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Application of Discrete Wavelet Transformation to Defect Detection in Truss Structures with Rigidly Connected Bars

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The structures examined in this paper are bridge-type trusses that were previously used as railway viaduct support structures. The considered trusses are modelled as 2D and 3D structures. The lower chord bar of the considered structure can be loaded by external forces located outside the rigid nodes (the points where truss bars are connected). Hence, in the numerical experiment in terms of 2D approach, the truss structure consists of the set of two-node beam finite elements with three degrees of freedom per node and exact shape functions. According to 3D approach, the truss is described as the set of two-node beam elements with six degrees of freedom per node. Axial and twisting displacements of the element are described by linear shape functions and the bending is described by polynomials of the third order corresponding to Euler-Bernoulli beam fields of deformation. The defect (damage) in truss structure is modelled as the local stiffness reduction of one or two lower chord bars. The analysis of a structural response is carried out using the discrete wavelet transformation (DWT). The aim if this work is to detect the localization of damage provided that it exists in the considered structure and to examine whether the DWT will prove to be the effective tool to defect detection. It is expected that the disturbance of the response signal will appear in the vicinity of the point where the defect exists. The family of Daubechies 4 wavelet is implemented. Numerical investigation is executed based on signal analysis of structural static response. Some numerical examples are presented.

Key words: bridge truss structures, defect detection, discrete wavelet transform, finite element method.

1. Introduction

The matter of non-destructive damage detection plays an important role in monitoring of engineering structures. This issue has been the subject of studies undertaken by many scientists who proposed some approaches based on, e.g., evolutionary algorithms [1], load optimization [2], information on natural frequencies [3], heat transfer [4], inverse analysis [5, 6] and artificial neural networks [7, 8]. Other, relatively recent techniques, which can be efficiently used for damage detection, are wavelet transformation (WT) [9], and its discrete form- discrete wavelet transformation (DWT) [10, 11]. Damage detection in truss structures has been recently quite intensively studied, introducing, e.g., static tests [12], vibration analysis [13], wavelet transform [14] and wavelet transform combined with strain mode method [15]. In the paper, the damage detection and localization in truss structures is presented. The types of considered trusses refer to the historical structures of Town-type (Fig. 1) and the Howe-type structures (Fig. 2), which are both generally characterized by high stiffness.

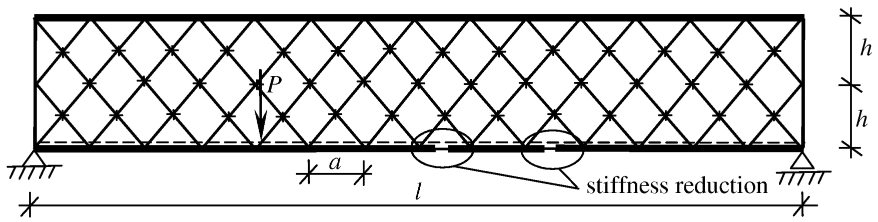

Fig. 1. The Town-type truss.

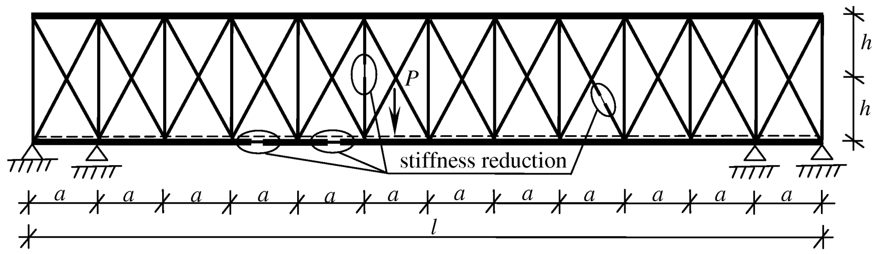

Fig. 2. The Howe-type truss.

The defects are introduced as local stiffness reduction of the selected single bars. Displacements and angles of rotation are analyzed as the structural response signals. A one-dimensional discrete wavelet transformation is used for signal decomposition. An evident disturbance of the transformed signal observed in the place of defects location is expected as the result of signal processing. Considered trusses are modelled by the finite element method as two- and three-dimensional models of the structure.

Any signal can be portrayed as a sum of sinusoidal signals. This fact is the basis for a widely used Fourier analysis. In this paper, we implement another kind of analysis of wavelet transform, in which for the representation of the signal, a linear combination of wavelet functions is used. In contrast to Fourier transform, wavelets are localized in the time and frequency domain. Therefore, they are well suited to deal with signals that have discontinuities. The principles of wavelet transformation were widely described in [11].

2. Defect detection in truss structures – problem formulation

The main aim of this study is to detect and locate the defect provided that it exists in the considered structure. The numerical task is conducted based on signal analysis of structural, static or dynamic response. Numerical models of two-dimensional (Figs. 3, 4 and 5) and three-dimensional structures (Fig. 9), loaded with a concentrated force or the set of two concentrated forces, moving along the lower chord, are studied. The lower chord of truss structure is considered as the element where the damage may occur. The 2D truss is modelled as the set of two-node finite elements with three degrees of freedom per node and exact shape functions. The axial displacement of the element is characterized by linear shape functions and bending is described by polynomials of the third order corresponding to the Euler-Bernoulli beam fields of deformation. The 3D truss is defined by two-node finite elements with six independent degrees of freedom per node (three translational displacements and three angles of rotation). For the axial and bending deformations exact shape functions are introduced. The twisting deformation is described by a linear shape function.

For the simplified Town-type truss, defects are introduced at two cross-sections of the lower chord bars (Figs. 3a and 3b).

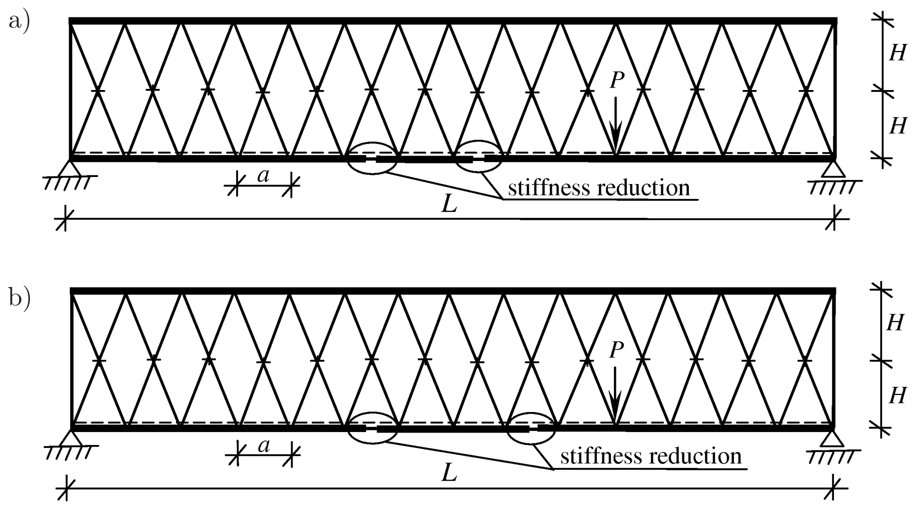

Fig. 3. Considered 2D simplified Town-type truss model.

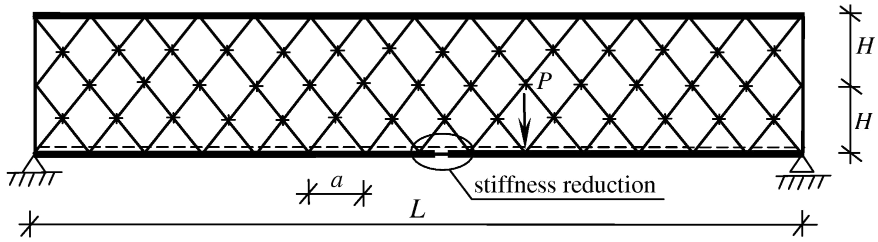

FIG. 4. Considered 2D Town-type truss model.

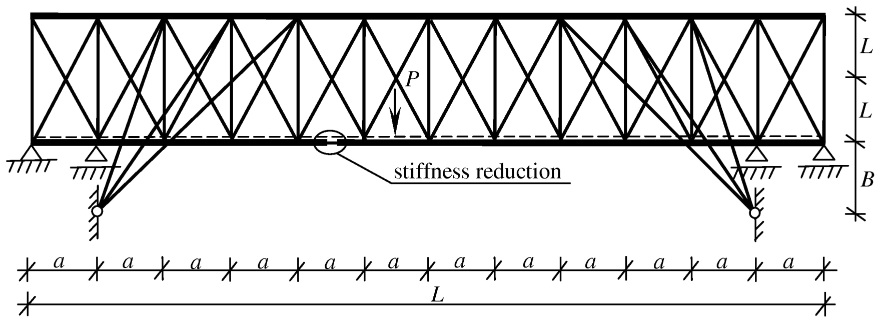

FIG. 5. Considered 2D Howe-type truss model (with additional bar support).

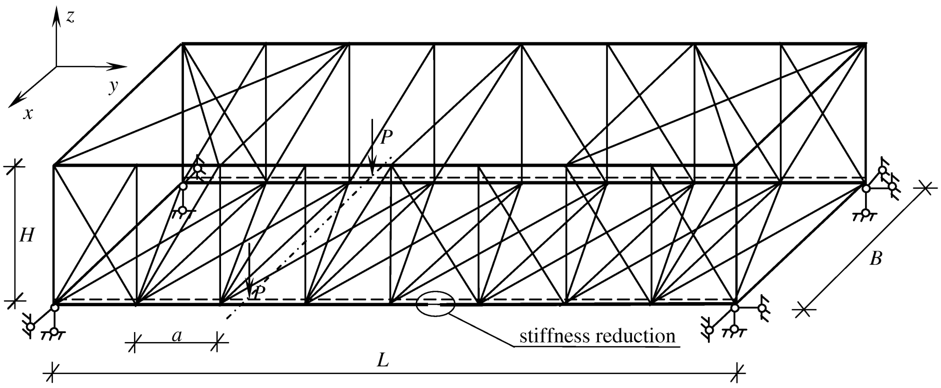

FIG. 6. Considered 3D railway bridge truss model.

A defect in the element $(i - j)$ is modelled as the local stiffness reduction. The defective elements are the lower chord bars of both 2D trusses (Fig. 7a) and 3D truss (Fig. 7b). The weakened cross-section of the bar is shown in Fig. 7c. The length of the damaged part of an element is $a_{\mathrm{d}} = 0.01 \cdot a$. The rectangular cross-section of truss bars is assumed. The principal axes 1 and 2 are shown in Figs. 7d and 7e.

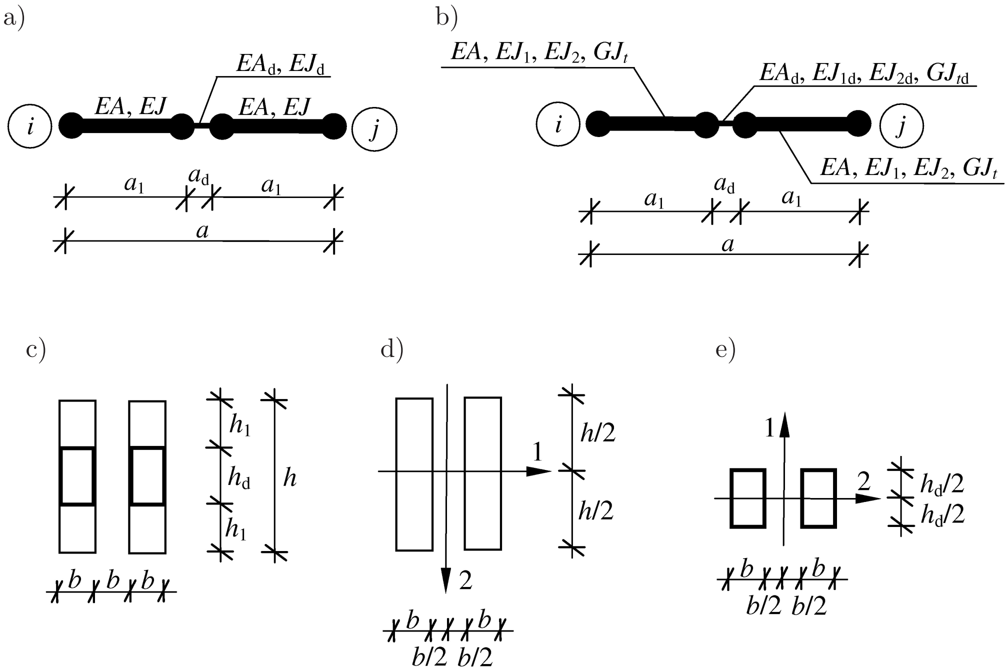

FIG. 7. Defective elements of the lower chord of the truss structure between nodes 'i' and 'j'.

3. Numerical examples

All structures are made of steel with Young's modulus $E = 205$ GPa and Poisson's ratio $v = 0.3$.

The static schemes of 2D truss structures, shown in Figs. 3–5, are considered. For upper and lower chord bar: $2 \times b \times h$; for diagonals and verticals: $b \times h$, where $b = 5.0$ cm and $h = 40.0$ cm. For damaged lower chord bar: $2 \times b_{\mathrm{d}} \times h_{\mathrm{d}}$, where $b_{\mathrm{d}} = b$ and $h_{\mathrm{d}} = 15.0$ cm. Geometric characteristics of all 2D truss elements are given below.

- Truss with undamaged upper chord: $A = 400.0$ cm^2, $J = 53\,333.3$ cm^4;
- Truss with undamaged lower chord: $A = 400.0$ cm^2, $J = 53\,333.3$ cm^4;
- Damaged lower chord element: $A_{\mathrm{d}} = 150$ cm^2, $J_{\mathrm{d}} = 2812.5$ cm^4;
- Diagonals: $A_{\mathrm{s}} = 200.0$ cm^2, $J_{\mathrm{s}} = 26\,666.7$ cm^4;
- Vertical struts: $A_{\mathrm{v}} = 200.0$ cm^2, $J_{\mathrm{v}} = 26\,666.7$ cm^4.

The dimensions of Town-type trusses (Fig. 3 and Fig. 4) are: $L = 14 \cdot a$, $a = 1.0$ m, $H = 1.0$ m.

The dimensions of Howe-type truss (Fig. 5) are: $L = 12 \cdot a$, $a = 2.0$ m, $H = 1.5$ m, $B = 2.0$ m.

The static scheme of 3D bridge truss structure is shown in Fig. 6. The rectangular cross-section of truss bars is assumed. For upper and lower chord bar: $2 \times b \times h$; for diagonal and horizontal vertical struts: $b \times h$, where $b = 5.0$ cm and $h = 40.0$ cm. The damaged lower chord bar has the same dimensions as in a 2D case. Geometric characteristics of the truss elements are given below.

- Truss with undamaged upper chord bars (axis 1 is perpendicular to yz plane):

  $A = 400.0$ cm^2, $J_1 = 53\,333.3$ cm^4, $J_2 = 10\,833.3$ cm^4, $J_t = 3333.3$ cm^4;

- Truss with undamaged lower chord bars (axis 1 is perpendicular to yz plane):

  $A = 400.0$ cm^2, $J_2 = 53\,333.3$ cm^4, $J_2 = 10\,833.3$ cm^4, $J_t = 3333.3$ cm^4;

- Damaged lower chord element (axis 1 is entered in yz plane):

  $A_{\mathrm{d}} = 150.0$ cm^2, $J_{1\mathrm{d}} = 4062.5$ cm^4, $J_{2\mathrm{d}} = 2812.5$ cm^4, $J_{t\mathrm{d}} = 1250.0$ cm^4;

- Diagonals located in vertical plane yz (axis 1 is perpendicular to yz plane):

  $A_{\mathrm{S}} = 200.0$ cm^2, $J_{1\mathrm{S}} = 26\,666.7$ cm^4, $J_{2\mathrm{S}} = 416.7$ cm^4, $J_{t\mathrm{S}} = 1666.7$ cm^4;

- Vertical struts located in vertical plane yz (axis 1 is perpendicular to yz plane):

  $A_{\mathrm{V}} = 200.0$ cm^2, $J_{1\mathrm{V}} = 26\,666.7$ cm^4, $J_{2\mathrm{V}} = 416.7$ cm^4, $J_{t\mathrm{V}} = 1666.7$ cm^4;

- Longitudes and diagonals located in horizontal plane xy (axis 1 is perpendicular to xy plane):

  $A_{\mathrm{H}} = 200.0$ cm^2, $J_{1\mathrm{H}} = 26\,666.7$ cm^4, $J_{2\mathrm{H}} = 416.7$ cm^4, $J_{t\mathrm{H}} = 1666.7$ cm^4.

The dimensions of the 3D truss (Fig. 6) are: $L = 8 \cdot a$, $a = 2.0$ m, $B = 5.0$ m, $H = 8.0$ m.

3.1. Example 1 – Town-type 2D simplified truss

In this section, the truss structure presented in Fig. 3a is considered. The defect is introduced as the lower chord bar stiffness reduction in the vicinity of the selected cross-section (Fig. 7a). Measurement points are assumed at the nodes which are located near the damaged lower chord element. Extensive analysis of the influence of a measurement point position was carried out in [11]. The truss structure is loaded with moving concentrated force $P = 100$ kN.

Let the displacements and angles of rotation be measured in equal intervals – N times. The aim of this analysis is to evaluate the influence of the number of measurements on defect detection effectiveness. An attempt is made to assess the minimal number of measurements required for proper data processing, while preserving the requirement that the number of data N for DWT must be a power of two. Let the defects be introduced in cross-sections of two neighbouring bars of the lower chord and the measurement point located at node 'j' between these elements (Fig. 8).

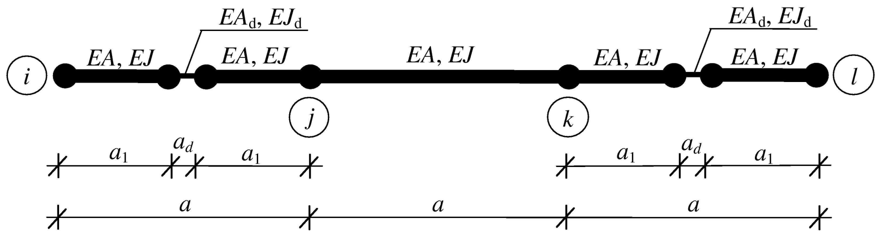

FIG. 8. Defects introduced in two lower chord bars.

Detail 1 of discrete wavelet transformation of the vertical displacements of one node for $N = 256$ and $N = 128$ (N – number of measurements) respectively is depicted in Fig. 8. The analogous decomposition was performed for structural response signal expressed in angles of rotation. The results of calculations are presented in Figs. 9 and 10.

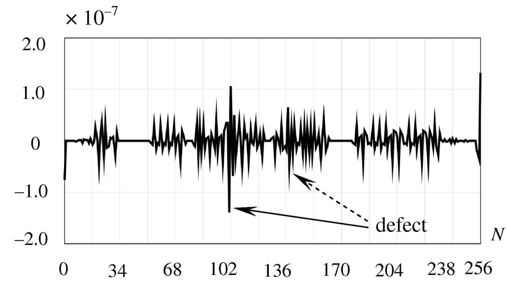

FIG. 9. DWT (Daubechies 4, detail 1) of vertical displacements; $N = 256$; measurement point located at node 'j'.

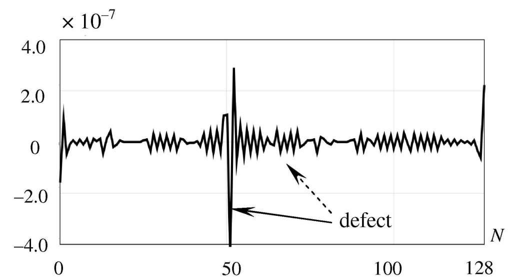

FIG. 10. DWT (Daubechies 4, detail 1) of vertical displacements; $N = 128$; measurement point located at node 'j'.

Let the measurement point be located at node 'k' (Fig. 8). The results of calculations are presented in Fig. 11 for $N = 256$ and in Fig. 12 for $N = 128$ (N – number of measurements).

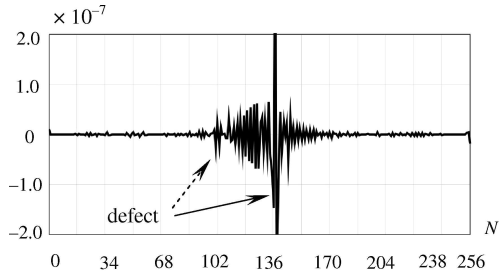

FIG. 11. DWT (Daubechies 4, detail 1) of rotation angle; $N = 256$; measurement point located at node 'k'.

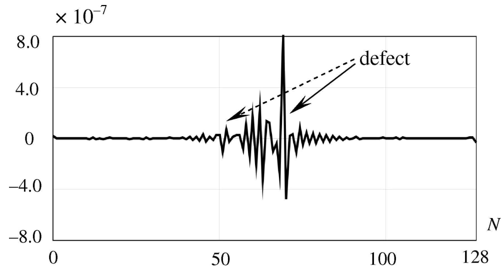

FIG. 12. DWT (Daubechies 4, detail 1) of rotation angle; $N = 128$; measurement point located at node 'k'.

It can be seen that in case of 256 measurements at least one defect was properly detected. For a smaller number of measurements ($N = 128$) the same was observed (Figs. 10 and 12).

Let the defects be introduced at cross-sections of two bars of the lower chord and the measurement point to be located at node 'j' between these elements (Fig. 13).

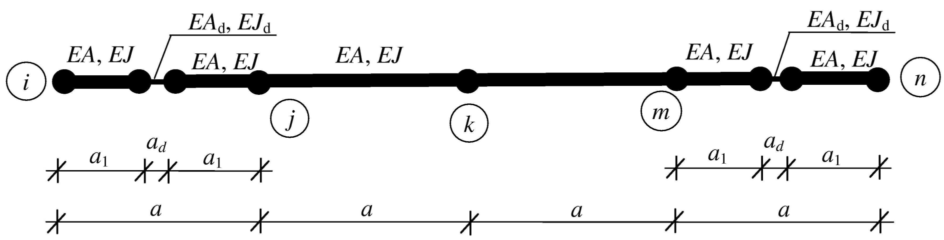

FIG. 13. Defects introduced in two lower chord bars.

The results of calculations are presented in the Fig. 14 for $N = 256$. For the measurement point located at node 'm' (Fig. 13), the results of calculations are presented in Fig. 15 for $N = 256$.

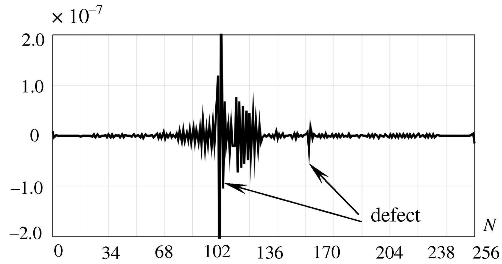

Fig. 14. DWT (Daubechies 4, detail 1) of rotation angle; $N = 256$; measurement point located at node 'j'.

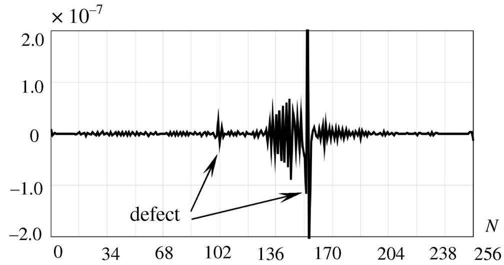

Fig. 15. DWT (Daubechies 4, detail 1) of rotation angle; $N = 256$; measurement point located at node 'm'.

3.2. Example 2 – Town-type 2D truss

The truss structure presented in Fig. 4 is considered in this section. The defect is introduced as the lower chord bar stiffness reduction at one selected cross-section. Measurement point is connected to the node 'i' located near the damaged lower chord element (Fig. 7a) [11]. The truss is loaded with moving concentrated force $P = 100$ kN. The results of calculation are presented in Fig. 16 for $N = 256$.

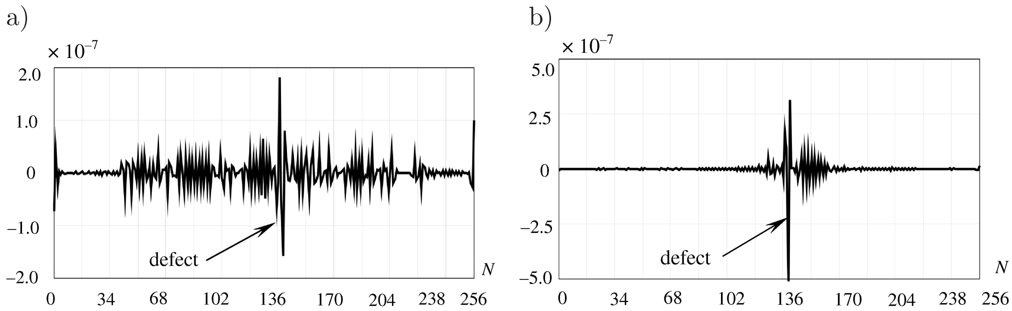

Fig. 16. DWT (Daubechies 4, detail 1) of: a) vertical displacements, b) rotation angle; $N = 256$ [11].

3.3. Example 3 – Howe-type 2D truss

Here, the truss structure presented in Fig. 5 is considered. The damage is introduced in the lower chord bar at one selected cross-section and the measurement point is located at nodes 'i', 'j' and 'k' near the damaged lower chord bar (Fig. 17).

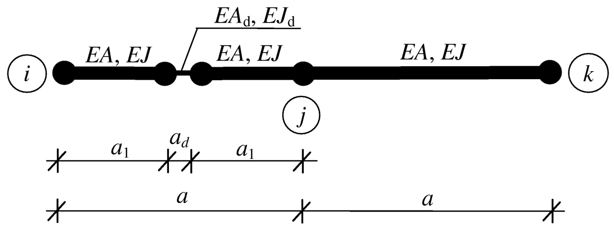

FIG. 17. Defected element of the lower chord bar.

The truss is loaded with moving concentrated force $P = 100$ kN. The results of calculations for $N = 256$ and $N = 128$ are presented in the Figs. 18–21 respectively.

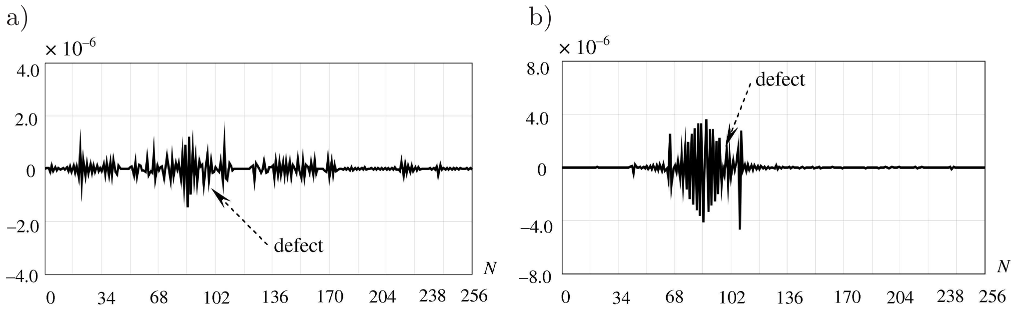

FIG. 18. DWT (Daubechies 4, detail 1) of: a) vertical displacements, b) rotation angle; $N = 256$; measurement point located at node 'i'.

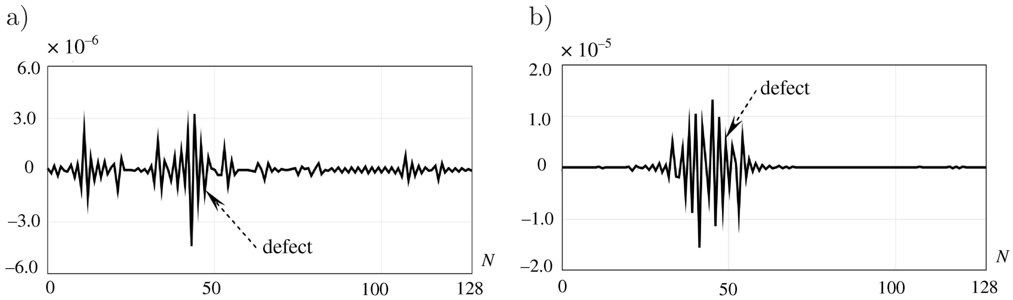

FIG. 19. DWT (Daubechies 4, detail 1) of: a) vertical displacements, b) rotation angle; $N = 128$; measurement point located at node 'i'.

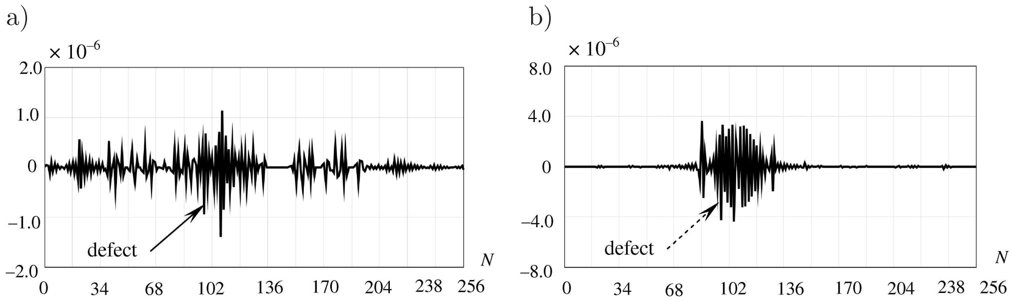

Fig. 20. DWT (Daubechies 4, detail 1) of: a) vertical displacements, b) rotation angle; $N = 256$; measurement point located at node 'j'.

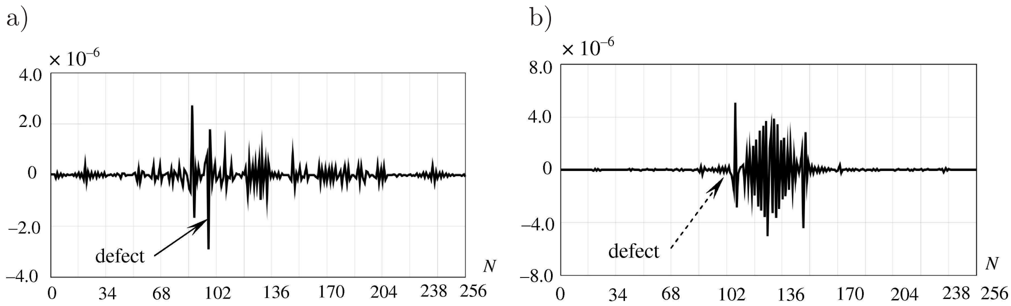

Fig. 21. DWT (Daubechies 4, detail 1) of: a) vertical displacements, b) rotation angle; $N = 256$; measurement point located at node 'k'.

3.4. Example 4 – 3D bridge truss

The truss structure presented in Fig. 9 is considered in this part of the paper. The damage is introduced as the lower chord bar stiffness reduction at one selected cross-section and the measurement point is located at nodes 'i', 'j' and 'k' near the damaged lower chord bar (Fig. 22). The truss is loaded with the set of two moving concentrated forces, each $P = 100$ kN. The results of calculations are presented in Figs. 23–25 respectively.

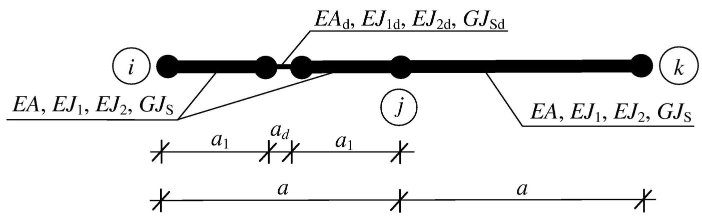

Fig. 22. Defected element of the lower chord bar.

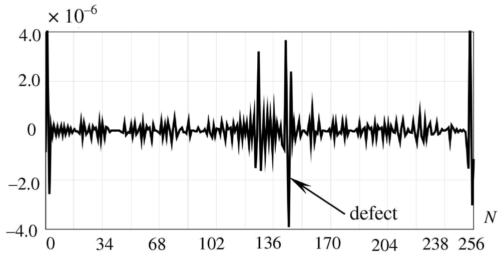

FIG. 23. DWT (Daubechies 4, detail 1) of vertical displacements; $N = 256$; measurement point located at node 'i'.

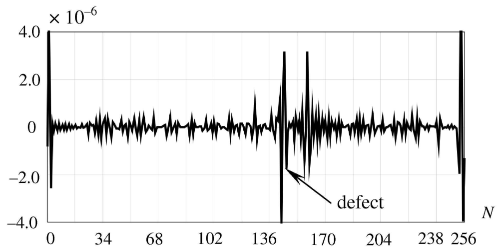

FIG. 24. DWT (Daubechies 4, detail 1) of vertical displacements; $N = 256$; measurement point located at node 'j'.

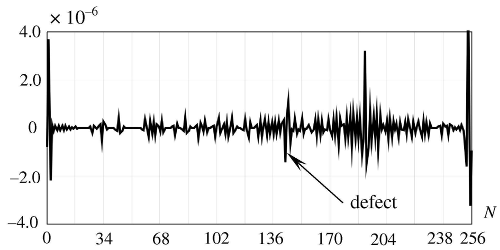

FIG. 25. DWT (Daubechies 4, detail 1) of vertical displacements; $N = 256$; measurement point located at node 'k'.

4. Concluding remarks

This paper is devoted to damage detection in bridge trusses using discrete wavelet transformation. The family of Daubechies 4 wavelets was used. The structural response signal was expressed in vertical displacements or angle of rotations for one selected node of the truss structure. Additional noise that simulates measurement errors has not been introduced. The lower chord of the truss was loaded directly with concentrated force. In all the cases the measurement point was located at a node near the defective bars because this was the only way to make defects localization visible. This occurs because wavelets are sensitive to bending effects of the truss' lower chord. These effects are strongly damped by vertical struts and diagonals at some distance from the damaged bar. In some cases of this type of problems, better results are obtained using discrete wavelet transformation of angle of rotation measured at selected node.

Additionally, the influence of the location of measurement point with respect to defective elements was studied. The presented method allowed to detect more than one defect but these defects have to be located close to each other. In addition, the method discovered small disturbances in the response signal of the defective structure and did not require the reference to a signal from undamaged structure; however its applicability has some limitations.

For 2D structure, the resulting scope for effective observation of defects from one measurement point is equal to about twenty five percent of the lower chord of the structure. One observation (measurement) point was enough to diagnose two sections of the truss lower chord (the length of diagnosed section is equal to $2a$). For 2D and 3D structures the analysis did not include localization of defects in upper chords, verticals, longitudes and diagonal struts of the trusses. Diagonals and vertical struts bring disruptions to the response signal of the construction and this is the impediment in the application of DWT to detect any defects in structures. The presented analysis revealed limited usefulness of DWT applied to study truss structures. In other structures such as beams [10, 18] and plates [4, 19] DWT showed a very high effectiveness hence the authors decided to check the efficiency of wavelets for more complicated structure. Nevertheless, some defects could not be localized in the lower chord bars.

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