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PROGRESSIVE FAILURE MODELLING OF TRANSVERSE CRACKING IN CROSS-PLY LAMI NATESWITH DOUBLE-EDGE-SEMICIRCULAR NOTCHES

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A local-global progressive failure finite element (FEM) model was created in combination with the maximum strain criterion on an individual layer scale to study the formation of transverse cracks in cross-ply laminates with double-edge-semicircular notches. It is assumed that the laminate is made of homogeneous orthotropic layers before the occurrence of the matrix failure. Then, the average reduced layer stiffness as a function of the applied load is separated from the reduced laminate stiffness. Additionally, a maximum strain strength criterion is applied on a very local scale of a single layer to induce the local matrix failure. The initiation, propagation and multiplication of the transverse cracks starting from the notch are predicted in this way. The influences of the laminar thickness and the notch-width aspect ratios were also studied. Furthermore, the stress distribution and redistribution caused by the initiation and propagation of the transverse cracks were calculated. In addition to the stress concentration caused by the circular notch, an extra stress concentration in the 0° layers is induced by the transverse cracks in the 90° layers. This stress redistribution will in turn initiate more local failures in the 0° layers or at the interface between the longitudinal and transverse layers. The simulated results show a good agreement with experimental observations.
Key words: cross-ply laminate, notch effects, transverse crack, finite element analysis.

1. Introduction

A number of different forms of matrix-dominated subcritical damage takes place before final catastrophic failure in a fibre-reinforced polymer composite laminate under loading. One of them is intralaminar matrix failure, e.g. the matrix cracks observed in the 90° plies in cross-ply laminates. Usually this is the first type of failure in polymer matrix composites. These micro-scale cracks (displacement discontinuity in a layer) actuate local stress concentration and stress redistribution on a laminar scale or between the laminae (three-dimensionally), and cause global stiffness reduction and strength decrease in conjunction with several others failure mechanisms. Geometrical notches may radically quicken this form of subcritical matrix damages through the notch-provoked stresses. Such matrix-dominated damages in composites have been a subject of extensive investigation in the field of composite materials.

Various forms of shear-lag models [1-6] were used by GARRETT and BAILEY, PARVIZI, GARRETT and BAILEY, MASTERS and REIFSNIDER, LAWS and DVORAK and HAN and HAHN. The main objectives of their investigations were to understand either the (global) uniform distribution of the transverse cracks or the stiffness reduction caused by the multiple occurrence of transverse cracks, or both. A resistance curve for mixed modes (Mode I and Mode II), was proposed and used by HAN and HAHN [6] to study the occurrence of increasing numbers of transverse cracks in balanced symmetric laminates under general in-plane loads. LAWS and DVORAK [4] attempted to solve the statistically progressive transverse cracking by examining the crack density growth as a function of the applied, monotonously increasing load. The shear-lag models lead to an essential understanding of the load transfer mechanism in composites, and show satisfactory agreement with tests. It is assumed that the load transfer occurs through shear stresses only at the interface between two fibres or two layers. This is a severe simplification of the real stress state and it neglects the shear stress through the thickness in transverse layers, and hence predicts a higher stiffness. Further, shear-lag models are less favourable when complex forms of failure are studied, e.g. in a case with notched laminates.

A great number of two-dimensional (2D) (plane strain/plane stress/generalized plane strain) finite element models (FEM) [7-11] has been constructed to simulate 2D responses of laminates with or without notches. Obviously, 2D FEM models have their inevitable shortcomings in handling the damages caused by the stresses related to the third direction, and sometimes this kind of stresses play an important role, e.g. for the case of delamination. First of all, a local failure in a laminate is a laminar thickness and/or stacking-sequence-related damage process. A 2D FEM model cannot include such three-dimensional (3D) effects. Then, the matrix-dominated subcritical damages in multidirectional laminates usually occur simultaneously in each layer, but not across the entire thickness until the final failure. These micro-scale occurrences create local displacement discontinuities between the laminae, which cannot be taken into account in a 2D model. In short, the mechanical stresses in a damaged laminate are three-dimensional, even under the simplest tensile load. Moreover, a geometrical notch can intensify the three-dimensional stresses and complicate the three-dimensional stress ditribution. Therefore, three-dimensional FEM modelling are preferential in the failure assessment in a notched laminate. Naturally, a 3D FEM model has its limitation, mainly due to its costs: it is much more complicated and takes much more computation time. Nevertheless, this disadvantage is becoming less significant with the rapid development of the calculation capacity of computers and growing user-friendly interfaces of software.

Several fracture mechanics-based models [12-15] were developed to predict the (final) failure of composite laminates with or without notches. WADDOUPS *et al.* [14] applied linear elastic fracture mechanics (LEFM) to notched laminates and developed a two-parameter model to predict the notched strength of composite laminates. According to their model, a damage zone of intense energy stands next to the notch. Fracture occurs as the material in the damaged zone is loaded to its critical level. A criterion using a fracture toughness parameter, K_Q^0, which serves as the fracture toughness of the 0° plies, was proposed by VAIDYA *et al.* to predict the final fracture strength of laminates containing an artificial crack. A notched strength model based on damage parameters, which uses the notch tip stress as the failure index, was developed by KORTSCHOT *et al.* One of their important conclusions is that the matrix-dominated subcritical damages must be considered in developing the models to predict the notched strength of composite laminates.

The present paper describes a numerically based stress approach to study the formation and multiplication of transverse cracks in cross-ply laminates with double-edge-semicircular notches (DEN). The construction of a three-dimensional FEM model, in combination with a progressive failure model, is concisely described. Several FEM parameters which affect the accuracy of the simulation were examined. The onset and growth of transverse matrix failure in 90° layers of the cross-ply laminates with DEN was then simulated by using the model. The stress distribution and its evolution with the growth of transverse cracks were evaluated. The influences of the notch size and of the lamina thickness on the initiation and propagation of the notch-controlled transverse cracking were also investigated.

2. Material modelling

2.1. *Initial elastic properties*

The typical initial elastic properties of the unidirectional (UD) laminae (E-glass reinforced polyester), with an average fibre volume of 55%, are

$$E_L = 42\,\text{GPa}, \quad E_T = 14\,\text{GPa}, \quad G_{LT} = 5.6\,\text{GPa}, \quad \nu_{LT} = 0.26, \quad \nu_{TT} = 0.45,$$

where L and T represent the longitudinal direction and the transverse direction of the laminae, respectively.

2.2. Global stiffness reduction in a single layer

In the region far from the notch, the transverse matrix cracking occurs in the same manner as that in the unnotched laminate. As a result of the transverse cracking, the stiffness degenerates with the same tendency as that in the unnotched laminates. Fig. 1 presents the layer-scale input stress-strain of the 90° layers in the field far from the notch.

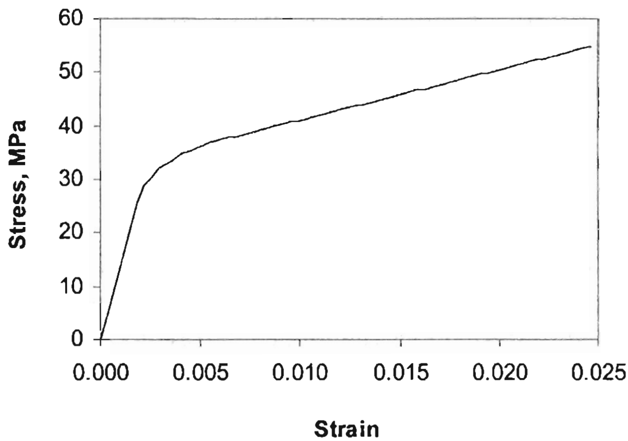

FIG. 1. The input stress-strain of the transverse layers.

2.3. Local damage modelling

2.3.1. Strength criterion The corresponding local matrix damage originating from the notch is induced by the application of a strength criterion or a group of strength criteria in the area near the notch in a single layer. The observed constant strain from unnotched laminates at which the first transverse crack (FTC) occurs, was used locally in the 90° layers in the area at the notch root as the critical

$$\varepsilon_{cr} = \varepsilon_{max}^{90} \tag{2.1}$$

strength parameter of the maximum strain criterion, see Eq. (2.1), where $\varepsilon_{max}^{90} =$ 0.002. Local transverse damage occurs while Eq. (2.1) is satisfied. The residual (initial) state of the material is implicitly involved in this critical material property in this case.

2.3.2. Local stiffness degradation Upon the occurrence of local transverse damage in the area near the notch root, the orthotropic material moduli of the cracked 90° layers at the integration points reduce to a group of lower values, 10% of the original values in the present calculations, as shown in Fig. 2 (a). Fig. 2 (b) presents the local material response of the 90° layers in the loading direction to the input shown in Fig. 2 (a). This is an approximate evaluation of the stiffness degradation caused by the transverse matrix cracking. First of all, the modulus reduction in the fibre direction of the transverse layers does not occur if the stress in this direction is tensile. On the other side, it is troublesome in reality to get the real material parameters of the damaged material. However, the present degradation assumption of all moduli does not induce much error because the stiffness degradation occurs only in the damaged area in the transverse layers, and the critical stress in the fibre direction is far from that reached in this transverse layer anyhow. A more complete model, treating other damage modes as well, should of course contain a proper separate degradation routine for every stiffness constant. This still remaines to be modelled.

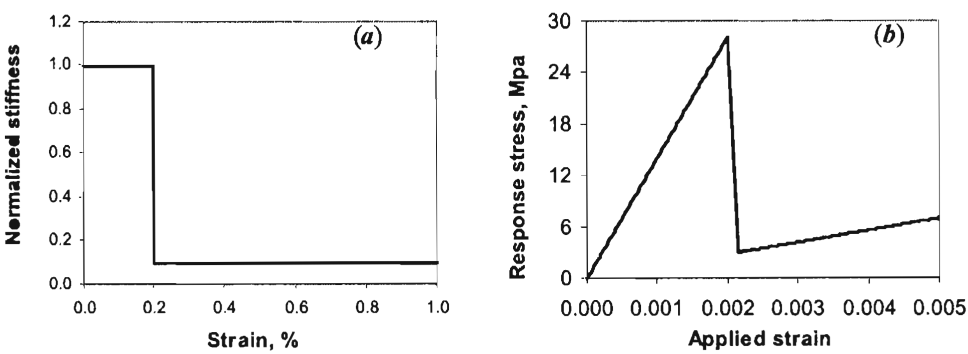

FIG. 2. (a) the normalized stiffness reduction of the 90° layers, (b) the local response of the 90° layers to the applied strain.

3. 3D FEM MODEL CONSTRUCTION AND FEM PARAMETER STUDY

3.1. 3D FEM model construction

The goal of the 3D FEM simulations is to understand the evolution of the local transverse cracking in the range near the notch root and to evaluate the local and global stress redistribution caused by the transverse cracks. Fig. 3 shows one eighth of a double edge notched (DEN) cross-ply laminate and the coordinate system used in the simulations. In Fig. 3, 2R/W is defined as the notch-width aspect ratio, and L and t represent the half-length and the half-thickness of the laminate, respectively. For a more accurate calculation of the stresses at the notch boundary and at free edges, 4 layers of 20-node brick elements in the thickness (z, 3) direction with 27 Gaussian integration points, were used to model one half of the laminate. The mesh is first constructed in the mid-plane (z=0), and then expanded in z (3) – direction according to the thicknesses of the laminae. The global displacements were applied through one of the boundary conditions. A commercial FEM package MARC 7.2, running on a Silicon Graphics Origin 200 workstation, was used to perform the simulations.

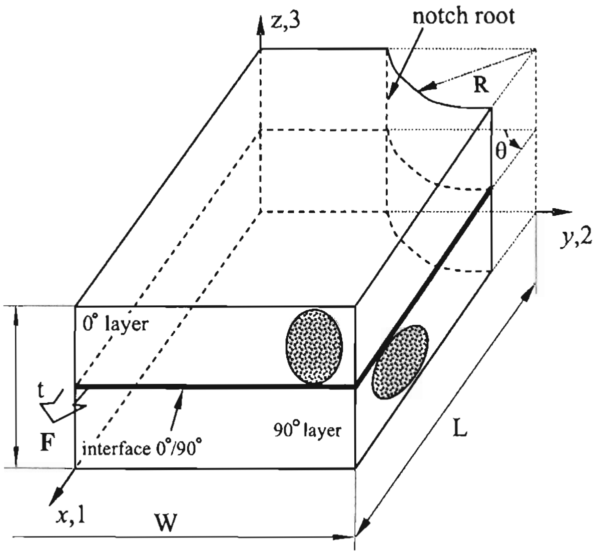

FIG. 3. The geometry and the coordinate system used.

3.2. FEM parameter study

The three-dimensional FEM model was optimized through a careful study of (1) mesh density, (2) mesh orientation, and (3) element layer number in the thickness directions. It was found that a medium mesh density, with an element number of about 400 in each layer, gives acceptable consistency of transverse crack growth and calculation time less than 24 hours in a progressive failure simulation. The mesh orientation affects the growth direction of the transverse cracks, as seen in Fig. 4, which is in practice predominated by the fibre direction of the 90° plies. This influence can be partly eliminated by the choice of an improved mesh orientation, as seen in Fig. 4 (b). A coarse mesh with an inadequate mesh orientation may lead to an overestimation of the damage area caused by transverse cracking. Therefore, the mesh refinement has to be carefully handled to avoid overestimation of the damage area and to avoid the deviation of transverse crack growth. The influence of the number of elements per layer in the thickness direction on the simulated transverse crack growth is small if four or more elements are applied in half of the thickness, as shown in Fig. 5.

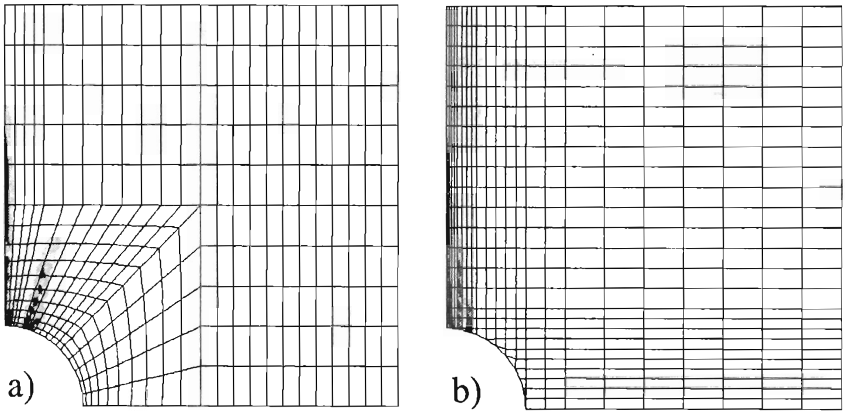

FIG. 4. Mesh orientation dependence of simulated transverse cracks: (a) unoptimized mesh, (b) optimised mesh. Simulations for a 2 mm laminate, at an applied strain 0.0014 and viewed from the symmetric surface. The darker areas represent the damaged parts in the transverse layers

3.3. *Type of simulations and damage characterization*

Two types of simulations were carried out:

- Simulation of local transverse matrix failure without considering the far-field material degradation in 90° layers. In this case, the highest applied global strain is relatively low, 0.2%.
- Simulation of the matrix-dominated failure including the far-field material degradation in 90° layers to examine the global effects, e.g. the maximum stress concentration. In this case, the highest applied strain is 1.0%.

The stress ratios, K_{ij}, defined in Eq. (3.1) are used to represent the local stress state in terms of the far-field average (applied) stress.

$$K_{ij} = \frac{\sigma_{ij}^{l}}{\sigma_{ave}^{\infty}} \qquad (i,j = 1,2 \text{ and } 3), \tag{3.1}$$

where $\sigma_{ij}^{l}(i,j = 1,2$ and 3, respectively) and σ_{ave}^{∞} refer to one of the local stress components and the far-field (applied) average stress, respectively. For the calculation of the stress concentration factor, the far-field stress is calculated by using Eq. (3.2):

$$\sigma_{ave}^{\infty} = \frac{1}{A_0}\int_0^{A_0} \sigma_{11} ds \tag{3.2}$$

where A_0 is the unnotched cross-sectional area of the laminate. In addition to the stress concentration, the FTC onset strain and FTC growth, as well as the

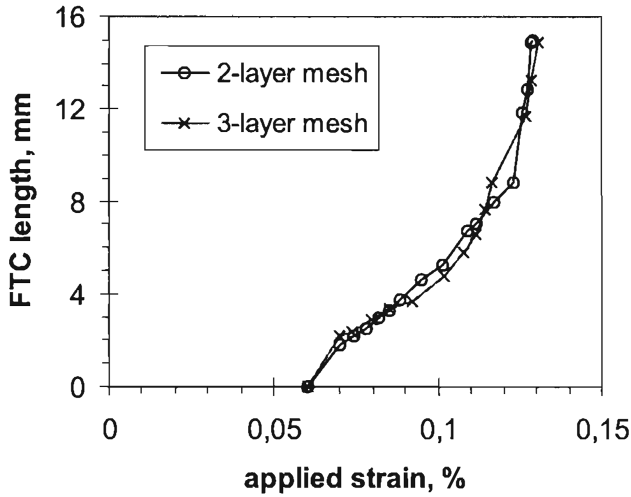

FIG. 5. The influence of the element number over the thickness of each lamina on the simulated FTC growth in a laminate with 2t = 6 mm and 2R/W = 0.25 (R = 5 mm). Here 2/3-layer mesh means that the 0-layers and 90-layers are respectively modelled by using 2 elements or 3 elements in the thickness direction.

transverse crack multiplication were inspected as a function of the applied load.

4. TRANSVERSE CRACKING IN CROSS-PLY LAMINATES WITH DEN

4.1. Overview of transverse cracking

The essential objective of the present modelling is to examine the onset, propagation and multiplication of the local transverse matrix failures in the area around the notch under quasi-static tensile load. As an overview, Fig. 6 presents a simulated damage pattern of a notched cross-ply laminate at about 40 MPa tensile stress. The darker areas represent the damaged parts in 90° layers where extra strain is induced. Such an area may be considered to be analogous to a crack. The transverse cracking is highly localized and characteristic for such a notched plate: the first several cracks initiate always from the notch, although the transverse cracks in other effectively loaded parts take place more or less randomly. One of the important attributes of the present modelling is that the model predicts, instead of consecutive damage areas, a separate transverse damage pattern parallel to the fibre direction in 90° layers. Extra local deformation is induced by the material damage in transverse layers.

In contrast with the cracking pattern following from the present modelling, a transverse matrix failure pattern from a reference [11] is illustrated in Fig. 7. The shadowed areas denote the matrix failure in the 90° layers of laminate

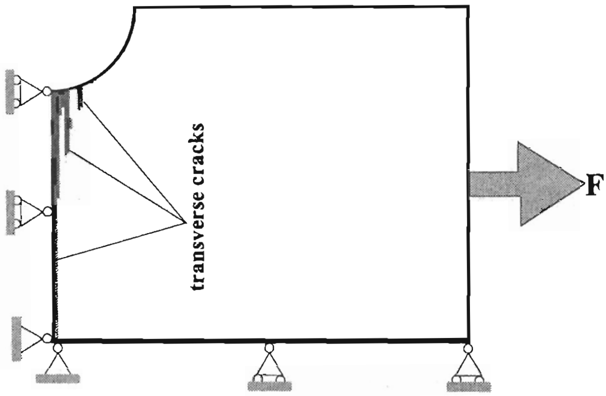

FIG. 6. Simulated transverse matrix failure pattern viewed from the 90° side at about 40 MPa.

$[0/90/\pm 45]_s$. Obviously, the mesh is relatively coarse and hence the predicted transverse cracking pattern is far from the reality. This is partly due to the limits of the calculation capacity available, e.g. a huge amount of computer resources is needed in a complete 3D simulation of matrix failures. Further, the model gives no evidence of a single damage mode or damage pattern. It predicts consecutive areas which represent the damage of the transverse layers in the laminate. Clearly, this kind of failure evaluation is a relatively rough estimation and it is likely to ignore local details of the matrix-dominated failures.

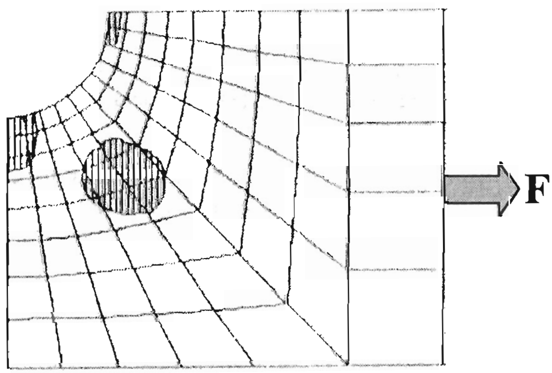

FIG. 7. Predicted damage pattern in 90° ply of AS4/3502 $[0/90/\pm 45]_s$ laminate at 276 MPa, TAN [11].

A result of the matrix cracking in 90° layers is a local and global stress redistribution in the laminate. Fig. 8 illustrates the evolution of the stress concentration in 0° layers with the development of transverse cracking. The initial part of Fig. 8 (a) is then magnified in Fig. 8 (b). The maximum stress concentration in 0° layers as defined in Eq. (3.1) is about 5.6 during elastic deformation, while the corresponding value raises to about 6.5 after FTC started, and it is still growing up with the development of transverse cracking in the transverse layers. This stress concentration may eventually reach a value as high as 7.8 after the transverse cracks are fully developed in the transverse layers. This local stress escalation in 0° plies causes an extra local deformation of the load bearing layers. This extra deformation will be reflected back into the transverse layers and, in turn, drive the expansion of transverse cracks. Obviously, this extra stress concentration depends on the thickness ratio between the 90° layers and the 0° layers. There is no analytical solution to predict the additional stress concentration caused by transverse cracking. Nevertheless, it should be noted that the effects of other damage modes on this stress concentration is not yet included, which might act adversely on it in reality. It should also be mentioned that the increase of the stress concentration factor is a kind of "locally accumulative effect" of all transverse cracks.

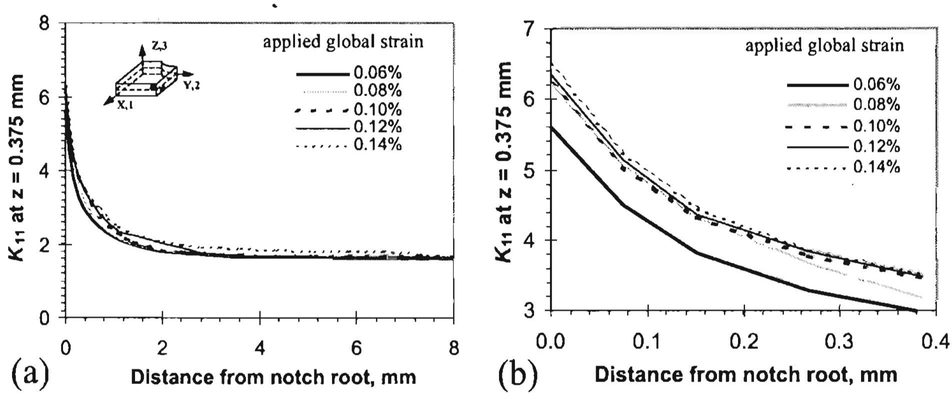

FIG. 8. (a). Stress concentration (at x = 0) and redistribution at the mid-surface of the 0° ply as a function of transverse cracking, 2t = 1 mm and 2R/W = 0.2. (b). Enlarged initial part of (a).

4.2. FEM Simulation of transverse crack growth

4.2.1. Transverse laminar thickness effects It is expected that the transverse laminar thickness will exert substantial influences on the transverse cracking, its initiation and multiplication. All of the simulations indicate that the initial length of the FTC is more or less proportional to the thickness, although they have the same local critical strain of 0.2%. The simulated FTC length at its initiation (at an average applied strain 0.0667% at $L = 10$ mm) is 0.15, 0.19 and 0.71 mm for $2t = 1, 2$ and 6 mm laminates, respectively. Fig. 9 presents three strain contourbands of ε_{11} from three separate simulations for laminates $2t = 1$ mm, 2 mm and 6 mm at an average applied strain of about 0.11%, to show the laminar thickness effects on the FTC initiation and transverse crack multiplication. The different behaviours of FTC growth are again shown in Fig. 10 (a). It can be seen from both Fig. 9 and 10 that the FTC length growth is remarkably affected by the laminar thickness: the thicker the laminate, the higher the FTC growth rate. It was also found in the simulations that the numbers of separate transverse cracks in about the same distance (about 1.8 mm) are 3, 2 and 1 for $2t = 1, 2$ and 6 mm laminates, respectively. This implies that a transverse crack in a thicker transverse lamina has a bigger influence zone than that in a thinner one. However, the thickness does not influence the strain at which the FTC starts, as illustrated in Fig. 10 (a), which gives the FTC onset global strains (at $L = 10$ mm) 0.0653%, 0.0654% and 0.0666% for laminates with $2t = 1$ mm, 2 mm and 6 mm and 2R/W=0.4, respectively. The influences of the transverse crack growth on the global stress and local stress are illustrated in Fig. 10 (b) for 1 mm laminate. The local stress in the area between two damaged neighbour areas in the midplane of the 90° ply remains roughly constant after the occurrence of the FTC, and the local stress at interface between the 0° and the 90° plies fluctuates considerably with the development of transverse cracks. The global stress shows some non-linearity with the occurred events in the transverse plies near the notch.

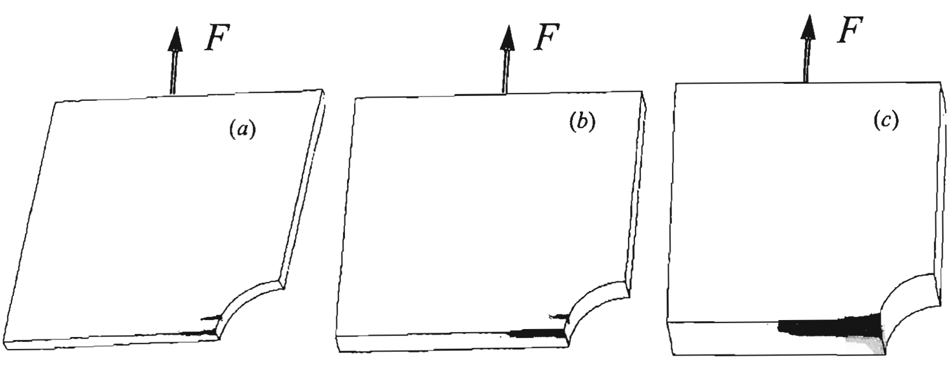

FIG. 9. Transverse cracking patterns at about the same applied strain, $\epsilon_{app} = 0.11$ (a) 2t = 1 mm, (b) 2t = 2 mm and (c) 2t = 6 mm, views of the lower symmetric parts.

4.2.2. Notch size effects on transverse cracking The transverse crack multiplication is drastically influenced by the notch aspect ratio. Fig. 11 presents the transverse crack multiplication in notched laminate with an aspect ratio (2R/W) 0.1, 0.2 and 0.5, respectively, at an average global strain of 0.14% (at $L = 10$ mm). It can be seen that the plate with a large notch size (2R/W=0.5) shows a different transverse crack multiplication behaviour: the third transverse

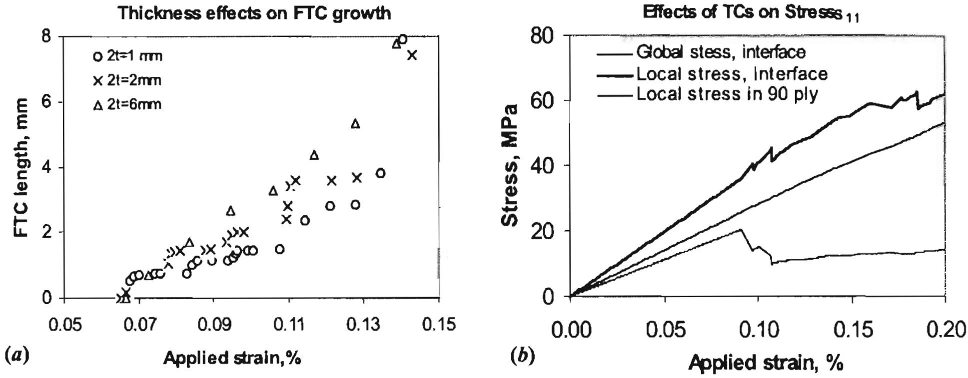

FIG. 10. Simulated transverse crack growth with the influence of the laminate thickness, (a) thickness effects on FTC growth, (b) effects of transverse cracks on s11, $2t = 1$ mm.

crack occurs later but grows much quicker than the first and the second ones. In fact, the stress distribution at the position (1, y, 0.125) (in 90° layer) of the third transverse crack, as shown in Fig. 12 (a), already explains such an unstable growth. It can be seen that the stress distribution is flat, and there is a lower stress area caused by the FTC and the second transverse crack (STC). This means that an unstable transverse crack growth will be inevitable. It is also expected that the FTC growth will adversely be influenced by the initiation of the STC, as well as the third transverse crack, as shown in Fig. 12 (b), the FTC growth is delayed by the initiation of the STC the third transverse crack.

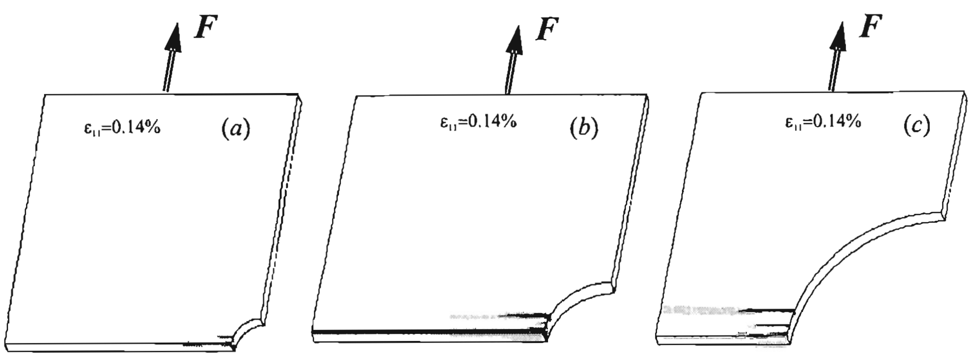

FIG. 11. Notch size effects on the multiplication of transverse cracks for 2t = 1 mm Laminate at $\epsilon_{app} = 0.14$ %: (a). 2R/W = 0.1, (b). 2R/W = 0.2, (c). 2R/W = 0.5, views of the lower symmetric parts.

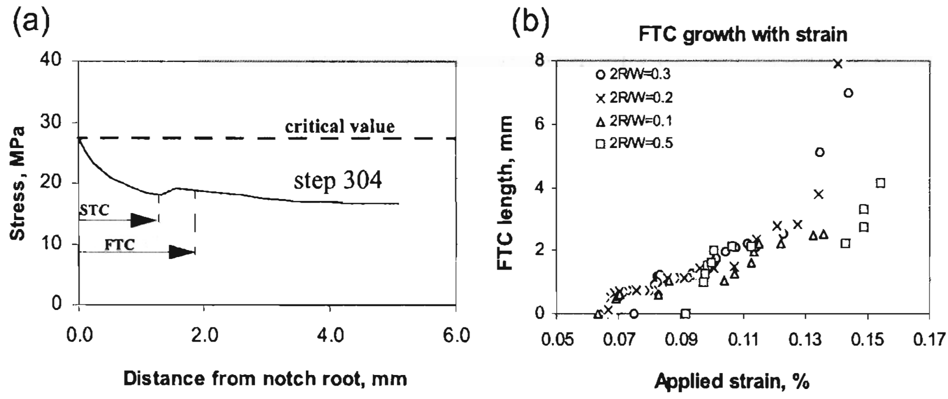

FIG. 12. (a) stress distribution at (1,y, 0.125) for $2t = 1$ mm and 2R/W=0.5. (b) transverse damage growth with the infuences of the notch sizes.

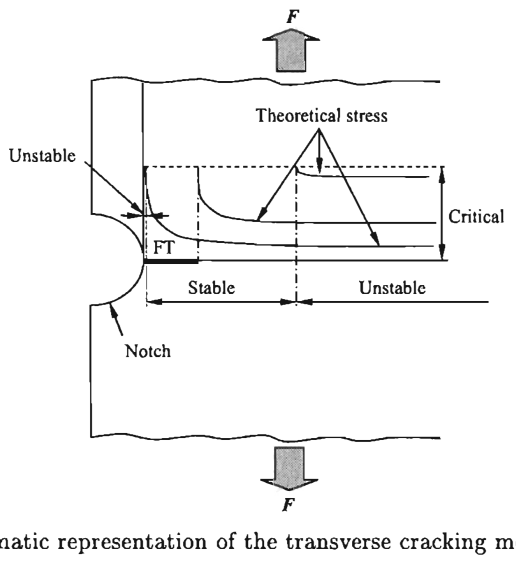

FIG. 13. Schematic representation of the transverse cracking mechanisms.

4.3. *Transverse cracking mechanisms*

The three-segment growth behaviour of the formation of transverse cracks starting from the notch can qualitatively be interpreted by the mechanisms shown in Fig. 13, after realizing the FEM calculations. The initial unstable growth occurs due to the local stress redistribution caused by the FTC in addition to the notch intensified stresses, as seen in Fig. 8. The stable growth is then expected to take place with the increase of the applied load – the failure index curve moving gradually upwards to meet the critical strength line and to authorize a stable propagation of the FTC. Inevitably, the unstable growth occurs as the parallel part of the failure index curve reaches the critical laminar strength (the dashed line), if the width of the sample is large enough. Obviously, the shadowed area in Fig. 13 can never be attained because of the critical strength of the material. In fact, the growth of matrix cracks, in terms of transverse cracking and longitudinal splitting, in a cross-ply laminate **without notch** is mostly unstable as it is loaded in one of the two fibre directions (a uniform distribution of stresses or strains, therefore, the failure index curve in an unnotched laminate sample is a line parallel to the critical strength curve). On the other hand, the failure index curves of a notched laminate can be changed with the occurrence of damage in the material, as shown in Fig. 8, and with positions, as seen in Fig. 12 (a).

5. Comparison with tests

5.1. FTC onset strain

The theoretical stress concentration factor can be used to calculate the FTC onset strain for a notched laminate with infinite width. A previous investigation showed [16] that the maximum stress concentration in the 90° layer is $K_t^\infty = 3.58$ in such a notched laminate during elastic deformation. Then combining the critical strain given in Eq. (2.1), the FTC onset strain can be calculated, $\varepsilon_{FTC}^\infty = 0.0559\%$. The FEM simulation predicts a FTC onset strain 0.0565% for the laminates with 2R/W=0.4 at $L = 25$ mm. This agrees quite well with the theoretical result.

5.2. Local transverse cracking pattern

Figures 14 (a) and 14 (b) present the similarity between the simulated transverse cracking pattern and the experimental observation: (a) is a transverse damage state simulated from the progressive failure model at an applied strain 0.13%, (b) is a recorded picture from a test at an applied strain 0.14% at $L = 25$ mm, both for the 6 mm laminate with 2R/W=0.25.

5.3. Propagation and multiplication of transverse cracks

The FEM simulation predicts a stable transverse damage growth, as observed in the tests. Fig. 15 shows a comparison between the simulated transverse damage growth and the experimental observation. The simulation predicts a similar tendency of the FTC initiation and its stable growth. As discussed above, the transverse crack numbers at about the same distance (about 1.8 mm) are 3, 2 and 1 for $2t = 1, 2$ and 6 mm laminates, respectively. This gives a transverse crack density of 1.7, 1.1 and 0.55 per millimetre, respectively. These values agree approximately with the experimentally observed values of the saturated transverse

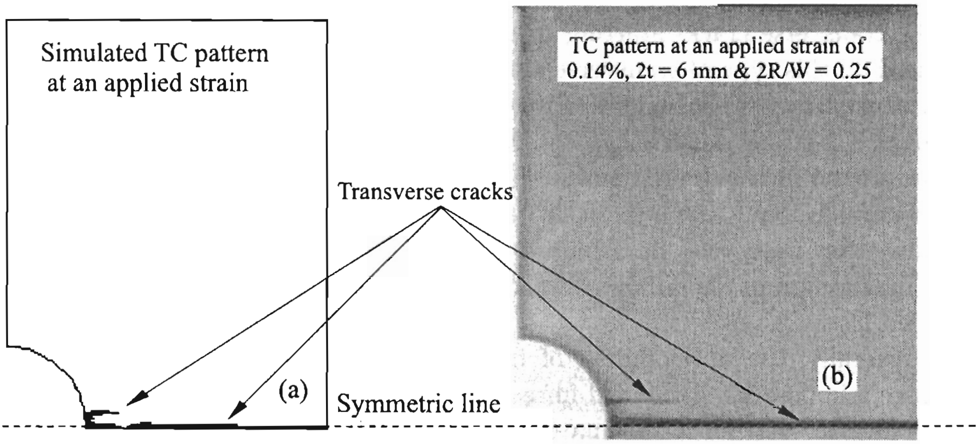

FIG. 14. Similarity of the simulated transverse cracking and the experimental observation: (a). simulated cracking pattern at $\epsilon_{app} = 0.13$ %, (b). observed cracking pattern at $\epsilon_{app} = 0.14$ %.

crack density, which are about 1.62, 1.02 and 0.48 per millimetre for $2t = 1, 2$ and 6 mm, respectively.

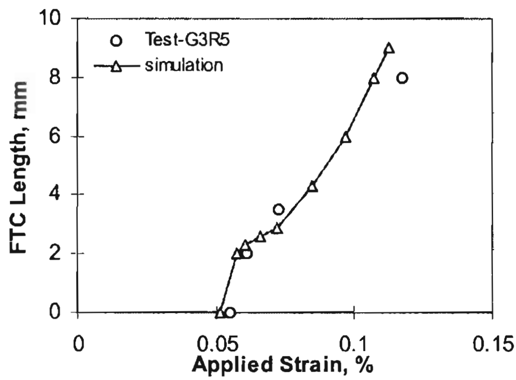

FIG. 15. Comparison of a transverse crack growth between simulation and test.

6. Discussions and Conclusions

An analysis is made, regarding a case of matrix-dominated failure at a rather low load level. In fact, no arguments are given about other modes of matrix failures and the ultimate failure of a notched laminate till now. This is due to the fact that the transverse cracking occurs at an extremely low load level, and the notch, through its enhanced local stress/strain field, accelerates the occurrence of the transverse cracks starting from the notch. The attention is preliminarily focused on this type of transverse matrix failure. Further, the property degradation caused by diverse modes of matrix-dominated failures must be taken into account if a higher load is applied. Moreover, other forms of matrix-dominated failures must also be taken into account in the finite element model, e.g. notch-induced splits and notch-induced delamination, if higher numerical loading is applied. Naturally, the final failure of a notched laminate will be affected by the matrix-dominated failures occurred before. This circumstance remains to be probed in the future.

Generally, the development of transverse cracks starts at material defects such as microcracks, debonded fibre-matrix interfaces and broken fibres, or even local uneven distribution of materials. Residual stresses will contribute to this process [17]. As load increases, these defects act as local stress/strain raisers, and integrate to devise a detectable transverse crack. This type of matrix cracks occurs more or less randomly over the whole effectively loaded area. However, the fixed onset position of the FTC from the notch root does confirm that the sites of stress/strain concentration can definitely be a type of initiators. No matter where they start, these microcracks grow consistently along the interface between fibre and matrix or inside the matrix, due to the fact that the interface and the matrix are much weaker than the fibre. Therefore, the propagation direction of a matrix crack in laminates is highly regulated by the fibre direction in each ply.

The constructed numerical strength model is capable of taking both local and global matrix failures into account on a local scale. Particularly, the simulation here gives a sense of the evolution of transverse cracking controlled by a maximum strain criterion under a non-uniform tensile stress field. The agreement between the simulation and the experimental observation insinuates the likelihood of the applicability of the present method in modelling other forms of matrix-dominated failures in composites. The evolution of the stress concentration and the local stress redistribution indicate the dependence of the local stress/strain upon the development of the matrix failures. The calculation also confirms that the elastic solution of stress/strain concentration will become invalid after the occurrence of the FTC, which takes place at considerably low load level. The influences of the laminar thickness and the notch-width aspect ratio on the transverse cracking behaviour are eventually the effects of these parameters on the relevant stress/strain components.

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