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THE SUBSTANTIAL BALANCE OF INTERNAL ENERGY (IE) FOR EXTENSIVE QUANTITIES AS VIEWED BY THE BALANCE THEORY

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The article presents the balance axiom and the basic balance leading to the referential balances of a scalar and vector extensive quantity (EQ). Next the substantial balance of internal energy (IE) has been done, defining the production part which is the sum of IE creation and annihilation, as well as the exchange part which is the sum of IE inflow and outflow through the boundaries of a substantial system. The substantial balance of internal energy (IE) achieved in this way has been reduced to the balance under the conditions of a reversible change, to obtain finally the balance of IE, commonly referred to as the first principle of thermodynamics for extensive quantities (EQs).

1. The balance axiom and the elementary balance of extensive quantities (EQs)

Before formulating the balance axiom, we will define the notion of EQ production and transfer. The production is the result of two processes: those of EQ creation (generation) and EQ annihilation (ebbing away). EQ creation is always greater than zero but its annihilation – always smaller. EQ production, however, can be either greater or smaller than zero, depending on which of the production elements is dominant. Thus we can write the following equation:

$$\text{creation} > 0 \; + \; \text{annihilation} < 0 = \text{production} \gtrless 0 \,.$$

Similarly, the exchange is the result of two processes, namely, those of EQ inflow and outflow. EQ inflow is always positive but EQ outflow – negative. EQ exchange, however, can be either positive or negative, depending on which of the exchange elements is dominant. So we can put it down as follows:

$$\text{inflow} > 0 \; + \; \text{outflow} < \text{exchange} \gtrless 0 \,.$$

With the above considerations in mind, we can now formulate the balance axiom of extensive quantities (EQs). The balance axiom says that a change in the extensive quantities (EQs) stored in the balance system can be made only either as a result of EQ production within the balancing system or as a result of EQ exchange through the boundaries of the balance system, or as a result of the two processes combined.

In the next part of our discussion, we will relate the quantities in question to generalised motion, i.e. referential velocity $\overline{w}$. The referential elements of volume, surface, line will be distinguished by an arrow symbol $\to \overline{w}$ below the proper letter.

Figure 1 will be helpful in formulating the basic balance for EQ, as it describes the referential balance system $\underset{\to\overline{w}}{V}$

Inside the referential balance system embedded in the exterior, limited by the referential surface $\underset{\to\overline{w}}{S}$ creation and annihilation take place, and so do the EQ inflow and outflow from the balance system.

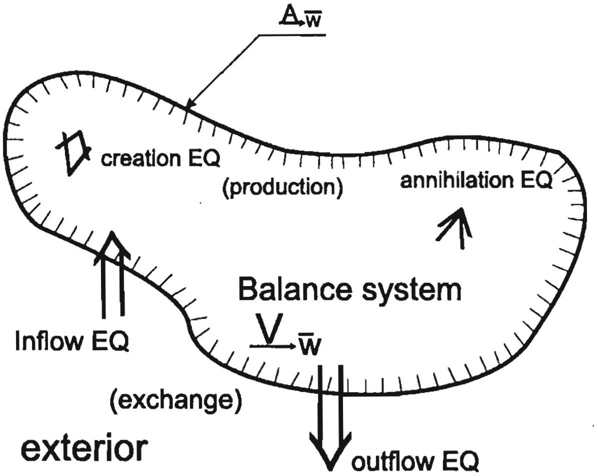

FIG. 1. Referential balance system.

Considering the balance axiom formulated above, we will now write the basic balance for EQs.

$$\left\{\begin{array}{l}\text{Exchange}\\ \text{of EQs storage}\\ \text{storedinsystem}\end{array}\right\} = \left\{\begin{array}{l}\text{Production of}\\ \text{EQ storage}\\ \text{withinsystem}\end{array}\right\} + \left\{\begin{array}{l}\text{Exchange EQ storage}\\ \text{between system and exterior}\\ \text{through system boundaries}\end{array}\right\}$$

Denoting by capital letters the following:

S, $\overline{S}$ – storage of a scalar and vector EQ, respectively,
P, $\overline{P}$ – production velocity of a scalar and vector EQ,
τ, $\overline{\tau}$ – exchange velocity of a scalar and vector EQ, respectively,

we will obtain a differential equation of the basic balance for a scalar EQ

$$dS = Pdt + \tau dt \tag{1.1}$$

and for a vector EQ

$$d\overline{S} = \overline{P}dt + \overline{\tau}dt\,. \tag{1.2}$$

The individual elements of the above balances denote:

dS, $d\overline{S}$ – storage exchange of a scalar and vector EQ, respectively,
Pdt, $\overline{P}dt$ – storage production of a scalar and vector EQ, respectively,
τdt, $\overline{\tau}dt$ – storage exchange of a scalar and vector EQ, respectively.

The storage (S, $\overline{S}$) and storage production (Pdt, $\overline{P}dt$) are non-referential in character. However, storage change (dS, $d\overline{S}$) becomes a referential increment.

Storage exchange ($\tau\, dt$, $\overline{\tau}\, dt$) is also a referential increment. So the basic balances for a scalar and a vector EQ will look respectively as follows:

$$\underbrace{\underbrace{\underset{\longrightarrow \overline{w}}{dS}}_{storage\ change} = \underbrace{Pdt}_{storage\ production} + \underbrace{\underset{\rightarrow \overline{w}}{\tau}\, dt}_{storage\ exchange}}_{of\ a\ scalar\ EQ} \tag{1.3}$$

$$\underbrace{\underbrace{\underset{\longrightarrow \overline{w}}{d\overline{S}}}_{storage\ change} = \underbrace{\overline{P}dt}_{storage\ production} + \underbrace{\underset{\rightarrow \overline{w}}{\overline{\tau}}\, dt}_{storage\ transfer}}_{of\ a\ vector\ EQ} \tag{1.4}$$

Dividing both sides of the two balances by time differential dt, we will obtain respectively, a referential balance of a scalar EQ

$$\underbrace{\underbrace{\frac{dS}{dt}\overline{w}}_{\substack{velocity\ of\\ storage\ change}} = \underbrace{P}_{\substack{velocity\ of\\ storage\ production}} + \underbrace{\underset{\rightarrow \overline{w}}{\tau}}_{\substack{velocity\ of\\ storage\ exchange}}}_{of\ a\ scalar\ EQ} \tag{1.5}$$

and a referential balance of a vector EQ

$$(1.6)\qquad \underbrace{\underbrace{\frac{d\overline{S}}{dt}_{\overline{w}}}_{\substack{\textit{velocity of}\\ \textit{storage change}}} = \underbrace{\overline{P}}_{\substack{\textit{velocity of}\\ \textit{storage production}}} + \underbrace{\overline{\tau}_{\to\overline{w}}}_{\substack{\textit{velocity of}\\ \textit{storage exchange}}}}_{\textit{of a vector EQ}}$$

2. The substantial balance of internal energy (IE) storage

Internal energy (IE) is a scalar extensive quantity (EQ), whose storage change is described by dependence (1.3) being the referential balance of a scalar extensive quantity (EQ). In contrast, for instance, to the energy of an electromagnetic field, the seat of internal energy is a substance. If we identify referential velocity $\overline{w}$ with substantial velocity $\overline{u}$ related to a substantial area within which every point moves in the same way as the substance, i.e. the substantial velocity of a local and momentary mass centre

$$(2.1)\qquad \overline{w} = \overline{u}\,,$$

then the referential balance of a scalar EQ described by dependence (1.3) will assume the form of the substantial balance of a scalar EQ.

$$(2.2)\qquad dS_{\longrightarrow\overline{u}} = P dt + \tau_{\to\overline{u}} dt\,.$$

The substantial balance of a scalar EQ is characterised by the fact that the substantial surface is impervious to the mass flow. Thus the substantial balance is a balance with a constant mass storage i.e. it is related to a closed system. Figure 2 shows substantial area $V_{\to\overline{u}}$ limited by substantial surface $S_{\to\overline{u}}$ and on this basis we will illustrate the substantial balance of internal energy for a closed system.

Following the substantial balance of an extensive quantity (EQ) described by relation (2.2), we will perform the balance of internal energy (IE) in a substantial system, identifying the scalar extensive quantity (EQ) with internal energy (IE). Thus we can write a substantial change of the storage of a scalar EQ as a substantial change of the IE storage. The left-hand side of balance (2.2) will be as follows:

$$(2.3)\qquad dS_{\longrightarrow\overline{u}} = dE_I\,.$$

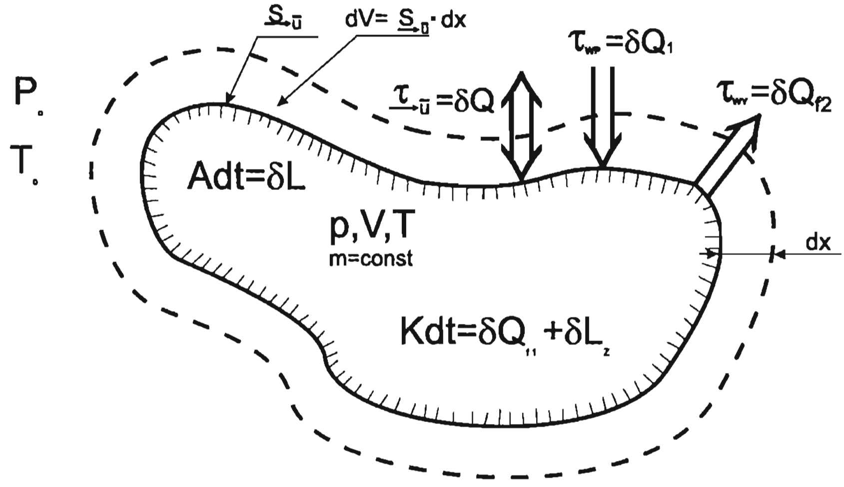

FIG. 2. Schematic diagram of the substantial balance of internal energy.

The production of a scalar EQ storage within a substantial system is equal to the sum of the creation of a scalar EQ storage and the annihilation of a scalar EQ storage. Thus we have

$$Pdt = Kdt + Adt \tag{2.4}$$

where:

K	–	creation velocity of a scalar EQ storage,
A	–	annihilation velocity of a scalar EQ storage,
Kdt	–	creation of a scalar EQ storage,
Adt	–	annihilation of a scalar EQ storage.

The production of internal energy (IE) stored in a substantial system in accordance with relation (7) will be presented as the sum of the creation and annihilation parts of EQ storage. The creation of EQ storage results from the quantity of friction heat created in the system, and the storage of work done by generalised forces and generalised coordinates. The part of the created friction heat which remains in the system influences the change in the EQ storage stored in this system. The basic heat increment created in the system is defined by the relation

$$\delta Q_f = \delta Q_{f1} + \delta Q_{f2} \tag{2.5}$$

where:

δQ_{f1}	–	basic increment of friction heat within a substantial balance area,
δQ_{f2}	–	basic increment of friction heat flowing out of the system into the exterior through the substantial surface.

Next we will define the basic increment of work storage created by generalised forces

$$\delta L_z = \sum_{\alpha=1}^{\gamma+1} \left(\overline{F}_\alpha \cdot d\overline{X}_\alpha\right). \tag{2.6}$$

In the above relation, $\overline{F}_\alpha$ denotes a generalised force, and $\overline{X}_\alpha$ – a generalised coordinate. The basic increment of external work storage, defined by relation (9), is the second component of the EQ creation part in a substantial system. Thus we can express the creation part by the following relation:

$$K dt = \delta Q_{f_1} + \delta Q_{f_2} + \delta L_z = \delta Q_f + \delta L_z . \tag{2.7}$$

The annihilation part of internal energy storage brings about its conversion into absolute work storage. Thus the annihilation part which by definition is smaller than zero is defined by the following dependence:

$$A dt = -\delta L \tag{2.8}$$

The production of internal energy (IE) storage within the substantial balance system, in accordance with relations (7), (10) and (11), is expressed by the relation

$$P dt = \delta Q_f + \delta L_z - \delta L . \tag{2.9}$$

The exchange of a scalar EQ storage through the substantial $\underset{\to\overline{u}}{S}$ surface of substantial area $\underset{\to\overline{u}}{V}$ is equal to the sum of the inflow of a scalar EQ from the exterior to the system and the outflow of a scalar EQ from the system to the exterior.

$$\underset{\to\overline{u}}{\tau} dt = \underset{\longrightarrow\overline{u}}{\tau_{WP}} \cdot dt + \underset{\longrightarrow\overline{u}}{\tau_{WY}} dt , \tag{2.10}$$

where:

- $\underset{\longrightarrow\overline{u}}{\tau_{WP}}$ – the inflow stream of a scalar EQ storage,
- $\underset{\longrightarrow\overline{u}}{\tau_{WY}}$ – the outflow stream of a scalar EQ storage,
- $\underset{\longrightarrow\overline{u}}{\tau_{WP}}$ – the inflow of a scalar EQ storage,
- $\underset{\longrightarrow\overline{u}}{\tau_{WY}}$ – the outflow of a scalar EQ storage.

Identifying the extensive quantity (EQ) in the substantial balance of a scalar extensive quantity storage (2.2) with internal energy (IE), we will define the exchange elements of internal energy storage. The inflow of heat through the substantial surface to the substantial balance system equals

$$\underset{\longrightarrow\overline{u}}{\tau_{WP}} \cdot dt = \delta Q_1, \tag{2.11}$$

where δQ_1 is the inflow of a basic increment of heat through the substantial surface.

The outflow of heat through the substantial surface $\underset{\rightarrow\overline{u}}{S}$ from the balance system is equal to

$$\underset{\longrightarrow\overline{u}}{\tau_{WP}} \cdot dt = -\delta Q_2 \tag{2.12}$$

Thus, in accordance with relation (2.10) and after the consideration of results (2.11) and (2.12), the exchange of internal energy (IE) storage through the substantial surface equals

$$\underset{\longrightarrow\overline{u}}{Tdt} = \delta Q_1 - \delta Q_2\,. \tag{2.13}$$

The difference between the basic increment of heat δQ_1 flowing in through the substantial surface to the substantial balance system and the basic increment of heat δQ_{f_2} flowing out of the substantial balance system, constitutes the value δQ of the basic heat transferred between the system and the exterior.

$$\delta Q = \delta Q_1 - \delta Q_{f_2}\,. \tag{2.14}$$

Hence according to (2.13) and after considering (20), the exchange of internal energy (IE) storage through the substantial surface will be as follows:

$$\underset{\rightarrow\overline{u}}{\tau} \cdot dt = -\delta Q\,. \tag{2.15}$$

Taking account of relations (2.3), (12) and (2.15) in the referential balance of a scalar extensive quantity (EQ) (2.2), we will obtain the equation defining the balance of internal energy (IE) for a substantial system.

$$\underbrace{\underbrace{dE_I}_{change} = \underbrace{\delta Q_f - \delta L + \delta L_z}_{production} + \underbrace{dQ}_{exchange}}_{of\ IE\ storage}\,. \tag{2.16}$$

Considering the definition of basic work,

$$\delta L_{el} = \delta L - \delta L_z\,, \tag{2.17}$$

the balance of internal energy storage for a substantial system will be as follows:

$$\underbrace{\underbrace{dE_I}_{change} = \underbrace{\delta Q_f - \delta L_{el}}_{production} + \underbrace{dQ}_{exchange}}_{of\ IE\ storage} . \tag{2.18}$$

On the basis of the above balance we can formulate the theorem of the substantial balance of internal energy (IE). It says that a basic change in the internal energy stored in a substantial system is caused, firstly, by its production within the system, which is equal to the difference between basic increments of friction heat and basic work, and secondly, by the transfer through system's boundaries, which is equal to the basic increments of heat exchanged between the system and the exterior.

Taking account of the definition of absolute work in balance (2.16)

$$\delta L = pdV \tag{2.19}$$

and of external work defined by relation (9), we will obtain the substantial balance of IE storage which can be transcribed in an extended form as

$$\underbrace{\underbrace{dE_I}_{change} = \underbrace{\delta Q_f - pdV + \sum_{\alpha=1}^{\gamma+1} \left(\overline{F}_\alpha \cdot d\overline{X}_\alpha\right)}_{production} + \underbrace{dQ}_{exchange}}_{of\ IE\ storage} . \tag{2.20}$$

Making the assumption that the basic increment of external work equals zero,

$$\delta L_z = 0 \tag{2.21}$$

that is to say, assuming that there is no influence of the exterior on the system, we will obtain the most popular form of the substantial balance of internal energy storage,

$$\underbrace{\underbrace{dE_I}_{change} = \underbrace{\delta Q_f - pdV}_{production} + \underbrace{dQ}_{exchange}}_{of\ IE\ storage} . \tag{2.22}$$

3. Formulation of the first principle of thermodynamics for extensive quantities (EQs)

The substantial balance of internal energy constitutes a basis for formulating the first principle of thermodynamics. The first principle of thermodynamics is the substantial balance of internal energy carried out under the conditions of a reversible thermodynamic process, that is to say, under the conditions of a thermodynamic equilibrium of the process which goes on without friction,

$$\delta Q_f = 0\,. \tag{3.1}$$

Thus in compliance with equation (2.18) and condition (32), the substantial balance of internal energy under the conditions of a reversible transfer, i.e. the first principle of thermodynamics, will be of the following form:

$$\underbrace{\underbrace{dE_I}_{change} = \underbrace{-\delta L_{el}}_{production} + \underbrace{\delta Q}_{exchange}}_{of\ IE\ storage}\ . \tag{3.2}$$

On the basis of the above balance, we can formulate the theorem called the first principle of thermodynamics which says that the exchange of internal energy (IE) stored in a substantial system, under the conditions of a reversible thermodynamic process, is caused, firstly, by the production equal to the basic increment of basic work taken with a minus sign, and secondly, by the exchange equal to the basic increment of heat exchanged between the system and the exterior, through the system boundaries. Taking account of the relations defining basic work (23), absolute work (29) and external work (9) in balance (3.2), we will obtain the balance describing the first principle of thermodynamics in an extended form

$$\underbrace{\underbrace{dE_I}_{change} = \underbrace{-pdV + \sum_{\alpha=1}^{\gamma+1} \left(\overline{F}_\alpha \cdot d\overline{X}_\alpha\right)}_{production} + \underbrace{dQ}_{exchange}}_{of\ IE\ storage}\ . \tag{3.3}$$

Making the assumption that the basic increment of external work is equal to zero (30), that is assuming that there is no external influence on the system, we will obtain the most popular form of the first principle of thermodynamics

$$\underbrace{\underbrace{dE_I}_{change} = \underbrace{-pdV}_{production} + \underbrace{\delta Q}_{exchange}}_{of\ IE\ storage}\ . \tag{3.4}$$

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