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THE INFLUENCE OF INTRALAMINAR DAMAGE ON MECHANICAL PROPERTIES OF COMPOSITE LAMINATES

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In the paper the damage of fiber-reinforced polymeric matrix laminates is considered with the aim to examine the change of their mechanical properties. The damage mode in the form of intralaminar cracks is described in terms of the damage tensor by Vakulenko and Kachanov. The crack discontinuity parameter is estimated in the frames of the linear elastic fracture mechanics. To derive the constitutive relations, which take into account stress, strain and developing damage, an approach based on polynomial invariant functions and irreducible integrity basis by Adkins is employed. Theoretical results are compared with the experimental data obtained for carbon/epoxy laminates.

NOTATIONS

(1, 2) material Cartesian coordinate system for a single ply,
(x, y) reference Cartesian coordinate system for a single ply,
β averaged crack opening factor,
ε_{rst} the Ricci symbol,
ν_{12} the major Poisson ratio of a ply in on-axis configuration,
ρ_m average crack density,
ρ_{ult} ultimate average cracks density,
θ_m the angle between the material axis and the reference axis,
a_i, b_i material parameters,
A_2, A_6, A_8, A_{10} material parameters for damaged laminate,
$\mathbf{A}$, $\mathbf{A}^o$, $\mathbf{A}^d$ the global extensional: total stiffness matrix, stiffness matrix for "virgin" laminate and stiffness matrix of the damage state influence on a laminate stiffness,
$\mathbf{b}$ displacement jump vector across a crack surface S,
c_i, f_i material constants,
c_m an imaginary strip width,
$\mathbf{C}$, $\mathbf{C}^o$, $\mathbf{C}^d$ the stiffness matrix for on-axis ply: total matrix, matrix for "virgin" ply and matrix of the damage state influence, on the stiffness of a ply,
$\mathbf{d}'$, $\mathbf{d}$ the second order damage tensor,
e_{ij}, a_{ij} second order kinematic matrices in the Adkins approach,

E_1, E_2	the longitudinal and transverse Young's moduli of the on-axis ply,
E_x^L, E_y^L, G_{xy}^L, ν_{xy}^L	the current engineering constants of a laminate,
$f(t_m/c_m)$	the finite width correction factor for stress intensity factor K_I,
F_{ult}, σ_{ult}, $\varepsilon_{\text{Lult}}$, $\varepsilon_{\text{Tult}}$	ultimate tensile load, stress, longitudinal and transverse strain,
G_{12}	the shear modulus of a ply in on-axis configuration,
$\mathbf{n}$	a unit outward normal to the surface S,
PR	abbreviation for the term "Poisson's ratio",
s_m	cracks spacing along the observation section,
S	the projection of crack surface on its "midplane",
$\mathbf{S}$	the compliance matrix,
t_m	thickness of the m-th ply,
U_n^o, U_n^d	material coefficients for the *on-axis* ply due to Tsai–Pagano method,
v_m	the volume fraction of the m-th ply,
V	the volume of representative composite sample,
V_i^o	coefficients depending on volume fraction and plies orientation,
V_k^d	coefficients depending on volume fraction, damaged plies orientation and cracks density,
YM	abbreviation for the term "Young's modulus".

1. Introduction

One of the most important questions in the behaviour of structural materials is how their strength and stiffness characteristics are influenced by the defects and different damage mechanisms, which initiate and develop as a result of the applied load. In isotropic and homogenous materials, in most cases the deterioration problem can be formulated in terms of fracture toughness i.e. the resistance to growth of single crack. Fracture mechanics provides now very efficient methods to solve the problems of this class.

In composite materials the deterioration process is much more complex, due to the existence of at least two different constituents. The main feature of composite material deterioration is multiplicity of cracks of various shapes and forms, which depend not only on the constituents' properties, but also on their geometrical arrangement. Damage of composite materials, expressed in terms of fracture mechanics is, at the present time, in relatively early stage and has a limited usage, mainly due to sophisticated solutions of the crack problem in anisotropic body. However, significant progress must be noticed in the last years, due to application of specialized, commercial numerical codes and methods [1–2].

Basic approaches used in the analysis of laminates damage are: shear lag approach [3–5], self-consistent approach [6–7], micro-macro approach based on

Mori–Tanaka average model [8–9], minimum complementary potential energy method [10], variational mechanics approach [11–12], approach based on the average crack opening displacement [3, 5, 13–15], probabilistic approach [8, 16], homogenization methods [17–18].

In the present paper the continuum damage mechanics (CDM) approach is applied (see e.g. [19–22]). The criterion for classifying any work as belonging to CDM is the description of crack fields in terms of damage measures associated with CDM concepts. Taking into account anisotropic properties of composite materials, the second order damage tensor is often used (see e.g. [23–24]) in damage analysis.

2. Problem formulation

The mechanisms of composite laminates damage are well known from the paper by Reifsnider *et al.* [25] (review of Reifsnider contribution to the composite materials mechanics is given in [26]). For the laminate under fatigue load they specified the following consecutive damage stages: intralaminar matrix cracking, cracks coupling and interfacial debonding, delamination, large scale fiber breaking and at last – formation of a failure path leading to the total material deterioration. It should be pointed out that not all of the specified mechanisms must necessarily occur. Besides, some of them can cover more or less wide range of laminate "life period" – it depends mainly on the laminate ply stacking sequence and on the applied load.

For laminates with off-axis plies being separated by on-axis plies under monotonically increasing tensile load, the predominant mechanism of laminate deterioration is intralaminar transverse matrix cracking [27–28]. That phenomenon is observed in a wide range of applied load. The delamination and fibers breaking occurred nearly simultaneously with specimen failure. This observation has found confirmation in author's experiments, carried out on symmetrical cross-ply and angle-ply specimens [29].

Macroscopically observed (with use of e.g. optical methods) and measured (e.g. in standard tensile test) effect of gradual material deterioration is the change of mechanical properties, namely strength and stiffness characteristics of a laminate.

The present paper deals with intralaminar damage mechanism with the aim to provide a description of the changes of mechanical properties of the composite laminate due to the developing damage process. It was reported by the author in several papers [29, 31–34].

The analysis is confined to the laminates having combinations of unidirectional, cross-ply and angle-ply orientations under monotonically increasing tensile load.

In order to evaluate the influence of damage state on changes of the composite stiffness, the four basic steps have to be done, namely: modelling of intralaminar damage, derivation of constitutive equations, which take into account stress, strain and damage, calculation of damaged laminate stiffness, and finally – experimental verification of theoretical predictions. Each step needs specific approach and relevant tools, to be passed.

Laminate's transverse matrix cracking is described in the frame of CDM approach with use of tensorial damage representation by VAKULENKO and KACHANOV [35–36].

The constitutive relations for damaged composite unidirectional lamina are derived in the frame of an approach based on polynomial representation of stress tensor as a function of the internal state variables [37–39]. Using those relations and employing concepts of the Classical Laminates Theory (CLT), the changes of engineering constants of a laminate are obtained, as the final results.

They are related to the author's experimental data obtained for a carbon /epoxy composite.

3. INTRALAMINAR MATRIX CRACKING

The first stage of damage development is dominated by intralaminar matrix cracking in off-axis plies. It begins at relatively low level of the tensile load and results in a roughly periodic array of cracks with nearly parallel midplanes, evenly distributed within the lamina – see Figs. 1 and 2. Intralaminar cracks span the entire width of a composite specimen. This feature allows for their "eye" observations and is of primary importance, when experimental verification of theoretical model is considered.

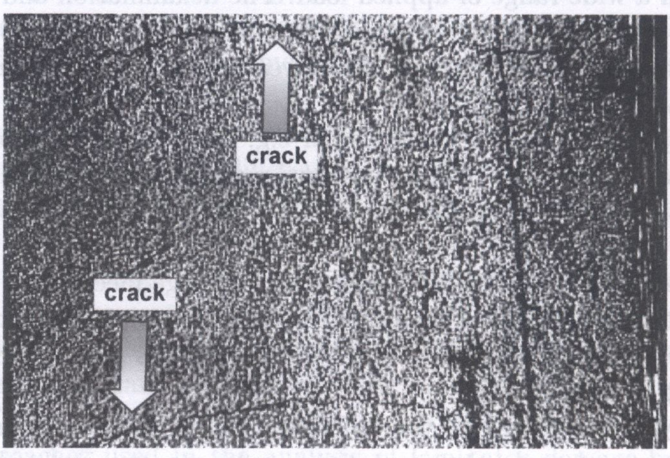

FIG. 1. Transverse matrix cracking in [0, 90_8, 0] cross-ply laminate.

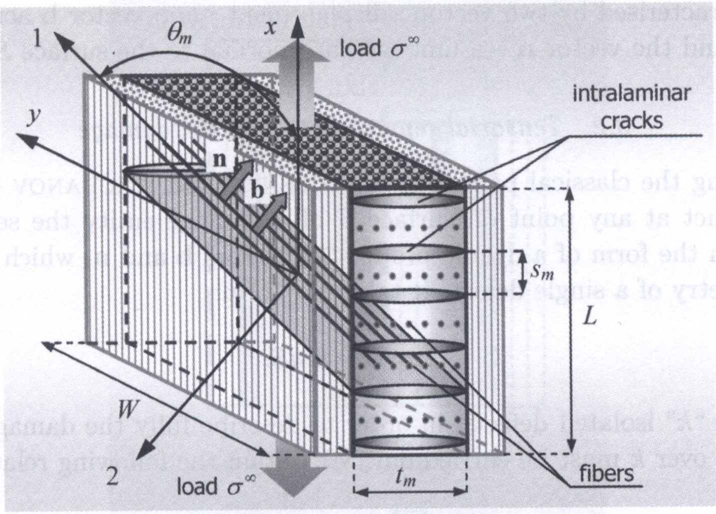

FIG. 2. Orientation of intralaminar matrix cracks within *off-axis* ply.

The important factor determining the cracking process and crack density is the ply orientation angle θ_m, i.e. the angle between the material axes (1, 2) and any arbitrary reference axes (x, y) – see Fig. 2.

Almost regular pattern of cracks within an individual ply, possibility of initiation of similar patterns within the other off-axis plies, as well as very small crack size, make the approach based on CDM, dealing with continuous description of the discreet crack field, an effective tool in the analysis of composite's intralaminar damaging [19].

3.1. *CDM approach for composite body*

The CDM approach to the cracked composite materials is based on an element of the volume of composite, containing a representative sample of damage entities. By the damage entity we understand a single structural change, which in composite materials made of brittle constituents, under mechanical loads, can take a form of the matrix crack. The collection of damage entities of the same or similar geometrical features is called damage mode. In laminated fiber composites, under tensile load, one can observe mainly intralaminar cracks. Depending on the layers sequence in a given laminate, we can distinguish several damage modes. The set of all damage modes is called damage.

Let us consider the two states of a body, namely the initial "virgin" state and the actual one – the state in which internal damage is developing in the body in response to the applied load. The transition from the first to the second state can be characterised by two vectors: displacement jump vector $\mathbf{b}$ across a crack surface S and the vector $\mathbf{n}$ – a unit outward normal to the surface S.

3.2. Tensorial representation of the damage

Following the classical papers by VAKULENKO and KACHANOV [35–36] one can construct at any point on surface S of a damage entity the second order tensor $\mathbf{d}'$ in the form of a dyadic product of vectors $\mathbf{b}$ and $\mathbf{n}$, which defines the local geometry of a single defect. It takes the form:

$$\mathbf{d}' = \mathbf{b} \otimes \mathbf{n}\, dS.$$

If there are "k" isolated defects, in order to describe fully the damage field, the summation over k must be carried out. We obtain the following relation:

$$\mathbf{d}' = \sum_k \mathbf{b}_k \otimes \mathbf{n}_k\, dS_k.$$

For the transition from discrete to the continuous model, the averaging procedure must be employed, by means of averaging the damage field over a volume V, containing representative sample of "k" damage modes.

Confining further analysis to the normal discontinuities (cracks in Mode I), we get after transformations the following relation:

$$\langle \mathbf{d}' \rangle_n = \mathbf{d} = \frac{1}{V} \sum_k \int_{S_k} \beta_k\, \mathbf{n}_k \otimes \mathbf{n}_k\, dS_k.$$

Symbol "$\langle\ \rangle$" denotes an average over volume V, surface S is understood as the projection of a crack surface on a crack "midplane", a multiplier β_k – as an averaged crack opening factor, depending on the crack geometry.

Factor β_k is calculated in the frame of linear elastic fracture mechanics (LEFM) with the use of averaging procedure. Possibility and admissibility of such an approach was discussed by Varna *et al.* [5]. In order to estimate β_k, additional assumptions and approximations, as well as a concept of "imaginary strip" described in [31], are also employed.

Let us introduce an "imaginary" strip formed by the considered m-th ply with cracks and neighbouring plies – see Fig. 3.

It is assumed, that the strip satisfies the requirement of symmetry with respect to the vertical symmetry axis of the cracks contained within the m-th ply. Furthermore, the choice of a strip must take into account the stacking sequence of a laminate. In order to satisfy both requirements, the class of laminates has to be confined to laminates with undamaged outer plies, since only then $t_m < c_m$

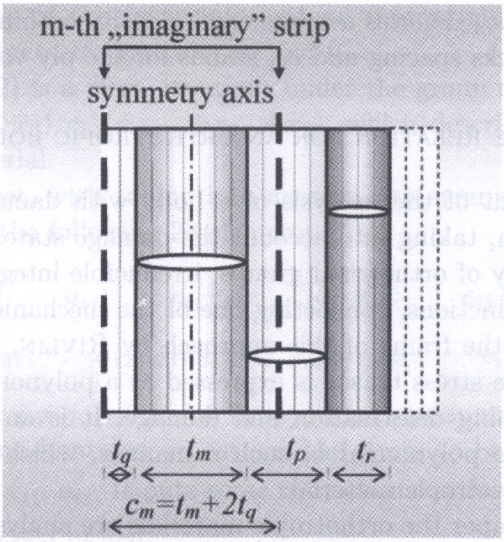

FIG. 3. Construction of imaginary strip.

and the solution derived from LEFM employed in the present paper is reasonable (the analogous observation relates also to the last right-hand damaged ply).

In order to divide a laminate into "imaginary" strips it is assumed that each m-th damaged ply, being equivalent to the m-th damage mode, is associated with m-th strip. It is "constructed" from the m-th ply with cracks of length t_m and the adjacent plies (or their parts) of the same thickness t_q (requirement of symmetry), irrespective of their mechanical state i.e. damaged or undamaged state. It means that interaction between the cracks within neighbouring plies is neglected. Note that cracks within one ply are also not assumed to interact one with each other.

The proposed procedure allows, to some extent, to take into account the constraint effect of neighbouring plies on the cracking process developing in a given ply, contrary to e.g. the TALREJA concept [41] of a crack surface activity vector $\mathbf{a}$.

Using geometrical relations arising from Fig. 2, we get finally for the m-th intralaminar damage mode, within the layer in its *on-axis* configuration, the following form of the only non-zero component of the damage tensor:

$$d_{22}^{m} = d_{2}^{m} = \frac{\pi t_m}{4E_2} \rho_m \nu_m f\left(\frac{t_m}{c_m}\right) \sigma^{\infty}. \tag{3.1}$$

E_2 denotes here the transverse Young's modulus of a ply, c_m is an imaginary strip width, $f(t_m/c_m)$ denotes the finite width correction factor for the stress intensity factor K_I, $\rho_m = 1/s_m$ denotes average crack density within the m-th damaged ply, s_m denotes cracks spacing and v_m stands for the ply volume fraction.

4. Constitutive relation for an orthotropic body with damage

The crucial point of the analysis of a body with damage is to derive the constitutive relation, taking into account the damage state. It can be achieved by employing theory of orthogonal groups, irreducible integrity basis and polynomial invariants functions, connecting one of the mechanical tensors with the remaining ones. In the frame of this approach by Rivlin, Ericksen [42] and Adkins [37–38], the stress tensor is expressed as a polynomial in the elements of the tensors defining deformation and damage. It is an important issue to construct a suitable polynomial in such a manner, which includes symmetry properties of an anisotropic material.

In the present paper the orthotropic materials are analysed, since composite laminates generally belong to this class of materials. The implementation of Adkins's approach to the orthotropic composite laminates is presented in details in paper [34] by the author. Here, just for clarity of presentation, only key points of the analysis are given.

Following the Rivlin and Ericksen approach – the stress tensor components for orthotropic material are expressed as follows:

$$\sigma^{ij} = f^{ij}\,(e_{rs},\, a_{pq})\,; \qquad (f^{ij} = f^{ji})\,, \tag{4.1}$$

where σ^{ij} is the stress tensor referred to the rectangular Cartesian system x^i, e_{ij}, a_{ij} are symmetrical matrices formed in Cartesian system x^i (they are called by Adkins [38] "kinematic" matrices) and the functions f^{ij} are polynomials in the arguments indicated. The coefficients, which appear in these polynomials, are material parameters, which do not depend on the position through the body as well as upon any deformation.

For the orthotropic body and two symmetrical second order kinematic matrices e_{ij}, a_{ij}, Adkins [37] derived the specific form of Eq. (4.1), namely:

$$\sigma^{ij} = A^{ij}_{tt}\,\Theta_{tt} + \varepsilon_{rst}\,\varepsilon_{rst}\,A^{ij}_{rs}\Big[P^{(1)}_{rs;t} + P^{(2)}_{rs;t} + Q^{(1)}_{rs;t} + Q^{(2)}_{rs;t} + R_{rs;t}\Big] \tag{4.2}$$

where

$$P^{(1)}_{rs,t} = e_{rs}\Theta^{(1)}_{tt}\,, \qquad P^{(2)}_{rs,t} = a_{rs}\Theta^{(2)}_{tt}\,, \qquad Q^{(1)}_{rs,t} = e_{rt}e_{ts}\Theta^{(3)}_{tt}\,,$$

$$Q^{(2)}_{rs,t} = a_{rt}a_{ts}\Theta^{(4)}_{tt}\,, \qquad R_{rs,t} = e_{rt}a_{ts}\Theta^{(5)}_{tt} + e_{st}a_{tr}\Theta^{(6)}_{tt}\,.$$

The term ε_{rst} denotes the Ricci symbol and A^{ij}_{tt} is equal to 1 or 0 depending on indices combination. The functions Θ_{tt} and $\Theta^{(k)}_{tt}$ ($k = 1,\ldots, 6$) are invariant polynomial functions of the invariants system appropriate to the case where stress depends on kinematic matrices e_{ij} and a_{ij}.

The relation (4.2) is a form, invariant under the group of orthogonal transformations $(\bar{x}_1, \bar{x}_2, \bar{x}_3) = (\pm x_1, \pm x_2, \pm x_3)$, which describes the orthotropic symmetry of a material.

The invariants set, with taking into account the symmetry of matrices e_{ij} and a_{ij}, consists of the following 23 elements:

$$
\begin{aligned}
&e_{i\,i}; \quad a_{i\,i}; \quad e_{ij}\,e_{j\,i}; \quad a_{ij}\,a_{j\,i}; \quad e_{ij}\,a_{j\,i};\\
&e_{12}\,e_{23}\,e_{31}; \quad a_{12}\,a_{23}\,a_{31}; \quad e_{ij}\,e_{j\,k}\,a_{ki}; \quad e_{ij}\,a_{j\,k}\,a_{ki}
\end{aligned}
\tag{4.3}
$$

and: $i, j, k = 1, 2, 3$; $i \neq j$, $i \neq k$, $j \neq k$; i, j, k are not summed.

In successive considerations we assume the stress to be linearly dependent upon both matrices e_{ij}, a_{ij}. It puts some restrictions on functions $Q^{(1)}_{rs\,;\,t}$, $Q^{(2)}_{rs\,;\,t}$, $\Theta^{(5)}_{tt}$ and $\Theta^{(6)}_{tt}$, discussed in [34].

Taking into account the above considerations, Eq. (4.2) can be transformed to the following constitutive relation for orthotropic laminate ply in Cartesian, material axes x_i:

$$
\sigma_{ij} = A_{ij\,t\,t}\,\Theta_{tt} + \varepsilon_{rst}\,\varepsilon_{rst}\,A_{ij\,r\,s}\,\left[P^{(1)}_{rs;t} + P^{(2)}_{rs;t} + R_{rs;t}\right]. \tag{4.4}
$$

Let us further identify the symmetrical matrices e_{ij} and a_{ij} as respectively, the strain tensor ε_{ij} and the damage tensor d_{ij}. An assumption of linear dependence of stress on kinematic matrices is then equivalent to the assumption of strains and damage being small quantities.

4.1. *Stiffness matrix for the single damaged ply in the material axis*

We confine subsequent analysis to the in-plane behaviour of a laminate ply. Thus, the stress, strain and damage tensors in Voigt's notation are as follows:

$$
\boldsymbol{\sigma} = \begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{bmatrix} = \begin{bmatrix}\sigma_{1}\\ \sigma_{2}\\ \sigma_{6}\end{bmatrix}, \qquad \boldsymbol{\varepsilon} = \begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{12}\end{bmatrix} = \begin{bmatrix}\varepsilon_{1}\\ \varepsilon_{2}\\ \varepsilon_{6}\end{bmatrix}, \qquad \mathbf{d} = \begin{bmatrix}d_{11}\\ d_{22}\\ d_{12}\end{bmatrix} = \begin{bmatrix}d_{1}\\ d_{2}\\ d_{6}\end{bmatrix}.
$$

From Eq. (4.4), after some calculations we derive the stresses:

$$
\sigma_1 = \Theta_{11}, \qquad \sigma_2 = \Theta_{22}, \qquad \sigma_6 = \varepsilon_6\,\Theta^{(1)}_{33} + d_6\,\Theta^{(2)}_{33}. \tag{4.5}
$$

Now, one must derive the polynomial functions Θ_{11}, Θ_{22}, $\Theta^{(1)}_{33}$ and $\Theta^{(2)}_{33}$, which in general are the functions of invariants set (4.3), but for analyzed in-plane case only the chosen invariants, specified in [34], are of interest.

Taking into account the assumed linearity relations – the relevant polynomials are the following:

$$\Theta_{11} = a_1\,\varepsilon_1 + a_2\,\varepsilon_2 + (a_3\,d_1 + a_4\,d_2)\,\varepsilon_1 + (a_5\,d_1 + a_6\,d_2)\,\varepsilon_2 + a_7\,d_6\,\varepsilon_6 + a_8, \tag{4.6}$$

$$\Theta_{22} = b_1\,\varepsilon_1 + b_2\,\varepsilon_2 + (b_3\,d_1 + b_4\,d_2)\,\varepsilon_1 + (b_5\,d_1 + b_6\,d_2)\,\varepsilon_2 + b_7\,d_6\,\varepsilon_6 + b_8, \tag{4.7}$$

$$\Theta^{(1)}_{33} = c_1\,d_1 + c_2\,d_2 + c_3, \qquad \Theta^{(2)}_{33} = f_1\,\varepsilon_1 + f_2\,\varepsilon_2 + f_3. \tag{4.8}$$

The coefficients a_1–a_8, b_1–b_8, c_1–c_3, f_1–f_3 are material parameters. Polynomial functions Θ_{11}, Θ_{22}, $\Theta^{(1)}_{33}$, $\Theta^{(2)}_{33}$ are not independent one from the other, as the stress derived with use of these functions must satisfy the constitutive equation of general form:

$$\sigma_i = C_{ij}\,\varepsilon_j = C^o_{ij}\,\varepsilon_j + C^d_{ij}\,\varepsilon_j, \qquad i,j = 1,2,6, \tag{4.9}$$

where C_{ij} denotes the stiffness matrix. It can be decomposed into two constituent matrices – $\mathbf{C}^o$ and $\mathbf{C}^d$. The first one relates to undamaged, "virgin" state of ply, while the second matrix characterises the change of the m-th ply stiffness due to damage.

Stiffness matrix for the in-plane case has the following components (in Voigt's notation):

$$\mathbf{C} = \begin{bmatrix} C_{11} & C_{12} & C_{16} \\ C_{12} & C_{22} & C_{26} \\ C_{16} & C_{26} & C_{66} \end{bmatrix}.$$

We can now calculate stresses from Eq. (4.9) and then compare derived formulas with those given by Eq. (4.5). After calculations, with use of polynomial functions (4.6), (4.7) and (4.8) we obtain both the stiffness matrices.

Matrix $\mathbf{C}^o$ can be also easily found in textbooks on composite mechanics, e.g. [30, 40]. It takes the following form:

$$\mathbf{C}^o = \begin{bmatrix} K\,E_1 & K\,\nu_{21}\,E_1 & 0 \\ K\,\nu_{21}\,E_1 & K\,E_2 & 0 \\ 0 & 0 & G_{12} \end{bmatrix}; \qquad K = \frac{1}{1-\nu_{12}\,\nu_{21}}.$$

Note that we derived the stiffness matrix for undamaged material with elements $C^o_{16} = 0$, $C^o_{26} = 0$. This result was expected, since it is typical for orthotropic laminate in principal material axes (1, 2) (so-called *special orthotropy*).

It has been shown in [32–33] that for the intralaminar damage, being of interest, the only non-zero damage tensor component is d_2 (see Eq. (3.1)). Therefore, the stiffness matrix (so-called *reduced* stiffness matrix) associated with the damage state in the m-th damaged ply, expressed in principal material axes takes the form:

$$\mathbf{C}^{dm} = \begin{bmatrix} A_6 & A_2 & 0 \\ A_2 & A_{10} & 0 \\ 0 & 0 & A_8 \end{bmatrix} d_2^m. \tag{4.10}$$

Let us notice that we obtain $C_{16}^{dm} = 0$, $C_{26}^{dm} = 0$, thus the initial special orthotropy is still retained in the presence of damage in the on-axis ply, being under consideration.

In order to determine matrix (4.10) – four unknown material parameters, namely A_2, A_6, A_8 and A_{10} must be involved in further analysis.

The result obtained here, from the purely formal point of view is similar to that one derived by TALREJA [43] from considerations based on thermodynamics with internal state variables introduced by COLEMAN and GURTIN [44]. Therefore, the mathematical approach used in the present paper has also a physical background.

4.2. The stiffness matrix for a damaged laminate

The stiffness matrices $\mathbf{C}^o$ and $\mathbf{C}^d$ derived for a single, respectively, "virgin" and damaged ply in on-axis configuration are a basis for evaluation of transformed stiffness matrices for a ply in any reference coordinate system (x, y).

Taking into consideration the fact, that laminate consists of many layers with different orientation in relation to the reference coordinate system (x, y), both matrices have to be transformed from material axes (1, 2) to reference axes (x, y). After transformation – with the use of the standard TSAI and PAGANO procedure described e.g. in [40] – we get the transformed reduced stiffness matrix for any constituent ply.

The next step is to derive the stiffness matrix for a laminate being a collection of plies, some of which can be damaged, while the remaining ones can be still intact.

In order to simplify the analysis, as well as to have possibility to compare theoretical predictions with experimental data, the class of laminates being of interest was restricted to the symmetrical laminates. Besides, to derive laminate's engineering characteristics it was necessary to consider unidirectional tensile load only. Under those limitations the problem is confined to the analysis of extensional stiffness matrix $\mathbf{A}$. It was calculated in the frame of the CLT.

The global extensional stiffness matrix $\mathbf{A}$ (here we use a "normalized" matrix i.e. divided by laminate thickness t) can be decomposed into two matrices, namely: matrix $\mathbf{A}^o$ for "virgin" material and matrix $\mathbf{A}^d$ describing the influence of the damage state developing within some layers on the stiffness of a laminate. Thus, we have:

$$\mathbf{A}/t = \mathbf{A}^o + \mathbf{A}^d = \overline{\mathbf{T}}^o\mathbf{P}^o + \overline{\mathbf{T}}^d\overline{\mathbf{P}}^d \tag{4.11}$$

where transformation matrices have the following form:

$$\overline{\mathbf{T}}^o = \begin{bmatrix} U_1^o & V_1^o & V_2^o \\ U_1^o & -V_1^o & V_2^o \\ U_4^o & 0 & -V_2^o \\ U_5^o & 0 & -V_2^o \\ 0 & 1/2\,V_3^o & V_4^o \\ 0 & 1/2\,V_3^o & -V_4^o \end{bmatrix}, \qquad \overline{\mathbf{T}}^d = \begin{bmatrix} \overline{U}_1 & V_1^d & V_2^d \\ \overline{U}_1 & -V_1^d & V_2^d \\ \overline{U}_4 & 0 & -V_2^d \\ \overline{U}_5 & 0 & -V_2^d \\ 0 & 1/2\,V_3^d & V_4^d \\ 0 & 1/2\,V_3^d & -V_4^d \end{bmatrix},$$

$$\mathbf{P}^o = \begin{bmatrix} 1 \\ U_2^o \\ U_3^o \end{bmatrix}, \qquad \overline{\mathbf{P}}^d = \begin{bmatrix} V_0^d \\ \overline{U}_2 \\ \overline{U}_3 \end{bmatrix}.$$

The coefficients V_i^o $(i = 1, 2, 3, 4)$ depend on volume fraction and orientation angle θ_m of each laminate's constituent layer, whereas V_k^d $(k = 0, 1, 2, 3, 4)$ depend on volume fraction, orientation angle θ_m and cracks density within the damaged layers only.

The coefficients U_n^o and $\overline{U}_n^d$ $(n = 1, 2, 3, 4, 5)$ are purely material parameters and depend on the four standard independent engineering constants for *on-axis* ply i.e. E_1, E_2, G_{12}, ν_{12} in the first case and depend on "new" material parameters A_2, A_6, A_8, A_{10} in the second case. The matrix $\mathbf{A}^d$ is unknown as long as parameters A_2, A_6, A_8 and A_{10} remain not determined in appropriate tests.

Note that for an orthotropic laminate in virgin state, to get full information on laminate stiffness – the four on-axis constants are needed. When the damage is included, we also need to know four constants, which however can not be derived from the tests carried out on a single ply (like in the first case), but on a laminate as a whole. The first crack in a single ply means its final fracture and the damage in such a sense as in the present paper can not be defined. Therefore, the plies stacking sequence must be chosen in such a way which makes the calculations possible, but on the other hand, as easy as possible.

5. Engineering constants for a damaged laminate

In order to determine the engineering constants, the compliance matrix $\mathbf{S}$ must be determined. For symmetrical laminates, coupling and bending stiffness matrices are equal zero, therefore stiffness and compliance matrices satisfy the "standard" relation:

$$\mathbf{S} = \mathbf{A}^{-1}\, t \equiv \mathbf{A}'\, t \quad .$$

The current engineering constants of a laminate can be directly derived from compliance matrix by use of the relations, given in [40]:

$$\begin{aligned} E_x^L &= \frac{1}{t}\,\frac{A_{11}A_{22} - A_{12}^2}{A_{22}}, & E_y^L &= \frac{1}{t}\,\frac{A_{11}A_{22} - A_{12}^2}{A_{11}}, \\ G_{xy}^L &= \frac{1}{t}\,A_{66}, & \nu_{xy}^L &= \frac{A_{12}}{A_{22}}. \end{aligned} \tag{5.1}$$

The above equations are adequate for any laminate configuration. For specific class of laminates, i.e. cross-ply laminates, they can be expressed in closed form and therefore, they have been used in calculation of "new" material parameters A_2, A_6, A_8 and A_{10}.

5.1. *Cross-ply laminates*

It has been previously mentioned that, in order to calculate unknown parameters, laminate stacking sequence must be chosen in a specific way. The simplest laminate configuration, which allows reaching this goal, is a cross-ply laminate with 0° ply volume fraction v_0 and 90° ply volume fraction v_{90}, since in such a laminate the damage develops in 90° ply only. The matrices $\mathbf{A}^o$ and $\mathbf{A}^d$ take then the following forms:

$$A^o = \begin{bmatrix} K\,(\nu_0 E_1 + \nu_{90} E_2) \\ K\,(\nu_0 E_2 + \nu_{90} E_1) \\ G_{12} \\ 0 \\ 0 \\ K\,E_1\,\nu_{21} \end{bmatrix}, \qquad A^d = \begin{bmatrix} A_{10} \\ A_6 \\ A_8 \\ 0 \\ 0 \\ A_2 \end{bmatrix} V_0^d.$$

For cross-ply laminates, by use of Eq. (5.1), after a number of straightforward but rather tedious calculations, we finally get the desired constants in the following form:

$$E_x^L = E_x^{oL} + V_0^d\left[A_{10} + A_6\left(\nu_{xy}^{oL}\right)^2 - 2A_2\nu_{xy}^{oL}\right], \tag{5.2}$$

$$E_y^L = E_y^{oL} + V_0^d\left[A_6 + A_{10}\left(\nu_{yx}^{oL}\right)^2 - 2A_2\nu_{yx}^{oL}\right], \tag{5.3}$$

$$v_{xy}^L = v_{xy}^{oL} + \frac{1 - \nu_{xy}^{oL}\nu_{yx}^{oL}}{E_y^{oL}}\,V_0^d\left[A_2 - A_6\,\nu_{xy}^{oL}\right], \tag{5.4}$$

$$G_{xy}^L = G_{xy}^{oL} + V_0^d A_8. \tag{5.5}$$

The factor V_0^d takes into account the volume fraction of the m-th damaged ply and the damage within this ply; it is expressed as follows:

$$V_0^d = \sum_{m=1}^{M} v_m \, d_2^m .$$

5.2. *Material characteristics for a damaged laminate*

The equations set (5.2)–(5.5) can be solved with respect to parameters A_2, A_6, A_8, A_{10}. All the needed quantities one can determine from the tensile test of a laminate specimen. The three unknown constants A_2, A_6, A_{10} have been determined in unidirectional tensile test carried out on specimens of code $[0/90_3]_s$, manufactured from carbon/epoxy composite. Note that the testing procedure must allow for continuous measurements of cracks within the 90° ply. The details of specimens' preparation and their testing are given in the next chapter.

The constant A_8 relating to a shear modulus only, has not been determined. Besides, it was assumed that transverse Young's modulus was constant, because transverse cracks within 90° ply do not produce the change of transverse stiffness. The procedure for the determining the parameters A_2, A_6, A_{10} is shown in Fig. 4.

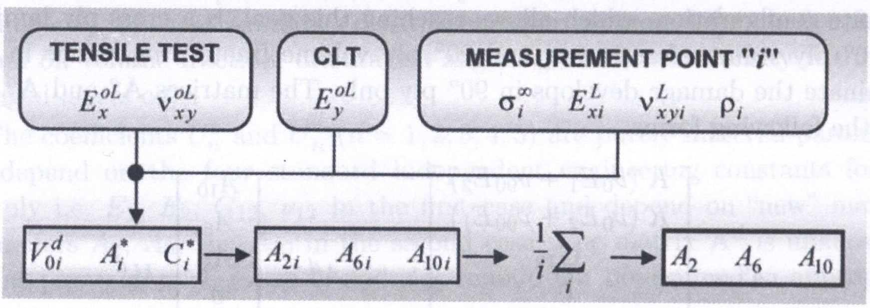

FIG. 4. The procedure of calculation of material parameters.

The following values of material parameters were found: $A_2 = -192.0$ GPa, $A_6 = -34.7$ GPa, $A_{10} = -258.0$ GPa. These values make possible calculations of the current engineering constants of any laminate by use of Eq. (5.1).

6. Testing procedure

6.1. *Material and specimens*

Specimens, shown in Fig. 5, have been manufactured from unidirectional prepreg CFRE tape Vicotex NCHR 174B. Characteristics of a lamina in its principal material axis are given in Table 1.

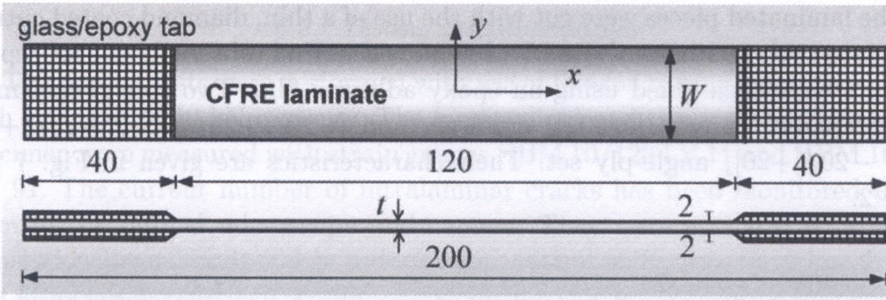

FIG. 5. The specimen.

Table 1. Characteristics of Vicotex NCHR 174B unidirectional lamina.

Stiffness characteristics				Strength characteristics					Coefficients of linear expansion	
E_1 [GPa]	E_2 [GPa]	G_{12} [GPa]	ν_{12}	X_t [MPa]	X_c [MPa]	Y_t [MPa]	Y_c [MPa]	S [MPa]	α_1 [1/°C]	α_2 [1/°C]
137.0	10.0	4.8	0.3	1531	1390	41	145	98	3.1 10^{-7}	3.1 10^{-5}

Rectangular laminate pieces, after forming the desired stacking sequence have been laminated according to the composite manufacturer recommendations. We used a standard testing machine equipped in a heating device and thermocouple controlled by microcomputer system. All parameters of the lamination process are shown in Fig. 6.

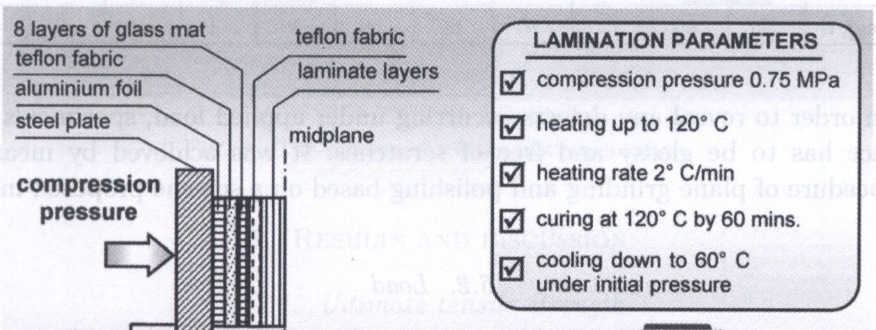

FIG. 6. Lamination of the specimen material.

The laminated pieces were cut with the use of a thin, diamond coated cutting wheel into two specimens of size 25×200 mm each. End tabs made of glass/epoxy laminate were then glued using an epoxy adhesive film. Two sets of specimens were prepared, namely $[0/90_n]_s$ cross-ply and $[-20/\ +20/\ -\alpha_2/\ -20/\ +20/\ +\alpha_2/\ -20/\ +20]_s$ angle-ply set. Their characteristics are given in Fig. 7 and Table 2.

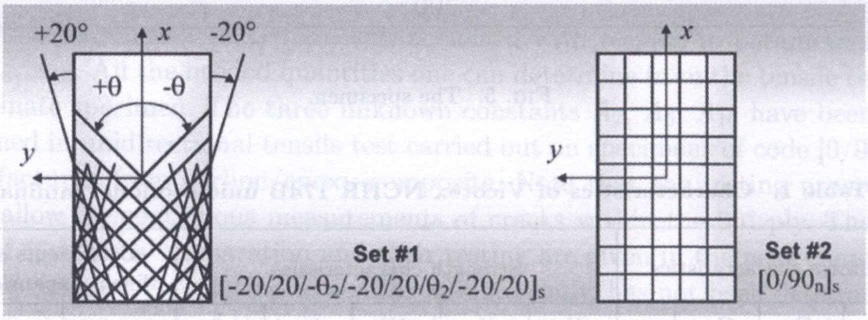

FIG. 7. Two sets of test specimens.

Table 2. Geometrical characteristics of test specimens.

Geometry	Specimens set # 1							Specimens set # 2			
	A	B	C	D	E	G	H	1	2	3	4
t [mm]				2.46				0.48	0.74	0.98	1.22
W [mm]	24.38	23.74	24.66	24.54	24.75	23.56	23.46	24.86	24.96	24.88	24.98
α [deg], n	90	80	75	70	60	50	40	1	2	3	4

In order to reveal any defects occurring under applied load, specimen's edge surface has to be glossy and free of scratches. It was achieved by means of a procedure of plane grinding and polishing based on a scheme proposed in [45].

6.2. Load

In all tests, the monotonically increasing tensile load was applied to the specimens. The programmed and automatically controlled load rate was kept constant up to the final failure of each specimen and was equal to 0.2 kN/min (approx. 2% of ultimate load/min) for set # 2 and 0.5 kN/min (approx. 1.5% of ultimate load/min.) for set # 2. The chosen, relatively slow rate allowed to count the number of cracks at any, almost fixed load level.

6.3. *Testing instrumentation*

The tension tests were performed with use of Instron testing machine equipped with the head of 50 kN capacity. The longitudinal and transverse strains on the specimen were measured with strain gauges HBM 10/120 LY 11 and HBM 10/120 XY 91. The current number of intralaminar cracks has been monitored *in situ* through the optical microscope and camera. They were attached to a special translation beam, equipped in a device for vertical shift, necessary for observation of a chosen reference section of specimen's side edge. This instrumentation allows for almost continuous measurements of cracks number, though it is connected with some disadvantages caused by lack of automated system of data registration and analysis. Configuration of the testing instrumentation is shown in Fig. 8.

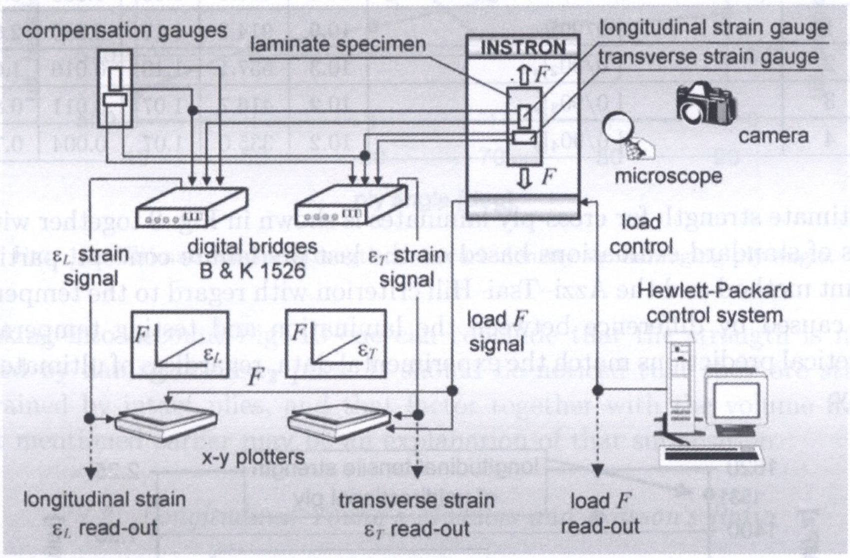

FIG. 8. The scheme of testing instrumentation.

7. Results and discussion

7.1. *Ultimate tensile strength*

In Table 3 the ultimate load, stress, longitudinal and transverse strains and average crack density are given for both specimens sets.

Ultimate crack density for set # 2 depends strongly on 90° ply thickness, however the ultimate tensile load is nearly the same for all four specimens, since it is determined mainly by undamaged, outer 0° plies. Thus, specimen longitudinal strength is not significantly influenced by damage of the 90° ply.

Table 3. Ultimate tensile load F_{ult}, stress σ_{ult}, longitudinal strain $\varepsilon_{\text{L ult}}$, transverse strain $\varepsilon_{\text{T ult}}$ and average crack density ρ_{ult} for specimens set #1 and set #2.

Specimen			Ultimate characteristics				
Set	Name	Code	F_{ult} [kN]	σ_{ult} [MPa]	$\varepsilon_{\text{L ult}}$ [%]	$\varepsilon_{\text{T ult}}$ [%]	ρ [1/mm]
1	A	$[-20/20/90_2/-20/20/90_2/-20/20]_s$	35.0	582.7	0.82	0.148	1.310
	B	$[-20/20/-80_2/-20/20/80_2/-20/20]_s$	33.9	580.0	0.85	0.166	0.960
	C	$[-20/20/-75_2/-20/20/75_2/-20/20]_s$	35.1	579.0	0.83	0.219	0.812
	D	$[-20/20/-70_2/-20/20/70_2/-20/20]_s$	34.6	573.6	0.86	0.271	0.573
	E	$[-20/20/-60_2/-20/20/60_2/-20/20]_s$	36.5	599.5	0.93	0.495	0.440
	G	$[-20/20/-50_2/-20/20/50_2/-20/20]_s$	34.3	592.2	0.99	0.816	0.333
	H	$[-20/20/-40_2/-20/20/40_2/-20/20]_s$	34.5	598.1	1.00	1.298	0.125
2	1	$[0/90]_s$	10.9	914.3	1.12	0.045	2.020
	2	$[0/90_2]_s$	10.3	557.1	1.10	0.016	1.083
	3	$[0/90_3]_s$	10.2	416.7	1.07	0.011	0.860
	4	$[0/90_4]_s$	10.2	335.0	1.07	0.004	0.710

Ultimate strength for cross-ply laminates is shown in Fig. 9 together with the results of standard evaluations based on the last ply failure concept, partial ply discount method and the Azzi–Tsai–Hill criterion with regard to the temperature effect caused by difference between the lamination and testing temperatures. Theoretical predictions match the experimental data, regardless of ultimate crack density.

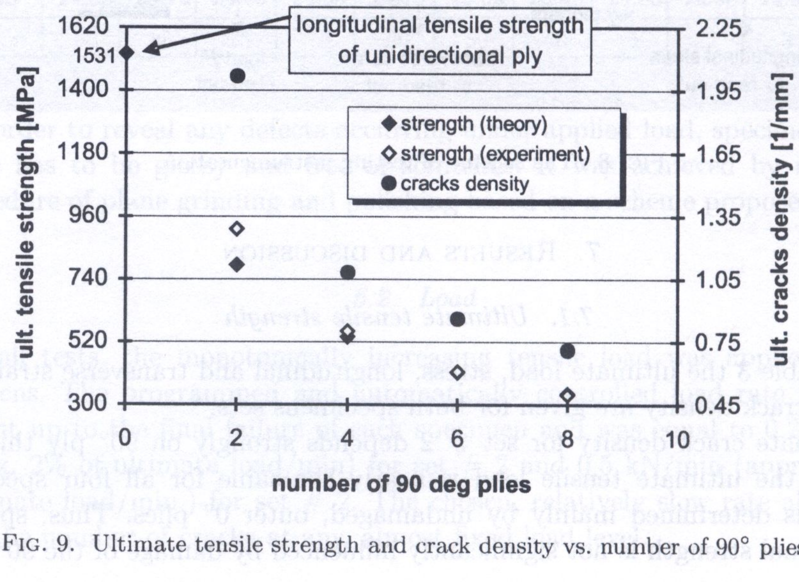

FIG. 9. Ultimate tensile strength and crack density vs. number of 90° plies.

The ultimate stresses for set # 1 are slightly different, what reflects the angle dependence of ultimate strength, but differences are less than 4.5%. It is probably caused by the fact, that volume fraction of damaging plies is 0.4, while it is 0.6 in case of untouched $\pm 20°$ plies. Strength dependence on average ultimate crack density is shown in Fig. 10.

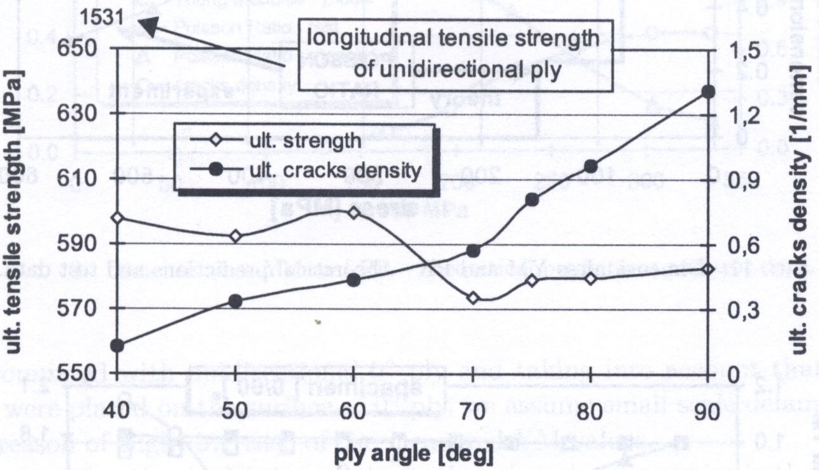

FIG. 10. Ultimate tensile strength and crack density vs. damaging ply angle.

Taking into account Fig. 10 one can conclude that the strength is not influenced by damage of $\pm\alpha_2$ plies. It should be noticed that they are strongly constrained by intact plies, and that factor together with the volume fraction aspect mentioned earlier may be an explanation of that supposition.

7.2. *Longitudinal Young's Modulus and Poisson's Ratio*

An example of the standard procedure for estimation of a laminate stiffness changes, based on strength analysis [30], is shown in Fig. 11. Partial ply discount method (PPDM), last ply failure concept and Azzi–Tsai–Hill criterion are used in calculations. The substantial differences – both quantitative and qualitative – between the longitudinal Young's modulus (abbreviated to YM) and Poisson's ratio (abbreviated to PR), calculated on PPDM basis and those taken from tests are easily visible. It is a general observation that PPDM underestimates the stiffness of a damaged laminate. The other observation is that PPDM leads to a somewhat unreasonable prediction of "step" change of the engineering constants instead of the gradual as it is observed in tests.

In Figs. 12 and 13 dimensionless (ratio of actual value to the initial one) YM and PR, as well as cracks density are plotted as functions of the applied load for all four cross-ply specimens.

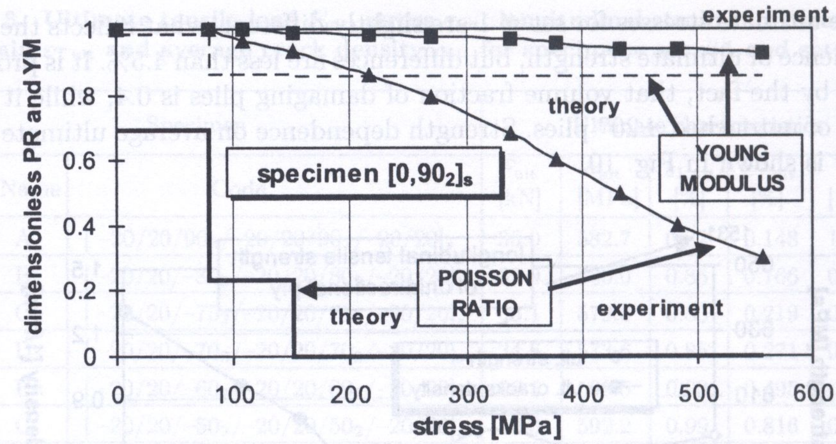

FIG. 11. Dimensionless YM and PR – theoretical predictions and test data.

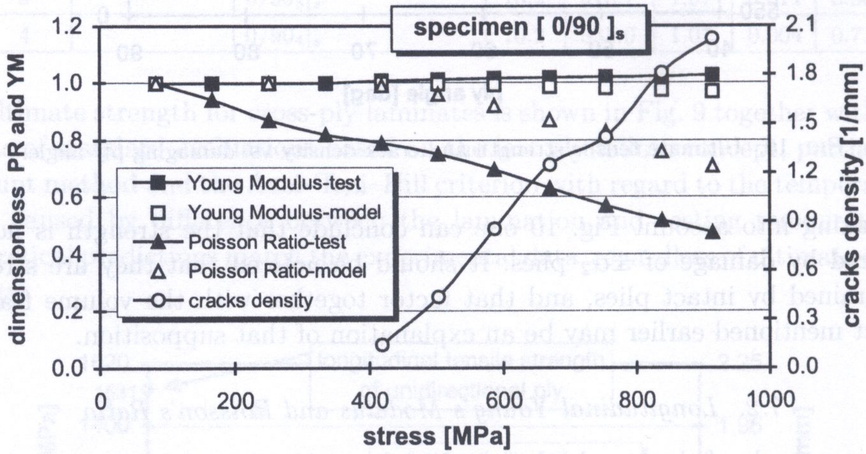

FIG. 12. Dimensionless YM and PR – theoretical predictions and test data.

The reduction in PR is as big as 50% for $[0/90]_s$ specimen and 90% for $[0/90_4]_s$. It follows from the very beginning of the loading process, but more detailed analysis of experimental data show that PR is reduced more when the load increase is accompanied by growth of cracks density.

For the specimen $[0/90]_s$ we have noticed – instead of the expected reduction – a very small (max. 2.5%) increase of YM. We observed at crack tips very short interlaminar cracks at 0/90 boundary and the resulting partial delamination at specimen free edges, which has intensified with increasing multitude of cracks. Recalling that longitudinal YM of cross-ply specimen is decreased by 90° ply

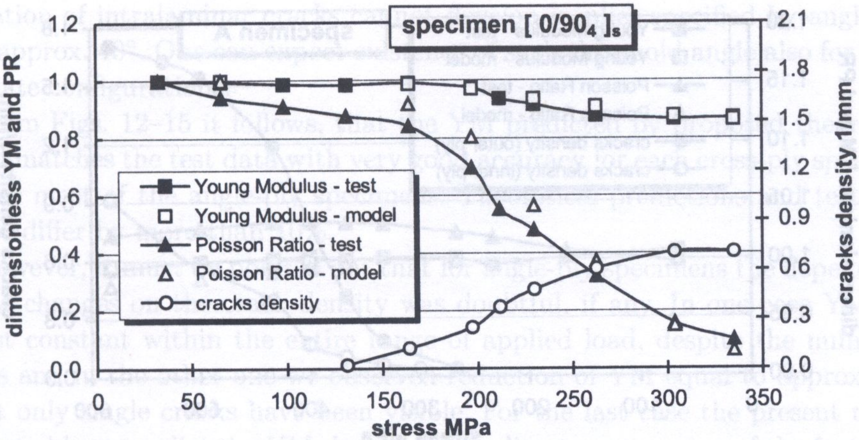

FIG. 13. Dimensionless YM and PR – theoretical predictions and test data.

when compared with unidirectional 0° ply and taking into account that strain gauges were placed on the surface of 0° ply, we assume small scale delamination as the reason of slight increase of the measured YM values.

Let us notice that reduction of the engineering characteristics is the largest for the thicker specimen, while the number of cracks in this specimen is the smallest. Thus, stiffness changes are not only damage, but also damaged ply volume fraction – dependent. One can observe also that the onset stress for matrix cracking initiation is inversely related to the thickness of cracking 90° ply (it is confirmed e.g. in [27]). It allows to conclude, that it is reasonable to divide 90° plies of cross-ply laminates into groups of thin plies, instead of forming one thick block of 90° orientation.

The mechanical behaviour of the specimens set # 1 is in general quite different when compared with the set # 2. Dimensionless YM, PR and crack density for specimens A and H are shown, respectively, in Figs. 14 and 15.

In all cases the measured PR increases with the applied load and the resulting crack density increase. It is different than in cross-ply specimens. Instead of the expected reduction of PR accompanying the damage growth, we observed its progressive growth. In case of specimen A – maximum growth of PR was equal to approx. 3.5% and crack density was the greatest, while in case of specimen H in which we observed only single cracks it was as big as 18%. From the above considerations it follows, that growth of PR is caused by laminate layout and damage mechanism possibly different from that being considered. Intralaminar cracks could only reduce this growth, but they were not able to cause absolute reduction of PR. Results reported in [4] for laminates stacking sequence $[\Theta_1/\Theta_2]_s$ confirm this phenomenon.

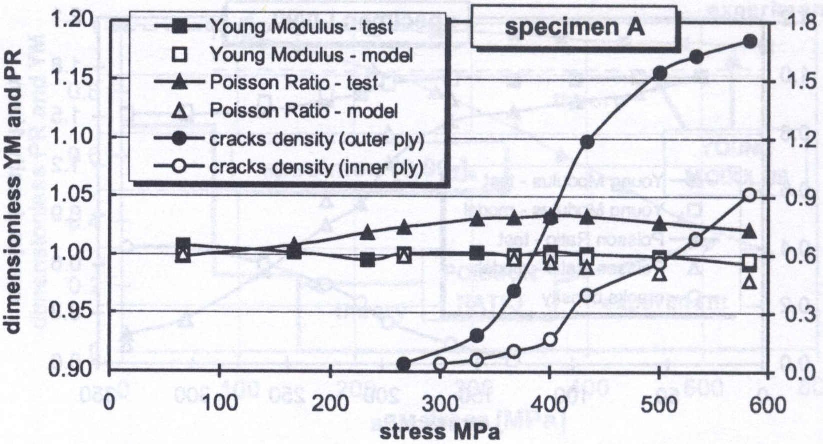

FIG. 14. Dimensionless YM, PR and crack density vs. stress for set # 1 specimen A.

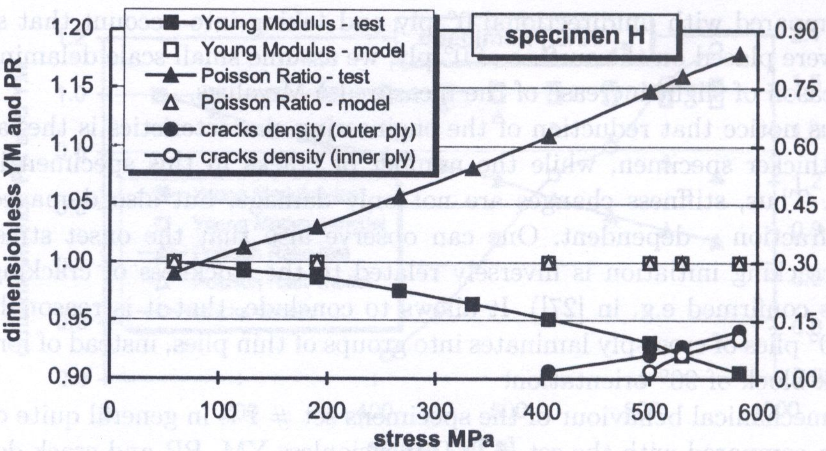

FIG. 15. Dimensionless YM, PR and crack density vs. stress for set # 1 specimen H.

For specimens A, YM is nearly constant and equal to the initial value within the entire range of applied load, despite the numerous cracks. Thus, actual values of the modulus do not depend on damage state. For specimen H we observe a reduction of YM equal to approx. 9%. However, it cannot be explained by damage growth, as crack density in specimen H is the smallest in comparison with the remaining specimens of set # 1.

Let us notice that we never observed cracks in fixed plies of ±20° orientation. Within plies ±40° (Fig. 15) we found only individual cracks, thus one can conclude that in the considered case the damage mechanisms connected with formation of intralaminar cracks cannot develop in plies specified by angle less than approx. 40°. One can expect existence of such threshold angle also for other laminate configurations.

From Figs. 12–15 it follows, that the YM predicted by proposed theoretical model matches the test data with very good accuracy for each cross-ply specimen and for most of the angle-ply specimens. Theoretical predictions and test data do not differ by more than 10%.

However, it must be pointed out that for angle-ply specimens the dependence of YM changes on the crack density was doubtful, if any. In one case YM was almost constant within the entire range of applied load, despite the numerous cracks and in the other one we observed reduction of YM equal to approx. 9%, whilst only single cracks have been visible. For the last case the present model was not able to predict the YM change. It is a direct consequence of the fact, that for a low crack density (or no crack at all) – a model must deliver the same result as for a nearly virgin material (or entirely virgin) and in fact, it does. Therefore, the observed mismatch does not mean that the model is unreasonable.

Fitting of the calculated and measured PR is in general not as good as in the case of YM, but is significantly better than that given by PPDM. For cross-ply specimens the maximum difference between predictions given by theoretical model and test results is reaches approx. 30%, but in most cases is much less. For most angle-ply specimens, instead of expected PR reduction (predicted for the applied model), its progressive increase was observed, but maximum differences have not been drastically big, as they were within the range 3÷18 %.

8. Conclusions

The influence of intralaminar cracks in carbon/epoxy laminates of two orientations on the strength and engineering characteristics has been investigated. Theoretical analysis and experimental data allow to conclude that:

- longitudinal strength is not considerably influenced by damage of 90° plies in cross-ply laminates and $\pm\alpha_2$ plies in the other orientation being considered,
- for cross-ply laminates both YM and PR are reduced due to a damage development in 90° ply. PR is much more sensitive to crack density than YM,
- in order to avoid the significant stiffness reduction in cross-ply laminates – 90° plies should not be formed in thick blocks,
- for angle-ply laminates being considered, influence of developing damage on the engineering characteristics is insignificant, if any. Despite the damage development, the growth of PR was observed, while reduction of YM was

noticed only for specific orientations and probably was caused by reasons other than intralaminar cracks,

- stiffness changes not always may be an appropriate measure of damage state in composite laminates, contrary to the isotropic materials, for which it is generally accepted measure,
- the results obtained from applied theoretical model are in good agreement with experimental data especially for cross-ply laminates – it can be seen as confirmation of correctness of the theoretical description.

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