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NONLINEAR FLEXURAL AND TORSIONAL VIBRATIONS OF AXIALLY MOVING ORTHOTROPIC WEB

K. MARYNOWSKI [1], Z. KOŁAKOWSKI [2]

[1] **Department of Dynamics of Machines,**
e-mail: `kmarynow@ck-sg.p.lodz.pl`

[2] **Department of Strenght of Materials and Structures,**
Technical University of Łódź,
1/15 Stefanowskiego Str., 90-924 Łódź,
e-mail: `kola@orion.p.lodz.pl`

A new approach to the analysis of the dynamic behaviour of an axially moving orthotropic web is presented. Mathematical model of the moving web system constitutes two nonlinear, coupled equations governing the transverse displacement and stress function. The results of numerical investigations show the solutions to the linearized and nonlinear problems. Free vibrations of the web with different initial streses are analyzed. The effect of orthotropy factor and axial transport velocity on transverse and torsional vibrations are presented

1. Introduction

Axially moving webs in the form of thin, flat rectangular shape materials with small flexural stiffness occur in the industry as band saw blades, power transmission belts, magnetic tapes and paper webs. Excessive vibrations of moving webs increase the defects and can lead to failure of the web. In the paper and textile industries involving motion of thin materials, stress analysis in the moving web is essential for the control of wrinkle, flutter and sheet break. Although the mechanical behaviour of axially moving materials has been studied for many years, little information is available on the nonlinear dynamic behaviour and stress distribution in the axially moving orthotropic web.

A lot of earlier works in this field focussed on dynamic investigations of string-like and beam-like axially moving isotropic systems (e.g. [2, 3]). In order to consider the dynamical behaviour of moving materials coupled with surrounding fluid, one often ignores the boundary layer shear forces and introduces different additional mass expressions for the Coriolis and centrifugal inertia terms [1]. For the purpose of attenuation and guidance, coupling problems between the moving material and fluid-air bearings have been studied [4, 5, 9].

Recent works in this field analysed the equilibrium displacement and stress distribution in nonlinear model of an axially moving web [6, 10], the wrinkling phenomenon and stability of the linear model of an axially moving isotropic plate [8], nonlinear vibrations of power transmission belts [11] and stress distribution in an axially moving plate [12]. The importance and practical applications of stress analysis of moving materials were discussed in [7], where the authors applied the Kirchhoff thin plate theory for isotropic materials and Lagrangian kinematic descriptions, in order to avoid convective terms.

On the other hand, it is well known that many materials traditionally considered as isotropic, exhibit some degree of anisotropy due to the working processes. Also the growing interest in composite materials demands a better understanding of the strength of materials anisotropic by design. The aim of this paper is to analyse the nonlinear dynamic behaviour of an axially moving orthotropic web.

The nonlinear model of axially moving orthotropic web has been derived in [13]. One of the principal goals of this paper is to investigate the flexural and torsional vibrations of axially moving web with different initial streses in subcritical and supercritical regions of the transport speed. Numerical investigations have been carried out for a thin orthotropic steel plate.

2. Mathematical model of the moving web system

An elastic moving web of the length l is considered. The web moves at velocity c which may change in time. The co-ordinate system and geometry of the considered physical model are shown in Fig. 1.

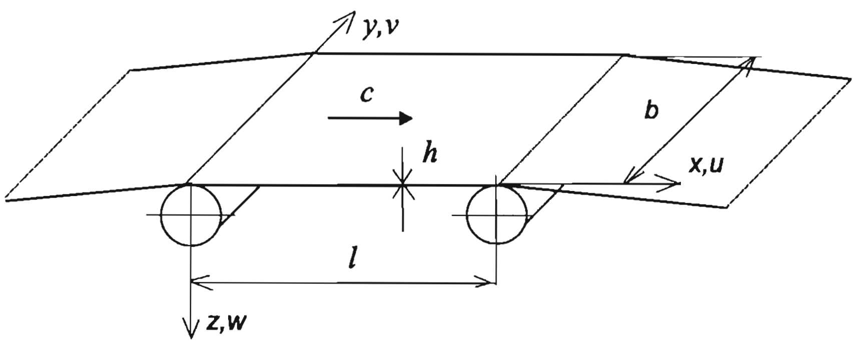

FIG. 1. Axially moving web.

The dynamic analysis is carried out using the thin-walled plate model. The considered web is composed of plane rectangular plate segments, with principal axes of orthotropy parallel to their edges. Such model enables a dynamic analysis of the web with various material properties and parameters. The governing equations of motion for the i-th plate segment were derived in [13] and have the following form:

$$\begin{aligned}(2.1)\qquad &\rho_i h_i(-w_{i,tt} - 2cw_{i,xt} - c_{,t}w_{i,x} - c^2 w_{i,xx}) - \beta_i\, w_{i,t} - \beta_i\, c\, w_{i,x} + M_{xi,xx}\\ &+2M_{xyi,xy} + M_{yi,yy} + q_i + (N_{xi}w_{i,x})_{,x} + (N_{yi}w_{i,y})_{,y} + (N_{xyi}w_{i,x})_{,y}\\ &+(N_{xyi}w_{i,y})_{,x} = 0,\end{aligned}$$

$$\begin{aligned}(2.2)\qquad &\rho_i h_i(-u_{i,tt} - c_{,t} - 2cu_{i,xt} - c_{,t}u_{i,x} - c^2 u_{i,xx}) - \beta_i\, c - \beta_i\, u_{i,t}\\ &- \beta_i\, c\, u_{i,x} + N_{xi,x} + N_{xyi,y} = 0,\end{aligned}$$

$$\begin{aligned}(2.3)\qquad &\rho_i h_i(-v_{i,tt} - 2cv_{i,xt} - c_{,t}v_{i,x} - c^2 v_{i,xx}) - \beta_i\, v_{i,t}\\ &- \beta_i\, c\, v_{i,x} + N_{xyi,x} + N_{yi,y} = 0.\end{aligned}$$

where: N_{xi}, N_{yi}, N_{xyi} – in-plane stress resultants for the i-th plate,
q_i – transverse loading of the i-th plate,
u_i, v_i, w_i – displacement components of the i-th plate middle surface,
β_i – damping coefficient,
ρ_i – mass density of the i-th plate.

Neglecting the phenomenon of elastic wave propagation in the $x - y$ plane for $c = 0$ and $t =$ const the governing equations (3.1) and (3.2) can be replaced by the nonlinear compatibility equation in the following form:

$$(2.4)\qquad \frac{1}{E_i\, h_i\, \eta_i}\left[\Phi_{i,xxxx} + \Phi_{i,xxyy}\left(\frac{E_i \eta_i}{G_i} - 2\,\nu_i\,\eta_i\right) + \eta_i\;\Phi_{i,yyyy}\right] = w^2_{i,xy} - w_{i,xx}\, w_{i,yy},$$

where: Φ – the Airy stress function, which satisfies the conditions

$$(2.5)\qquad N_{xi} = \Phi_{\mathrm{i,yy}};\quad N_{yi} = \Phi_{\mathrm{i,xx}};\quad N_{xyi} = -\Phi_{\mathrm{i,xy}}.$$

After substituting (2.5) into (2.1), the nonlinear mathematical model of the moving web system has the following form:

$$\begin{aligned}(2.6)\qquad &\rho_i h_i(-w_{i,tt} - 2cw_{i,xt} - c_{,t}w_{i,x} - c^2 w_{i,xx}) - \beta_i\, w_{i,t} - \beta_i\, c\, w_{i,x} + M_{xi,xx}\\ &+ 2M_{xyi,xy} + M_{yi,yy} + q_i + \Phi_{i,yy}w_{i,xx} - 2\,\Phi_{i,xy}\, w_{i,xy} + \Phi_{i,xx}\, w_{i,yy} = 0,\end{aligned}$$

$$(2.7)\qquad \frac{1}{E_i\, h_i\, \eta_i}\left[\Phi_{i,xxxx} + \Phi_{i,xxyy}\left(\frac{E_i \eta_i}{G_i} - 2\,\nu_i\,\eta_i\right) + \eta_i\;\Phi_{i,yyyy}\right] = w^2_{i,xy} - w_{i,xx}\, w_{i,yy}.$$

The kinematic and static continuity conditions at the junctions of adjacent plates are given in [13]. The exact dynamic solutions, satisfying the nonlinear, coupled equations (3.4) and (3.5) probably cannot be determined in a closed form. To solve this nonlinear model, in the paper [14] the numerical method of Unger's transition matrix and Godunov's orthogonalization procedure have been used. This approximate solution has been used in numerical investigations presented below.

3. Numerical results and discussion

Numerical investigations have been carried out for a steel web. The following parameters have been assumed: length l = 1m, width b = 0.2 m, thickness h = 0.0015 m, mass density ρ = 7800 kg/m^3, Young's modulus along the x - axis E_x= 0.2 10^{12}N/m^2, Young's modulus along the y - axis E_y= 0.167 10^{12}N/m^2, shear modulus G= 6.5 10^{10} N/m^2, Poisson's ratio ν = 0.3, orthotropy factor η = 0.836.

In numerical analysis, free vibrations of the web with two different initial stresses has been investigated. Both the constant (dashed) and parabolic (solid) axial stress plots are shown in Fig. 2.

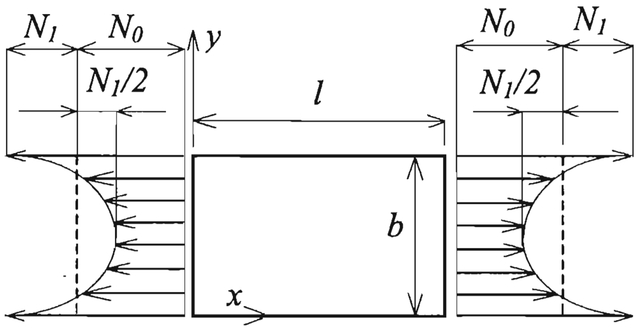

FIG. 2. Two kinds of axial stresses.

At first, the linearized undamped system with constant initial stress (N_0= 2500 N/m) was investigated. Figure 3 shows the modes of two lowest flexural eigenfrequencies (ω_{11} and ω_{21}) and two lowest torsional natural frequencies (ω_{12} and ω_{22}).

Let σ and ω denote the real part and the imaginary part of the eigenvalues, respectively. The positive value of σ indicates instability of the system and ω is natural frequency of the web. To show dynamic behaviour of the web, the lowest three flexural (solid line) and two torsional (dotted line) natural frequencies versus the transport velocity are shown in Fig. 4.

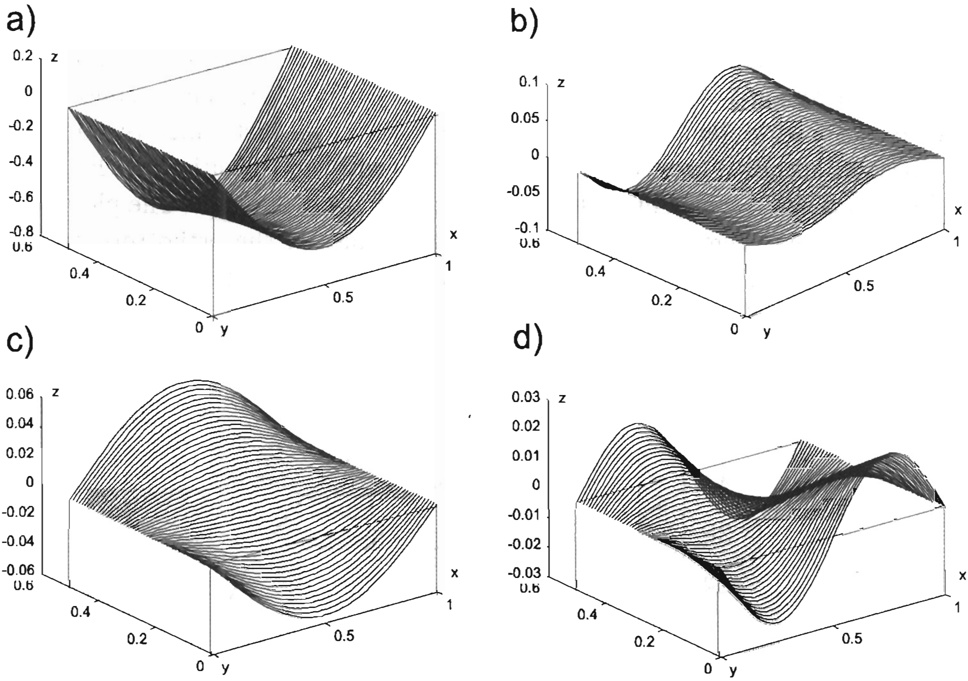

FIG. 3. Non-trivial equilibrium positions of axially moving web: a) ω_{11}, b) ω_{21}, c) ω_{12}, d) ω_{22}.

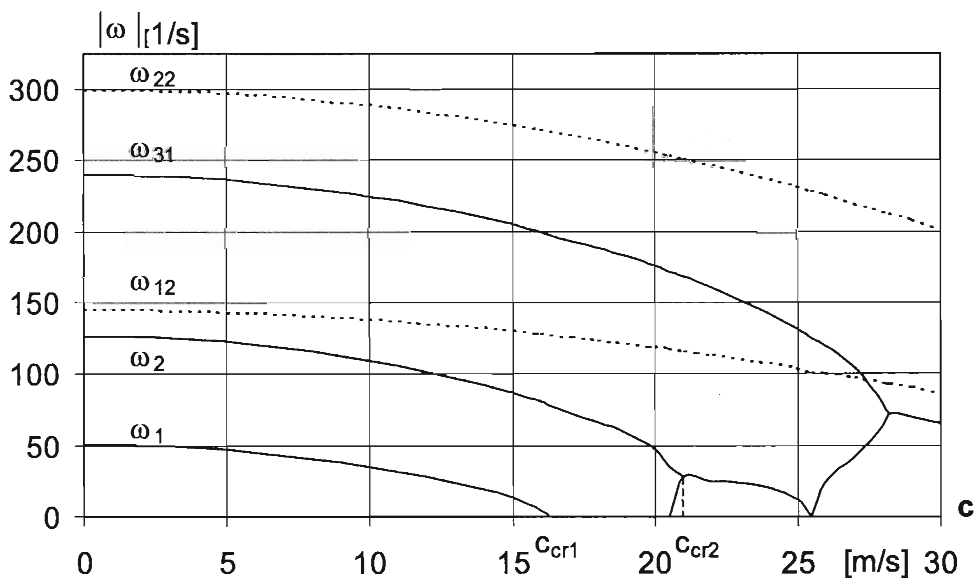

FIG. 4. Natural frequencies (N_0=2500 N/m).

In supercritical transport speeds ($c > c_{cr1}$), at first the web experiences a divergent instability (the fundamental mode with non-zero σ and zero ω), and next a flutter instability (non-zero σ and non-zero ω). The second critical transport speed is denoted by c_{cr2}. Between these two instability regions there is a second stability area where $\sigma = 0$. The width and position of the second stable region are dependent on the orthotropy factor of the web. The plots of the lowest transverse eigenfrequencies for various values of the orthotropy factor η are shown, in Fig. 5 – in undercritical, and in Fig. 6 – in supercritical transport speeds, respectively.

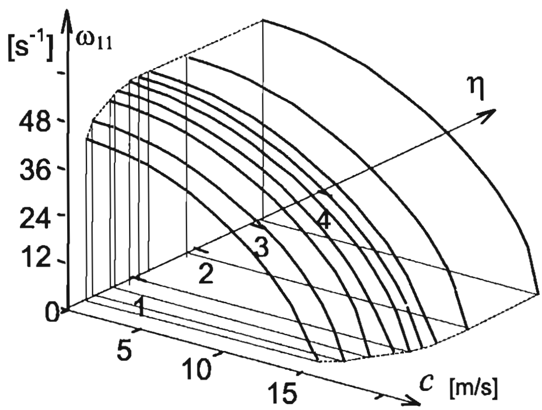

FIG. 5. Lowest flexural eigenfreqency (N_0 = const).

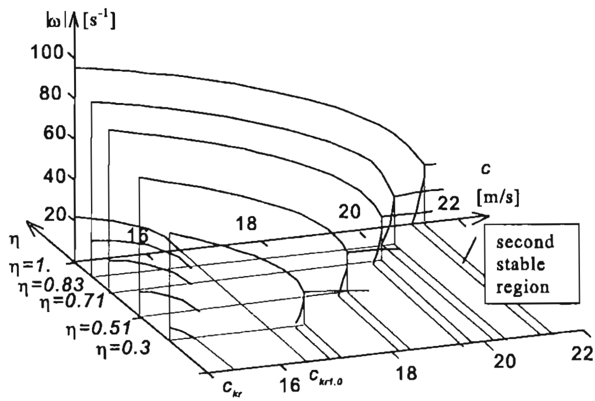

FIG. 6. Lowest flexural eigenfrequencies (N_0 = const).

Furthermore, the linearized undamped system with parabolic initial stress, which is shown in Fig. 2, was investigated. The lowest four flexural and torsional natural frequencies as functions of the transport velocity for constant and parabolic initial stresses are shown in Fig. 7 and Fig. 8, respectively. Parabolic initial stress shifts the torsional natural frequencies towards higher transport velocities.

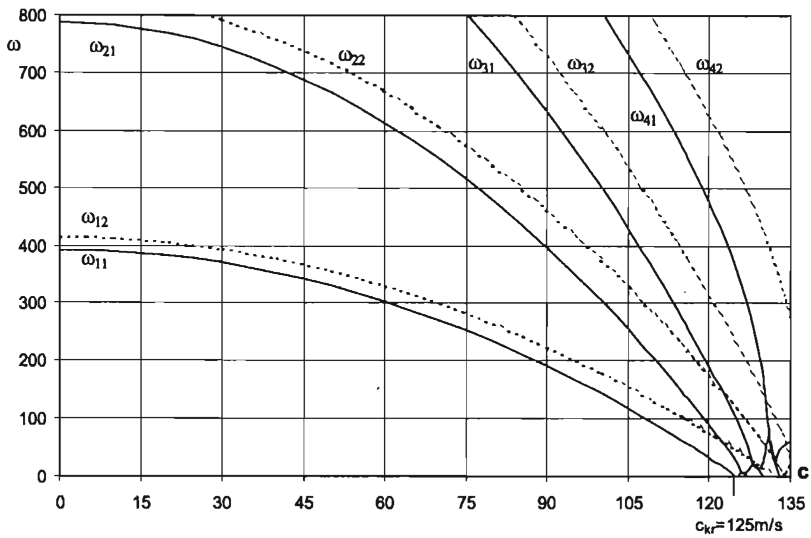

FIG. 7. Flexural (–) and torsional (- -) natural frequencies ($N_0 = 182 \times 10^3$ N/m, $N_1 = 0$).

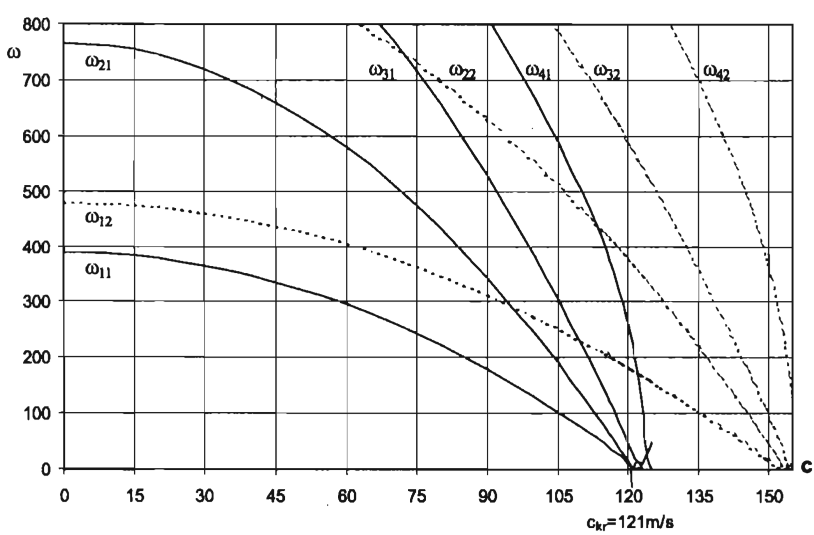

FIG. 8. Flexural (–) and torsional (- -) natural frequencies ($N_0 = 182 \times 10^3$ N/m, $N_1 = 182 \times 10^3$ N/m).

Next the nonlinear undamped model with $N_0 = 2500$ N/m $= const$ was investigated. Numerical results in the form of the phase and transverse vibrations diagrams are shown in Figs. 9, 10 and 11. In these figures the quantity q_1 represents the first generalized coordinate of flexural vibrations, and q_2 represents its

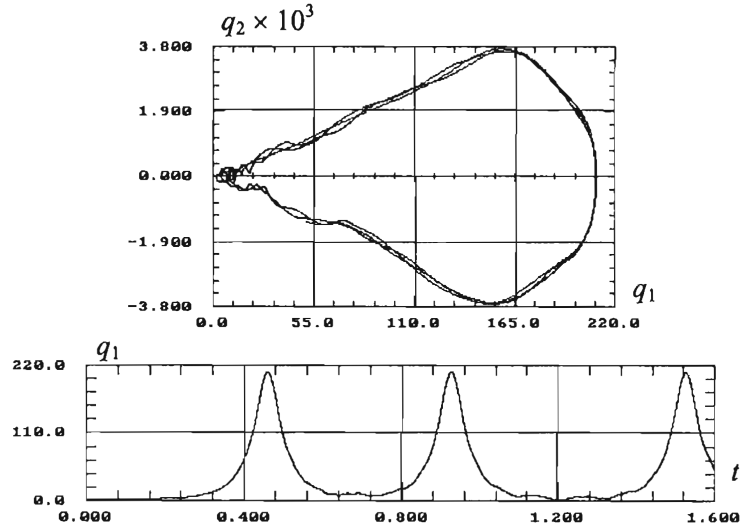

FIG. 9. Phase and vibration plots (c =20 m/s).

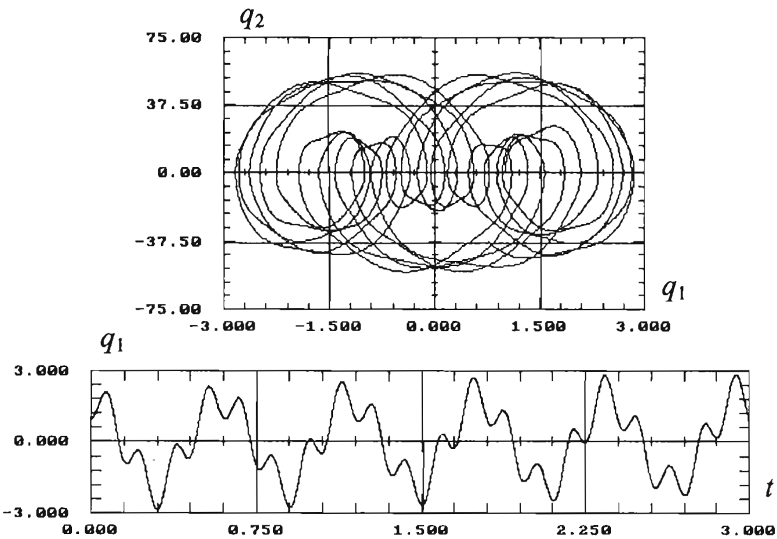

FIG. 10. Phase and vibration plots (c =20.5 m/s).

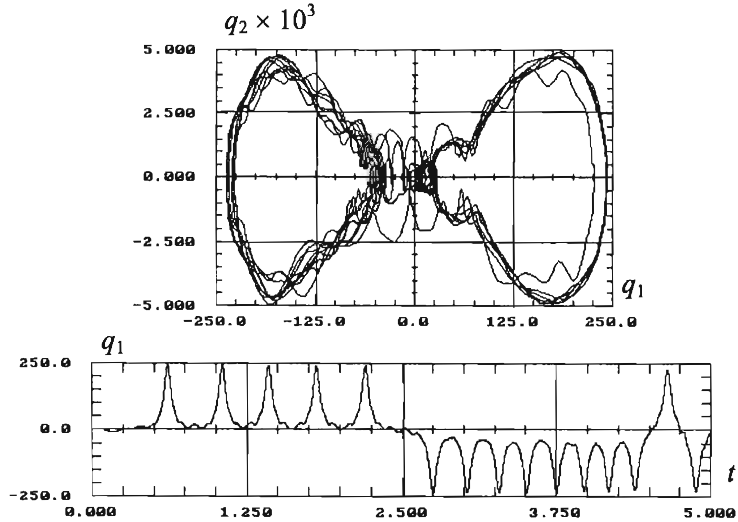

FIG. 11. Phase and vibration plots (c =21.1 m/s).

time derivative. In divergence instability region of the linearized system one can observe nonlinear vibrations which characterize the stable limit cycle (Fig. 9). In the second stable region of linearized system, the local motion between the coexisting equilibrium positions occurs (Fig. 10). Above this area, the nonlinear system experiences a global motion between new equilibrium positions with stable limit cycles (Fig. 11).

4. CONCLUSIONS

In the paper, nonlinear flexural and torsional free vibration of an axially moving web with two different axial stresses were analyzed. The nonlinear equation of motion was derived on the basis of the thin plate model of axially moving web. Numerical investigations have been carried out for steel web. The linearized mathematical model analysis shows in the subcritical region of transport speed for the constant axial tension of the web, the lowest transverse natural frequency decreases during the axial velocity increase. At the first critical transport speed the fundamental eigenfrequency vanishes indicating divergence instability.

In the supercritical region of transport speed, at first the web experiences the divergent instability and next the flutter instability above the second critical speed. The second stable region of the linearized system may appear above the first critical transport speed. The critical transport speed value and the position of the second stability region are dependent on the orthotropy factor of the web. In the range $\eta < 1$, the decreasing of the orthotropy factor diminishes the critical axial speed.

Dynamic analysis of the linearized system with a parabolic initial stress, was investigated as well. This form of initial stress moves the torsional natural frequencies towards higher transport speeds.

Dynamic analysis of the nonlinear system with constant axial stress shows in supercritical transport speed region that the non-trivial equilibrium positions bifurcate from the straight configuration of the web, and global motion between coexisting equilibrium positions occurs.

5. Acknowledgement

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