ENGINEERING TRANSACTIONS • *Engng. Trans.* • **51**, 1, 47–61, 2003
Polish Academy of Sciences • Institute of Fundamental Technological Research
10.24423/engtrans.500.2003

OPTIMISATION REGARDING LOCAL STABILITY AND STRENGTH OF LONG MULTILAYERED PLATES SUBJECTED TO COMPRESSION

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In this paper the method of optimal selection of orthotropic layers thickness is presented. The considerations concern long rectangular plates subjected to uniform compression in the direction of longer edges. Such plates (or cylindrical panels) of increased local buckling resistance could be applied as components of thin-walled columns of closed cross-sections.

1. Introduction

In many modern structures their light weight together with high carrying capacity is one of the fundamental demands. The designers know that this is the feature of thin-walled structures, in which the high strength properties of a material can be better exploited. However, safe work of these structures could be threatened by the possibility of buckling, often at small effective stress. Particularly, this concerns a thin element such as plate or shell subjected to compression.

In long thin-walled columns, many different buckling modes can occur, such as global flexural buckling, torsional or flexural-torsional one, local buckling, lateral buckling. As the effect of interaction of two (or more) buckling modes mentioned above, the interactive buckling can also appear [5, 6]. Therefore, dimensioning of sub-components in thin-walled structures should be determined by stability constraints in addition to strength and stiffness constraints. The designers should look for such thin-walled structures, which - at the same weight – will exhibit greater buckling resistance.

Nowadays, with progress in the materials technology, these plates (or panels) can be manufactured as solid composite or multi-layered (sandwich and laminated) ones. Many works has been devoted to the problem of reducing the structural weight of sandwich members (e.g. [4, 10]). Despite the gradual growth of interest in the optimisation of composite structures, the number of available investigations is small in comparison with the literature concerning the optimisation of isotropic structures. The problem of maximising the failure loads for composite rectangular plates and cylindrical shells subjected to compression was investigated in papers [8, 9].

In long compressed elements (multi-layered plates or cylindrical panels) made of orthotropic (composite) materials of high orthotropy ratio, there are also possibilities of increasing the buckling resistance at a constant weight (constant plate thickness). As it is known, the high orthotropy ratio is characteristic for fibrous composites stiffened in one direction with glass, kevlar, spectra or thornel fibres. Extensive review of mechanical properties, thermal expansion coefficients, fracture and strength data for advanced composite materials can be found in [1, 2].

The local buckling stress of long rectangular orthotropic plates (single-layered) with principal axes of orthotropy parallel to the plates edges, subjected to compression in the direction of longer edges, depends in a significant way ([3, 11]) on the square root of the product of plate bending stiffness in the principal directions of orthotropy ($\sqrt{D_1 \cdot D_2}$). The increase of $\sqrt{D_1 \cdot D_2}$ value in the single-layered plate, made of the orthotropic material of given elastic properties ($E_1, E_2, \nu_{12}, \nu_{21}$), is only possible when the plate thickness increases, and so does the weight.

In case of multilayered plates, made of the same material and of the same thickness as a single-layered plate, the possibility of increasing the $\sqrt{D_1 \cdot D_2}$ value (thus the buckling stress value) exists, if the thickness of each layer and the arrangement of layers (e.g. the principal axes of orthotropy in neighbouring layers are inclined by 0 and 90 degrees to the plate axes) are properly selected.

In this work the method of optimal selection of arrangement and of thickness of orthotropic layers in a long rectangular plate, subjected to compression, is presented. This optimal selection is conducted regarding local stability and strength of a multilayered plate.

2. Formulation of the problem

Let us consider a rectangular multilayered plate of a length l much greater than its width b ($l >> b$). The plate of a total thickness t consists of an odd number of layers arranged symmetrically towards the plate thickness. Principally, in the analysed plates, all layers are made of the same orthotropic material, of high orthotropy ratio. Only the three-layered plate is treated more generally.

The principal axes of orthotropy of layer material are parallel to the plate edges. Under such assumption the principal directions of orthotropy in neighbouring layers have to be alternated at 90 degrees. The middle layer may be manufactured from the same material as the remaining ones or from the isotropic material. It should be mentioned that the application of isotropic layer makes sense only in a plate of 5 layers at least. In this case it can be the thin layer of a large value of tensile stiffness ($K_{11} = K_{22}$) or the slightly thicker layer, increasing plate bending stiffness, made of light material.

The arrangement of layers in the plate is shown in Fig.1.

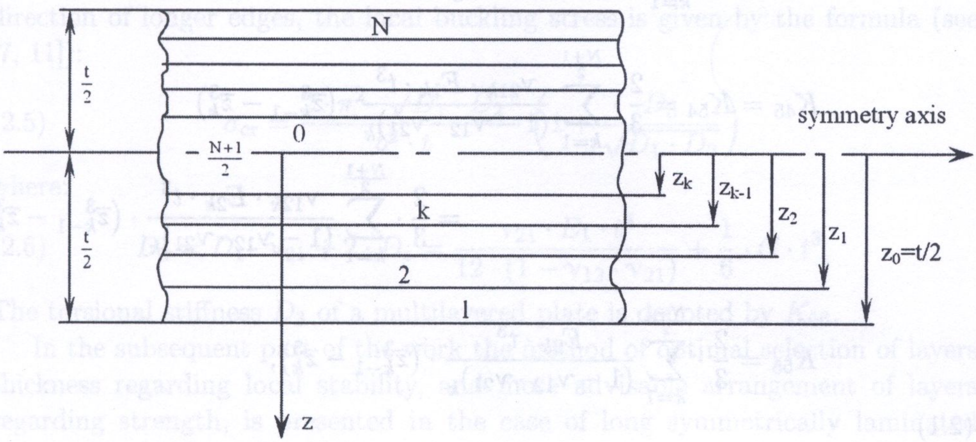

FIG. 1. Arrangement of layers.

For symmetrically laminated plates (the term "symmetrically laminated" refers to plates in which every lamina above the plate midplane has a corresponding lamina located at the same distance below the plate midplane, with the same thickness and material properties [3]), the stiffness coefficients K_{ij} are determined by the relationships resulting from the formulae given in the work of Jones [3]:

$$K_{11} = 2 \cdot \sum_{k=1}^{\frac{N+1}{2}} \frac{E_{1k} \cdot t}{(1 - \nu_{12} \cdot \nu_{21})_k} \cdot (\overline{z}_{k-1} - \overline{z}_k),$$

$$K_{12} = K_{21} = 2 \sum_{k=1}^{\frac{N+1}{2}} \frac{\nu_{21k} \cdot E_{2k} \cdot t}{(1 - \nu_{12} \cdot \nu_{21})_k} \cdot (\overline{z}_{k-1} - \overline{z}_k)$$

$$= 2 \cdot \sum_{k=1}^{\frac{N+1}{2}} \frac{\nu_{12k} \cdot E_{1k} \cdot t}{(1 - \nu_{12} \cdot \nu_{21})_k} \cdot (\overline{z}_{k-1} - \overline{z}_k),$$

$$K_{22} = 2 \cdot \sum_{k=1}^{\frac{N+1}{2}} \frac{E_{2k} \cdot t}{(1 - \nu_{12} \cdot \nu_{21})_k} \cdot (\overline{z}_{k-1} - \overline{z}_k),$$

$$K_{33} = 2 \cdot \sum_{k=1}^{\frac{N+1}{2}} G_k \cdot t \cdot (\overline{z}_{k-1} - \overline{z}_k),$$

$$K_{44} = \frac{2}{3} \sum_{k=1}^{\frac{N+1}{2}} \frac{E_{1k} \cdot t^3}{(1 - \nu_{12} \cdot \nu_{21})_k} \cdot (\overline{z}^3_{k-1} - \overline{z}^3_k),$$

$$K_{45} = K_{54} = \frac{2}{3} \cdot \sum_{k=1}^{\frac{N+1}{2}} \frac{\nu_{21k} \cdot E_{1k} \cdot t^3}{(1 - \nu_{12} \cdot \nu_{21})_k} \cdot (\overline{z}^3_{k-1} - \overline{z}^3_k)$$

$$= \frac{2}{3} \cdot \sum_{k=1}^{\frac{N+1}{2}} \frac{\nu_{12k} \cdot E_{2k} \cdot t^3}{(1 - \nu_{12} \cdot \nu_{21})_k} \cdot (\overline{z}^3_{k-1} - \overline{z}^3_k),$$

$$K_{55} = \frac{2}{3} \cdot \sum_{k=1}^{\frac{N+1}{2}} \frac{E_{2k} \cdot t^3}{(1 - \nu_{12} \cdot \nu_{21})_k} \cdot (\overline{z}^3_{k-1} - \overline{z}^3_k),$$

(2.1)

$$K_{66} = \frac{2}{3} \cdot \sum_{k=1}^{\frac{N+1}{2}} G_k \cdot t^3 \cdot (\overline{z}^3_{k-1} - \overline{z}^3_k),$$

where: N – number of plate layers,

E_{1k} – elastic modulus of k-th layer in the direction of compression (along the longer edge of a plate),

E_{2k} – elastic modulus of k-th layer in the direction perpendicular to the direction of compressive force,

ν_{12k}, ν_{21k} – corresponding Poisson's ratios of k-th layer,

$\overline{z}_k = \dfrac{z_k}{t}$ – dimensionless co-ordinate of k-th layer (see Fig.1),

$\overline{z}_{\frac{N+1}{2}} = 0$ (co-ordinate of a midplane).

For each orthotropic layer the following relation is fulfilled:

(2.2) $$E_{1k} \cdot \nu_{21k} = E_{2k} \cdot \nu_{12k}.$$

For a single-layered plate we have:

(2.3)
$$K_{44} = D_1 = \frac{E_1 \cdot t^3}{12 \cdot (1 - \nu_{12} \cdot \nu_{21})},$$
$$K_{55} = D_2 = \frac{E_2 \cdot t^3}{12 \cdot (1 - \nu_{12} \cdot \nu_{21})}.$$

If the neighbouring layers k and $k+1$ of a plate are made of the same orthotropic material and are inclined by 90 degrees, the relations between elastic moduli and Poisson's ratios, for these layers, are as follows:

$$E_{1k} = E_{2k+1}, \quad E_{2k} = E_{1k+1}, \quad \nu_{12k} = \nu_{21k+1}, \quad \nu_{21k} = \nu_{12k+1}. \tag{2.4}$$

For a long orthotropic plate (single-layered) subjected to compression in the direction of longer edges, the local buckling stress is given by the formula (see [7, 11]):

$$\sigma_{\mathrm{cr}} = \frac{2 \cdot \pi^2 \cdot \sqrt{D_1 \cdot D_2}}{b^2 \cdot t} \cdot \left(1 + \frac{D_3}{\sqrt{D_1 \cdot D_2}}\right) \tag{2.5}$$

where:

$$D_3 = D_1 \cdot \nu_{21} + 2 \cdot D_s = \frac{\nu_{21} \cdot E_1 \cdot t^3}{12 \cdot (1 - \nu_{12} \cdot \nu_{21})} + \frac{1}{6} \cdot G \cdot t^3. \tag{2.6}$$

The torsional stiffness D_3 of a multilayered plate is denoted by K_{66}.

In the subsequent part of the work the method of optimal selection of layers thickness regarding local stability, and more advisable arrangement of layers regarding strength, is presented in the case of long symmetrically laminated plate subjected to compression in the direction of longer edges.

3. Solution of the problem for symmetrically laminated plate

The bending stiffness coefficients K_{44}, K_{55} for multilayered plates of symmetric arrangement of orthotropic layers are given by formulae (2.1). In order to obtain – for considered plates - the largest value of local buckling stress (2.5), the parameters $\overline{z}_k$ of a plate $(0 \leq \overline{z}_k \leq 0,5)$ should be selected in such a way that the value $\sqrt{K_{44} \cdot K_{55}}$ reaches the maximum. The detailed analysis of multilayered plates made of the same orthotropic material shows that the maximum value of $\sqrt{K_{44} \cdot K_{55}}$ is reached when plates are "isotropic" in bending, it means when

$$K_{44} = K_{55}. \tag{3.1}$$

From the above condition the following relation is obtained:

$$\sum_{k=1}^{\frac{N+1}{2}} \frac{(E_{1k} - E_{2k}) \cdot t^3}{1 - \nu_{12k} \cdot \nu_{21k}} \cdot \left(\overline{z}_{k-1}^3 - \overline{z}_k^3\right) = 0, \tag{3.2}$$

where: $\overline{z}_0 = \dfrac{1}{2}$, $\overline{z}_{\frac{N+1}{2}} = 0$.

It can be noticed that for plates of large number of layers, the analysis becomes impossible (there is only one equation). Thus, further considerations are limited to plates of three, five, and seven layers.

3.1. Three-layered plate

In this case the analysis is simple because the layers thickness is determined by one parameter $\overline{z}_1 = \dfrac{z_1}{t}$, only. The plate is symmetrically laminated, so we have: $E_{11} = E_{13}$, $E_{21} = E_{23}$, $\nu_{211} = \nu_{213}$, $\nu_{121} = \nu_{123}$, $G_1 = G_3$.

Let us assume that the middle layer (k=2) and the exterior layers (k=1 and k=3) are made of different materials of orthotropy ratios:

$$\alpha_1 = \alpha_3 = \frac{E_{21}}{E_{11}}, \quad \alpha_2 = \frac{E_{12}}{E_{22}}. \tag{3.3}$$

The parameters β and γ, shortening the notation, are introduced

$$\beta = \frac{E_{22}}{E_{11}}, \quad \gamma = \frac{1 - \nu_{121} \cdot \nu_{211}}{1 - \nu_{122} \cdot \nu_{212}}, \tag{3.4}$$

which characterise the ratios of material properties of the middle layer to those of exterior layers. Usually, the parameter γ is close to unity.

From the condition $K_{44} = K_{55}$ we have:

$$\overline{z}_1 = \left(2 \cdot \sqrt[3]{1 + \frac{1 - \alpha_2}{1 - \alpha_1} \cdot \beta \cdot \gamma} \right)^{-1} . \tag{3.5}$$

If the considered plate consists of layers made of the same material (principal axes of orthotropy of the middle layer are inclined by 90 degrees to the exterior layer axes), the formula (3.5) takes a form:

$$\overline{z}_1 = \left(2 \cdot \sqrt[3]{2} \right)^{-1} \cong 0.4 . \tag{3.6}$$

Therefore the optimal plate, regarding its stability, consists of layers of thickness:
exterior layers $t_1 = t_3 \approx 0.1t$, middle layer $t_2 = 2z_1 \approx 0,8t$.

The buckling stress for a 3-layered plate, found from the relation (2.5), corresponds to the average stress uniformly distributed throughout the plate thickness, so the sectional critical force equals:

$$N_{\text{cr}} = \sigma_{\text{cr}}^{\text{av}} \cdot t. \tag{3.7}$$

The critical sectional force can be expressed by the dimensionless critical strain ε_{cr} and by equivalent elastic modulus E_z of a three-layered plate, in the direction of compression:

$$N_{\text{cr}} = E_z \cdot \varepsilon_{\text{cr}} \cdot t. \tag{3.8}$$

The critical force N_{cr} is carried out by all layers at the same strain ε_{cr}.

From the condition of equilibrium of sectional forces, the following relation is obtained:

$$2 \cdot E_{11} \cdot \varepsilon_{cr} \cdot t_1 + E_{12} \cdot \varepsilon_{cr} \cdot t_2 = E_z \cdot \varepsilon_{cr} \cdot t, \tag{3.9a}$$

or

$$E_{11} \cdot \varepsilon_{cr} \cdot (t - 2 \cdot z_1) + E_{12} \cdot \varepsilon_{cr} \cdot 2 \cdot z_1 = E_z \cdot \varepsilon_{cr} \cdot t. \tag{3.9b}$$

Using the expressions (3.9b) and (3.5), the formula for modulus E_z of a three-layered plate is found:

$$E_z = \left(1 - \frac{1 - \alpha_2 \cdot \beta}{\sqrt[3]{1 + \frac{1-\alpha_1}{1-\alpha_2} \cdot \beta \cdot \gamma}}\right) \cdot E_{11}. \tag{3.10}$$

When $\alpha_1 = \alpha_2 = \alpha, \beta = 1$ and γ=1, then:

$$E_z = \left(1 - \frac{1 - \alpha}{\sqrt[3]{2}}\right) \cdot E_{11}. \tag{3.11}$$

Knowing that

$$\varepsilon_{cr} = \frac{\sigma_{cr}}{E_z}, \tag{3.12}$$

one can determine the stresses in particular layer as:

$$\begin{aligned} \sigma_1 = \sigma_3 = E_{11} \cdot \varepsilon_{cr} = \frac{E_{11}}{E_z} \cdot \sigma_{cr}, \\ \sigma_2 = E_{12} \cdot \varepsilon_{cr} = \frac{E_{12}}{E_z} \cdot \sigma_{cr}. \end{aligned} \tag{3.13}$$

It follows from the above expressions, that the stresses in layers are inversely proportional to the equivalent modulus E_z, so the smallest stress corresponds to the largest value of E_z.

For a three-layered plate made of the same material ($\alpha_1 = \alpha_2 = \alpha$, $\beta = 1$, $\gamma = 1$ and $G_1 = G_2 = G$), the equivalent modulus (3.11) increases together with parameter α. Thus, the layers should be arranged in such a way that the parameter $\alpha > 1$, so:

$$\alpha_1 = \frac{E_{21}}{E_{11}} > 1 \Rightarrow E_{21} > E_{11}, \quad \alpha_2 = \frac{E_{12}}{E_{22}} > 1 \Rightarrow E_{12} > E_{22},$$

what means that in the exterior layers the elastic modulus in the direction perpendicular to the direction of compression should be larger than the elastic modulus in the direction of compression. In the thick middle layer ($t_2 = 0.8t$), the situation is opposite.

For a three-layered plate, in which the properties of layers are characterised by the following parameters: $\alpha_1 = \alpha_2 = \alpha, \gamma = 1, \beta = n\alpha$, the modulus E_z is described as:

$$E_z = \left(1 - \frac{1 - n \cdot \alpha^2}{\sqrt[3]{1 + n \cdot \alpha}}\right) \cdot E_{11}. \tag{3.14}$$

The modulus E_z reaches the extreme values when $\dfrac{dE_z}{d\alpha} = 0$. From this condition the quadratic equation is obtained:

$$(6 \cdot n - 1) \cdot n \cdot \alpha^2 + 6 \cdot n \cdot \alpha + 1 = 0, \tag{3.15}$$

the solution of which has the form:

$$\alpha = \frac{3 + \sqrt{3 + \frac{1}{n}}}{1 - 6 \cdot n}. \tag{3.16}$$

The parameter α has a physical meaning if $0 < n < \dfrac{1}{6}$.

The results of calculations of plate parameters, for $0.01 \leq n \leq 0.12$, are given in Table 1.

Table 1.

	n	$\alpha = \dfrac{E_{21}}{E_{11}}$	$\beta = n \cdot \alpha$	$\alpha \cdot \beta = \dfrac{E_{12}}{E_{11}}$	$\bar{E} = \dfrac{E_z}{E_{11}}$	$\bar{z}_1 = \dfrac{z_1}{t}$
1	0.01	13.998	0.1399	1.9567	1.9158	0.4786
2	0.02	11.682	0.2336	2.7293	2.6125	0.4662
3	0.03	11.009	0.3303	3.6362	3.4877	0.4546
4	0.04	10.910	0.4364	4.7610	4.3333	0.4431
5	0.05	11.137	0.5568	6.2015	5.4880	0.4314
6	0.06	11.617	0.6970	8.0969	6.9499	0.4192
7	0.07	12.341	0.8638	10.6605	8.8499	0.4063
8	0.0769	13.000	1.0000	13.0000	10.5244	0.3969
9	0.08	13.340	1.0672	14.2373	11.3913	0.3925
10	0.09	14.688	1.3219	19.4163	14.9077	0.3776
11	0.10	16.514	1.6514	27.2708	19.9808	0.3613
12	0.11	19.051	2.0956	39.9217	27.7062	0.3431
13	0.12	22.738	2.7285	62.0393	40.3639	0.3224

The optimal material parameters and the optimal thickness of layers for the considered plates are given in 8-th row of Table 1 ($n = 0.0769, \beta$=1). It shows that the optimal orthotropy ratio α equals 13, and the equivalent modulus $E_z \approx 10.5E_{11}$ at $E_{21} = 13E_{11}$. When the layers are properly arranged, the stresses in particular layers are contained in the intervals:

$$0.0248 \cdot \sigma_{cr}^{av} \leq \sigma_1 \leq 0.524 \cdot \sigma_{cr}^{av},$$
$$1.021 \cdot \sigma_{cr}^{av} \leq \sigma_2 \leq 1.537 \cdot \sigma_{cr}^{av}.$$

In case of a three-layered plate with an isotropic middle layer, the condition $K_{44} = K_{55}$ can be only fulfilled when the exterior layers are also isotropic.

3.2. Five-layered plate

In case of a 5-layered plate, two parameters $\overline{z}_1$ and $\overline{z}_2$ have to be known in order to determine the layers thickness. The relation connecting these parameters can be obtained from the condition $K_{44} = K_{55}$. For a plate in which all layers are made of the same orthotropic material (the second and the fourth layer are inclined by 90 degrees to the axes of remaining layers), the relation between parameters $\overline{z}_1$ and $\overline{z}_2$ has the following form:

$$\overline{z}_1 = \frac{1}{2\sqrt[3]{2}} \cdot \sqrt[3]{1 + 16 \cdot \overline{z}_2^3}. \tag{3.17}$$

The optimal thickness of layers "1" and "2", for a given value of middle layer thickness found from the formula (3.17), are presented in Fig. 2.

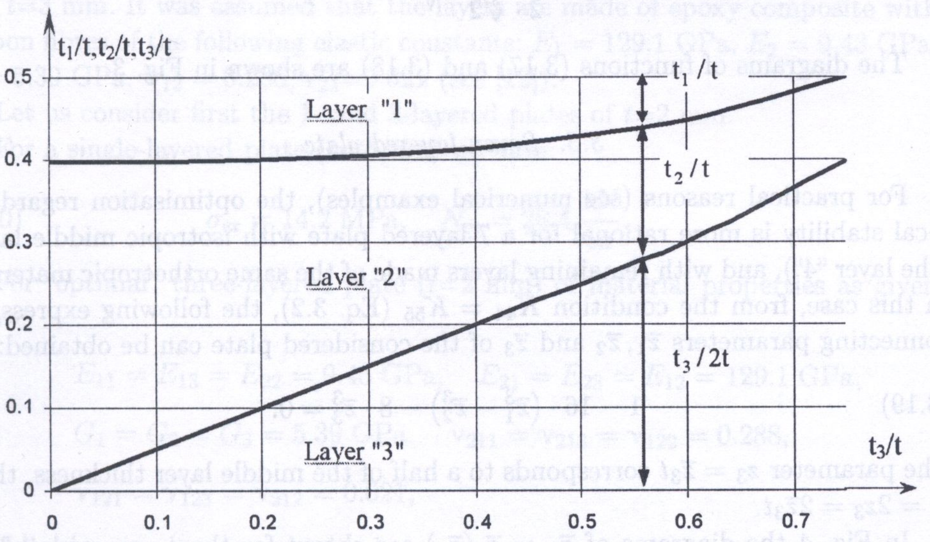

FIG. 2. Optimal thickness of layers – five-layered plate.

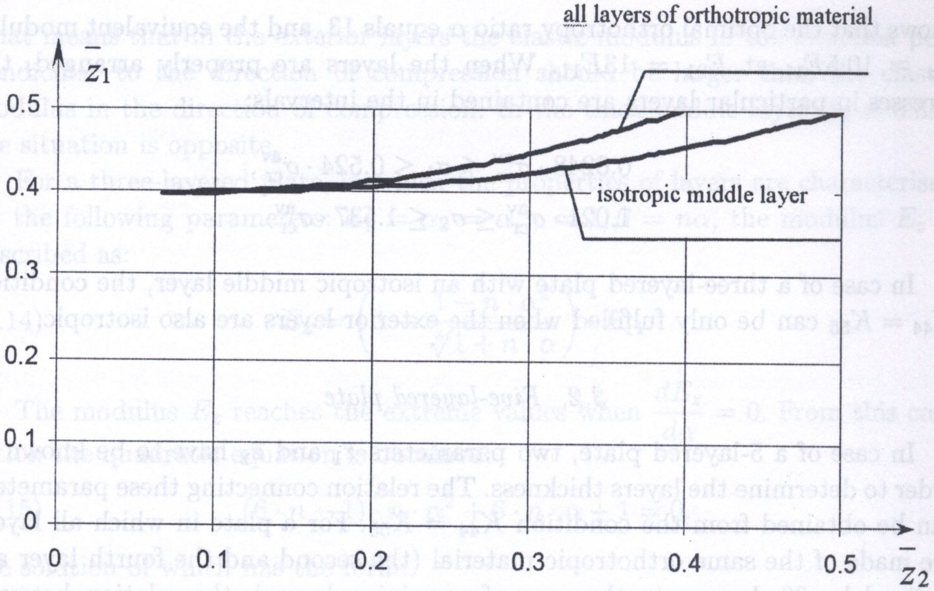

FIG. 3. Relation between parameters $\overline{z}_1$ and $\overline{z}_2$ described by Eq. (3.17) and Eq. (3.18).

When the middle layer of a 5-layered plate is isotropic, and the remaining layers are made of the same orthotropic material, the Eq. (3.17) becomes:

$$\overline{z}_1 = \frac{1}{2\cdot\sqrt[3]{2}}\cdot\sqrt[3]{1+8\cdot\overline{z}_2^3}. \tag{3.18}$$

The diagrams of functions (3.17) and (3.18) are shown in Fig. 3.

3.3. *Seven-layered plate*

For practical reasons (see numerical examples), the optimisation regarding local stability is more rational for a 7-layered plate with isotropic middle layer (the layer "4"), and with remaining layers made of the same orthotropic material. In this case, from the condition $K_{44} = K_{55}$ (Eq. 3.2), the following expression connecting parameters $\overline{z}_1, \overline{z}_2$ and $\overline{z}_3$ of the considered plate can be obtained:

$$1 - 16\cdot\left(\overline{z}_1^3 - \overline{z}_2^3\right) - 8\cdot\overline{z}_3^3 = 0. \tag{3.19}$$

The parameter $z_3 = \overline{z}_3 t$ corresponds to a half of the middle layer thickness, thus $t_4 = 2z_3 = 2\overline{z}_3 t$.

In Fig. 4 the diagrams of $\overline{z}_1 = \overline{z}_1(\overline{z}_2)$ are shown for three assumed values of the parameter $\overline{z}_3$=0.125, 0.25, and 0.375. Reading the diagrams, one should remember that $\overline{z}_1 \geq \overline{z}_2$ and $\overline{z}_2 \geq \overline{z}_3$.

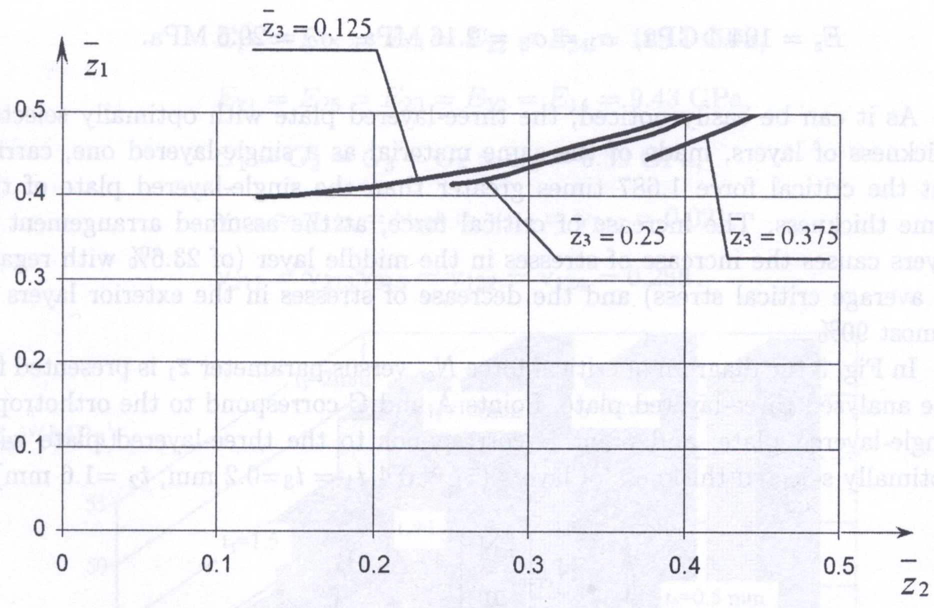

FIG. 4. Diagrams of $\bar{z}_1 = \bar{z}_1(\bar{z}_2)$.

3.4. Numerical examples

The calculations were conducted for plates built of 1, 3, 5 and 7 layers. The geometrical parameters were taken as follows: l=1500 mm, b=150 mm, t=2 mm and t=3 mm. It was assumed that the layers are made of epoxy composite with carbon fibres of the following elastic constants: $E_1 = 129.1$ GPa, $E_2 = 9.43$ GPa, $G = 5.39$ GPa, $\nu_{12} = 0.288, \nu_{21}=0.021$ (see [12]).

Let us consider first the 1- and 3-layered plates of t=2 mm.

For a single-layered plate (t=2 mm) we have:

$$\sigma_{\rm cr} = 14.2 \text{ MPa}, \quad N_{\rm cr} = 28.4 \, \frac{\text{kN}}{\text{m}}. \tag{3.20}$$

For "optimal" three-layered plate (t=2 mm) of material properties as given below:

$$E_{11} = E_{13} = E_{22} = 9.43 \text{ GPa}, \quad E_{21} = E_{23} = E_{12} = 129.1 \text{ GPa},$$

$$G_1 = G_2 = G_3 = 5.39 \text{ GPa}, \quad \nu_{211} = \nu_{213} = \nu_{122} = 0.288,$$

$$\nu_{121} = \nu_{123} = \nu_{212} = 0.021,$$

the following results were obtained:

$$\sigma_{\rm cr}^{\rm av} = 23.96 \text{ MPa}, \quad N_{\rm cr} = 47.92 \, \frac{\text{kN}}{\text{m}},$$

$$E_z = 104.4 \text{ GPa}, \quad \sigma_1 = \sigma_3 = 2.16 \text{ MPa}, \quad \sigma_2 = 29.6 \text{ MPa}.$$

As it can be easily noticed, the three-layered plate with optimally selected thickness of layers, made of the same material as a single-layered one, carries out the critical force 1.687 times greater than the single-layered plate of the same thickness. The increase of critical force, at the assumed arrangement of layers causes the increase of stresses in the middle layer (of 23.6% with regard to average critical stress) and the decrease of stresses in the exterior layers of almost 90% .

In Fig. 5 the diagram of critical force N_{cr} versus parameter $\overline{z}_1$ is presented for the analysed three-layered plate. Points A and C correspond to the orthotropic single-layered plate, and point B corresponds to the three-layered plate with optimally selected thickness of layers ($\overline{z}_1 \cong 0.4, t_1 = t_3$=0.2 mm, t_2 =1.6 mm).

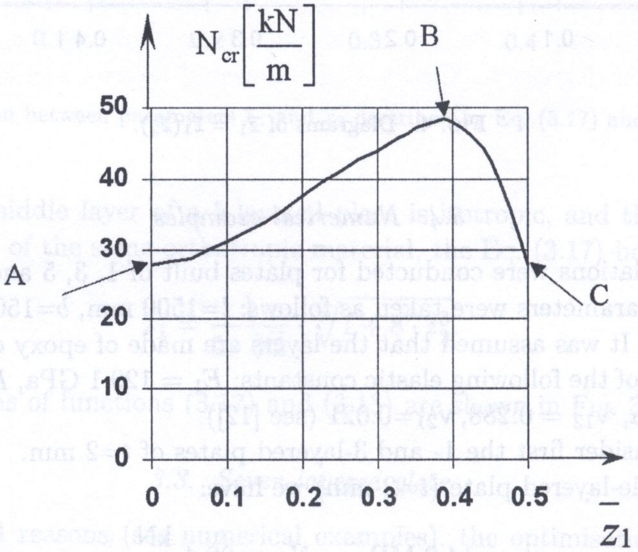

FIG. 5. Critical force N_{cr} versus parameter $\overline{z}_1$ for the analysed three-layered plate.

Let us assume that in the analysed, optimal (regarding local stability), three-layered plate the axes of orthotropy in all layers have been inclined by 90 degrees. Then the value of critical force remains unchanged but the value of equivalent elastic modulus E_z and values of layer stresses are not the same. This results in very unfavourable (regarding strength) increase of compressive stress in exterior layers – up to 378% of the average critical stress.

The values of average critical stress for five-layered plates of total thickness t=3 mm are shown in Fig. 6. The material properties of layers were taken as follows:

$$E_{11} = E_{15} = E_{13} = E_{22} = E_{24} = 129.1 \text{ GPa},$$

$$E_{21} = E_{25} = E_{23} = E_{12} = E_{14} = 9.43 \text{ GPa},$$

$$G_1 = G_2 = G_3 = G_4 = G_5 = 5.39 \text{ GPa},$$

$$\nu_{121} = \nu_{123} = \nu_{125} = \nu_{212} = \nu_{214} = 0.021,$$

$$\nu_{211} = \nu_{213}\nu_{215} = \nu_{122} = \nu_{124} = 0.288\,.$$

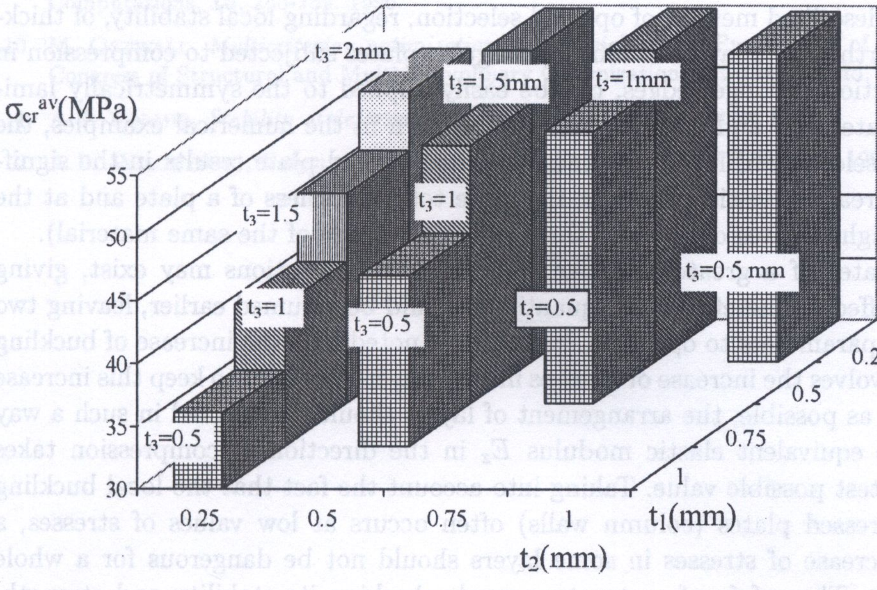

FIG. 6. Average critical stress for 5-layered plate of different thicknesses of layers ($t_1 = t_5, t_2 = t_4$).

The maximum value of critical average stress equals $\sigma_{\text{cr}}^{\text{av}} = 54.73$ MPa. For the assumed arrangement of layers the stresses in particular layers are:

$$\sigma_1 = \sigma_3 = \sigma_5 = 78.88 \text{ MPa}, \quad \sigma_2 = \sigma_4 = 5.76 \text{ MPa}.$$

The inclination of orthotropy axes (in all layers) by 90 degrees to the previous configuration gives the results:

$$\sigma_1 = \sigma_3 = \sigma_5 = 10.42 \text{ MPa}, \quad \sigma_2 = \sigma_4 = 142.65 \text{ MPa}.$$

For a three-layered plate (t=3 mm), designed as optimal ($\bar{z}_1 \cong 0.4$, $t_1 = t_3$=0.3 mm, t_2 =2.4 mm), the average critical stress also equals $\sigma_{\text{cr}}^{\text{av}} = 54.73$ MPa, but the stresses in layers are:

$$\sigma_1 = \sigma_3 = 4.9 \text{ MPa}, \quad \sigma_2 = 66.93 \text{ MPa}.$$

During the analysis of five- and seven-layered plates, made of the same orthotropic material, it is seen that the optimal plate regarding local stability is the three-layered one. Therefore in the analysis of 5- and 7-layered plates, the isotropic middle layer was taken into account, which can be treated as a spacer for exterior layers of large strength.

4. Conclusions

The described method of optimal selection, regarding local stability, of thickness of orthotropic layers in long rectangular plates subjected to compression in the direction of longer edges, can be easily applied to the symmetrically laminated plates of 3 and 5 layers. As it was shown in the numerical examples, the optimal selection of layers thickness in three-layered plate results in the significant increase of critical force at the same total thickness of a plate and at the same weight (plates of one and three layers are made of the same material).

In plates of a greater number of layers, many solutions may exist, giving similar effects. Therefore some quantities should be assumed earlier, leaving two or three parameters to optimise. It should be noted that the increase of buckling stress involves the increase of stresses in some layers. In order to keep this increase as small as possible, the arrangement of layers should be selected in such a way that the equivalent elastic modulus E_z in the direction of compression takes the greatest possible value. Taking into account the fact that the local buckling of compressed plates (column walls) often occurs at low values of stresses, a slight increase of stresses in some layers should not be dangerous for a whole structure. The safefy of a structure needs checking its stability and strength. In multilayered structures the condition of strength should be fulfilled by each layer, and the condition of stability (if delamination does not occur) – by the whole structure (plate, wall, thin-walled strut, shell, etc.).

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Received October 8, 2001.