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PROPERTIES OF VERY SMALL HELMHOLTZ RESONATORS WITH REGARD TO WALL PRESSURE MEASUREMENTS

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Measurements of wall pressure fluctuations require pressure sensors of very small dimensions and very high sensitivity. Frequency characteristics of very small resonators used for mesuring the wall pressure fluctuations are investigated. The investigations are performed both theoretically and experimentally.

1. Introduction

Measurements of wall pressure fluctuations require pressure sensors of very small dimensions(1) and high sensitivity (~ 1 mV/μb). Piezo-microphones may have the desired small dimensions, yet the resulting sensitivity is often too low. On the other hand, condenser microphones offer the desired sensitivity, but their dimensions are often too large.

The influence of the microphone diameter can be reduced by mounting diaphragms in front of the condenser microphone membrane. A diaphragm with the necessary free space in front of the membrane forms a small Helmholtz resonator, whose frequency response must be known with regard to the spectrum of the wall pressure fluctuations.

Since the classical theory of Helmholtz resonators is not sufficient for very small resonators, the frequency characteristics of small resonators were investigated experimentally and theoretically. This report will be confined to these investigations; measurements of wall pressure fluctuations themselves will not be dealt with.

2. Experimental facility

The arrangement chosen for the investigation of the Helmoholtz resonators consists basically of the following: The enclosure consisted of a hollow cylinder with an inner diameter of $2R=15$ mm. Its exchangeable top plate had a thickness of $l=1$ to 8 mm. In the centre of the plate was a bore hole between $2r_0=0.5$ and 3 mm, which formed the neck of the resonator. The 1/2″ and occasionally the 1/4″ microphones from Brüel and Kjaer were mounted concentrically into the movable cylindrical plugs which formed the bottom plate. For the measurements, the distance between the plug and top plate, i.e., the resonator height, is varied between $h=0.1$

(1) Smaller than the lowest wavelength to be measured.

and 3 mm (accurate to within 0.03 mm). The joints between the plug and the cylinder bore were made by O-rings, which initially gave rise to widely scattered measurement results. Even very small damage to the rings, later reduced by constructive measures but not definitely eliminated, caused leaks which increased the resonator damping in some cases by a multiple and slightly reduced resonator frequency.

The resonance frequency f_r and the power factor Q were determined by two methods. With the first method, the resonance characteristic of the Helmholtz resonator at constant pressure (~ 5 μbar) and with variable frequency (0.2 to 5 kHz) was recorded using a Brüel and Kjaer beat frequency oscillator, type 1013, and a Brüel and Kjaer level recorder, type 2305, and the resonance frequency f_r was determined using a Hewlett-Packard counter, type 5233 L. The power factor Q was determined within an accuracy of ± 0.3 dB and the frequency f_r within 1 %. In the second, less exact procedure, a dying-out oscillation was generated by a pressure shock. Here, in addition to the oscillation of the Helmholtz resonator, the resonance frequency of the neck air column was frequently observed, weakened by the frequency response of the microphone; it was approximately ten times higher than the Helmholtz resonator frequency. A quantitative investigation of this oscillation was not carried out with regard to the changes in the experimental facility it would have required. The results of both procedures for f_r and Q are comparable within the measuring accuracy. Only the measurements with a neck length of $l = 8$ mm were quantitatively analyzed, since here the effects at the neck ends in relation to the processes in the neck have merely the character of a correction. Since an analysis of the Helmholtz resonator frequency characteristics showed that they correspond well to those of the linear oscillation equation, the relationships between f_0 (non--attenuated resonance frequency), f_r, Q, q (quality factor) and ρ^+ (loss coefficient) resulting from this equation were used to evaluate the measurements:

$$Q = (1+2q^2)/\sqrt{1+4q^2}, \quad q = \omega/\rho^+, \quad f_r/f_0 = \sqrt[4]{1+Q^{-2}}.$$

3. Theoretical aspects

3.1. *Equation of motion*

Assuming that the mass is concentrated in the resonator neck, that the displacement x of the air particles in the neck is everywhere the same and that the spring forces are represented by the volume of the enclosure, the classical equation of motion for the Helmholtz resonator is expressed by:

$$\ddot{X} + \rho_f \dot{X} + c^2 F/(Vl)\, X = pF/(\rho l)$$

($X = Fx$, $F = \pi r_0^2$ neck cross-section, l neck length, c velocity of sound, $V = \pi R^2 h$ volume of the enclosure, p pressure amplitude, ρ density, $\rho_f = \omega/q$ loss coefficient). Instead of l, we take the corrected value $l_x = l + \pi r_0/2$ [1]. With the acoustic conductance $G = \pi r_0^2/l_x$, the classical theory yields then for the non-attenuated resonance frequency $f_0 = c\,(G/V)^{0.5}/(2\pi)$ and for the power factor $Q \approx q = 2\pi\,(V/G^3)^{0.5}$.

Comparison of the theoretical and experimental values of Q for the small resonators showed, however, that practically all the experimental data lay below the theoretical values; the power factor differed in some cases by a factor of 10 or more. It was possible to bring the theoretical and experimental results into better agreement by employing the following corrections, even though some of these are quite rough estimates; some of the corrections were taken directly from the literature and, in some cases, expanded, and some were newly developed.

3.2. *Volume corrections*

3.2.1. Corrections due to the stiffness of the microphone membrane. According to [2], p. 24 – 25, the membrane of the 1/2″ microphone has a stiffness of $\mathrm{St} \approx 17 \cdot 10^7$ dyne/cm^5. Thus an additional volume, the so-called equivalent volume $V_{eq} = \kappa \cdot p_0/\mathrm{St} \approx 8.5$ mm^3, must be added to the original volume V_0 (p_0 air pressure). This corresponds to an additional enclosure height of 0.048 mm for the most frequently used enclosure diameter of 15 mm. The above-quoted stiffness is a limiting value for low frequencies, but up to 5 kHz, the complexity of the stiffness may be neglected (see [2] for details). Due to the high stiffness of the 1/4″ microphone membrane, its equivalent volume can be neglected here.

3.2.2. Corrections due to the neck volume. Neglecting the neck volume V_N gives a lower boundary and adding V_N to V_0 gives an upper boundary for the effective resonator volume. Here, the mean value 0.5 V_N was added to V_0. For a neck length of 8 mm, this corresponds to an additional enclosure height of $4(r_0/R)^2$ mm.

3.2.3. Corrections due to the adiabatic-isothermal transition. In the classical resonator theory, adiabatic change of pressure is assumed; when the enclosure has very small dimensions, however, the pressure changes are isothermal, so that the acoustical impedance of the enclosure, $Z_0 = \kappa\ p_0/(\omega V_0)$, falls to $p_0/(\omega\ V_0)$ and the resonance frequency is reduced by the factor $\kappa^{-0.5}$. The complex values for the volume correction factor F_V in the region of transition from adiabatic to isothermal can be derived as a function of $(f_r\ \rho c_p/\lambda)^{0.5}\ h$, see [3]. Further frequency corrections, e.g. those in [4], have not been considered because of their minor influence.

3.3. *Corrections of the power factor Q*

As it was mentioned above, the measured values of the power factor Q have mostly 1/10 of the values according to the classical theory, which considers radiation losses only but does not account for the friction losses in the neck and in the enclosure.

3.3.1. The loss coefficient of the neck. The loss coefficient of the neck ρ_N, which was calculated approximately in [5] by assuming $r_0 \gg \sqrt{\nu/\omega}$, was determined anew using the equation of the oscillating flow through a circular tube, neglecting compressibility and heat conduction:

$$u_t - \nu(u_{rr} + u_r/r) = \frac{1}{\rho}\,\frac{p}{l}, \qquad p = p' \cos \omega t .$$

(The following functions p_0, q_0, q_1 consist of sums, products and quotients of Kelvin functions, see [6], p. 379 ff). According to H. SAMULON [5], the loss coefficient due to friction in the neck ρ_N is defined by:

$$\rho_N = W_f / m\overline{\overline{u^2}},$$

W_f is the mean value of the power due to friction forces in the neck.

With

$$m\overline{\overline{u^2}} = \pi r_0^2 \, l\rho \frac{x_0 \, p_0 - 2q_0}{2x_0 \, p_0} \left(\frac{1}{\rho} \frac{p'}{l\omega} \right)^2$$

and

$$W_f = 2\pi\mu l \frac{1}{t_0} \int_0^{t_0} dt \int_0^{r_0} u_r^2 \, r dr = \pi\mu l x_0 \, q_1 / p_0 \left(\frac{1}{\rho} \frac{p'}{l\omega} \right)^2, \qquad x_0 = r_0 \sqrt{\omega/\nu} \, ,$$

the loss coefficient becomes

$$\rho_N = 2\omega q_1 / (x_0 \, p_0 - 2q_0) \quad \text{and the quality factor}$$

$$q_N = \omega / \rho_N = (x_0 \, p_0 - 2q_0)/(2q_1);$$

$$q_N = x_0^2/6 - x_0^4/360 \ldots \quad \text{for } x_0 \to 0,$$

$$q_N = x_0/\sqrt{2} - 0.5 \quad \text{for } x_0 \to \infty.$$

3.3.2. The loss coefficient of the enclosure. The calculation of the loss coefficient due to friction in the enclosure ρ_E is based on the simplifying assumptions that the velocity field in the enclosure may be approximated by the incompressible velocity field between two infinitely large parallel walls with the distance $h = 2d$, and that the velocity is only a function of the distance y from the wall. Thus the calculation is based on the equation

$$u_t - \nu u_{yy} = C \cos(\omega t) \, .$$

With $x_0 = d\sqrt{\omega/\nu}$, the quality factor is expressed by

$$q_E = \omega/\rho_E = \frac{2x_0 \left(\cos(\sqrt{2}\, x_0) + \cos(\sqrt{2}\, x_0) \right) - \sin(\sqrt{2}\, x_0) - \sin(\sqrt{2}\, x_0)}{\sin(\sqrt{2}\, x_0) - \sin(\sqrt{2}\, x_0)},$$

$$q_E = 0.4 d^2 \, \omega/\nu + \ldots \quad \text{for} \quad x_0 \to 0,$$

$$q_E \approx d\sqrt{2\omega/r} - 1 \quad \text{for} \quad x_0 \to \infty.$$

4. COMPARISON OF THEORETICAL AND EXPERIMENTAL VALUES

For comparison of the theoretical and experimental values, see Table 1 and Fig. 1.

The resonance frequency f_r was calculated using the formula

$$f_r = \frac{c}{2\pi} \frac{r_0}{R} \sqrt{\frac{1}{l_x h_x F_V}} \sqrt[4]{1 - Q^{-2}}$$

Table 1. The calculated and measured quality factors q: Ø neck diameter, q_N neck quality factor, q_E enclosure q.f., q_R radiation q.f., q_C calculated resonator q.f., q_M measured resonator q.f. (neck length 8 mm, encloseure diameter 15 mm)

h (mm)	Ø 3 mm					Ø = 1 mm			
	q_N	q_E	q_R	q_C	q_M	q_N	q_E	q_C	q_M
0.1						10.8	1.02	1.02	2.43
0.15						10.5	2.06	1.72	3.27
0.20	47.4	5.39	99	4.61		10.1	3.10	2.37	3.96
0.30	45.6	8.23	107	6.55	6.35	9.42	4.98	3.25	4.77
0.40	44.0	10.9	115	8.10	8.40	8.82	6.45	3.72	5.11
0.60	41.8	15.8	130	10.6	11.2	8.09	9.31	4.32	5.38
0.80	39.4	20.3	143	12.3	13.2	7.59	11.9	4.63	5.43
1.0	37.7	24.5	156	13.6	14.6	7.16	14.3	4.76	5.38
1.5	34.8	34.3	183	15.8	16.9	6.44	19.8	4.85	5.31
2.0	32.6	43.1	208	17.0	17.4	6.00	24.9	4.82	5.08
2.5	30.9	51.3	231	15.7	17.7	5.64	29.7	4.73	4.87
3.0	29.6	59.3	250	18.3	17.4	5.37	34.2	4.63	4.67
3.5	28.5	66.8	270	18.6	17.4	5.14	38.5	4.52	4.55

h (mm)	Ø = 2 mm					Ø = 0.5 mm			
	q_N	q_E	q_R	q_C	q_M	q_N	q_E	q_C	q_M
0.1	29.3	1.71	237	1.60	1.80	3.32	0.47	0.42	1.70
0.15	28.6	3.24	248	2.88	2.96	3.13	0.95	0.73	1.79
0.20	27.7	4.66	265	3.93	4.22	2.99	1.51	1.00	1.80
0.30	26.3	7.05	293	5.46	6.20	2.78	2.78	1.39	1.82
0.40	25.1	9.24	332	6.62	7.51	2.50	3.92	1.53	1.79
0.60	23.4	13.3	368	8.31	9.22	2.21	5.80	1.60	1.67
0.80	22.1	17.1	414	9.41	10.2	1.98	7.46	1.56	1.57
1.0	21.0	20.5	455	10.1	10.9	1.81	8.96	1.51	1.47
1.5	19.2	28.6	543	11.3	11.5	1.51	12.4	1.35	1.27
2.0	17.9	35.8	620	11.7	11.5	1.31	15.3	1.21	1.12
2.5	17.0	42.7	688	11.9	11.5	1.16	18.1	1.09	0.98
3.0	16.3	49.2	750	12.0	11.4	1.01	20.3	0.96	0.88
3.5	15.6	55.4	810	12.0	11.1				

with

$$l_x(\text{cm}) = l + 0.5\pi r_0\,, \qquad h_x(\text{cm}) = h + 0.0048 + 0.4 r_0^2/R^2\,.$$

With larger neck diameters, the measured values lie, on the average, somewhat above the calculated frequency values, contrary to the case of small neck diameters. No satifactory explanation for the discrepancies, which lie within 10%, has as yet been found.

The calculated resulting quality factor q_C of the Helmholtz resonator is based on the equation:

$$q_C^{-1} = q_R^{-1} + q_N^{-1} + q_E^{-1}\,;$$

q_R, the quality factor due to radiation, is neglected for $q_R > 100\, q_C$. The calculated value q_C and the measured value q_M of the resulting quality factor agree rather well for larger resonator heights, where q_C is essentially determined by the friction in the neck. For the small resonator heights, the measured values, however, are

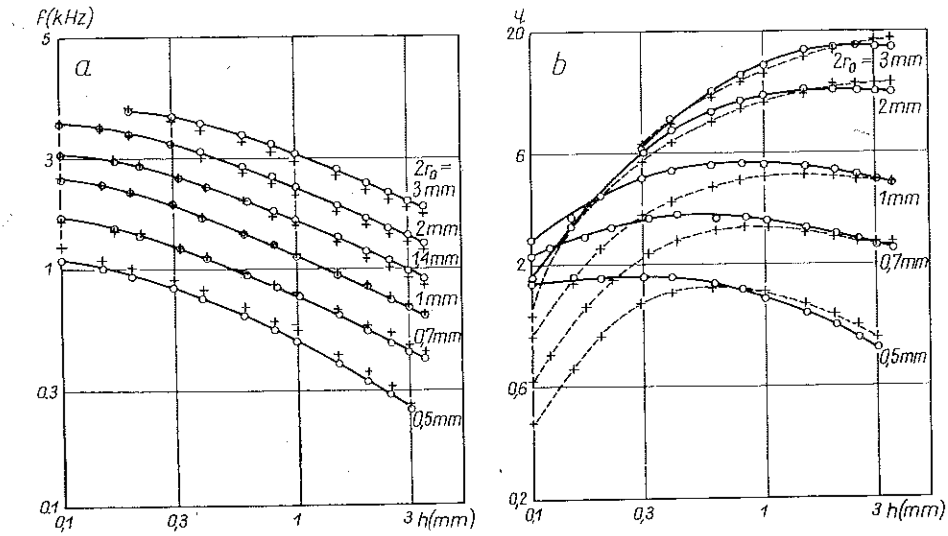

FIG. 1a. The calculated + and measured ○ resonance frequency f_r as function of the resonator height for an enclosure diameter of 15 mm and length of 8 mm with the neck diameter $2r_0$ as parameter

FIG. 1b. The calculated - - - and measured —— quality facotr q as function of the resonator height for an enclosure diameter of 15 mm and a neck length of 8 mm with the neck diameter $2r_0$ as parameter

considerably higher in the case of small neck diameters. Yet the values of q_C and q_M are in better agreement when the roughly estimated, certainly too small values of q_E are used than when q_E is neglected.

An improvement of the calculation of the both loss coefficients due to friction in the neck and in the enclosure will be dealt with in the future.

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Streszczenie

WŁASNOŚCI BARDZO MAŁYCH REZONATORÓW HELMHOLTZA PRZY POMIARACH CIŚNIENIA NA ŚCIANKACH ZBIORNIKÓW

Zbadano charakterystyki częstotliwości rezonatorów Helmholtza o bardzo małych wymiarach przeznaczonych do pomiaru fluktuacji ciśnienia na ściankach zbiorników. Badania mają charakter zarówno teoretyczny jak i doświadczalny.

Резюме

СВОЙСТВА ОЧЕНЬ МАЛЫХ РЕЗОНАТОРОВ ГЕЛЬМГОЛЬЦА ПРИ ИЗМЕРЕНИЯХ ДАВЛЕНИЯ НА СТЕНКАХ РЕЗЕРВУАРОВ

Исследованы частотные характеристики резонаторов Гельмгольца очень малых размеров, предназначенных для измерения флуктуации давления на стенках резервуаров. Исследования имеют так теоретический, как и экспериментальный характер.

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