Engineering Transactions, **66**(3): 301–324, 2018, doi: 10.24423/EngTrans.401.20180928
Polish Academy of Sciences • Institute of Fundamental Technological Research (IPPT PAN)
National Engineering School of Metz (ENIM) • Poznan University of Technology

Research Paper

Thermal Instability Analysis of an Elastico-Viscous Nanofluid Layer

Ramesh CHAND[1)], Gian C. RANA[2)], Dolors PUIGJANER[3)]

[1)] *Department of Mathematics*
Government College
Nurpur, Himachal Pradesh, India
e-mail: rameshnahan@yahoo.com

[2)] *Department of Mathematics*
NSCBM Govt. College Hamirpur
Himachal Pradesh, India
e-mail: drgcrana15@gmail.com

[3)] *Department d' Enginyeria Informatica i Matematiques*
Universitat Rovira i Virgili, Adva, Paisos Catalans
26, Tarragona, Catalonia, Spain
e-mail: dolors.puigjaner@urv.cat

The purpose of this paper is to study the thermal instability analysis of an elastico-viscous nanofluid layer heated from below. The Rivlin-Ericksen type fluid model is used to describe the rheological behavior of an elastico-viscous nanofluid. The linear stability criterion for the onset of both stationary and oscillatory convection is derived by applying the normal model analysis method. The presence of nanoparticles enhances the thermal conductivity of the fluid, and the model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. The effect of the physical parameters of the system, namely the concentration Rayleigh number, Prandtl number, capacity ratio, Lewis number, and kinematic visco-elasticity coefficient, on the stability of the system is numerically investigated. In addition, sufficient conditions for the non-existence of oscillatory convection are reported.

Key words: nanofluid; oscillatory convection; Rivlin-Ericksen fluid; thermal instability; viscoelasticity.

Notations

a – wavenumber,
c – specific heat,
d – thickness of the horizontal layer,
D_B – diffusion coefficient [m^2/s],

D_T – thermophoretic diffusion coefficient,
$\mathbf{g}$ – acceleration due to gravity [m/s^2],
F – kinematic visco-elasticity parameter,
k – thermal conductivity [W/(m·K)],
Le – Lewis number,
n – growth rate of disturbances,
N_A – modified diffusivity ratio,
N_B – modified particle-density ratio,
p – pressure [Pa],
Pr – Prandtl number,
$\mathbf{q}$ – Darcy velocity vector [m/s],
Ra – Rayleigh number,
Ra$_c$ – critical Rayleigh number,
Rm – density Rayleigh number,
Rn – concentration Rayleigh number,
t – time [s],
T – temperature [K],
(u, v, w) – Darcy velocity components,
(x, y, z) – space co-ordinates [m].

Greek symbols
α – thermal expansion coefficient [1/K],
φ – nanoparticle volume fraction,
κ – thermal diffusivity,
μ – viscosity,
μ' – visco-elasticity,
ρ – density of fluid [kg/m^3],
ρ_p – nanoparticle mass density [kg/m^3],
ω – dimensional frequency.

Superscripts
$'$ – non-dimensional variables,
$''$ – perturbed quantity.

Subscripts
p – particle,
f – fluid,
b – basic state,
0 – lower boundary,
1 – upper boundary,
H – horizontal plane.

1. Introduction

The principle of thermal instability is an important phenomenon that has applications in many different areas such as geophysics, food processing, oil reservoir modeling, thermal insulator design and nuclear reactors, among others. To date, thermal instability problems related to different types of fluids and geometric configurations have been extensively studied. The thermal instability of a Newtonian fluid under a wide range of hydrodynamic and hydromagnetic assumptions was discussed in detail by Chandrasekhar [1]. The thermal instability of a Maxwellian visco-elastic fluid in the presence of a magnetic field was analyzed by Bhatia and Steiner [2]. Scanlon and Segel [3] investigated the effect of suspended particles on the onset of convection in a horizontal layer uniformly heated from below, and they found that the critical Rayleigh number decreases when suspended particles are present. These authors concluded that the destabilization effect of suspended particles was due to the fact that the heat capacity of the pure fluid was supplemented by the particles.

Much research in recent years has focused on the study of nanofluids for applications in several industries such as the automotive, pharmaceutical or energy supply industries. A nanofluid is a colloidal suspension of nanosized particles, that is, particles the size of which is below 100 nm, in a base fluid. Common fluids such as water, ethanol or engine oils are typically used as base fluids in nanofluids. Among various nanoparticles that have been used in nanofluids, oxide ceramics such as Al_2O_3 or CuO, nitride ceramics such as AlN or SiN and several metals such as Al or Cu can be found. The understanding of the so-called anomalous increase in thermal conductivity of nanofluids has generated considerable research interest since the term nanofluid was first coined by Choi [4].

Xuan and Li [5] investigated convective heat transfer and flow features of the Cu-water nanofluid, and they observed that the heat transfer coefficient of the nanofluid was larger than that of the base fluid under the same Reynolds number. Buongiorno [6] proposed that the absolute nanoparticle velocity can be viewed as the sum of the base fluid velocity and a relative slip velocity. After analyzing the effect of the following seven slip mechanisms: inertia, Brownian diffusion, thermophoresis, diffusiophoresis, the Magnus effect, and fluid drainage and gravity, he concluded that Brownian diffusion and thermophoresis are the dominant slip mechanisms in the absence of turbulent eddies.

The onset of convection in a horizontal layer heated from below (the Bénard problem) for a nanofluid was studied by Tzou [7]. Alloui *et al.* [8] performed an analytical and numerical study of a natural convection problem in a shallow cavity filled with a nanofluid and heated from below. These authors reported that the presence of nanoparticles in a fluid reduced the strength of the flow field, and these reductions were especially relevant at low values of the Rayleigh number. Furthermore, they found that there is an optimum nanoparticle volume fraction, which depends on both the type of nanoparticles and the Rayleigh number, at which the heat transfer through the system is maximum. Yadav *et al.* [9] used a Galerkin method to study the onset of convection in the Rayleigh-Bénard problem with nanofluids, and they reported that the joint behavior of Brownian motion and thermophoresis of nanoparticles reduced the critical Rayleigh number by one order of magnitude as compared to fluids without nanoparticles. A considerable number of thermal instability problems in a horizontal layer of porous medium saturated by a nanofluid have also been numerically and analytically investigated by Kuznetsov and Nield [10–12], Nield and Kuznetsov [13–15] and Chamkha and Rashad [16], Chand [17], Chand and Rana [18–22], Yadav *et al.* [23], Chand *et al.* [22–26], and Rana *et al.* [27].

All the studies referred above dealt with Newtonian nanofluids. However, the growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology, and petroleum industry has attracted widespread interest in the study on non-Newtonian nanofluids. Although experiments performed by Toms and Strawbridge [28] revealed that the behavior of a dilute solution of methyl methacrylate in n-butyl acetate agrees well with the theoretical model of Oldroyd [29], it is widely known that there are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or by Oldroyd's constitutive relations. One such type of fluids, which has relevance in chemical technology, is the Rivlin-Ericksen elastico-viscous fluid. Rivlin and Ericksen [30] proposed a theoretical rheological model to describe the behavior of this type of elastico-viscous fluids.

The thermal instability problem in the Rivlin-Ericksen elastico-viscous fluids under a considerable amount of different hydrodynamic and hydromagnetic assumptions was studied by Prakash and Chand [31, 32], Rana and Chand [33], Sharma and Rana [34], Gupta and Sharma [35], Chand and Rana [36], and Rana [37]. Recently, Chand and Rana [38], Rana *et al.* [39], Rana and Chand [40], Sheu [41], and Chand and Rana [42] used the Oldroyd-B fluid model to describe the rheological behavior of the nanofluid in their investigation about thermal instability in a porous medium layer saturated with a viscoelastic fluid.

The growing number of applications of nanofluids, which include several fields, motivated the current study. In this study, our main aim is to study the thermal instability problem in a horizontal layer of a Rivlin-Ericksen elastico-viscous nanofluid.

2. Mathematical formulations

We consider an infinite horizontal layer of a Rivlin-Ericksen elastico-viscous nanofluid of thickness d, bounded by the planes $z = 0$ and $z = d$. The layer is heated from below with the gravity force $\mathbf{g} = (0, 0, -g)$ aligned in the z direction, as shown in Fig. 1. The temperature T and the volumetric fraction of nanoparticles φ, at the lower (upper) boundary, are assumed to take constant values

T_0 and φ_0 (T_1 and φ_1), respectively. We know that keeping a constant volume fraction of nanoparticles at the horizontal boundaries will be almost impossible in a realistic situation.

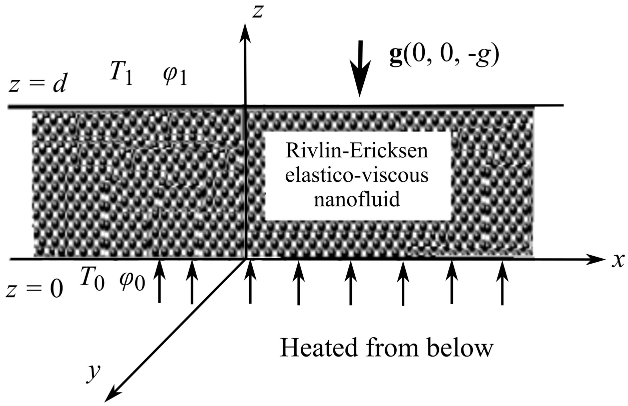

Fig. 1. Schematic sketch of the physical situation.

3. Assumptions

The mathematical equations describing the physical model are based on the following assumptions:

1) all thermophysical properties, except for the density in the buoyancy term, are constant (the Boussinesq hypothesis);
2) base fluid and nanoparticles are in the thermal equilibrium state;
3) nanoparticles are spherical;
4) nanofluid is incompressible and laminar;
5) negligible radiative heat transfer.

3.1. Governing equations

The conservation equations of mass and momentum for an incompressible Rivlin-Ericksen elastico-viscous fluid (Chandrasekhar [1], Prakash and Chand [31] and Rana [37]) are

$$\nabla \cdot \mathbf{q} = 0, \tag{3.1}$$

$$\rho \frac{\mathrm{d}\mathbf{q}}{\mathrm{d}t} = -\nabla p + \rho \mathbf{g} + \left(\mu + \mu' \frac{\partial}{\partial t} \right) \nabla^2 \mathbf{q}, \tag{3.2}$$

where $\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + (\mathbf{q} \cdot \nabla)$ stands for convection derivative, $\mathbf{q} = q(u, v, w)$ is the Darcy velocity vector, p is the hydrostatic pressure, $\mathbf{g} = (0, 0, -g)$ is the acceleration due to gravity, and μ and μ' are the viscosity and the visco-elasticity respectively.

The ρ density of the nanofluid can be written as in BUONGIORNO [6]:

$$\rho = \varphi\rho_p + (1-\varphi)\,\rho_f, \tag{3.3}$$

where φ is the volume fraction of nanoparticles, ρ_p is the density of nanoparticles, and ρ_f is the density of the base fluid. Following TZOU [7] and KUZNETSOV and NIELD [10] we approximate the density of the nanofluid by that of the base fluid, that is, we consider $\rho = \rho_f$. Now by introducing the Boussinesq approximation for the base fluid, the specific weight $\rho\mathbf{g}$ in Eq. (3.2) becomes

$$\rho\mathbf{g} \cong \left\{\varphi\rho_p + (1-\varphi)\left[\rho\left(1-\alpha\left(T-T_0\right)\right)\right]\right\}\mathbf{g}, \tag{3.4}$$

where α is the coefficient of thermal expansion.

Thus, the equation of motion (3.2) for a Rivlin-Ericksen elastico-viscous nanofluid can be written as

$$\rho\frac{\partial\mathbf{q}}{\partial t} = -\nabla p + \left\{\varphi\rho_p + (1-\varphi)\left[\rho\left(1-\alpha\left(T-T_0\right)\right)\right]\right\}\mathbf{g} + \left(\mu + \mu'\frac{\partial}{\partial t}\right)\nabla^2\mathbf{q}. \tag{3.5}$$

The continuity equation for the nanoparticles (BUONGIORNO [6]) is

$$\frac{\partial\varphi}{\partial t} + \mathbf{q}\cdot\nabla\varphi = D_B\nabla^2\varphi + \frac{D_T}{T_1}\nabla^2 T. \tag{3.6}$$

The energy equation for the nanofluid (BUONGIORNO [6]) is

$$\rho c\left(\frac{\partial T}{\partial t} + \mathbf{q}\cdot\nabla T\right) = k\nabla^2 T + (\rho c)_p\left(D_B\nabla\varphi\cdot\nabla T + \frac{D_T}{T_1}\nabla T\cdot\nabla T\right), \tag{3.7}$$

where ρc is the heat capacity of the base fluid, $(\rho c)_p$ is the heat capacity of nanoparticles, and k is the nanofluid thermal conductivity.

The boundary conditions are

$$w = 0, \qquad T = T_0, \qquad \varphi = \varphi_0 \qquad \text{at} \qquad z = 0, \tag{3.8$_1$}$$

$$w = 0, \qquad T = T_1, \qquad \varphi = \varphi_1 \qquad \text{at} \qquad z = 1. \tag{3.8$_2$}$$

We introduce non-dimensional variables as

$$(x', y', z') = \frac{1}{d}(x, y, z), \qquad (u', v', w') = \frac{d}{\kappa}(u, v, w), \qquad t' = \frac{\kappa}{d^2}t,$$

$$p' = \frac{d^2}{\mu\kappa}p, \qquad \varphi' = \frac{(\varphi-\varphi_0)}{(\varphi_1-\varphi_0)}, \qquad T' = \frac{(T-T_0)}{(T_0-T_1)},$$

where $\kappa = \frac{k}{\rho c}$ is the thermal diffusivity of the fluid.

Using the scales defined above and considering that, in the spirit of Boussinesq approximation, temperature gradients in the dilute suspension of nanoparticles are small enough to linearize Eq. (3.5) by neglecting the term involving the product of φ and T (Tzou [7]) Eqs. (3.1), (3.5)–(3.7) in non-dimensional form can be written as (dropping the primes for simplicity)

$$\nabla \cdot \mathbf{q} = 0, \tag{3.9}$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial \mathbf{q}}{\partial t} = -\nabla p + \left(1 + F\frac{\partial}{\partial t}\right)\nabla^2 \mathbf{q} - \mathrm{Rm}\,\widehat{e}_z + \mathrm{Ra}\,T\widehat{e}_z - \mathrm{Rn}\,\varphi\widehat{e}_z, \tag{3.10}$$

$$\frac{\partial \varphi}{\partial t} + \mathbf{q}\cdot\nabla\varphi = \frac{1}{\mathrm{Le}}\nabla^2\varphi + \frac{N_A}{\mathrm{Le}}\nabla^2 T, \tag{3.11}$$

$$\frac{\partial T}{\partial t} + \mathbf{q}\cdot\nabla T = \nabla^2 T + \frac{N_B}{\mathrm{Le}}\nabla\varphi\cdot\nabla T + \frac{N_A N_B}{\mathrm{Le}}\nabla T\cdot\nabla T, \tag{3.12}$$

where non-dimensional parameters are:

$\mathrm{Pr} = \dfrac{\mu}{\rho\kappa}$ is the Prandtl number,

$\mathrm{Le} = \dfrac{\kappa}{D_B}$ is the Lewis number,

$F = \dfrac{\mu'\kappa}{\mu d^2}$ is the kinematic visco-elasticity parameter,

$\mathrm{Ra} = \dfrac{\rho g \alpha d^3 (T_0 - T_1)}{\mu\kappa}$ is the Rayleigh number,

$\mathrm{Rm} = \dfrac{\rho_p \varphi_0 + \rho(1-\varphi_0) g d^3}{\mu\kappa}$ is the density Rayleigh number,

$\mathrm{Rn} = \dfrac{(\rho_p - \rho)(\varphi_1 - \varphi_0) g d^3}{\mu\kappa}$ is the concentration Rayleigh number,

$N_A = \dfrac{D_T (T_0 - T_1)}{D_B T_1 (\varphi_1 - \varphi_0)}$ is the modified diffusivity ratio, and

$N_B = \dfrac{\rho_p c_p (\varphi_1 - \varphi_0)}{\rho c}$ is the modified particle-density ratio.

The dimensionless boundary conditions are

$$w = 0, \qquad T = 1, \qquad \varphi = 0 \qquad \text{at} \qquad z = 0, \tag{3.13$_1$}$$

$$w = 0, \qquad T = 0, \qquad \varphi = 1 \qquad \text{at} \qquad z = 1. \tag{3.13$_2$}$$

3.2. Basic solutions

Following Kuznetsov and Nield [10–12] and Sheu [41] we assume a quiescent basic state that verifies

$$u = v = w = 0, \qquad p = p(z), \qquad T = T_b(z), \qquad \varphi = \varphi_b(z). \tag{3.14}$$

Therefore, when the basic state defined in Eq. (3.14) is substituted into Eqs. (3.9)–(3.12), these equations reduce to

$$0 = -\frac{\mathrm{d}p_b(z)}{\mathrm{d}z} - \mathrm{Rm} + \mathrm{Ra}\, T_b(z) - \mathrm{Rn}\, \varphi_b(z), \tag{3.15}$$

$$\frac{\mathrm{d}^2\varphi_b(z)}{\mathrm{d}z^2} + N_A \frac{\mathrm{d}^2 T_b(z)}{\mathrm{d}z^2} = 0, \tag{3.16}$$

$$\frac{\mathrm{d}^2 T_b(z)}{\mathrm{d}z^2} + \frac{N_B}{\mathrm{Le}} \frac{\mathrm{d}\varphi_b(z)}{\mathrm{d}z} \frac{\mathrm{d}T_b(z)}{\mathrm{d}z} + \frac{N_A N_B}{\mathrm{Le}} \left(\frac{\mathrm{d}T_b(z)}{\mathrm{d}z} \right)^2 = 0. \tag{3.17}$$

Using boundary conditions $(3.13)_1$ and $(3.13)_2$ the solution of Eq. (3.16) is

$$\varphi_b(z) = -N_A T_b + (1 - N_A)\, z + N_A. \tag{3.18}$$

By substituting expression (3.18) into Eq. (3.17), we obtain

$$\frac{\mathrm{d}^2 T_b(z)}{\mathrm{d}z^2} + \frac{(1 - N_A)\, N_B}{\mathrm{Le}} \frac{\mathrm{d}T_b(z)}{\mathrm{d}z} = 0. \tag{3.19}$$

The solution of differential Eq. (3.19) with boundary conditions $(3.13)_1$ and $(3.13)_2$ is

$$T_b(z) = \frac{1 - \mathrm{e}^{-(1-N_A)N_B(1-z)/\mathrm{Le}}}{1 - \mathrm{e}^{-(1-N_A)N_B/\mathrm{Le}}}. \tag{3.20}$$

Since, according to Sheu [41], the exponents in Eq. (3.20) are small for most typical nanofluids the exponential function in Eq. (3.20) is expanded into the power series, and all the terms except for the first order one are neglected. Therefore, a good approximation for the basic state solution is $T_b = 1 - z$ and $\varphi_b = z$.

3.3. Perturbation solutions

To study the stability of the system, we superimposed infinitesimal perturbations on the basic state, so that

$$\mathbf{q}\,(u,v,w) = 0 + q''\,(u,v,w)\,, \qquad T = (1-z) + T'', \qquad \varphi = z + \varphi'', \qquad p = p_b + p''. \tag{3.21}$$

Introducing Eq. (3.21) into Eqs. (3.8)–(3.11), linearizing the resulting equations by neglecting nonlinear terms that are the product of prime quantities and dropping the primes ($''$) for convenience, the following equations are obtained:

$$\nabla \cdot \mathbf{q} = 0, \tag{3.22}$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial \mathbf{q}}{\partial t} = -\nabla p + \left(1 + F\frac{\partial}{\partial t}\right)\nabla^2 \mathbf{q} + \mathrm{Ra}\,T\widehat{e}_z - \mathrm{Rn}\varphi\widehat{e}_z, \tag{3.23}$$

$$\frac{\partial \varphi}{\partial t} + w = \frac{1}{\mathrm{Le}}\nabla^2\varphi + \frac{N_A}{\mathrm{Le}}\nabla^2 T, \tag{3.24}$$

$$\frac{\partial T}{\partial t} - w = \nabla^2 T + \frac{N_B}{\mathrm{Le}}\left(\frac{\partial T}{\partial z} - \frac{\partial \varphi}{\partial z}\right) - \frac{2N_A N_B}{\mathrm{Le}}\frac{\partial T}{\partial z}. \tag{3.25}$$

Boundary conditions for Eqs. (3.22)–(3.25) are

$$w = 0, \qquad T = 0, \qquad \varphi = 0 \qquad \text{at} \qquad z = 0,\ 1. \tag{3.26}$$

Please note that as the parameter Rm is not involved in Eqs. (3.22)–(3.26) it is just a measure of the basic static pressure gradient. The six unknowns: u, v, w, p, T, and φ can be reduced to three by operating Eq. (3.23) with $\widehat{e}_z \cdot curl\, curl$, which yields

$$\frac{1}{\mathrm{Pr}}\frac{\partial}{\partial t}\nabla^2 w - \left(1 + F\frac{\partial}{\partial t}\right)\nabla^4 w = \mathrm{Ra}\nabla_H^2 T - \mathrm{Rn}\nabla_H^2 \varphi, \tag{3.27}$$

where ∇_H^2 is the two-dimensional Laplace operator on the horizontal plane, that is, $\nabla_H^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}$.

4. Normal modes

We express the disturbances in normal modes of the form

$$[w,\ T,\ \varphi] = [W(z),\ \Theta(z),\ \Phi(z)]\exp\left(ik_x x + ik_y y + \omega t\right), \tag{4.1}$$

where k_x and k_y are the wave numbers in the x and y direction, respectively, and ω is the growth rate of the disturbances.

Substituting the identities in Eq. (4.1) into Eqs. (3.24)–(3.27) we obtain the following eigenvalue problem:

$$(D^2-a^2)\left((1+\omega F)(D^2-a^2)-\frac{\omega}{\mathrm{Pr}}\right)W-a^2\mathrm{Ra}\Theta+a^2\mathrm{Rn}\Phi=0, \tag{4.2}$$

$$W-\frac{N_A}{\mathrm{Le}}(D^2-a^2)\Theta-\left(\frac{1}{\mathrm{Le}}(D^2-a^2)-\omega\right)\Phi=0, \tag{4.3}$$

$$W+\left(D^2-a^2-\omega+\frac{N_B}{\mathrm{Le}}D-\frac{2N_AN_B}{\mathrm{Le}}D\right)\Theta-\frac{N_B}{\mathrm{Le}}D\Phi=0, \tag{4.4}$$

$$W=0,\qquad D^2W=0,\qquad \Theta=0,\qquad \Phi=0\qquad \text{at}\qquad z=0 \tag{4.5}$$

and

$$W=0,\qquad D^2W=0,\qquad \Theta=0,\qquad \Phi=0\qquad \text{at}\qquad z=1, \tag{4.6}$$

where $D\equiv\frac{\mathrm{d}}{\mathrm{d}z}$ and $a=\sqrt{k_x^2+k_y^2}$ is the dimensionless horizontal wave number.

5. Linear stability analysis

Let us consider solutions W, Θ and Φ of the form

$$W=W_0\sin(\pi z),\qquad \Theta=\Theta_0\sin(\pi z),\qquad \Phi=\Phi_0\sin(\pi z). \tag{5.1}$$

Substituting Eq. (5.1) into Eqs. (4.2)–(4.4) and integrating each equation from $z=0$ to $z=1$, we obtain the following linear system:

$$\begin{bmatrix} -J\left((1+\omega F)J+\frac{n}{\mathrm{Pr}}\right) & a^2\mathrm{Ra} & -a^2\mathrm{Rn} \\ 1 & -(\mathrm{J}+\omega) & 0 \\ 1 & \frac{N_A}{\mathrm{Le}}J & \frac{J}{\mathrm{Le}}+\omega \end{bmatrix}\begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix},$$

where $J=\pi^2+a^2$

The above linear system has a non-trivial solution if and only if

$$\mathrm{Ra}=\frac{1}{a^2}\left((J+\omega)\left(J^2+\left(J^2F+\frac{J}{\mathrm{Pr}}\right)\omega\right)\right)-\frac{\left(\frac{N_A}{\mathrm{Le}}J+J\right)+\omega}{\frac{J}{\mathrm{Le}}+\omega}\mathrm{Rn}. \tag{5.2}$$

Setting $n=i\omega$ (where ω is a real dimensional frequency) in Eq. (5.2), we obtain

$$\mathrm{Ra}=\Delta_1+i\omega\Delta_2, \tag{5.3}$$

where

$$\Delta_1 = \frac{J}{a^2}\left(J^2 - \left(JF + \frac{1}{\mathrm{Pr}}\right)\omega^2\right) - \frac{\frac{J^2}{\mathrm{Le}^2}\left(N_A + \mathrm{Le}\right) + \omega^2}{\left(\frac{J}{\mathrm{Le}}\right)^2 + \omega^2}\mathrm{Rn}, \tag{5.4}$$

and

$$\Delta_2 = \frac{J^2}{a^2}\left(1 + JF + \frac{1}{\mathrm{Pr}}\right) - \frac{\frac{J}{\mathrm{Le}} - J\left(\frac{N_A}{\mathrm{Le}} + 1\right)}{\left(\frac{J}{\mathrm{Le}}\right)^2 + \omega^2}\mathrm{Rn}. \tag{5.5}$$

Since Ra is a physical quantity, it must be real. Hence, from Eq. (5.3) we can conclude that either $\omega = 0$ (steady onset) or $\omega \neq 0$ and $\Delta_2 = 0$ (oscillatory onset).

6. Stationary convection

For stationary convection $n = 0$ ($\omega = 0$), Eq. (5.2) reduces to

$$(\mathrm{Ra})_s = \frac{\left(\pi^2 + a^2\right)^3}{a^2} - \mathrm{Rn}\left(\mathrm{Le} + N_A\right). \tag{6.1}$$

Equation (6.1) expresses the Rayleigh number as a function of the dimensionless horizontal wave number and the parameters Rn, Le, N_A. Since the elastico-viscous parameter F is not present in Eq. (6.1) it may be concluded that in the stationary case ($n = 0$) the Rivlin-Ericksen elastico-viscous fluid behaves like an ordinary Newtonian fluid. Please note that Eq. (6.1) is identical to that obtained by Sheu [42] in the absence of porous medium and Chand and Rana [18] in the absence of Darcy-Brinkman porous medium.

The critical cell size at the onset of instability is obtained by minimizing Ra with respect to wave number a. Thus, the critical cell size must satisfy $\left(\frac{\partial \mathrm{Ra}}{\partial a}\right)_{a=a_c} = 0$, which gives

$$2\left(a_c^2\right)^3 + 3\pi^2 a_c^2 - \pi^6 = 0. \tag{6.2}$$

From Eq. (6.1), we obtain

$$a_c = \frac{\pi}{\sqrt{2}} = 2.22144. \tag{6.3}$$

The corresponding critical Rayleigh number Ra_c for steady onset is

$$(\mathrm{Ra}_c)_s = \frac{27\pi^2}{4} - \mathrm{Rn}\left(\mathrm{Le} + N_A\right). \tag{6.4}$$

7. Oscillatory convection

For oscillatory convection ($\omega \neq 0$), we must have $\Delta_2 = 0$. Hence, the frequency of the oscillations is

$$\omega^2 = \frac{a^2}{J}\left(\frac{\frac{1}{\mathrm{Le}} - \left(\frac{N_A}{\mathrm{Le}} + 1\right)}{1 + JF + \frac{1}{\mathrm{Pr}}}\right)\mathrm{Rn} - \frac{J^2}{\mathrm{Le}^2}. \tag{7.1}$$

Equation (5.3) with $\Delta_2 = 0$ gives the following thermal oscillatory Rayleigh number:

$$\mathrm{Ra}_{\mathrm{osc}} = \frac{J}{a^2}\left(J^2 - \left(JF + \frac{1}{\mathrm{Pr}}\right)\omega^2\right) - \frac{\frac{J^2}{\mathrm{Le}^2}(\mathrm{Le} + N_A) + \omega^2}{\left(\frac{J}{Le}\right)^2 + \omega^2}\mathrm{Rn}. \tag{7.2}$$

Since oscillatory convection will only exist when positive values for ω^2 in Eq. (7.3) can be obtained, the following conditions:

$$\begin{aligned} &\mathrm{Rn} < 0 \qquad \text{and} \qquad \frac{1}{\mathrm{Le}} > \left(\frac{N_A}{\mathrm{Le}} + 1\right) \\ \text{or} \qquad &\mathrm{Rn} > 0 \qquad \text{and} \qquad \frac{1}{\mathrm{Le}} < \left(\frac{N_A}{\mathrm{Le}} + 1\right), \end{aligned} \tag{7.3}$$

are sufficient for the non-existence of oscillatory convection. Please note that the violation of expressions (7.3) does not necessarily imply the occurrence of oscillatory convection.

8. Results and discussions

Critical Rayleigh numbers for the onset of steady and oscillatory convection are given by Eqs. (6.4), (7.1), and (7.2), respectively. The critical Rayleigh value obtained for the onset of steady convection in the current Rivlin-Ericksen elastico-viscous fluid problem does not depend on viscoelastic parameters, and it takes the same value as the one obtained for an ordinary Newtonian fluid. Furthermore, the critical wave number a_c, defined by Eq. (6.3) at the onset of steady convection coincides with those reported by Tzou [7], Kuznetsov and Nield [12], and Chand and Rana [18]. Please note that this critical value does not depend on any thermophysical property of the nanofluid. Consequently, the combined behavior of Brownian motion and thermophoresis of nanoparticles does not change the cell size at the onset of steady instability and the critical cell size (a_c) is identical to the well known result for the Bénard instability with a regular fluid. In the absence of nanoparticles, that is $\mathrm{Rn} = N_A = 0$, the critical Rayleigh number takes the value $\mathrm{Ra}_c = \frac{27\pi^2}{4}$, which is exactly the critical

Rayleigh number for regular fluids see CHANDRASEKHAR [1]. Thus the combined effect of Brownian motion and thermophoresis of nanoparticles on the critical Rayleigh number is reflected in the second term in Eq. (6.4). From Eq. (6.4), it can be concluded that for the case of top-heavy distribution of nanoparticles ($\varphi_1 > \varphi$ and $\rho_p > \rho$), which corresponds to positive values of Rn, the value of the steady critical Rayleigh number for the nanofluid is smaller than that for an ordinary fluid, that is, steady convection sets earlier in these kinds of nanofluids than in an ordinary fluid. This implies that the thermal conductivity of ordinary fluids is higher than that of nanofluids with a top-heavy distribution of nanoparticles. On the contrary, for the case of bottom-heavy distribution of nanoparticles ($\varphi_1 < \varphi$ and $\rho_p > \rho$), which corresponds to the negative values of Rn, the value of the critical Rayleigh number for the nanofluid is larger than that for an ordinary fluid, that is, convection sets earlier in an ordinary fluid than in a nanofluid with a bottom-heavy distribution of nanoparticles. This implies that the thermal conductivity of this kind of nanofluids is higher than that of ordinary fluids.

The computations are carried out for different values of parameters considered in the range $10^2 \leq \mathrm{Ra} \leq 10^5$ (thermal Rayleigh number), $-10 \leq N_A \leq -1$ (modified diffusivity ratio), $10^2 \leq \mathrm{Le} \leq 10^4$ (Lewis number), $-10 \leq \mathrm{Rn} \leq 10$ (nanoparticles' Rayleigh number), $10^{-1} \leq F \leq 10$ (kinematic viscoelasticity parameter), and $10^{-2} \leq \mathrm{Pr} \leq 10^2$ (Prandtl number Pr) (NIELD and KUZNETSOV [15], CHAND and RANA [31], CHAND and RANA [38]).

The variation of the steady thermal Rayleigh number Ra_s as a function of the wave number for different sets of values for the physical parameters Le, Rn and N_A are shown in Figs. 2–4. The values of N_A and Rn are fixed to -5 and -1, respectively.

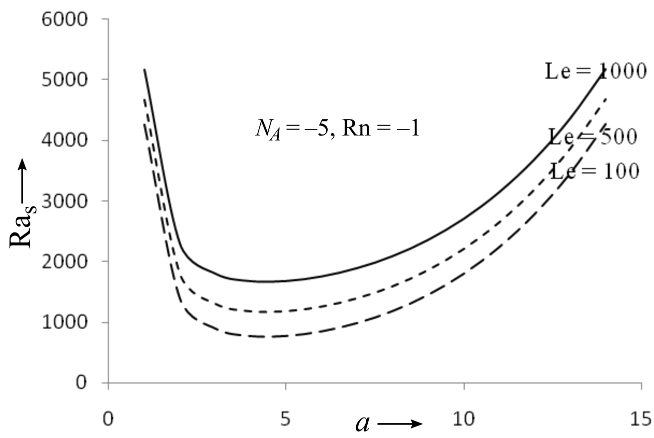

FIG. 2. Variation of stationary Rayleigh number with wave number for different values of Lewis number.

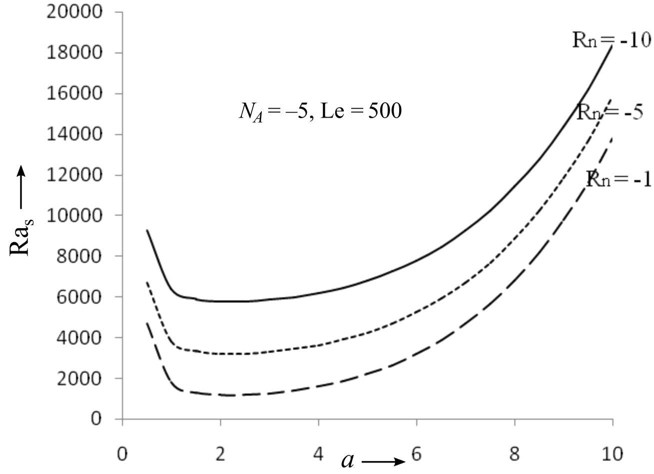

FIG. 3. Variation of stationary Rayleigh number with wave number for different values of concentration Rayleigh number.

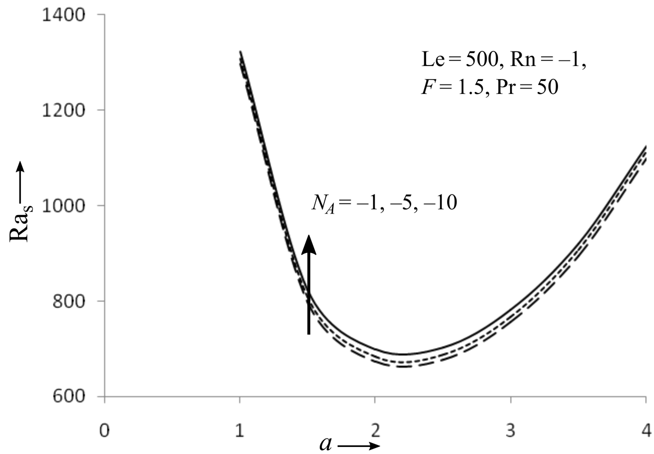

FIG. 4. Variation of stationary Rayleigh number with wave number for different values of modified diffusivity ratio.

The variation of Ra_s with the wave number for three different values of the Lewis number, namely Le = 100, 500 and 1000, is plotted in Fig. 2. The increase in the Rayleigh number, observed in Fig. 2, as the Lewis number is increased reveals that an increase in the Lewis number tends to delay the onset of steady convection.

Figure 3 represents the variation of stationary Rayleigh number Ra_s with wave number for the different value of concentration Rayleigh number, and it is found that the stationary Rayleigh number decreases with an increase in the value of the concentration Rayleigh number Rn, which implies that the concentration Rayleigh number destabilizes the stationary convection. The negative value of Rn indicates a bottom-heavy distribution while the positive value of Rn indicates a top-heavy distribution of nanoparticles. It is also observed that the stationary convection is possible for both bottom-heavy and top-heavy nanoparticles' distribution, and the stationary Rayleigh number is smaller for a top-heavy distribution as compared to that of a bottom-heavy distribution of nanoparticles.

Figure 4 represents the variation of the stationary Rayleigh number Ra_s as a function of the wave number for two values of modified diffusivity ratio ($N_A = -10$ and -1) when $\mathrm{Le} = 500$ and $\mathrm{Rn} = -1$. It is observed that the stationary Rayleigh number Ra_s increases with an increase in the value of N_A, which implies that a modified diffusivity ratio stabilizes the fluid layer. Nield and Kuznetsov [15] and Kuznetsov and Nield [12] showed that oscillatory instability is possible only for bottom-heavy nanoparticle distributions. For heavy nanoparticles ($\rho_p > \rho$), a bottom-heavy nanoparticle distribution is equivalent to a negative value of Rn. In such a case, the value of N_A will also be negative. From now only negative values of Rn are considered. Results observed in Figs. 2, 3 and 4 are consistent with those reported by Sheu [41] and Chand and Rana [18].

Figure 5 shows the variation of the oscillatory Rayleigh number with the wave number for the fixed value of the kinematic visco-elasticity parameter F. It is found that the oscillatory Rayleigh number increases with an increase in the value of the kinematic visco-elasticity parameter F. Thus, kinematic visco-elasticity parameter F has a stabilizing effect on the oscillatory convection.

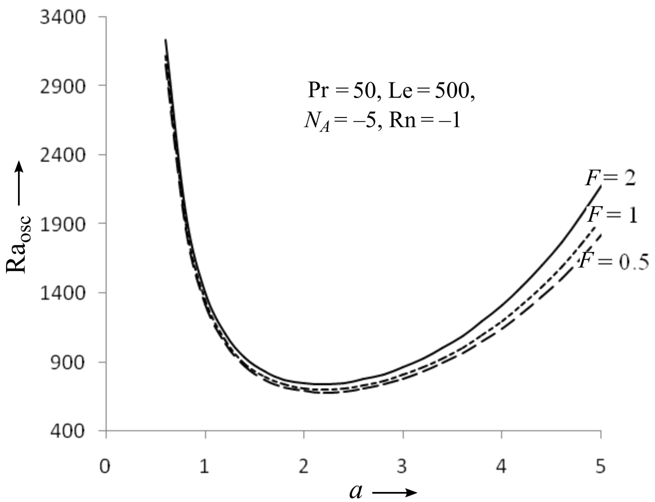

Fig. 5. Variation of oscillatory Rayleigh number with wave number for the different value of kinematic visco-elasticity parameter F.

Figure 6 shows the variation of the oscillatory Rayleigh number with wave number a for the fixed values of modified diffusivity ratio N_A, and it is found that the oscillatory Rayleigh number slightly decreases as the values of the modified diffusivity ratio N_A increase; thus a modified diffusivity ratio has a destabilizing effect on the oscillatory convection.

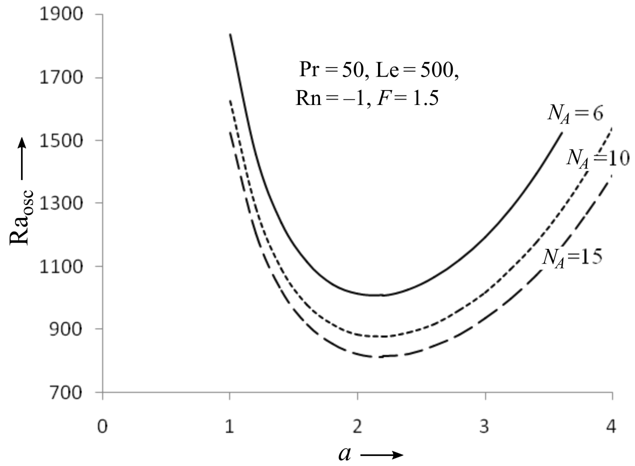

Fig. 6. Variation of oscillatory Rayleigh number with wave number for different modified diffusivity ratio.

Figure 7 shows the variation of the oscillatory Rayleigh number with wave number a for the fixed values of the Prandtl number, and it is found that the oscillatory Rayliegh number slightly decreases with an increase in the value of

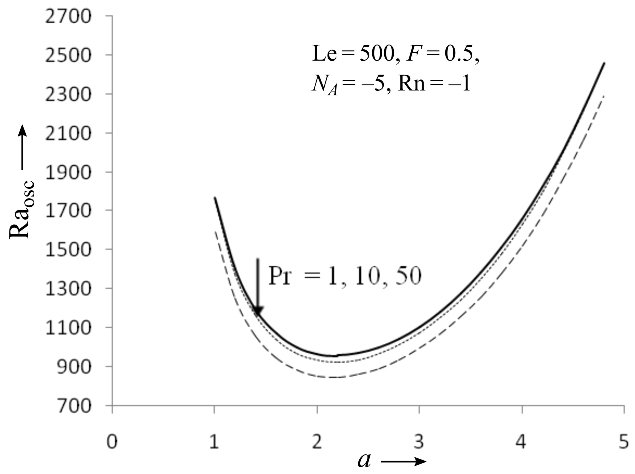

Fig. 7. Variation of oscillatory Rayleigh number with wave number for different Prandtl number.

the Prandtl number; thus Prandtl number has a destabilizing effect on the oscillatory convection.

Figure 8 shows the variation of the oscillatory Rayleigh number with wave number a for the fixed values of the concentration Rayleigh number, and it is observed that the oscillatory Rayleigh number slightly decreases with an increase in the values of the concentration Rayleigh number (for a bottom-heavy distribution of nanoparticles); thus a concentration Rayleigh number destabilizes the oscillatory convection. It is also observed that the oscillatory convection was not possible for the top-heavy distribution of nanoparticles.

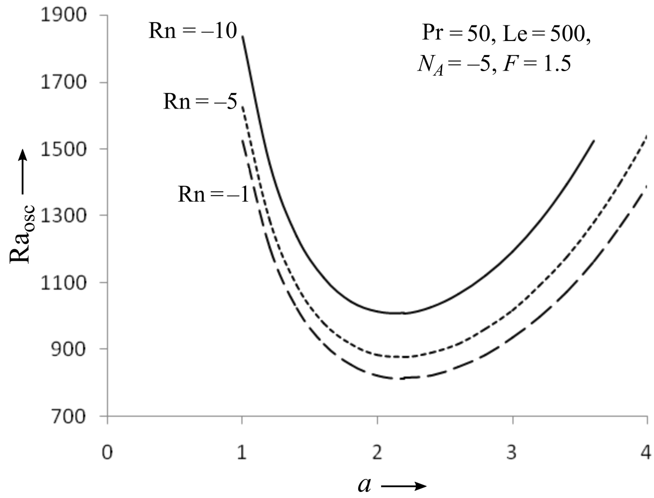

FIG. 8. Variation of oscillatory Rayleigh number Ra with wave number for different concentration Rayleigh number.

Figure 9 shows the variation in the oscillatory Rayleigh number with wave number a for the fixed values of the Lewis number, and it is found that the oscillatory Rayleigh number decreases with an increase in the values of the Lewis number. Thus the Lewis number has a destabilizing effect on the oscillatory convection.

Figure 10 shows the variation of both stationary Rayleigh number and oscillatory Rayleigh number with wave number for the fixed values of other parameters. It is found that the stationary Rayleigh number is higher than the oscillatory Rayleigh number. Thus the oscillatory instability could set in before the stationary instability, and this result suggests that the oscillatory convection might set in before the stationary convection.

From Figs. 5 to 9, it is observed that the kinematic visco-elasticity parameter F has a stabilizing effect while the rest of physical parameters, namely N_A, Pr, Rn and Le, have a destabilizing effect on the oscillatory convection, that is,

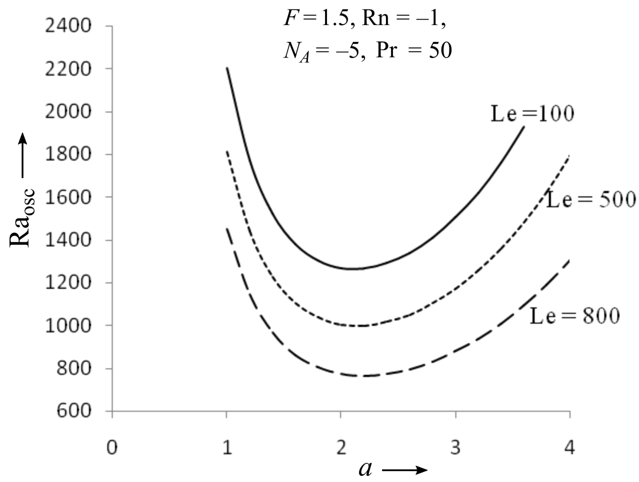

FIG. 9. Variation of oscillatory Rayleigh number Ra with wave number for different Lewis number.

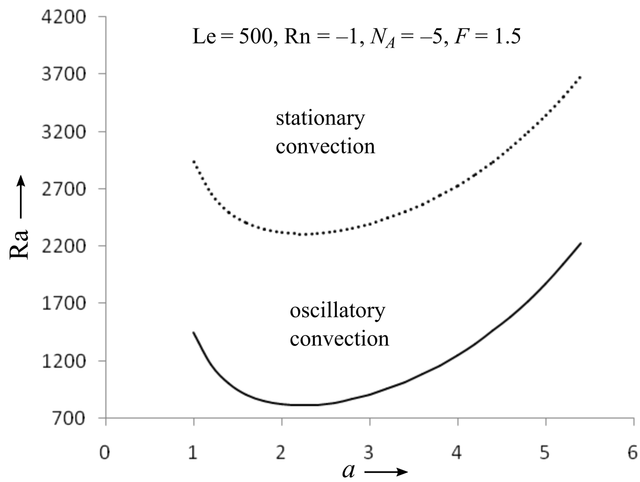

FIG. 10. Variation of stationary Rayleigh number and oscillatory Rayleigh number with wave number.

the onset of oscillatory convection is accelerated when one of these parameters is increased. Figure 10 shows the variation of both stationary and oscillatory Rayleigh numbers as the wave number is varied for a set of fixed values of the rest of the physical parameters.

In Fig. 10, it can be observed that the stationary Rayleigh number is higher than the oscillatory Rayleigh number along with the whole range of wave numbers investigated. This result suggests that the oscillatory convection might set in before the steady convection.

The variation of the critical oscillatory Rayleigh number with the kinematic visco-elasticity parameter F for different sets of values for the physical parameters Le, Rn, N_A and Pr is shown in Figs. 11–14. It is worth noting that for the set of physical parameters chosen in all these figures convection sets in with an oscillatory mode and it remains oscillatory until a certain (critical) value of the kinematic visco-elasticity parameter F is reached. At this critical value of F, con-

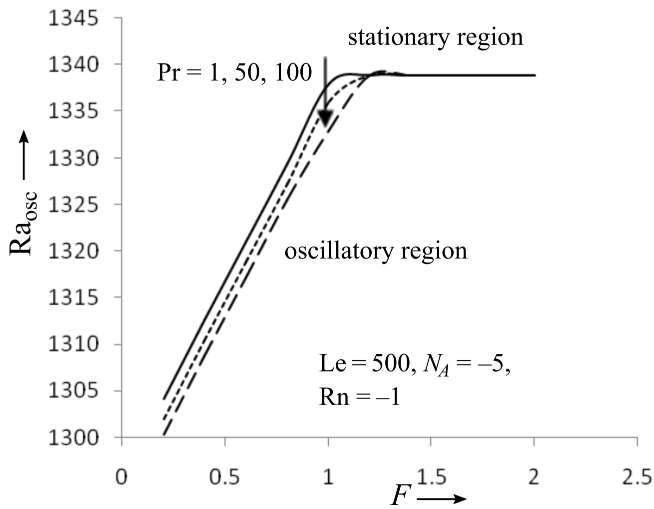

FIG. 11. Variation in the critical oscillatory Rayleigh number with a kinematic visco-elasticity parameter for different values of Prandtl number Pr.

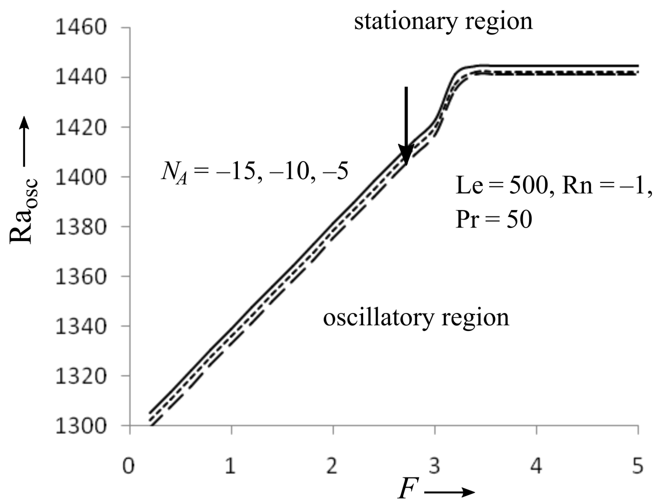

FIG. 12. Variation in the critical oscillatory Rayleigh number with a kinematic visco-elasticity parameter for different values of modified diffusivity ratio.

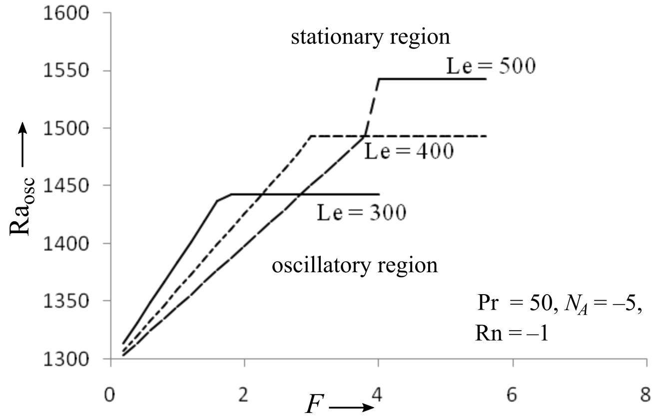

FIG. 13. Variation in the critical oscillatory Rayleigh number with a kinematic viscoelasticity parameter for different values of Lewis number Le.

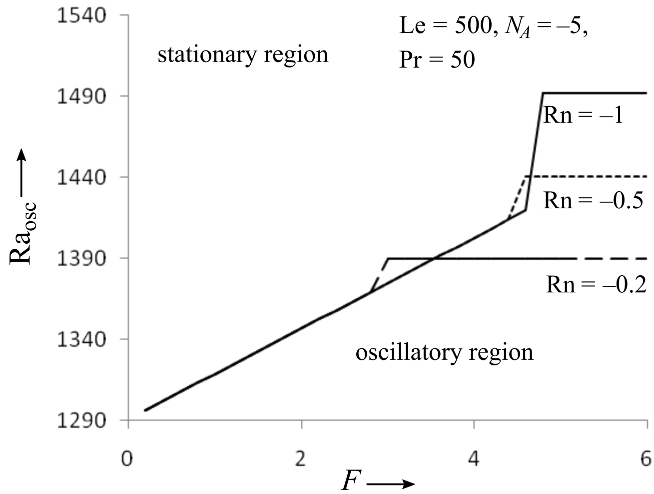

FIG. 14. Variation in the critical oscillatory Rayleigh number with a kinematic viscoelasticity parameter for different values of concentration Rayleigh number.

vection ceases to be oscillatory, and it becomes steady. This critical value of F, which depends on the other physical parameters: Le, Rn, N_A and Pr, determines the boundary between oscillatory and steady convection.

Figure 11 shows the effect of the Prandtl number Pr on the variation of the critical oscillatory Rayleigh number as a function or the kinematic viscoelasticity parameter F when $\mathrm{Le} = 500$, $N_A = -5$ and $\mathrm{Rn} = -1$. It was also found that the oscillatory Rayleigh number decreases with increases in the Prandtl number Pr, indicating a destabilizing effect of the Prandtl number on the oscillatory convection.

Figure 12 shows the effect of the modified diffusivity ratio N_A on the oscillatory Rayleigh number for the fixed values of other parameters. It is found that the oscillatory Rayleigh number slightly decreases with an increase in the values of the modified diffusivity ratio; thus modified diffusivity ratio has a destabilizing effect on the oscillatory convection.

Figure 13 shows the effect of the Lewis number on the oscillatory Rayleigh number for the fixed values of other parameters. It is found that Rayleigh number decreases with an increase in the values of the Lewis number; thus the Lewis number has a destabilizing effect on the oscillatory convection.

Figure 14 shows the effect of the concentration Rayleigh number on the oscillatory Rayleigh number for the fixed values of other parameters. It is found that Rayleigh number slightly decreases with an increase in the value of the concentration Rayleigh number, which implies that the concentration Rayleigh number has a destabilizing effect on the oscillatory convection.

The results presented in Figs. 5–14 are in good agreement with those obtained by Sheu [42].

9. Conclusions

The onset of both stationary and oscillatory convection for a Rivlin-Ericksen elastico-viscous nanofluid layer heated from below is investigated by using a linear stability analysis. For the case of stationary convection, the Rivlin-Ericksen nanofluid behaves like an ordinary Newtonian nanofluid. The effect of both the Lewis number Le and the modified diffusivity ratio N_A is to stabilize the stationary convection and to destabilize the oscillatory convection. The concentration Rayleigh number Rn destabilizes both stationary and oscillatory convection. The oscillatory convection is possible only for bottom-heavy nanoparticle distributions whereas stationary convection is possible for both bottom and top-heavy distributions of nanoparticles. The Prandtl number Pr destabilizes the oscillatory convection and does not affect stationary convection. The kinematic visco-elasticity parameter F stabilizes the oscillatory convection and does not affect stationary convection. Convection initially begins in the form of an oscillatory mode, and it remains oscillatory until certain (critical) value of the kinematic visco-elasticity parameter F is reached. At this critical value of F, convection ceases to be oscillatory, and stationary convection occurs. This critical value of F, which depends upon the rest of the parameters, namely Le, N_A and Rn, determines the boundary between oscillatory and stationary convection. The steady Rayleigh number is higher than the oscillatory Rayleigh number. Sufficient conditions for the non-existence of oscillatory convection are presented.

Acknowledgment

Authors would like to thank the learned referees for their valuable comments and suggestions for the improvement of the quality of the paper.

References

1. Chandrasekhar S., *Hydrodynamic and Hydromagnetic Stability*, Clarendon Press, Oxford, 1961.
2. Bhatia P.K., Steiner J.M., *Thermal instability of visco-elastic fluid in hydromagnetics*, *Journal* of *Mathematical Analysis* and Applications, **41**(2): 271–282, 1973.
3. Scanlon J.W., Segel L.A., *Some effect of suspended particles on onset of Bénard Convection*, Physics of Fluids, **16**(10): 1573–1578, 1973, doi: 10.1063/1.1694182.
4. Choi S., Eastman J.A., *Enhancing thermal conductivity of fluids with nanoparticles*, [in:] D.A. Siginer and H.P. Wang [Eds.], Developments and Applications of Non-Newtonian Flows, ASME FED, **66**: 99–105, 1995.
5. Xuan Y., Li Q., *Investigation of convective heat transfer and flow features of nanofluids*, ASME Journal of Heat Transfer, **125**(1): 151–155, 2003.
6. Buongiorno J., *Convective transport in nanofluids*, ASME Journal of Heat Transfer, **128**(3): 240–250, 2005.
7. Tzou D.Y., *Thermal instability of nanofluids in natural convection*, International Journal of Heat and Mass Transfer, **51**(11–12): 2967–2979, 2008.
8. Alloui Z., Vasseur, P., Reggio M., *Natural convection of nanofluids in a shallow cavity heated from below*, International Journal of Thermal Science, **50**(3): 385–393, 2011.
9. Yadav D., Agrawal G.S., Bhargava R., *Rayleigh-Bénard convection in nanofluid*, International Journal of Applied Mathematics and Mechanics, **7**: 61–76, 2010.
10. Kuznetsov A.V., Nield D.A., *Effect of local thermal non-equilibrium on the onset of convection in a porous medium layer saturated by a nanofluid*, Transport in Porous Media, **83**(2): 425–436, 2010.
11. Kuznetsov A.V., Nield D.A., *Thermal instability in a porous medium layer saturated by a nanofluid: Brinkman model*, Transport in Porous Media, **81**(3): 409–422, 2010.
12. Kuznetsov A.V., Nield D.A., *The onset of double-diffusive nanofluid convection in a layer of saturated porous medium*, Transport in Porous Media, **85**(3): 941–951, 2010.
13. Nield D.A., Kuznetsov A.V., *Thermal instability in a porous medium layer saturated by a nanofluid*, International Journal of Heat and Mass Transfer, **52**(25–26): 5796–5801, 2009.
14. Nield D.A., Kuznetsov A.V., *The onset of convection in a layer of cellular porous material: effect of temperature-dependent conductivity arising from radiative transfer*, Journal of Heat Transfer, **132**(7): 074503, 2010.
15. Nield D.A., Kuznetsov A.V., *The effect of vertical throughflow on thermal instability in a porous medium layer saturated by a nanofluid*, Transport in Porous Media, **87**(3): 765–775, 2011.

16. Chamkha A.J., Tashad A.M., *Natural convection from a vertical permeable cone in a nanofluid saturated porous media for uniform heat and nanoparticles volume fraction fluxes*, International Journal of Numerical Methods for Heat and Fluid Flow, **22**(8): 1073–1085, 2012.

17. Chand R., *On the onset of Rayleigh-Bénard convection in a layer of nanofluid in hydromagnetics*, International Journal of Nanoscience, **12**(06): 1350038, 2014.

18. Chand R., Rana G.C., *On the onset of thermal convection in rotating nanofluid layer saturating a Darcy-Brinkman porous medium*, International Journal of Heat and Mass Transfer, **55**: 5417–5424, 2012.

19. Chand R., Rana G.C., *Oscillating convection of nanofluid in porous medium*, Transport in Porous Media, **95**(2): 269–284, 2012.

20. Chand R., Rana G.C., *Hall effect on the thermal instability in a horizontal layer of nanofluid*, Journal of Nanofluids, **3**(3): 247–253, 2014.

21. Chand R., Rana G.C., *Thermal instability in a Brinkman porous medium saturated by nanofluid with no nanoparticle flux on boundaries*, Special Topics & Reviews in Porous Media: An International Journal, **5**(3): 277–286, 2014.

22. Chand R., Rana G.C., *Magneto convection in a layer of nanofluid in porous medium – a more realistic approach*, Journal of Nanofluids, **4**(2): 196–202, 2015.

23. Yadav D., Agrawal G.S., Bhargava R., *Thermal instability of rotating nanofluid layer*, International Journal of Engineering Science, **49**(11): 1171–1184, 2011.

24. Chand R., Rana G.C., Kumar S., *Variable gravity effects on thermal instability of nanofluid in anisotropic porous medium*, International Journal of Applied Mechanics and Engineering, **18**(3): 631–642, 2013.

25. Chand R., Rana G.C., Hussein A.K., *On the onset of thermal instability in a low Prandtl number nanofluid layer in a porous medium*, JAFM, **8**: 265–272, 2015.

26. Chand R., Rana G.C., Hussein A., *Effect of suspend ed particles on the onset of thermal convection in a nanofluid layer for more realistic boundary conditions*, International Journal of Fluid Mechanics Research, **42**: 375–390, 2015.

27. Rana G.C., Thakur R.C., Kango S.K., *On the onset of double-diffusive convection in a layer of nanofluid under rotation saturating a porous medium*, Journal of Porous Media, **17**(8): 657–667, 2015.

28. Toms B.A., Strawbridge D.J., *Elastic and viscous properties of dilute solutions of polymethyl methacrylate in organic liquids*, Transactions of the Faraday Society, **49**: 1225–1232, 1953.

29. Oldroyd J.G., *Non-Newtonian effects in steady motion of some idealized elastico-viscous liquids*, Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences, **245**(1241): 278–286, 1958.

30. Rivlin R.S., Ericksen J.L., *Stress deformation relations for isotropic materials*, Journal of *Rational* Mechanics and Analysis, **4**: 323–334, 1955.

31. Rivlin R.S., Ericksen J.L., *Stress-deformation relations for isotropic materials*, [in:] *Collected Papers of RS Rivlin*, pp. 911–1013, Springer, New York, NY, 1997.

32. Prakash K., Chand R., *Effect of kinematic visco-elasticity instability of a Rivlin-Ericksen elastico-viscous fluid in porous medium*, Ganita Sandesh, **14**: 1–8, 1999.

33. Prakash K., Chand R., *Thermal instability of Oldroydian visco-elastic fluid in the presence of finite Larmor radius, rotation and variable gravity field in porous medium*, Proceedings of the National Academy of Sciences, India Section A: Physical Sciences, **72**: 373–386, 2002.

34. Rana G.C., Chand R., *Effect of rotation on the onset of compressible Rivlin-Ericksen fluid heated from below saturating a Darcy-Brinkman porous medium*, Research Journal of Engineering and Technology, **3**(2): 76–81, 2012.

35. Sharma V., Rana G.C., *Thermosolutal instability of Rivlin-Ericksen rotating fluid in the presence of magnetic field and variable gravity field in porous medium*, Proceedings of National Academy of Science INDIA, **73**: 93–112, 2003.

36. Gupta U., Sharma G., *On Rivlin-Erickson elastico-viscous fluid heated and soluted from below in the presence of compressibility, rotation and Hall currents*, Journal of Applied Mathematics and Computing, **25**(1–2): 51–66, 2007.

37. Chand R., Rana G.C., *Dufour and Soret effects on the thermosolutal instability of Rivlin-Ericksen elastico-viscous fluid in porous medium*, Zeitschrift für Naturforschung A, **67**(12): 685–691, 2012.

38. Rana G.C., *Thermal instability of compressible Rivlin-Ericksen elastico-viscous rotating fluid permeated with suspended dust particles in porous medium*, International Journal of Applied Mathematics and Mechanics, **8**: 97–110, 2012.

39. Chand R., Rana G.C., *Thermal instability of Rivlin-Ericksen elastico-viscous nanofluid saturated by a porous medium*, ASME Journal of Fluids Engineering, **134**(12): 121203-1–121203-7, 2012, doi: 10.1115/1.4007901.

40. Rana G.C., Thakur R.C., Kango S.K., *On the onset of thermosolutal instability in a layer of an elastico-viscous nanofluid in porous medium*, FME Transactions, **42**: 1–9, 2014.

41. Rana G.C., Chand R., *Stability analysis of double-diffusive convection of Rivlin-Ericksen elastico-viscous nanofluid saturating a porous medium: a revised model*, Forsch Ingenieurwes, **79**: 87–95, 2015.

42. Sheu L.J., *Thermal instability in a porous medium layer saturated with a viscoelastic nanofluid*, Transport in Porous Media, **88**(3): 461–477, 2011.

43. Chand R., Rana G.C., *Instability of Walter's B' viscoelastic nanofluid layer heated from below*, Indian Journal of Pure & Applied Physics, **53**: 759–767, 2015.

Received July 12, 2016; accepted version June 4, 2018.