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ON THE LATE STAGE OF THE POINT EXPLOSION IN AN EXPONENTIAL MEDIUM AND SELF-SIMILAR RAIZER'S AND HAYES'S SOLUTIONS

A. A. MARKOV (MOSCOW)

The influence of the effects of counter pressure, gravity force and initial conditions on the asymptotic of the solution in upper region of the flow and under one is investigated.

It is shown that calculation of gas-dynamics of the late stage of the explosion (stage of acceleration of upper part of a shock) require the estimation of the interaction of upper and lower regions of the flow. The conjugation conditions of both regions are suggested and some examples of calculations for the plane shock are presented.

INTRODUCTION

We shall consider an adiabatic motion of an inviscid perfect gas caused by a point explosion in an exponential medium.

For the early stage, when the heterogeneity of the atmosphere and counter-pressure are unessential, the solution of the problem was obtained by L. I. SEDOV (self-similar solution) [1], and computations were developed by G. I. TAYLOR [2].

Original computations of the heterogeneity effects, based on the thin-layer model have been carried out by E. I. ANDRYANKIN, A. M. KOGAN, A. S. KOMPANEETS and V. P. KRAINOV [5].

Computations of the explosion in a non-uniform atmosphere assuming local-radiality of the flow were performed by D. D. LAUMBACH and R. F. PROBSTEIN [6].

Computations of the point explosion in exponential atmosphere with Sedov's solution as initial data have been performed by many authors. The influence of the counter-pressure, gravity-force effects and non uniformity of medium is described in monographs written by V. P. KOROBEINIKOV, I. S. MELNIKOVA and E. V. ROZANOV [3] and H. S. KESTENBOIM, G. S. ROSLAKOV and L. A. CHOODOV [4].

The shock going upward accelerates after it has travelled sufficiently far from the center of the explosion. After a short period, the accelerating shock leaves the atmosphere (the "break" of the atmosphere takes place).

If the initial energy of the explosion is sufficiently large, the descending shock remains strong for a period which is some scores of times longer than the „break" time.

In the papers reffered to, the calculations were performed only up to the time near the „break" time, when the descending shock had traversed a distance equal to approximately two heterogeneity scales.

In connection with the problem of the accurate study of strong shocks during a long period of time, including the stage of acceleration of an upper part of a shock, the „break" stage and the after-break stage, the limiting laws governing the propagation of strong shock waves in a medium with exponentially varying density and pressure are of interest. Plane self-similar rising and descending shocks have been studied by RAIZER [9, 10], curved self-similar shocks with exponentially varying ray-tube area — by Hayes.

In the present paper, it is shown that the self-similar solutions reffered to above are definite asymptotics of the exact solution of the problem in question. Self-similar solutions are valid in certain regions near the shock.

In the case of plane shocks, asymptotic expansions for gas dynamic functions have been constructed, the first terms of which coincide with the self-similar Raizer solutions. We have observed a non-uniform nature in the tendency of the solution to self-similar limits, which is in agreement with the results of the numerical analysis [7, 8]. It is shown that calculations of gas-dynamics of the late stage of an explosion requires estimation of the interaction between the upper and lower regions of the flow. The motion of a gas in these regions follows different scales.

The conditions for conjugation of the two regions are suggested and some examples of calculations presented.

1. LIMITING AND SELF-SIMILAR SOLUTIONS

1. First, let us consider a motion in an inviscid perfect gas caused by a plane explosion with energy E_0 per square unit. We assume that the atmosphere density and pressure depend on the space coordinate according to the exponential law:

$$\rho_0=\overset{*}{\rho_0}\, e^{-x/\Delta}, \quad p=\overset{*}{p_0}\, e^{-x/\Delta};$$

where $\overset{*}{\rho_0}$, $\overset{*}{p_0}$ — density and pressure at the point of blast, Δ — scale of heterogeneity.

Self-similar Raizer solutions [9, 10] describe the disturbed gas flow at late stages, when the shocks $x=\varphi^+(t)$ (rising shock) and $x=\varphi^-(t)$ (descending shock) have propagated sufficiently far, and the initial energy E_0 has dropped out of the characteristic parameters of the problem.

A priori, the velocity of the shock is assumed to follow the law

$$d\varphi^{\pm}(t)/dt=\alpha^{\pm}\Delta/t,$$

where $\alpha^{\pm}$ are the similarity parameters determined by the regularity conditions for the solution at the singular (saddle) point of the differential equation for the gas pressure.

It appears that the self-similar Raizer solutions [9, 10] are certain limits of the present solution satisfying the initial conditions.

Let us consider, for instance, a region near the rising shock $x=\varphi^+$. In the gas dynamic equations we shall proceed from the dimensionless variables (u, ρ, p, m, t) [7, 13] to the new variables $V(\eta^+, \tau^+)$, $R(\eta^+, \tau^+)$, $P(\eta^+, \tau^+)$, η^+, τ^+ and assume the numerical solution [7] for a sufficiently late stage as the initial condition of the problem (in detail see [13]). The initial condition is so taken, that the shock is initially of a self-propagating type; see also [11].

The formulae of transformation are as follows:

(1.1) $u=\dot{\varphi}^+V^+, \quad \rho=e^{-\varphi^+}R^+, \quad p=e^{-\varphi^+}(\dot{\varphi}^+)^2P^+, \quad \eta^+=me^{\varphi^+}, \quad \tau^+=\ln(\dot{\varphi}^+)^2.$

The variable η^+ changes in the interval $1\leqslant\eta^+\leqslant e^{\varphi^+-\varphi^-}$, where $\eta^+=1$ is the, coordinate of a shock $x=\varphi^+$.

Auxiliary time τ^+ changes in the interval $\tau_0\leqslant\tau^+<\infty$. In this case, we have $\tau^+\to\infty$ while $\dot{\varphi}^+\to\infty$ (acceleration of the shock).

The equations of motion in terms of variables (1.1) are presented as follows [index (+) is omitted in the interests of simplicity]:

$$(1.2)\quad \begin{gathered} 2\omega(\tau)\frac{\partial Z}{\partial\tau}+\eta\frac{\partial Z}{\partial\eta}+\frac{\partial V}{\partial\eta}+Z=0, \qquad 2\omega(\tau)\frac{\partial S}{\partial\tau}+\eta\frac{\partial S}{\partial\eta}+S(2\omega+\gamma-1)=0, \\ 2\omega(\tau)\frac{\partial V}{\partial\tau}+\eta\frac{\partial V}{\partial\eta}-\frac{\partial P}{\partial\eta}+\omega V+A_g e^{-\tau}=0, \\ P=Z^{-\gamma}S, \qquad Z=R^{-1}, \qquad \omega=\ddot{\varphi}/\dot{\varphi}^2. \end{gathered}$$

Here A_g is a numerical parameter, indicating the influence of the gravity force. In the system (1.2) a new unknow function of the variable τ has appeared — namely $\omega(\tau)$. This function determines the law of the shock propagation and will be found in accordance with the initial conditions and boundary conditions for the shock wave φ^+ simultaneously with the functions $Z(\eta,\tau)$, $V(\eta,\tau)$, $S(\eta,\tau)$. Note that for the variables (η,τ) the wave φ^+ has been stopped $(\eta^+=1)$ and the function $\omega(\tau)$ is determined by the characteristic condition along the characteristic of the second family $(d\eta^+/d\tau^+=\eta^+-\sqrt{a^+},\ a^+=\gamma P^+/Z^+)$, which reaches the shock wave $\eta^+=1$.

A self-similar solution [10] appears to be the following limit of the solution in question: η^+ is fixed, $\tau^+\to+\infty$

$$V\to V_0(\eta), \quad P\to P_0(\eta), \quad R\to R_0(\eta), \quad \omega\to\alpha^{-1} \quad (\eta=\eta^+),$$

where constant value α is a similarity parameter.

By an analogous procedure, the self-similar solution [9] (for descending shock φ^-) may be found. For this purpose, it is sufficient to introduce the new variables V, R, P, η^-, τ^- replacing the index (+) by index (−) in the formulae (1.1). Variable η^- changes in the interval $e^{-\varphi^++\varphi^-}\leqslant\eta^-\leqslant 1$, where $\eta^-=1$ is a coordinate of the shock wave φ^-. If $A_g=A_p=0$ (here A_p — the numerical parameter showing the influence of the counter-pressure effect, see [7]), then the auxiliary time $\tau^-=\ln(\dot{\varphi}^-)^2$ changes in the interval $-\infty<\tau^-\leqslant\tau_0^-$. The self-similar solution [9] appears to be the following limit: η^- is fixed, $\tau^-\to-\infty$, $A_p=A_g=0$, $V\to V_0^-(\eta^-)$, $P\to P_0^-(\eta^-)$, $R\to R_0^-(\eta^-)$.

Note, that $\omega(\tau^-)\to\tilde{\alpha}^{-1}$, where $\tilde{\alpha}$ is the similarity parameter of solution [9].

2. Now let us consider the axi-symmetrical adiabatic flow of an inviscid gas, caused by the point explosion with initial energy E_0, and take as an initial condition the numerical solution of this problem [4] for the late stage, when the acceleration of the upper part of a shock has taken place.

Let us consider an area near the upper part of a shock and introduce the new variables V, U, R, P, ξ, r, τ in a manner similar to that mentioned above. We have

$$(1.3)\quad \rho = Re^{-\varphi(r,t)}, \quad u = UN_r, \quad v = VN_x, \quad p = N^2 e^{-\varphi} P, \ \xi = x - \varphi, \ \tau = \ln N^2,$$

where N — the velocity of a shock, N_x, N_r are the vertical and radial components of vector-velocity N, $x = \varphi(r, t)$ — the boundary of the upper part of the shock.

The equations of motion in variables (1.3) are presented as follows:

$$(1.4)\quad \begin{aligned} &2\omega(r,\tau)\frac{\partial R}{\partial \tau} + \frac{\partial (W-1) R}{d\xi} + \nu(r,\tau)\frac{\partial UR}{\partial r} + \frac{\nu}{r} UR - R = 0, \\ &2\omega(r,\tau)\frac{\partial U}{\partial \tau} + (W-1)\frac{\partial U}{\partial \xi} + R^{-1}\frac{\partial P}{\partial \xi} + \nu U \frac{\partial U}{\partial r} - \frac{R^{-1}}{\varphi_r}\frac{\partial P}{\partial r} + \frac{\nu'}{\nu} U = 0, \\ &2\omega(r,\tau)\frac{\partial V}{\partial \tau} + (W-1)\frac{\partial V}{\partial \xi} + R^{-1}\frac{\partial P}{\partial \xi} + \nu U\frac{\partial V}{\partial r} + \frac{\alpha'}{\alpha} V + A_g e^{-\tau} = 0, \\ &2\omega\frac{\partial P}{\partial \tau} + (W-1)\frac{\partial P}{\partial \xi} + \gamma P\frac{\partial W}{\partial \xi} + \nu U\frac{\partial P}{\partial r} + \gamma\frac{\nu}{r}\frac{\partial (rU)}{\partial r} + (2\omega - 1)P = 0. \end{aligned}$$

Here $W = (V + \varphi_r^2 U)/(1 + \varphi_r^2)$; the parametric functions $\alpha(r,\tau)$, $\nu(r,\tau)$, $\alpha'(r,\tau)$, $\nu'(r,\tau)$, $\omega(r,\tau)$ determine the law of the motion for the part of the shock in question. The following relations hold:

$$\alpha = (1 + \varphi_r^2)^{-1}, \quad \nu = -\alpha\varphi_r, \quad \omega = \varphi_{tt}/\varphi_r^2 + \nu\varphi_{rt}/\varphi_t, \quad \nu' = \omega\nu - \alpha^2\varphi_{rt}/\varphi_t, \quad \alpha' = \nu'\varphi_r + \omega,$$

where $\varphi_t = \partial\varphi/\partial t$, $\varphi_{rt} = \partial^2\varphi/\partial r\,\partial t$ etc.

Let us consider the limit: r, ξ are fixed, $\tau \to \infty$, $\nu/r \to k(r)$,

$$(1.5)\quad \omega \to \omega_0(r), \quad \nu' \to \nu_0'(r), \quad (R, U, V, P) \to (R_0, U_0, V_0, P_0).$$

This limit contains as a specific case the self-similar Hayes solution [11] for curved shocks with exponentially varying ray-tube area according to the law: $A = A_0 e^{-k_0 x}$. Namely, the solution [11] is found from (1.5) if in addition we assume that $\omega_0 = \text{const}$, $k = k_0 - \text{const}$, R_0, U_0, V_0, P_0 are the functions of the only one variable, and further — $U_0(\xi) \equiv V_0(\xi)$.

For $k_0 = 0$, Raizer's solution [9] is obtained.

2. The study of the disturbances in the vicinity of some self-similar solutions of the second kind

The results obtained in this and the following paragraphs are concerned with the plane shock problem (see [2]).

1. Now we shall consider a small perturbation near the self-similar solution [10] (rising shock) and linearize these equations with respect to steady ($\tau^+ \to +\infty$) limit.

At the same time, we shall linearize the equations near the self-similar solution [9] (descending shock). We shall complement the linear equations by initial conditions and boundary conditions at the line $\eta^+ = 1$.

Each of the linear problems (for rising shock and descending one) has been strictly studied and theorems of unique solvability has been proved (in detail see [12 – 14]). Note that the parametric function $\omega_1(\tau)$, which determines the disturbance of the function $\omega(\tau)$ near the constant limit ω_0 is determined in a unique way by the characteristic relation along the second family $(d\eta/d\tau=\eta-\sqrt{a},\ a=\gamma P_0/Z_0)$ according to the initial conditions at the interval of $1\leqslant\eta^+\leqslant\eta_0^+$ for the rising shock and at the interval of $\eta_0^-\leqslant\eta^-\leqslant 1$ for descending shock. Here $\eta^\pm$ represent the rotation points of s-characteristics, $\eta_0^\pm$ are the roots of the equation $\eta^\pm-\sqrt{a^\pm}=0$.

2. To evaluate more precisely the disturbances when $|\tau^\pm|\to\infty$, we shall employ in the case of linear problems the Laplace transform for the variables $\tau^\pm$ and we shall obtain a system of ordinary differential equations with the spectral parameter λ. Apart from the required functions $\tilde{Z}^\pm(\eta^\pm,\lambda)$, $\tilde{V}^\pm$, $\tilde{S}^\pm$, $\tilde{P}^\pm$ of the variables $\eta^\pm,\lambda$, the spectral problems contain the unknown parametric functions $\Omega^\pm(\lambda)$ which are the Laplace images of the functions $\omega_1^\pm(\tau^\pm)$ (note: the index (+) stands for a rising and index (−) stands for a descending shock).

The system of differential equations for $\tilde{Z}$, $\tilde{V}$, $\tilde{S}$, $\tilde{P}$ is reduced to one equation of the second order for $\tilde{Z}$. The coefficient before the second derivative $d^2\tilde{Z}/d\eta^2$ vanishes at the point $\eta=\eta_0$. The parametric function $\Omega(\lambda)$ is determined in a unique manner by the regularity conditions of the solution $\tilde{Z}(\eta,\lambda)$ at the singular point η_0. Here we have the analogy with self-similar solutions of the second kind.

The solution of spectral problems is an analytic function of λ in the half-plane $\mathrm{Re}\,\lambda\geqslant\sigma^+$ for the rising shock and $\mathrm{Re}\,\lambda\leqslant\sigma^-$ for the descending shock. These solutions admit of analytical continuation over the boundaries $\mathrm{Re}\,\lambda=\sigma^\pm$ (in detail see [12, 13]) but they have known poles, which are the roots of some spectral equation $\Delta^\pm(\lambda)=0$. The estimation of these poles ascertained enabled us to construct the asymptotics for the time-dependent solution when $|\tau^\pm|\to 0$.

3. The asymptotics of accelerating and decelerating plane shock in an exponential medium

Using the inversion for the Laplace transform of the solution in the spectral problem for descending shock, we arrive at the asymptotic law for $\varphi^-(t)$:

$$(3.1)\quad \varphi=\varphi^-,\ \ddot{\varphi}/\dot{\varphi}^2=\omega_0+\omega_{-1}\dot{\varphi}^{-2}(t)+\omega_1\left(\dot{\varphi}^2(t)\dot{\varphi}^{-2}(0)\right)^{\lambda_1}+\omega_2\left(\dot{\varphi}^2(t)\dot{\varphi}^{-2}(0)\right)^{\lambda_2}+\ldots$$

Here, $\omega_0=\alpha^{-1}$, α — a similarity parameter of the solution [9], so that the first term at the right side of (3.1) corresponds to the self-similar law of the shock propagation; $\dot{\varphi}(0)$ is the shock velocity at the initial stage with evident inequality $\dot{\varphi}^2(t)\dot{\varphi}^{-2}(0)<1$; $\lambda_1,\lambda_2,\ldots$ are the roots of the spectral equation $\Delta^-(\lambda)=0$, which depends on the parameter $\gamma\ (\gamma=c_p/c_v)$; for $\gamma=2$ we found $\lambda_1=0.375$, $\lambda_2=0.5$; $\lambda_3=0.71$; ... . The term $\omega_{-1}\dot{\varphi}^{-2}(t)$ shows the influence of the counter-pressure and the gravity force.

The value ω_{-1} may be written in the form $\omega_{-1}=k_pA_p+k_gA_g$ with constant coefficients k_p, k_g. (For the case $\gamma=2$, the coefficients k_p, k_g were found in an explicit form). In the ca se of $A_p=A_g=0$, the formula (3.1) shows the rate of the tendency of the descending shock to the self-similar limit. The terms in (3.1) with coefficients $\omega_1, \omega_2, \ldots$ show the influence of the initial conditions.

If $A_p^2+A_g^1>0$, then the formula (3.1) is valid only while the descending shock is strong — i.e., $A_p\,\dot\varphi^{-2}(t)<1$, $A_g\,\dot\varphi^{-2}(t)<1$.

Analogous formulae for the velocity, density and pressure of a gas were found. The expansions have the same terms $(\dot\varphi^2(t)\,\dot\varphi^{-2}(0))^{\lambda_i}$, $i=1, 2, \ldots$ [see (3.1)], but the coefficients are functions of the variable η^{-1}. These functions show the non-uniformity in the tendency of the solution to the self-similar limit.

The analogous formulae were found for the rising shock. We obtain the power rate of increase of the disturbances in the vicinity of the self-similar solution [10] in terms of η^+, while the velocity of tending to self-similar limit with fixed η^+ is determined by the value $(\dot\varphi^2(t)\,\ddot\varphi^{-2}(0))^{-\lambda^*}$, $\varphi=\varphi^+$; $\lambda^*>0$; λ^* being the root of the spectral equation $\Delta^+(\lambda)=0$.

4. Application to numerical calculations

Following W. D. Hayes, we shall distinguish two problems.

1. A sufficiently weak explosion at a sufficiently low altitude.
2. A sufficiently strong explosion at a sufficiently high altitude.

Let us consider the second problem. The counter-pressure effects can be disregarded for a sufficiently long period, but the heterogeneity of the medium is essential.

The numerical computations of the problem, using the finite-difference method and comparative analysis of the other approximate methods, have been developed by Kestenboim, Roslakov and Choodov [4]. The calculations were carried out up to the moment when the descending part of the shock traversed the distance of about 2Δ (Δ is a heterogeneity scale); the rising part of the shock started to accelerate because of the decreasing of the upward density. The acceleration of the shock and the subsequent "break" of atmosphere constitute essential difficulties for numerical studies of the explosion. The results were obtained only till the moment preceding the "break" time, but the descending shock remains strong for a period which is some scores of times greater than the "break" time.

To perform the calculations after the "break", it is necessary to bound the domain of gas flow by the auxiliary boundary and to use certain boundary conditions.

The results for a plane explosion obtained above show that the disturbances which have appeared in the area $0<\eta^-\leqslant\eta_0^-$ cannot reach the shock φ^-, and the disturbances from the region $\eta_0^+\leqslant\eta^+$ cannot overtake the shock φ^+. For this reason, we can divide the flow into two zones such that the disturbances from the higher zone cannot reach the lower one. The boundary will be the limiting characteristics $\eta^-=\eta_0^-$. We can also separate the rising self-propagating shock by means of the boundary $\eta^+=\eta_0^+$ which is also a limiting characteristic.

Some computations at the late stage of the plane explosion (the stages preceding and following the "break") were performed. The shocks φ^+, φ^- were supposed to be strong ($A_p=0$).

The numerical calculations were performed by the finite-difference method [4] and use was made of the results refered to above concerning asymptotics (see also [13, 14]). We placed the auxiliary boundary $x=\varphi^*(t)$ above the limiting characteristics $\eta^-=\eta_0^-$. The evolution of the shocks φ^+, φ^- and boundary φ^* before the "break" time T for $\gamma=2$ and after the "break" time is shown in Fig. 1.

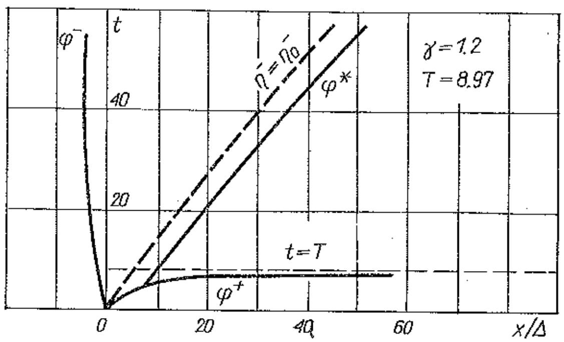

Fig. 1

The law of motion of the boundary was computed during the calculations. At the auxiliary boundary, use was made of the asymptotic conditions, using the similarity of the flow in the higher zone — that is, the partial derivatives of the functions Z^+ and P^+ [see (1.1), (1.2)] with respect to τ^+ were taken as zeros. In terms of Eulerian variables, these conditions can be written as follows:

$$x=\varphi^*(t)$$

(4.1)
$$p\left(\frac{\partial V}{\partial x}-1\right)+\frac{\partial p}{\partial x}(V-1)=0,$$

$$p\left[(2\omega-1)+\gamma\frac{\partial V}{\partial x}\right]+\frac{\partial p}{\partial x}(V-1)=0.$$

Here,

(4.2)
$$V(x,t)=u(x,t)/\dot{\varphi}^+(t), \qquad \omega=\ddot{\varphi}^+/(\dot{\varphi}^+)^2.$$

The velocity of gas $u(x,t)$ was calculated by the finite-difference approximation of the equation of gas motion. The function $V(x,t)$ [see (4.1), (4.2)] was determined according to (4.2) with the self-similar law for $\varphi^+(t)$ [10] — namely:

(4.3)
$$1/\dot{\varphi}^+=\begin{cases}(T-t)/\alpha, & \text{if} \quad t<T,\\ 0, & \text{if} \quad t\geqslant T,\end{cases}$$

where T is the „break" time, $T=t_0+\alpha/\dot{\varphi}^+(t_0)$, t_0 is the initial stage, for which the rising shock had to traverse a distance of about ten heterogeneity scales. The value ω [see (4.2)] was assumed to be constant ($\omega=1/\alpha$, α — similarity parameter of the solution [10]). The values of α for various γ were found by HAYES [11].

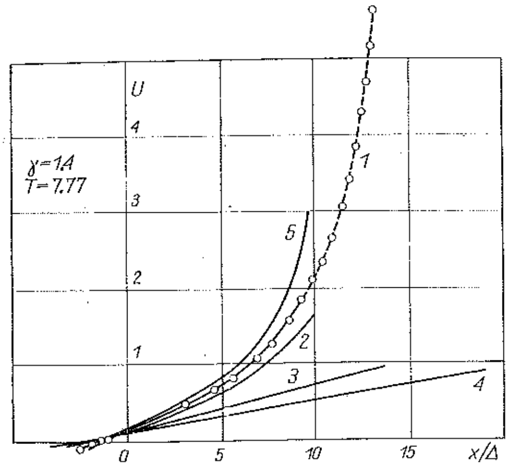

Fig. 2

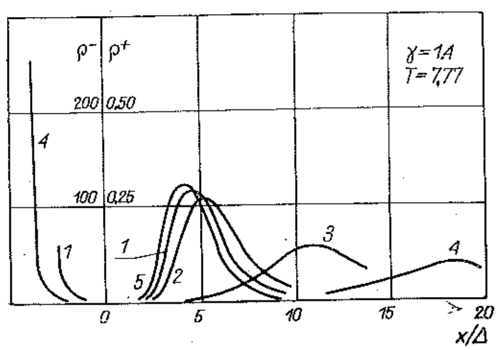

Fig. 3

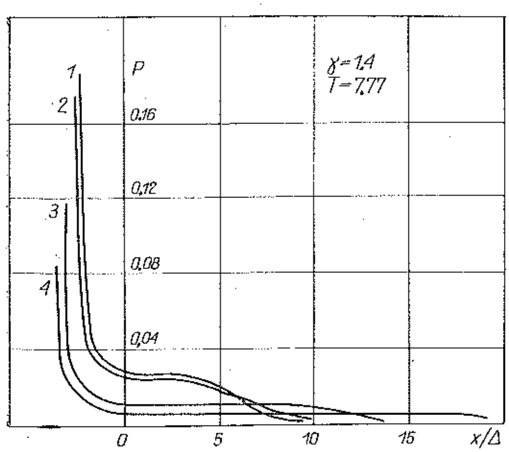

Fig. 4

Let us consider some results of calculations of the plane explosion model. Numerical results are shown for a period of time which is about four times that of the "break" time.

The velocity profiles of gas at various moments of time are shown in Fig. 2. The distributions of the velocity, density and pressure at the stage $t \approx 6.1$, for which the upper part of a shock had started to accelerate, were taken as initial data, and we started to employ auxiliary boundary conditions (4.1) – (4.3).

At a later stage ($t = 6.95$), the calculated velocity (line) was compared with the numerical solution [7]. Good agreement is seen in the whole area, $\varphi^- \leqslant x \leqslant \varphi^*$, for which the computations were made. The curves for $t = 7.78$ almost correspond to the "break" time ($T = 7.77$). The distributions of density (Fig. 3) and pressure (Fig. 4) are shown for the initial stage and for the after „break" stage. It is possible to observe the evolution of the rarefaction wave and decrease of the pressure. The distributions of gas-dynamic functions in the lower zone to the self-similar solution [9] with increasing accuracy.

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Streszczenie

O PÓŹNYM STADIUM EKSPLOZJI PUNKTOWEJ W OŚRODKU WYKŁADNICZYM I O SAMOPODOBNYCH ROZWIĄZANIACH RAIZERA I HAYESA

Rozważono problem gazodynamiki późnego stadium eksplozji punktowej w ośrodku wykładniczym. Omówione przepływy dotyczą płaskiego i osiowo-symetrycznego uderzenia o pionowej osi symetrii. Wspomniane wyżej rozwiązania samopodobne Raizera i Hayesa opisują przepływ w pobliżu osi symetrii w pewnym obszarze blisko uderzenia. Można te rozwiązania otrzymać jako pewien szczególny przypadek graniczny rozwiązania omawianego w niniejszej pracy, gdy pomocnicza zmienna czasowa τ ($\tau = \ln N^2$, N — prędkość propagacji uderzenia) zmierza do nieskończoności.

Sformułowano problem stateczności rozwiązań samopodobnych. Przedstawiono liniowe przybliżenie analizy tej stateczności posługując się metodą transformacji Laplace'a. Dokonano oceny widma w problemch ustalonych ($\tau = \infty$).

Rozpatrzono wpływ przeciwciśnienia, sił ciężkości i warunków początkowych na asymptotyczność rozwiązania w obszarze przepływu. Pokazano, że analiza gazodynamiczna w późnej fazie eksplozji (stadium przyśpieszenia górnej części uderzenia) wymaga oceny wzajemnego oddziaływania górnego i dolnego obszaru przepływu. Zaproponowano warunki sprzężenia obu obszarów i przedstawiono kilka przykładów obliczeń dla uderzenia płaskiego.

Резюме

О ПОЗДНОЙ СТАДИИ ТОЧЕЧНОГО ВЗРЫВА В ЭКСПОНЕНЦИАЛЬНОЙ СРЕДЕ И ОБ АВТОМОДЕЛЬНЫХ РЕШЕНИЯХ Ю. П. РАЙЗЕРА И У. Д. ХЕЙЗА

Рассмотрена проблема газодинамики поздной стадии точечного взрыва в экспоненциальной среде. Обсужденные течения касаются плоского и осесимметричного удара с вертикальной осью симметрии. Упомянутые выше автомодельные решения Райзера и Хейза описывают течения вблизи оси симметрии, в некоторой области близко удара. Эти решения можно получить как некоторый частный предельный случай решения обсуждаемого в настоящей работе, когда вспомагательная временная переменная τ ($\tau = \ln N^2$, N — скорость распространения удара) стремится к бесконечности.

Сформулирована проблема устойчивости автомодельных решений. Представлено линейное приближение анализа этой устойчивости, пользуясь техникой преобразований Лапласа. Произведена оценка спектра установившихся проблем ($\tau = \infty$).

Рассмотрено влияние противодавления, сил тяжести и начальных условий на асимптотику решения в области течения. Показано, что газодинамический анализ в поздной фазе взрыва (стадия ускорения верхней части удара) требует оценки взаимодействия верхнего и нижнего областей течения.

Предложены условия сопряжения обоих областей и представлено несколько приметов расчетов для плоского удара.

INSTITUTE FOR PROBLEMS IN MECHANICS
THE U.S.S.R. ACADEMY OF SCIENCES, MOSCOW

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