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Elastic Stresses in Thin-Walled Torsional Structures Designed with SADSF Method

Ireneusz MARKIEWICZ

Kielce University of Technology
al. Tysiąclecia Państwa Polskiego 7, 25-314 Kielce, Poland
e-mail: ireneusz.markiewicz@tu.kielce.pl

The paper presents general conclusions arising from FEM analyses of elastic properties of thin-walled structures designed with the application version of the statically admissible discontinuous stress fields (SADSF) method. The analyses have been carried out as a part of a large-scale research program, whose main objective is conclusive verification of practical usefulness of the SADSF method in design. The present state of development of the method's application software is so advanced that it allows one to design even very complicated thin-walled structures composed of plane elements. This is a particular class of structures to which the Saint Venant's principle is not applicable [3, 4], and the methods based on consecutive iterative improvements should be applied with due caution. In the SADSF method one does not use iterations, and the method can be applied already at the stage when only boundary conditions are known [3, 4]. Unfortunately, the method is an approximate one and does not apply to the elastic range of stress that usually exists in exploitation conditions.

On the basis of analyses carried out for several dozen cases of thin-walled structures designed with the SADSF method we can state, among other things, that in these structures there are dominating membrane states, deformations remain small, equivalent stress fields are well equalized also along free borders, stress concentrations are relatively low, and maximal levels of equivalent stresses are approximately the same in all component elements. It has also been proven that the structures designed with the SADSF method may have strength properties even several dozen times better that those of structures designed traditionally. The obtained conclusions are presented on the basis of three selected examples of original, which in this case have open-section structures (see e.g. [11]) designed to carry torsional load [3, 4].

Key words: design, thin-walled structures, limit load capacity, FEM analysis.

1. Introduction

The problem of designing structures that have good strength properties (good equalization of equivalent stress, low stress concentrations, *etc.*) and at the same time being light, have long been the centre of engineers' attention. The solutions satisfying these requirements are sought for in the class of thin-walled structures, and these are widely applied in practice (e.g., in automobiles and in various machine and building structures). However, they have specific properties. First of all, great sensitivity of their structural parameters to changes, and often apparently small ones, i.e. to the changes in number, spatial allocation and mutual connections of component elements [4]. In consequence, methods based on procedures of consecutive iterative improvements, among them also the advanced methods of so-called topological optimization [1], should be used for these structures with due care [3].

A method that does not rely on consecutive improvements and is easy to use for an engineer is one of Statically-Admissible Discontinuous Stress Fields (SADSF) [2, 3, 7, 8, 12–14]. The method is aimed at designing thin-walled structures composed of plane elements. One can use it already at the design stage, when only boundary conditions are known [3, 14].

A certain limitation of the SADSF method is that it is based on conclusions arising from the lower-bond theorem of limit analysis, and thus it is an approximate method. It assumes among other things, that one exclusively uses an rigid ideally-plastic model of material, a plane state of stress is realized in each component element, and that only the limit state of the structure is analysed which corresponds to the beginning of its collapse. For these reasons, since the method came into being in nineteen-sixties [12], researchers have carried out fragmentary investigations on properties of structures designed with the SADSF method. Initially, these were plane structures, later three-dimensional structures have also been examined [2–13]. The results of these investigations have shown good, sometimes even surprisingly good, load-carrying properties of SADSF structures in the whole range of applied loads including the elastic range – which usually is the range of working load. Unfortunately, because of a dissimilarity between the physical material models used for analyses in this range and those applied in the limit state, one can not prove that such properties would be encountered in all cases of structures designed with the SADSF method [2, 3, 12]. Nevertheless, having an adequately reach material from research confirming good properties of SADSF structures, one may regard such properties as the expected ones.

Therefore, the basic aim of the large-scale research program undertaken by the author has been recognition of actual properties of structures designed with the SADSF method and conclusive verification of the usefulness of the method in design. Having in mind large the scale of the investigations carried-out (a large number of structure cases) we have selected computational and experimental methods that are neither very onerous nor expensive.

In the course of the program we have investigated, among other things:

- distributions of elastic stress fields by means of FEM,
- development of plastic zones, examined using e.g. thermovision, and actual mechanisms of collapse and equilibrium paths,
- fatigue strength, determined by applying the local strain method.

Application of the FEM in the analyses of properties in the elastic range allowed us to relatively easily analyse practically all the interesting cases, known from literature [2–4, 14] of thin-walled structures designed with the SADSF method. Among them we have not found any negative example. Having at our disposal such an extensive research material we could assume that the good elastic properties, described in the following part of this work, are expected.

In this work, general conclusions arising from the aforementioned analyses are presented by example of three selected interesting structures whose proportions are similar to those of crossbars in carrying frames of vehicles (Fig. 1). The interesting detail is that, despite the fact that these structures consist of open sections they could be successfully designed for torsional loads. In this way it was proven once again that such profiles might have significant torsional rigidity, contrary to what was stipulated by simplified one-dimensional theories [3]. We have chosen these particular structures for presentation, the ones which belong to the "worst cases" encountered in our investigations. The reason is that in-plane bending loads appear there in most of the plane components, so that one can not expect good equalization of equivalent stress in large fragments of the structure's volume – contrary to what we observe in many other structures designed with the SADSF method (see for example [11]).

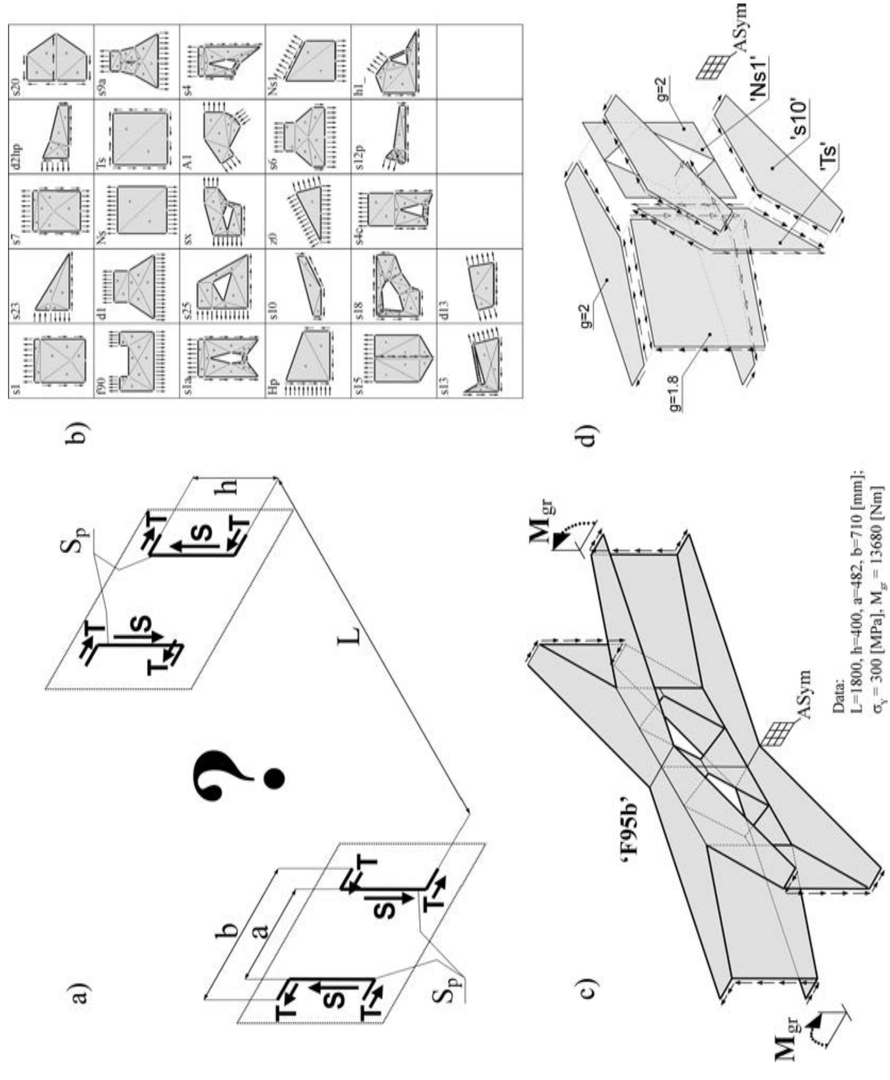
The structures were originally designed by W. Bodaszewski [3, 4] with the use of the software package SADSFaM1 developed by himself [3, 14].

An illustrative example of the formulation of a design problem is presented in Fig. 1a [3]. The only input data were: the limit load $\mathbf{S}$ and $\mathbf{T}$ applied to the segment S_p of the border S (which can be reduced to the limit moment $M_{gr} = S \cdot a - 2T \cdot h$), the geometry of the segment S_p (with dimensions: L, h, a, b, δ), and the plastic properties of the material of the sought-after structure (with yield point σ_Y).

The task was to design a statically-admissible stress field which satisfies the assumed boundary conditions, and – at each point of the structure – the assumed yield condition. Then, the contours of the field should be made identical to the contours of the sought-after structure [2, 3].

When using the application version of the method, one designs the structure by selecting ready-made particular solutions of the fields from a library (Fig. 1b), and connecting them while paying attention to satisfying the boundary conditions and equilibrium conditions at the places of connection. To perform this task, the designer should only know the fundamentals of statics and to show some inventiveness.

The solution to the problem from Fig. 1a, denoted as "F95b" (the same denotation as the file containing its data) is shown in Fig. 1c, while the method of


a)
b
a
h
L
S
T
S_p
?
b)
s1
s23
s7
d2hp
s20
f90
d1
Ns
Ts
s9a
s1a
s25
sx
A1
s4
Hp
s10
z0
s6
Ns1
s15
s18
s4c
s12p
h1
s13
d13
c)
'F95b'
M_{gr}
ASym
Data:
L=1800, h=400, a=482, b=710 [mm];
σ_Y = 300 [MPa], M_{gr} = 13680 [Nm]
d)
g=2
g=1.8
g=2
'Ns1'
's10'
'Ts'
ASym

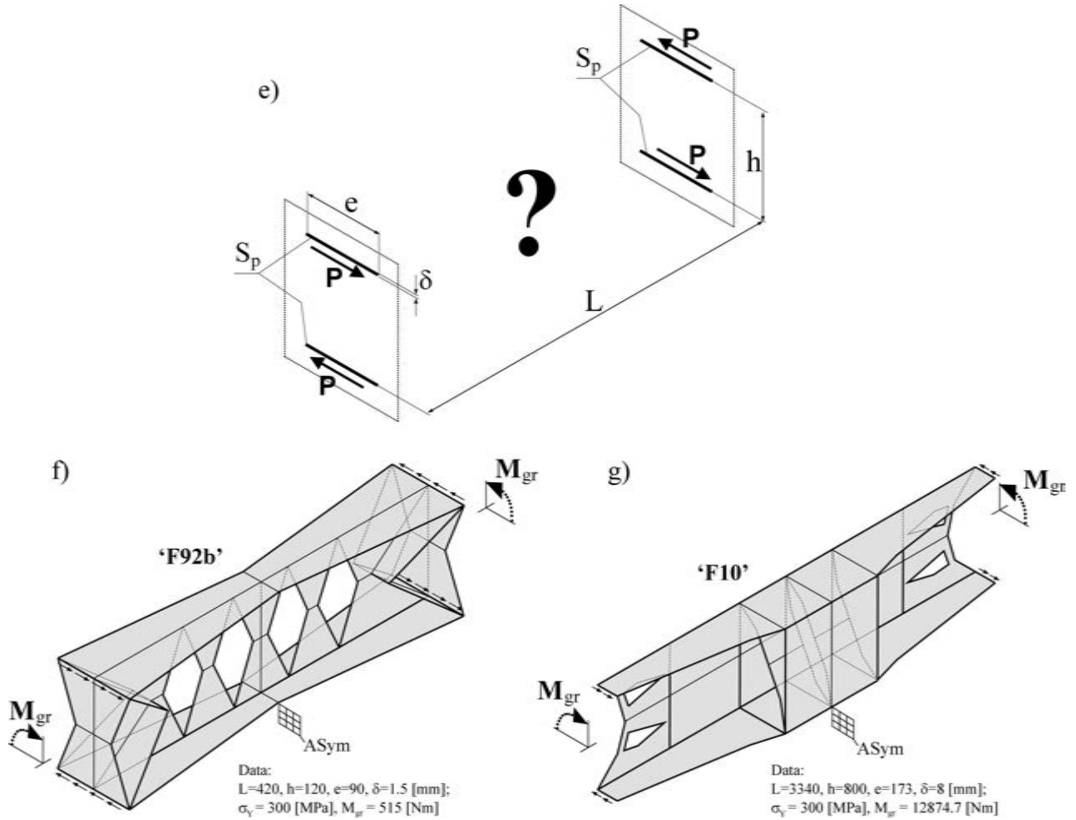


Fig. 1. Example formulations of design problems and contours of statically-admissible stress fields determining shape and dimensions of models of torsional structures analysed in work [4]: a) illustration of boundary conditions and problem formulation; b) library of ready-made solutions from the package SADSFaM1 [3, 14]; c) solution to the problem from Fig. 1a: statically-admissible complex spatial field, which determines contours of the structure; d) allocation of component library fields, and interactions between the fields in the antisymmetric half of the complex field; e) slightly different formulation of the design problem; f) solution obtained for boundary conditions given in Fig. 1e; g) yet another example of solution to the problem from Fig. 1e obtained for different values of border parameters.

selecting and assembling the component fields in its antisymmetric half is illustrated in Fig. 1d. In the flanges, fields of "s10" type are applied, which determine the shape and dimension of the elements. In order to transfer the forces **S**, and to satisfy the boundary conditions on the borders of oblique elements "s10", it was necessary to introduce oblique webs, in which one applies the fields of the "Ts" type that realise pure sheer. Considering equilibrium conditions on the borders joining the two fields, and assuming thickness of the oblique webs to be $g = 1.8$ [mm], one obtains the thickness of the flanges, $g = 2$ [mm]. In the web with holes one assumes a zero state of stress with fields of type "Ns1" and thickness $g = 2$ [mm].

Another illustrative example of formulation of design problem is presented in Fig. 1e, and two of its solutions, obtained for different values of border parameters, are shown in Figs. 1e and 1f. These solutions are denoted, respectively, "F92b" and "F10". The method of constructing such fields is described in works [3, 4].

2. COMPUTATIONAL MODELS

In the FEM analyses performed with the use of the CosmosM software package, we assumed, among other things:

- Small strains and linear-elastic physical model of material.
- Triangular shell elements type SHELL3 with 3 nodes and 6 degrees of freedom in a node.
- The average size of an element 3–5 times greater than its thickness.
- Loads introduced into the models in the form of torsional moments produced by forces Fy applied to the borders of holes in additional diaphragms "p1" (see Figs. 2a, 3a, and 5a); the nodes lying on the borders of holes in the additional diaphragms "p2" deprived of possibility of displacements Uy; fixed displacements Ux, Uy and rotations Ry, Rz of nodes lying in the centre of these diaphragms in order to exclude the possibility of rigid motion.
- The value of load equal to a half of the limit load-carrying capability assumed in the design with the yield point of $\sigma_Y = 300$ MPa; this means that, assuming ideal equalization of equivalent stress level, the equivalent stress would be equal to $\sigma_{eq} = 150$ MPa at each point of the analysed structure.
- The shapes and dimensions of the analysed models are almost ideally consistent with those from the solutions to the design problems; small corrections were only made in the vicinity of contour refractions, which were smoothed with Bezier curves drawn out of the original borders in order to prevent diminishing the assumed limit load-carrying capability [2]; the tasks of border correction have not been undertaken.

The assumed shell model allows only for approximate analysis of local three-dimensional states that arise, for example, in the areas of connection between plane component elements.

In this work, we restrict ourselves to the analyses of only the linear-elastic range of stress, because this is the typical exploitation range in this class of structures. The FEM analyses in elastic-plastic range will be the subject of a separate work.

3. General results of analyses

At the beginning, to facilitate reviewing main results of this investigation, we here summarize the conclusions, which seem to be the general as they frequently appear in the whole range of the already performed analyses:

1. Deformations remain small and membrane states dominate in the structure; equivalent stresses that arise due to bending states take relatively small values.
2. Stress concentrations are low, maximal values of equivalent stresses reach similar levels in all component elements.
3. Good equalization of equivalent stress fields in large fragments of the structures [11], and – in the case of structures built of elements bent in their planes – at least along free borders.
4. Strength properties of structures designed with the SADSF method are radically better than those of systems whose structures were not properly accommodated for the carried load and/or were designed with an intuitive approach.

4. Detailed results

The results are presented as object drawings which illustrate first the important details of the structures and shapes of the models resulting from the solutions to the design problems, and then distribution of equivalent stresses σ_{eq} (in the Huber-Misses sense) arising due to membrane and bending states.

4.1. Model based on solution "F95b"

The shape of the analysed model of the structure "F95b" and the assumed boundary conditions are presented in Fig. 2a.

As it results from the SADSF solution, in the plane ASym of the structure there are no reactions acting perpendicularly to this plane, so that bimoment does not appear there. However, the bimoment arises out of the ASym plane and its value increases with the distance from the symmetry plane reaching a maximum at points where oblique webs are fixed. Then, the bimoment value

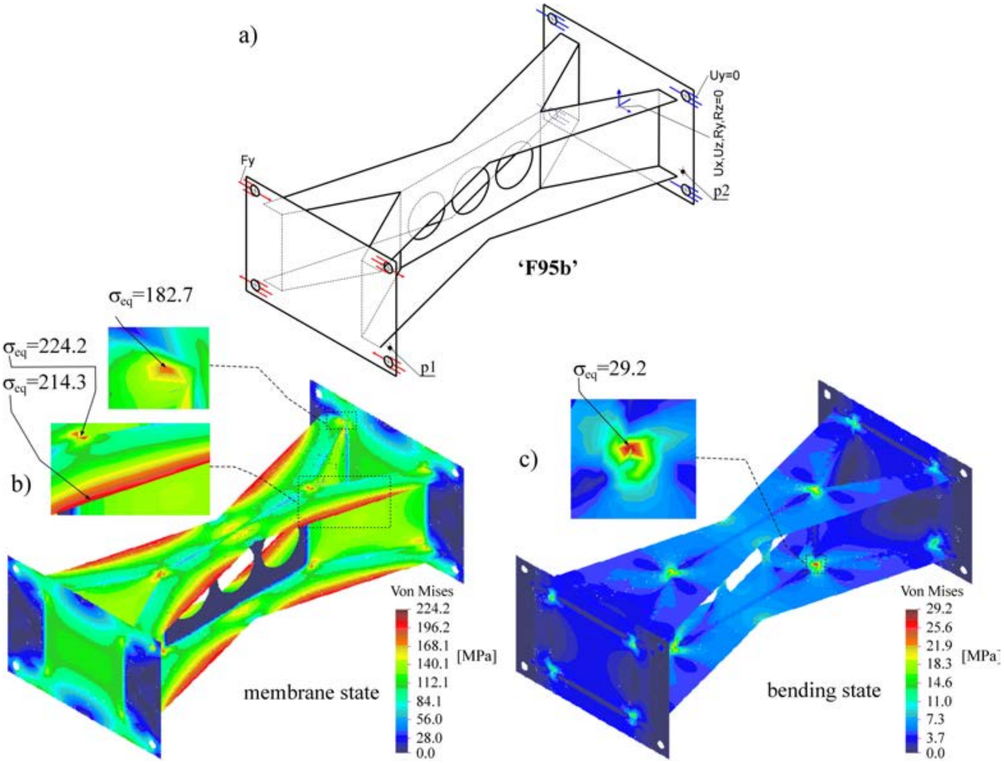


Fig. 2. Shape and boundary conditions assumed for model "F95b" in FEM analyses (a) and distributions of equivalent stresses calculated according to Huber-Misses criterion (b, c)

decreases and drops to zero in the terminal cross-sections of the diaphragms. It is worth noting that, in the vicinity of cross-sections with maximal bimoment values, there also appear the greatest stress concentrations in elastic state.

The distributions of equivalent stresses for membrane and bending states are presented in Fig. 2b and 2c. In Fig. 2b, we can see that:

- There appear small, local concentrations of equivalent stress at different places of the structure (i.e. at points marked with arrows, where $\sigma_{eq} = 224.2$ [MPa], $\sigma_{eq} = 214.3$ [MPa], and $\sigma_{eq} = 182.7$ [MPa]), however, the values of stresses at these points do not significantly differ from one another.
- Good equalization of equivalent stress along free borders of the flanges.
- An almost ideal level of stress equalization in oblique webs, where shearing stress has been assumed in the statically-admissible fields.
- Low equivalent stress areas are observed in the central web (with holes) where a zero state of stress has been assumed.

Consequently, in Fig. 2c one can see that maximal equivalent stress associated with bending state of stress reaches small values, and its maximal value equals only 13% (29.2/224.2) of the value of maximal stress associated with membrane state.

4.2. Model based on solution "F92B"

In the case of the structure model type "F92b", the shape and dimensions assumed for FEM analyses (Fig. 3a) were exactly the same as those obtained from the SADSF design problem (Fig. 1f). Modifications were only applied to the arrangement of holes in the web, for which a zero state of stress was assumed in the SADSF solution, and the web itself was introduced only to retain the constraints that maintained geometry of the structure.

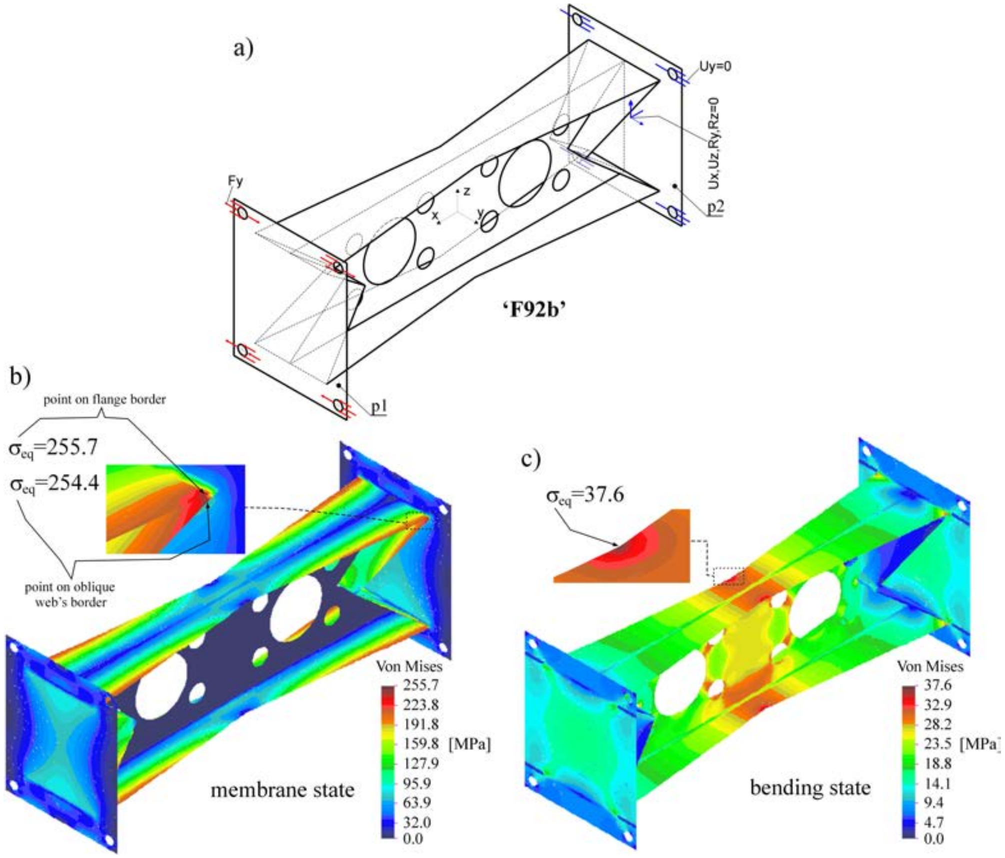


FIG. 3. Shape and boundary conditions assumed for model "F92b" in FEM analyses (a) and distributions of equivalent stresses calculated according to Huber-Misses criterion (b, c).

The structure was based on an **I**-section, and the effect of high rigidity was obtained thanks to the addition of oblique elements in the vicinity of the diaphragms. As follows from the analyses of interactions between component fields [3], these elements should be used for closing the bimoment reactions that arise due to asymmetry of the internal forces ASym (Fig. 1f) and increase with the distance from the plane of symmetry of the structure.

In this structure, all the elements (except the web) are kinds of membranes bent in their planes. In such elements, in the elastic range of load, there appears the axis of bending and strains increase with the distance from this axis. Then, one can expect their especially big discrepancies between the limit fields and elastic fields. However, it turned out that this structure exhibits several good properties, also in the membrane state. Among other things, there we have:

- Good equalization of equivalent stress along free borders and an almost identical maximal values of stress both in the flanges and in oblique elements (compare the values marked in Fig. 3b).
- Relatively low concentration of stresses (the maximal equivalent stress of 255.7 [MPa] is not much greater than that which we would obtain assuming ideal equalization of stress in the whole structure, i.e. stress value of 150 [MPa]).
- Areas with very low load in the web, where a zero state of stress was assumed.

Additionally, we found that the membrane state was the dominant one; the maximal equivalent stress due to bending (Fig. 3c) reached 37.6 [MPa], which was approximately 15% (37.6/255.7) of the maximal value obtained for the membrane state (Fig. 3b).

In order to demonstrate the quality of the presented solution, as well as the scale of possible improvements in strength properties resulting from the application of the SADSF method, we performed FEM analyses for a model of a regular **I**-section. The thickness of the sheet metal plates assumed for both models were the same. We also assumed constant width of the flanges, equal to the maximal width determined from statically admissible fields. The central web however, was without any holes, because the weight of the **I**-section model was assumed to be equal to that of the designed model. Boundary conditions are assumed the same as in the SADSF solution, however, the load value is assumed to be 14 times lower to make the maximum level of equivalent stress approximately the same in both models.

The distributions of the total equivalent stress fields obtained for both models are presented in Fig. 4. Such a great difference between load levels in the two models results first of all from the fact that in the structure from Fig. 4b designed with the SADSF method, the carried load mainly produces membrane state of stress, while in the structure from Fig. 4a the transmitted load must produce a bending state of stress which leads to low rigidity of the structure and generally to high level of equivalent stress [3]. In this model, maximal stress arising due

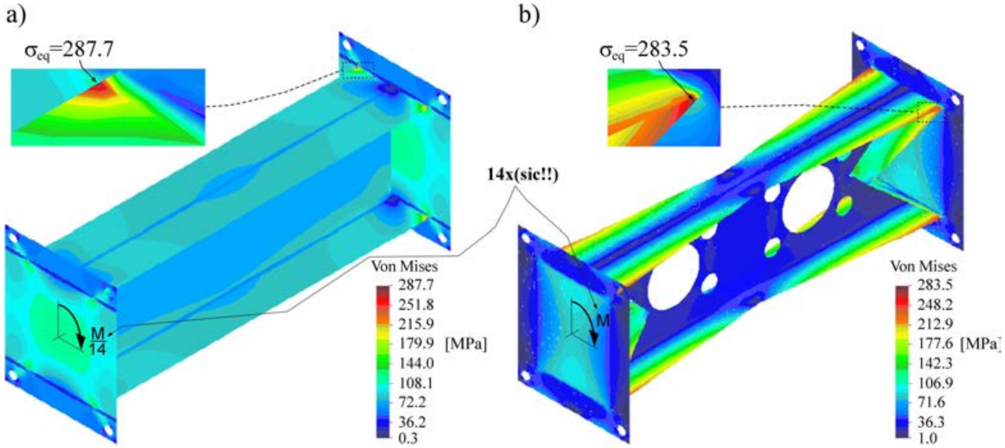


FIG. 4. Example comparison of distributions of total equivalent stresses in the model designed with the SADSF method (b) and in the model based on a regular **I**-section (a).

to membrane state equals 93.4 [MPa], while that associated with bending state is as high as 216.6 [MPa].

4.3. Model based on solution "F10"

The shape of the considered model "F10" and its boundary conditions are well illustrated with the drawings in Fig. 5a. The contour refractions on external borders and the holes in the central web were smoothed with Bezier curves.

This structure is based on a regular channel section, and the effect of high rigidity is achieved by very simple constructional means – by applying two additional plane elements welded into the section in its the central part. Thanks to such a structural design, we managed to "close" self-balancing bimoment systems, and in this way achieve radical increase in global rigidity of the structure as well as a significant decrease in the overall level of stress relative to the loads.

Also in this structure, one can not obtain well-equalized elastic equivalent stress in the flanges and in the additional oblique element because these parts are subjected to bending. In this case, in the membrane state we find (Fig. 5b and 5c):

- Good equalization of equivalent stress along free borders of the flanges.
- Almost the same values of maximal equivalent stresses in the flanges and in the additional oblique element.
- Good conformity between the results of FEM analyses and those of photoelastic examinations. An example juxtaposition of equivalent stress distribution and isochromatic fringes in the upper flange [6] is shown in Fig. 5b.

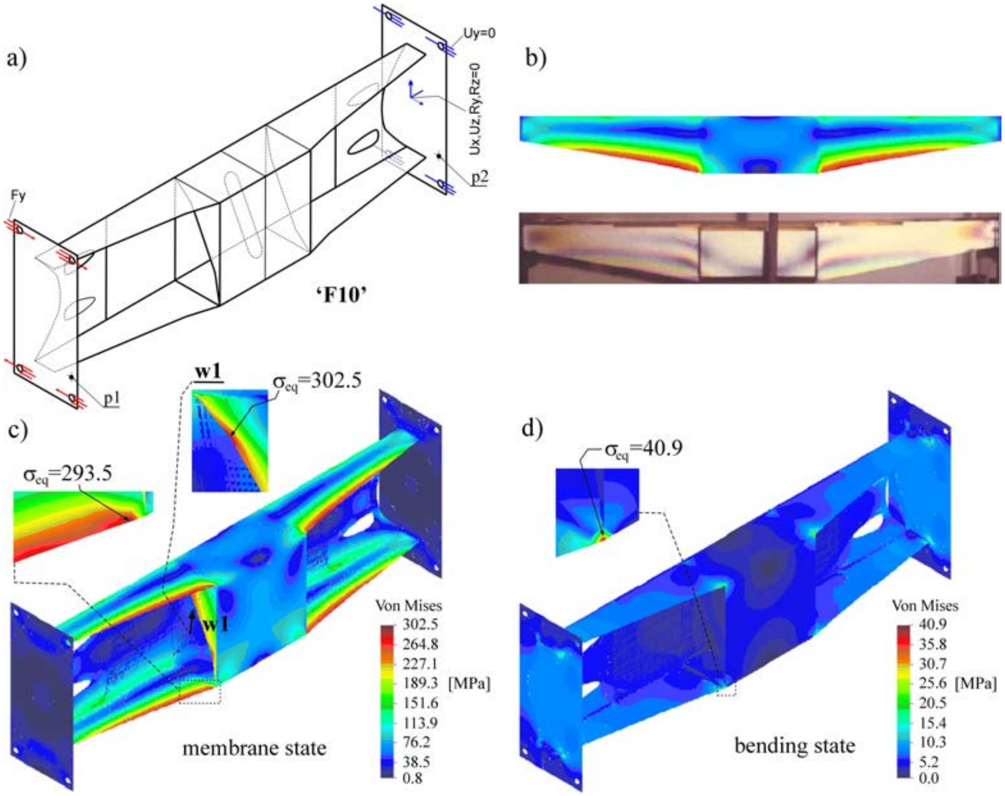


FIG. 5. Shape and boundary conditions assumed for model "F10" in FEM analyses (a), distributions of equivalent stresses calculated according to the Huber-Misses criterion (c, d), and field of isochromatic fringes registered in the flange (b) [6].

The maximal equivalent stress associated with the bending state (Fig. 5b) reaches locally only 14% of the value of equivalent stress related to the membrane state (40.9/302.5).

The essential advantage of systems designed with the SADSF method is that their structures are correctly chosen for the assumed loads, so that the loads are carried by the dominating membrane forces [3, 4]. One may ask however, "What happens if we change the structure obtained from the SADSF solution by removing one of its parts?". Let, for example, this part be the additional oblique element, whose mounting into the structure (by welding) is rather laborious.

This time, however, the load value was assumed 10 times lower in order to obtain the level of equivalent stress similar to that in the SADSF solution.

The distribution of equivalent stresses obtained for such a case is presented in Fig. 6. The bending state is the dominant one, here. Maximal stresses originating from the membrane state are small and constitute just 8.5% of the bending state stresses (28.0/331.9).

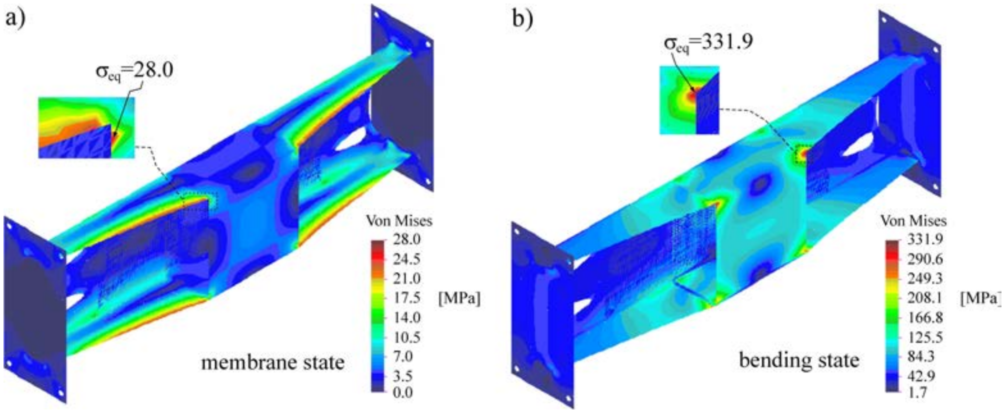


Fig. 6. Distribution of equivalent stresses in the structure modified by removing the additional oblique element and after a ten-fold decrease of the applied load.

5. Conclusions

The results obtained in this study confirm the enormous usefulness of the SADSF method in designing thin-walled structures and justify the need for popularizing this method in industry.

In all the cases analysed already – similarly as in the examples presented in this work – we have always found good and very good strength properties in elastic state. In consequence, the examined structures have also had good properties in the conditions of time-varying loads. Similar conclusions, based on other cases of structures designed with the SADSF method, have been repeatedly published in the relevant literature [2–13].

The scale of the analyses carried-out and the repeatability of the obtained results, allows us to treat the conclusions formulated in this work as general.

The examinations in the elastic-plastic range of stress have shown, among other things, an almost equal limit of load-carrying capabilities of all component elements of the structure, plasticization of vast portions of the structure's volume at the moment of collapse, and the prevalence of the membrane state within the whole range of the load values – up to the value not much smaller than the actual limit load. On the other hand, the collapse itself has always consisted in large, bending-type changes in geometry, and in all cases the actual limit load has turned out to be greater than that assumed in design.

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