.2001

OPTIMISATION OF FIVE-LINK SUSPENSION MECHANISM OF A VEHICLE SOLID TYRES [1]

A. GRZYB (*), M. KOWALSKI (**)

(*) CRACOW UNIVERSITY OF TECHNOLOGY, INSTITUTE OF RAIL VEHICLES
al. J. Pawła II 37, 31–864 Kraków, Poland

(**) CRACOW UNIVERSITY OF TECHNOLOGY, INSTITUTE OF AUTOMOBILES AND INTERNAL COMBUSTION ENGINES
al. J. Pawła II 37, 31–864 Kraków, Poland

The paper presents a method of constructing the optimum kinematic characteristics on the basis of five-link suspension. In order to solve this issue, the perturbation method of analysing nonlinear algebraic equations was used. After assuming the decision variables, defining objective functions and appropriate limitations, a sampling method of optimisation was used. Calculations enabled us to obtain a correction of characteristics of wheel steering and camber angles in comparison with initial characteristics. The obtained results indicate that it is possible to apply the above method to form the optimum characteristics of multi-link suspension mechanisms.

NOTATIONS

$A_j, j = 1, 2, ..., 5$	fixed points,
$B_k, k = 1, 2, ..., 7$	moving points,
$B_{k0}, k = 1, 2, ..., 7$	moving points in construction position,
$\mathbf{r}_{Aj}, \mathbf{r}_{Bk}, \mathbf{r}_{Bk0}$, $j = 1, ..., 5, k = 1, ..., 7$	vectors connecting the origin of co-ordinates $O(0,0,0)$ with an appropriate point of the mechanism,
$\mathbf{q}_k = \mathbf{r}_{Bk} - \mathbf{r}_{Bk0}$	displacement vectors; $\mathbf{q} = [q_{kx}, q_{ky}, q_{kz}]$,

[1]Paper financed by the Polish State Committee for Scientific Research (KBN) within the research grant PB 9T12C 008 14, realised in the years 1998–2000.

$\mathbf{r}_{AjBj}, \mathbf{r}_{BkBl}, \mathbf{r}_{Bk0Bl0}$, $j = 1, 2, ..., 5, k, l = 1, 2, ..., 7$	vectors connecting pairs of the mechanism points (points of the names that appear in the vector subscripts),
ε	perturbation parameter,
δ	wheel steering angle,
γ	wheel camber angle,
a_x, a_y, a_z	length of the ellipsoid half-shafts inside which new positions of fixed points are looked for,
w	coefficient defining the importance of selected component of objective function

1. Introduction

The requirements concerning stability and steerability, travel comfort, longitudinal and transverse camber of a vehicle, should be taken into account in the process of constructing solid tyres suspensions. It is also important that the construction parameters of optimally constructed suspensions are not changed during normal usage of the vehicle, and that they satisfy the imposed requirements. Multi-link suspensions are characteristic in comparison with other suspensions because the wheel load is transferred to the vehicle body through more than just three joints. Therefore, the loads of ball-and-socket joints and metal-rubber joints used to connect the link to cantilever of the wheel and the vehicle-body are smaller. Thanks to that, kinematic characteristics (mainly camber and steering angles) remain almost unchanged.

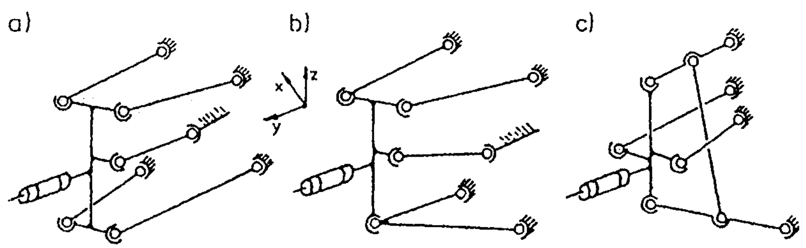


Fig. 1. Multi-link suspension mechanisms: a) five-link Mercedes, b) four-link Nissan, c) four-link with an additional joint BMW.

In mass-produced vehicles, multi-link suspensions were first used in the 1980's by Mercedes in the models of the vehicle-body W201 (190) and W124 (five link suspensions). Next, Nissan used four-link suspension in the models 300ZX and 240SX, BMW – four-link suspension in the model 850 and Volvo (model 760) – four link suspension with an additional joint between links (Integralachse) [17].

2. Aim and scope of work

Multi-criterion optimisation may be useful to construct solid tyres suspensions in view of the requirements presented in the Introduction. In order to solve certain problems connected with this idea, a dynamic model of complicated structure should be used in computer simulation of a vehicle motion. The consequent optimisation tasks can be solved with the help of genetic algorithms [20].

The above mentioned construction process can be divided into stages. In the paper by J. Knapczyk [9], a division into 5 stages can be found:

- structural synthesis during which a selection of number and type of links as well as moving fasteners is made,
- dimensional synthesis, the aim of which is to determine the range of values of dimensional parameters of a mechanism of fixed qualities, preserving the required limitations,
- elastokinematic and dynamic analysis which includes determination of displacements and loads inside the joints and determination of elastokinematic characteristics,
- modelling and computer simulation of the mechanism motion for kinematic and forced excitation, model parameters estimation, parametric sensitivity analysis, optimisation of construction and other parameters,
- strength analysis, stiffness, deformations, and then making technical documentation.

The problem of suspension optimisation with respect to its kinematic characteristics is solved first. At the next stages, the requirements concerning dynamics are taken into account (longitudinal and transverse camber, travel comfort).

J. M. Jimenez [7] proposed a synthesis method of five-link suspension mechanism, which takes advantage of the optimisation methods. Lengths of links and dimensions determining the cantilever position are assumed to be the design variables. Insignificant changes of wheel camber and insignificant side displacements are also required. However, the defect of this method is arriving at solutions for which collision of links might occur. Herein is presented a method which does not allow any collision thanks to the assumption that an appropriate initial selection of fixed and moving points (joints) has been made. Assuming appropriate regions (e.g. spheres or ellipsoids) in which fixed points (points of suspension fastening) are supposed to be placed in such a way as to form disconnected sets, it is possible to avoid collision of the links. In the case when the regions are not disconnected, it is necessary to apply additional conditions that would eliminate collisions.

Papers [1, 13] contributed to the issue of synthesis, kinematics and dynamics of multi-link suspension mechanisms. Mechanics of other mechanisms used in power transmission systems was approached in many papers, among which the following are worth mentioning: [10 – 12, 14 – 16, 18]. Optimisation of mechanisms is described in papers [2, 4, 5].

The primary objective of this paper is to present a method of analysis and optimisation of multi-link wheel suspensions with respect to the assumed kinematic characteristics: steering and camber wheel angles.

3. Method of analysis

The aim of kinematic analysis is to solve the system of equations of constraints. They are mainly nonlinear equations. The type of non-linearity of the equations and their number depend on the mechanism structure. In order to solve such a system of equations, the method of elimination or the numerical iterative methods (e.g. Newton-Raphson's or gradient methods) can be used.

The method of elimination involves reducing a system of equations to one multinomial equation with one unknown. This method leads to determining all the solutions and, additionally, it allows to avoid the singularities. However, the procedure of elimination is very complicated and the procedure of determining roots of the multinomial is sensitive to the calculation accuracy.

Numerical methods can be divided into dependent on or independent of the standard concerning the initial point distance. The Newton-Raphson method is one of the dependent ones while the perturbation method [3] is independent.

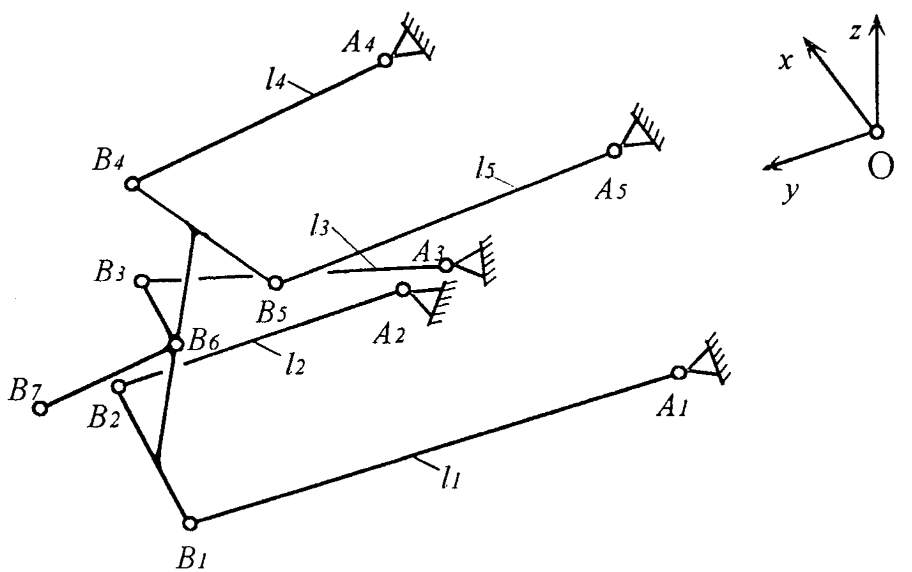


Fig. 2. Five-link suspension mechanism.

If the initial point in the Newton-Raphson or gradient methods is not placed near the final solution, the iterative process can be slow and its convergence – questionable. What is more, the iterative method determines only one (the closest to the initial one) out of many possible solutions. This solution does not make it easier to look for other ones. The procedure, even repeatedly applied, does not ensure that all the solutions are determined [9].

Five-link wheel suspension is presented in Fig. 2.

For the sake of the analysis, the suspension was replaced with a spatial mechanism without susceptible constraints. The motion of the assumed mechanism is limited by geometric constraints which are described by the following system of fourteen, nonlinear algebraic equations:

$$
\begin{aligned}
\mathbf{r}_{A1B1} \circ \mathbf{r}_{A1B1} - \mathbf{r}_{A1B10} \circ \mathbf{r}_{A1B10} &= 0,\\
\mathbf{r}_{A2B2} \circ \mathbf{r}_{A2B2} - \mathbf{r}_{A2B20} \circ \mathbf{r}_{A2B20} &= 0,\\
\mathbf{r}_{A3B3} \circ \mathbf{r}_{A3B3} - \mathbf{r}_{A3B30} \circ \mathbf{r}_{A3B30} &= 0,\\
\mathbf{r}_{A4B4} \circ \mathbf{r}_{A4B4} - \mathbf{r}_{A4B40} \circ \mathbf{r}_{A4B40} &= 0,\\
\mathbf{r}_{A5B5} \circ \mathbf{r}_{A5B5} - \mathbf{r}_{A5B50} \circ \mathbf{r}_{A5B50} &= 0,\\
\mathbf{r}_{B1B2} \circ \mathbf{r}_{B1B2} - \mathbf{r}_{B10B20} \circ \mathbf{r}_{B10B20} &= 0,\\
\mathbf{r}_{B1B3} \circ \mathbf{r}_{B1B3} - \mathbf{r}_{B10B30} \circ \mathbf{r}_{B10B30} &= 0,\\
\mathbf{r}_{B1B4} \circ \mathbf{r}_{B1B4} - \mathbf{r}_{B10B40} \circ \mathbf{r}_{B10B40} &= 0,\\
\mathbf{r}_{B1B5} \circ \mathbf{r}_{B1B5} - \mathbf{r}_{B10B50} \circ \mathbf{r}_{B10B50} &= 0,\\
\mathbf{r}_{B2B3} \circ \mathbf{r}_{B2B3} - \mathbf{r}_{B20B30} \circ \mathbf{r}_{B20B30} &= 0,\\
\mathbf{r}_{B2B4} \circ \mathbf{r}_{B2B4} - \mathbf{r}_{B20B40} \circ \mathbf{r}_{B20B40} &= 0,\\
\mathbf{r}_{B2B5} \circ \mathbf{r}_{B2B5} - \mathbf{r}_{B20B50} \circ \mathbf{r}_{B20B50} &= 0,\\
\mathbf{r}_{B3B4} \circ \mathbf{r}_{B3B4} - \mathbf{r}_{B30B40} \circ \mathbf{r}_{B30B40} &= 0,\\
\mathbf{r}_{B3B5} \circ \mathbf{r}_{B3B5} - \mathbf{r}_{B30B50} \circ \mathbf{r}_{B30B50} &= 0.
\end{aligned}
\tag{3.1}
$$

Examples of Eqs. (3.1) in the expanded form are presented in Appendix 1. In the above equations the symbol '$\circ$' denotes scalar vector multiplication. They express the (equated to zero) differences between distances of the points of the mechanism in the camber position and the corresponding distances of these points in the original position.

Equations (3.1) describe the mechanism configuration and are used to determine the (below defined) displacements $\mathbf{q}_k$ of points B_k, $k = 1, 2, ..., 5$, (for the sake of the analysis, displacement $\mathbf{q}_{1z} = s$ was assumed to be the given parameter which was changed in a particular range).

The following notations were assumed in these equations:

$A_j, j = 1, 2, ..., 5$ – fixed points (with respect to the system of co-ordinates connected with the vehicle body) which are centres of the joints that connect the suspension mechanism with the vehicle body;

$B_k, k = 1, 2, ..., 7$ – moving points that constitute centres of the joints that connect links (joints) with the wheel cantilever ($k = 1, ..., 5$) and cantilever points placed on the axis of wheel rotation;

$B_{k0}, k = 1, 2, ..., 7$ – moving points in construction position;

$\mathbf{r}_{Aj}, \mathbf{r}_{Bk}, \mathbf{r}_{Bk0}, j = 1, ..., 5, k = 1, ..., 7$ – vectors connecting the origin of co-ordinates $O(0,0,0)$ with the appropriate point of the mechanism;

$\mathbf{q}_k = \mathbf{r}_{Bk} - \mathbf{r}_{Bk0}$ – displacement vectors; $\mathbf{q}_k = [q_{kx}, q_{ky}, q_{kz}]$;

$\mathbf{r}_{AjBj}, \mathbf{r}_{BkBl}, \mathbf{r}_{Bk0Bl0}, j = 1, 2, ..., 5, k, l = 1, 2, ..., 7$ – vectors connecting pairs of the mechanism points (points of the names that appear in the vector subscripts).

Displacements $\mathbf{q}_6$, $\mathbf{q}_7$ of appropriate points B_6, B_7 constitute the solutions of systems of additional, linear equations. In previous papers, e.g. [6], linear equations that are difficult in the numerical analysis, were used for the analysis of these points motion.

In order to determine the position of the points, auxiliary vectors $\mathbf{n}$ and $\mathbf{n}_0$ are created and described by the equations:

$$\begin{aligned} \mathbf{n} &= \mathbf{r}_{B1B2} \times \mathbf{r}_{B1B3}, \\ \mathbf{n}_0 &= \mathbf{r}_{B10B20} \times \mathbf{r}_{B10B30}. \end{aligned} \tag{3.2}$$

To determine displacements $\mathbf{q}_6$, the following equations are used:

$$\begin{aligned} \mathbf{r}_{B1B6} \circ \mathbf{r}_{B1B2} - \mathbf{r}_{B10B60} \circ \mathbf{r}_{B10B20} &= 0, \\ \mathbf{r}_{B1B6} \circ \mathbf{r}_{B1B3} - \mathbf{r}_{B10B60} \circ \mathbf{r}_{B10B30} &= 0, \\ \mathbf{r}_{B1B6} \circ \mathbf{n} - \mathbf{r}_{B10B60} \circ \mathbf{n}_0 &= 0. \end{aligned} \tag{3.3}$$

To determine displacements $\mathbf{q}_7$, the following equations are used:

$$\begin{aligned} \mathbf{r}_{B1B7} \circ \mathbf{r}_{B1B2} - \mathbf{r}_{B10B70} \circ \mathbf{r}_{B10B20} &= 0, \\ \mathbf{r}_{B1B7} \circ \mathbf{r}_{B1B3} - \mathbf{r}_{B10B70} \circ \mathbf{r}_{B10B30} &= 0, \\ \mathbf{r}_{B1B7} \circ \mathbf{n} - \mathbf{r}_{B10B70} \circ \mathbf{n}_0 &= 0. \end{aligned} \tag{3.4}$$

Equations (3.2) and (3.3) in the expanded form are presented in Appendix 2.

As before, the symbol '$\circ$' denotes scalar vector multiplication while the symbol '$\times$' – their vector multiplication.

Firstly, the system of 14 nonlinear equations (3.1) is solved. For this purpose, the perturbation method presented in [3] and applied for suspension mechanics in

[6] is used. This method, in comparison with the Newton-Raphson's one, is not so sensitive to the distance of successive initial points which makes it useful for the optimisation methods. Moreover, it enables much faster calculations which is significant with respect to optimisation because of repeated, multiple solving of the changed mechanism.

Then, from systems of Eqs. (3.3) and (3.4), the co-ordinates of points B_6 and B_7 are calculated. Knowing these co-ordinates makes it possible to determine [17] the wheel steering angle from the equation

$$\delta = \operatorname{arctg}\left(\frac{x_{B6} - x_{B7}}{y_{B7} - y_{B6}}\right), \tag{3.5}$$

and the wheel camber angle from the equation

$$\gamma = \operatorname{arctg}\left(\frac{z_{B7} - z_{B6}}{\sqrt{\left[(x_{B7} - x_{B6})^2 + (y_{B7} - y_{B6})^2\right]}}\right). \tag{3.6}$$

In the analysis of additional points, solution of linear algebraic equations is carried out after substituting the value of s parameter (numerically), and not before the substitution (symbolically).

Evaluating the accuracy, we calculate the differences between distances of the mechanism points in extreme positions and the corresponding distances of the points in their original positions.

3.1. Basic assumptions of the perturbation method [3]

In the perturbation method, the left-hand parts of equations of general form:

$$f_j(u_1, u_2, ..., u_n) = 0, \qquad j = 1, 2, ..., n, \tag{3.7}$$

with the unknowns u_j (displacements q_{kx} , q_{ky} and q_{kz} of moving points B_k, $k = 1, 2, ..., 7$), are divided into linear f_{Lj} and nonlinear parts f_{Nj}:

$$f_{Lj}(u_1, u_2, ..., u_n) + f_{Nj}(u_1, u_2, ..., u_n) = 0,$$

where, in our case, the nonlinear parts are of the second degree because of the unknowns u_j. Perturbation parameter ε is introduced in the last system of equations which leads to the auxiliary system of equations:

$$g_j(\varepsilon, u_1, u_2, ..., u_n) = 0, \qquad j = 1, 2, ..., n, \tag{3.8}$$

where:

$$g_j(\varepsilon, u_1, u_2, ..., u_n) = f_{Lj}(u_1, u_2, ..., u_n) + \varepsilon f_{Nj}(u_1, u_2, ..., u_n).$$

For $\varepsilon = 1$ the system of Eq. (3.8) is identical with (3.7), while for $\varepsilon = 0$ the solution (3.8) is simple. Solutions of the system of auxiliary Eq. (3.8) is presented in the form of series with respect to the power ε:

$$u_j(\varepsilon) = \sum_{k=0}^{m} \varepsilon^k u_{jk}, \qquad j = 1, 2, ..., n, \tag{3.9}$$

assuming that for $m \to \infty$ series (3.9) have the radius of convergence greater than 1. After substituting the expression (3.9) to (3.8) we obtain:

$$g_j[\varepsilon, u_1(\varepsilon), u_2(\varepsilon), ..., u_n(\varepsilon)] = 0, \qquad j = 1, 2, ..., n.$$

After an additional assumption that the left-hand parts of the last system of equations can be expanded series with respect to ε powers, they are presented in the series:

$$\sum_{k=0}^{\infty} \varepsilon^k g_{jk} = 0, \qquad j = 1, 2, ..., n, \tag{3.10}$$

where g_{jk} is such a system of expressions that g_{j0} contains only u_{j0} unknowns, g_{j1} contains u_{j0} and u_{j1}, and generally g_{jk} contains u_{j0}, $u_{j1}, ..., u_{jk}$, where $j, k = 1, 2, ..., n$. In order to fulfil condition (3.10) it is enough to demand that the algebraic equations (linear ones in this case) be satisfied, each with n unknowns:

$$g_{jk} = 0, \qquad j, k = 1, 2, ..., n.$$

We obtain solutions of the system of Eq. (3.7) after substituting $\varepsilon = 1$ to expansions (3.9). They are as follows:

$$u_j = \sum_{k=0}^{m} u_{jk}, \qquad j = 1, 2, ..., n.$$

Using the above-presented method for the analysis of the mechanism in the camber position, we determine a new position which is the basis for determining the following position. Then we repeat the procedure and obtain a series of solutions which, after applying the spline functions, result in final solutions $\mathbf{q}_k(s)$ that describe displacements of points B_k, $k = 1, 2, ..., 7$, as functions of the given displacement $q_{1z} = s$.

In the case of the described suspension, the first equation of the system (3.1) reads as follows:

$$(x_{B1}-x_{A1})^2+(y_{B1}-y_{A1})^2+(z_{B1}-z_{A1})^2$$
$$-\Big[(x_{B10}-x_{A1})^2+(y_{B10}-y_{A1})^2+(z_{B10}-z_{A1})^2\Big]$$
$$=(x_{B1}-x_{A1})^2+(y_{B1}-y_{A1})^2+(z_{B1}-z_{A1})^2-l_1^2=0,$$

and after displacing point B_1 by the vector

$$\mathbf{q}_1=[q_{1x},q_{1y},q_{1z}],$$

it will take the form

$$[(x_{B1}+q_{1x})-x_{A1}]^2+[(y_{B1}+q_{1y})-y_{A1}]^2+[(z_{B1}+q_{1z})-z_{A1}]^2$$
$$-\Big[(x_{B10}-x_{A1})^2+(y_{B10}-y_{A1})^2+(z_{B10}-z_{A1})^2\Big]=0.$$

After transformation we obtain

$$\Big[(x_{B1}-x_{A1})^2+2(x_{B1}-x_{A1})q_{1x}+q_{1x}^2\Big]$$
$$+\Big[(y_{B1}-y_{A1})^2+2(y_{B1}-y_{A1})q_{1y}+q_{1y}^2\Big]$$
$$+\Big[(z_{B1}-z_{A1})^2+2(z_{B1}-z_{A1})q_{1z}+q_{1z}^2\Big]$$
$$-\big[(x_{B10}-z_{A1})^2+(y_{B10}-y_{A1})^2+(z_{B10}-z_{A1})^2\big]=0,$$

thus

$$(x_{B1}-x_{A1})^2+(y_{B1}-y_{A1})^2+(z_{B1}-z_{A1})^2$$
$$-\Big[(x_{B10}-x_{A1})^2+(y_{B10}-y_{A1})^2+(z_{B10}-z_{A1})^2\Big]$$
$$+2(x_{B1}-x_{A1})q_{1x}+2(y_{B1}-y_{A1})q_{1y}+2(z_{B1}-z_{A1})q_{1z}+q_{1x}^2$$
$$+q_{1y}^2+q_{1z}^2=0.$$

After following transformations we obtain an equation in the form that enables us to use the perturbation method:

$$2(x_{B1}-x_{A1})q_{1x}+2(y_{B1}-y_{A1})q_{1y}+2(z_{B1}-z_{A1})q_{1z}$$
$$+q_{1x}^2+q_{1y}^2+q_{1z}^2=0.$$

Once the perturbation parameter ε has been introduced and the equation divided into linear and nonlinear parts, we obtain:

$$2(x_{B1}-x_{A1})q_{1x}+2(y_{B1}-y_{A1})q_{1y}+2(z_{B1}-z_{A1})q_{1z}$$
$$+\varepsilon(q_{1x}^2+q_{1y}^2+q_{1z}^2)=0.$$

The remaining equations of the system (3.1) are transformed in a similar way.

4. Kinematics and optimisation of the mechanism

Numerical simulation of motion of the mechanism presented as a dynamic model makes it possible to examine the influence of suspension kinematics on stability and steerability. In this way, the kinematic characteristics of suspensions are determined. Knowing these characteristics and using a steering suspension [18] trajectories of points B_i, $i = 1, ..., 5$ can be determined. We can determine the initial positions of the suspension fixed points A_i and lengths of joints l_i on the condition that we know the trajectories of points B_i.

Optimisation involves determination of the solutions in the original position and then selection of new positions.

In the mechanism optimisation, co-ordinates of vectors $\mathbf{p}_j = [p_{jx}, p_{jy}, p_{jz}]$, $j = 1, ..., 5$, the origins of which are points A_j, and the ends – points A_{jz}, are the decision variables. The first stage of optimisation is to determine solutions of the mechanism in its original position. Then new positions are selected. There are certain limitations assumed in the optimisation and they are described by the following inequalities:

$$\frac{p_{jx}^2}{a_{jx}^2}+\frac{p_{jy}^2}{a_{jy}^2}+\frac{p_{jz}^2}{a_{jz}^2}-1\leq 0, \qquad j=1,...,5. \tag{4.1}$$

During the optimisation process points A_{jz} are determined. They constitute new, changed positions of points A_j of the investigated mechanism. For each j an appropriate inequality means that point A_{jz} is supposed to be found inside an ellipsoid of centre A_j and half-shafts a_{jx}, a_{jy}, a_{jz}, or in a particular case when $a_{jx} = a_{jy} = a_{jz} = R_j$, inside a sphere of radius R_j. The assumed limitations were used for applying spherical co-ordinates to transform every three random numbers of uniform distribution in the range (0, 1) to an appropriate point in the Cartesian space. In order to simplify the optimisation, the same R_j values were assumed for all the points and they were equal to $R_j = R = 15$ mm.

One of the criteria of optimisation is the sum of absolute values of ordinate differences (value of the wheel camber angle) of the demanded (ideal) and calculated (determined numerically) characteristics of the investigated mechanism, for the same abscissae $s = s_j$ (what means for the same mechanism, the deflection described by the variable $s \in \langle -100, 100 \rangle$). A sum concerning the characteristics of the wheel steering angle is assumed to be the second criterion.

Initially, we also used the integral criterion of curve matching but it turned out to be too time-consuming. However, it was possible to replace it with the above-mentioned sum of the point distances.

The objective function is assumed to be the weighted mean value of both criteria

$$fc = \sum_{j=1}^{ls} \left(w|\gamma(s_j) - \gamma_0(s_j)| + (1-w)|\delta(s_j) - \delta_0(s_j)| \right), \quad 0 < w < 1. \tag{4.2}$$

Since the obtained characteristics $\gamma(s)$ and $\delta(s)$ depend on the decision variables, the objective function is also dependent on them.

5. Numerical example

It is assumed in the numerical example that the aim is to tend to the following kinematic characteristics of suspension:

1) Wheel steering angle should be constant and equal to $\delta(s) = 0$.

2) Wheel camber angle should be defined by linear function $\gamma(s) = 0.0189 + 0.000529s$.

Weight coefficient in the objective function $w = 0.5$.

The theory of vehicle suspension construction proves that it is mostly advantageous if the wheel steering angle $\delta(s)$ does not change with the springing motion while the wheel camber angle performs linear run $\gamma(s)$ which makes it possible to avoid unwanted additional forces resulting from the curvilinear motion.

Co-ordinates of the initial positions of mechanism points have values that are close to those presented in [8]. The co-ordinates expressed in mm are as follows:

$$A_1 = (-105.0, 210.0, 225.0); \quad A_2 = (319.0, 500.0, 262.0);$$
$$A_3 = (214.0, 415.0, 297.0); \quad A_4 = (199.0, 448.0, 400.0);$$

$$A_5 = (0.0, 340.0, 410.0);$$

$$B_1 = (-43.0, 662.0, 208.0); \quad B_2 = (44.0, 632.0, 163.0);$$
$$B_3 = (141.0, 666.0, 269.0); \quad B_4 = (78.0, 662.0, 397.0);$$
$$B_5 = (-5.0, 666.0, 427.0); \quad B_6 = (-0.4, 605.0, 300.5);$$

$$B_7 = (0.0, 705.0, 302.0).$$

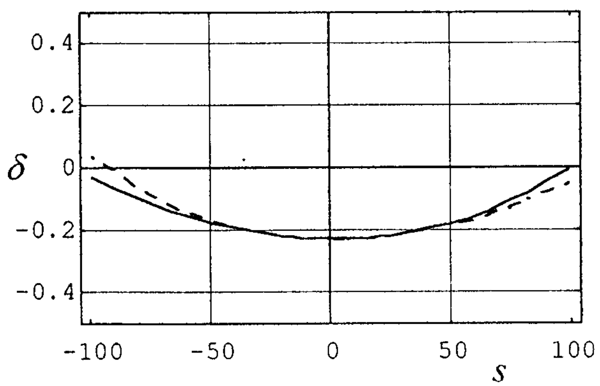
Since the sampling optimisation method was used in our case, selections were conducted in such a way as to obtain almost uniform distribution of the selected points inside the spheres of centres A_j, $j = 1, ..., 5$. After each selection of positions of the mechanism points, a solution of the system of 14 nonlinear, algebraic equations and of the additional equations was determined. Consequently, characteristics of the changed mechanism could have been calculated and then – the appropriate value of the objective function was found.

5000 selections were conducted, the result of which was about 25% correction in the examined cases. The co-ordinates of points after selection were rounded to integer number of millimetres. New, optimum co-ordinates of fixed points A_i of suspension are presented below:

$$A_1^z = (-109, 208, 239), \quad A_2^z = (317, 500, 251),$$
$$A_3^z = (217, 419, 302), \quad A_4^z = (193, 451, 403),$$
$$A_5^z = (3, 341, 422).$$

New value of the objective function was about 62.9% smaller than the initial value.

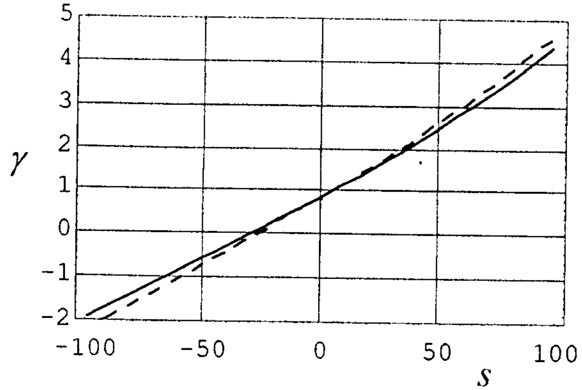
In order to check the correctness of the solution, another optimisation was carried out. The co-ordinates of points A_i obtained as a result of previous calculations were assumed to be the initial data. After conducting 5000 new selections, correction of the objective function values was not observed in any case. Figure 3 presents the characteristics of wheel steering angle $\delta(s)$ for the original mechanism and the one obtained as a result of optimisation.



- - - - - original mechanism ——— changed mechanism

FIG. 3. Characteristics of the change in steering angle according to the vertical displacement of point B7 for the original and changed mechanism.

Figure 4 presents characteristics of the wheel camber angle $\gamma(s)$ for the original mechanism and the one obtained as a result of optimisation.



- - - - - - original mechanism ——- changed mechanism

FIG. 4. Characteristics of the change in camber angles according to the vertical displacement of point B_7 for the original and changed mechanism.

6. Concluding remarks

This paper presents a method of constructing the characteristics of a five-link suspension. The method takes advantage of the perturbation method in order to solve the kinematics, in connection with the sampling method of optimisation. Moreover, a new, simpler set of equations (partly linear) was used for this kinematics analysis.

Weighted mean value of the absolute values of ordinate differences for the characteristics was used as a criterion of optimisation. The sum was sought for and obtained for a finite number of the considered points of characteristics. On the basis of calculations it was found that the criterion is of better accuracy in comparison with the integral criterion of curve matching. Since it is less time-consuming, it is more useful for optimisation issues where calculations connected with determining the values of the objective function are made repeatedly.

As it was mentioned before, the analysis of the complex system of 14 non-linear algebraic equations that describe geometric constraints of the investigated mechanism is time-consuming. The sampling method of optimisation requires to solve the system afresh for every newly selected point. Therefore, it seems recommendable to verify and use some other optimisation method which, for sufficient accuracy of the results, will not involve extensive calculations of the objective function values. Thus it is advisable to replace the sampling method of optimisation with such a method that would take advantage of the previously calculated values of this function. It is noticeable, however, that e.g. quickly operating gradient methods are of little use because of lack of analytic expressions describing the objective function derivatives.

The presented method is partially used in the research project by which the presented paper is financed. Additionally, it will be used in dynamic research, according to the stages depicted on page 2.

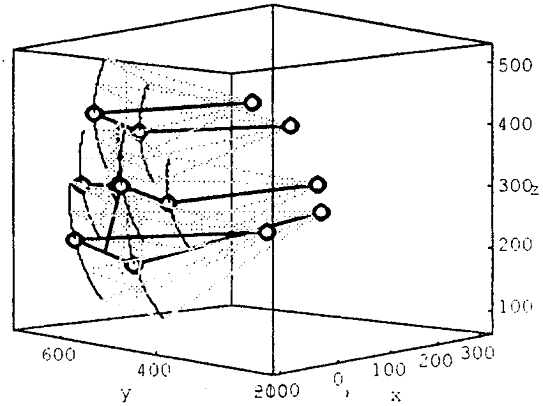


FIG. 5. Suspension mechanism and its moving point trajectories.

APPENDIX 1

Examples of equations: the first, sixth and last of the system of Eq. (3.1) in the expanded form:

$$(x_{B1} - x_{A1})^2 + (y_{B1} - y_{A1})^2 + (z_{B1} - z_{A1})^2 - \left[(x_{B10} - x_{A1})^2 + (y_{B10} - y_{A1})^2 + (z_{B10} - z_{A1})^2\right] = 0,$$

$$\cdots$$

$$(x_{B2} - x_{B1})^2 + (y_{B2} - y_{B1})^2 + (z_{B2} - z_{B1})^2 - \left[(x_{B20} - x_{B10})^2 + (y_{B20} - y_{B10})^2 + (z_{B20} - z_{B10})^2\right] = 0,$$

$$\cdots$$

$$(x_{B5} - x_{B3})^2 + (y_{B5} - y_{B3})^2 + (z_{B5} - z_{B3})^2 - \left[(x_{B50} - x_{B30})^2 + (y_{B50} - y_{B30})^2 + (z_{B50} - z_{B30})^2\right] = 0.$$

Appendix 2

Equations (3.2) in the expanded form:

$$\begin{aligned}\mathbf{n} = \mathbf{i}\,[&(y_{B2}-y_{B1})(z_{B3}-z_{B1})-(z_{B2}-z_{B1})(y_{B3}-y_{B1})]\\ +\mathbf{j}\,[&(z_{B2}-z_{B1})(x_{B3}-x_{B1})-(x_{B2}-x_{B1})(z_{B3}-z_{B1})]\\ +\mathbf{k}\,[&(x_{B2}-x_{B1})(y_{B3}-y_{B1})-(y_{B2}-y_{B1})(x_{B3}-x_{B1})],\end{aligned}$$

$$\begin{aligned}\mathbf{n}_0 = \mathbf{i}\,[&(y_{B20}-y_{B10})(z_{B30}-z_{B10})-(z_{B20}-z_{B10})(y_{B30}-y_{B10})]\\ +\mathbf{j}\,[&(z_{B20}-z_{B10})(x_{B30}-x_{B10})-(x_{B20}-x_{B10})(z_{B30}-z_{B10})]\\ +\mathbf{k}\,[&(x_{B20}-x_{B10})(y_{B30}-y_{B10})-(y_{B20}-y_{B10})(x_{B30}-x_{B10})],\end{aligned}$$

Equations (3.3) (similar to Eq. (3.4)) in the expanded form:

$$\begin{aligned}&(x_{B6}-x_{B1})(x_{B2}-x_{B1})+(y_{B6}-y_{B1})(y_{B2}-y_{B1})\\ &\qquad+(z_{B6}-z_{B1})(z_{B2}-z_{B1})\\ &\qquad-[(x_{B60}-x_{B10})(x_{B20}-x_{B10})+(_{B60}-y_{B10})(y_{B20}-y_{B10})\\ &\qquad+(z_{B60}-z_{B10})(z_{B20}-z_{B10})]=0,\end{aligned}$$

$$\begin{aligned}&(x_{B6}-x_{B1})(x_{B3}-x_{B1})+(y_{B6}-y_{B1})(y_{B3}-y_{B1})\\ &\qquad+(z_{B6}-z_{B1})(z_{B3}-z_{B1})\\ &\qquad-[(x_{B60}-x_{B10})(x_{B30}-x_{B10})+(y_{B60}-y_{B10})(y_{B30}-y_{B10})\\ &\qquad+(z_{B60}-z_{B10})(z_{B30}-z_{B10})]=0,\end{aligned}$$

$$\begin{aligned}&(x_{B6}-x_{B1})[(y_{B2}-y_{B1})(z_{B3}-z_{B1})-(z_{B2}-z_{B1})(y_{B3}-y_{B1})]\\ &+(y_{B6}-y_{B1})[(z_{B2}-z_{B1})(x_{B3}-x_{B1})-(x_{B2}-x_{B1})(z_{B3}-z_{B1})]\\ &+(z_{B6}-z_{B1})[(x_{B2}-x_{B1})(y_{B3}-y_{B1})-(y_{B2}-y_{B1})(x_{B3}-x_{B1})]\\ &-(x_{B60}-x_{B10})[(y_{B20}-y_{B10})(z_{B30}-z_{B10})-(z_{B20}-z_{B10})(y_{B30}-y_{B10})]\\ &-(y_{B60}-y_{B10})[(z_{B20}-z_{B10})(x_{B30}-x_{B10})-(x_{B20}-x_{B10})(z_{B30}-z_{B10})]\\ &-(z_{B60}-z_{B10})[(x_{B20}-x_{B10})(y_{B30}-y_{B10})-(y_{B20}-y_{B10})(x_{B30}-x_{B10})]=0.\end{aligned}$$

References

1. S. Dzierżek, *Influence of construction parameters on elastokinematic characteristics of a vehicle multi-link rear suspension*, Doctoral dissertation, Cracow University of Technology, Cracow 1995.
2. A. Grzyb, *Mechanics and optimisation of a chosen rod coupling for rail-vehicle drive systems*, Doctoral dissertation, Cracow University of Technology, Cracow 1975.
3. A. Grzyb, *On a Perturbation method for the analysis of the kinetostatics of mechanisms*, Akademie Verlag, ZAMM, Z. angew. Math. Mech., **72**, 6, 615–618, 1992.
4. A. Grzyb, *Kinematics and optimisation of a clutch in a power transmission system of locomotive*, Technical Magazine of Cracow University of Technology, Mechanics, z. 2-M, 49–63, 1997.
5. A. Grzyb, *Optimisation of a coupling of locomotive drive system*, 13^{th} Scientific Conference 'Rail Vehicles '98', Scientific Works of University of Technology No. 1392, Transport, 31, 93–101, Gliwice 1998.
6. A. Grzyb, J. Struski, *Methods of determining kinematics of multi-link solid tyre suspensions*, Scientific Conference: Interkonmot '98, Proc. of Scientific Commission of Motorization, Cracow Department of PAS, 16, 9–17, 1998.
7. J. M. Jimenez, *et. al*, *A Simple and general method for kinematic synthesis of spatial mechanism: an exhaustive numerical approach via mono-dimensional-search algorithm*, Trans ASME, J. of Mechanical Design, vol. 115, December. 1993.
8. J. Knapczyk, S. Dzierżek, *Analysis of displacements and forces of five-link mechanism of wheel leading*, Decrease in loss of energy in motor vehicles. Conference materials, Cracow 1992.
9. J. Knapczyk, *Methods of analysis of multi-link wheel suspensions*, Scientific Conference: Interkonmot '98. Proc. of Scientific Commission of Motorization Cracow Department of PAS, **16**, 19–29, 1998.
10. J. Madej, *Dynamic properties of drive systems incorporating flexible couplings with four tension elements*, Arch. of Mech. Engng., **2**, XXI, 305–323, 1974.
11. J. Madej, *Driving gears of rail-vehicles*, Publishing House of Warsaw University of Technology, Warsaw 1978.
12. J. Madej, *Mechanics of power transmission of rail-vehicles with electric traction engines*, PWN, Warsaw 1983.
13. J-P. Merlet, *Kinematic analysis of a rear suspension*, ISATA 29th International Symposium on Automotive Technology and Automation, Mechatronics - Advanced Development Methods and Systems for Automotive Products, Florence 1996.
14. J. Osiecki, *Equations of vibrations and analysis of the dynamic loading of a drive system with Alsthom-type couplings*, Nonlinear Vibrations Problems, **10**, 225–243, 1969.
15. J. Osiecki, *Structural and kinematic features of plane pull rod couplings for misaligned shafts*, Arch. of Mech. Engng., **3**, XVI, 379–397, 1969.
16. J. Osiecki, *Vibrations of mechanical systems with plane couplings compensating the misalignement of the shafts*, Arch. of Mech. Engng., **4**, XVI, 451–478, 1969.
17. J. Struski, *Kinematics of five-link suspension of rear drive wheels*, Development prospects of construction, technology and operation of motor vehicles and combustion engines, Monograph 210, Publishing House of Cracow University of Technology, Cracow 1996.

18. J. STRUSKI, W. WACH, *Modelling of dynamics of multi-link solid tyre suspensions*, Problems of safety in motor vehicles (conference materials), Kielce – Cedzyna, 18-19 Februar, 163–174.

19. Y. TSUKUDA, *et al.*, *Development of a new multi-link rear suspension*, SAE Paper, No. 881774.

20. KIKUO FUJITA, SHINSUKE AKAGI, NORIYASU HIROKAWA, TAKANORI HIRATA, *Design optimisation of multi-link suspension system for total vehicle handling and stability*, AIAA-98-478.

Received January 21, 2000.