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MAGNETOHYDRODYNAMIC OSCILLATORY FREE CONVECTION FROM A VERTICAL PLATE

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This paper investigates the problem of oscillatory free convection laminar boundary layer flows from a semi-infinite vertical flat plate in the case when the mean surface-temperature varies as a function of the distance from the leading edge and there is a transverse magnetic field imposed at the plate. The induced magnetic field and the viscous dissipation term have been neglected and the study of the oscillatory flow is restricted to a small amplitude only.

NOMENCLATURE

u, v velocity components,
x, y coordinates,
t time,
g acceleration due to gravity,
β coefficient of thermal expansion,
T temperature,
ν kinematic viscosity,
σ magnetic conductivity,
H magnetic field,
ρ density of the medium,
α thermal diffusivity,
T_m means steady temperature function,
ω frequency of oscillation,
$H(x)$ a function, Eqs. (2.4),
U velocity in outer flow,
ψ stream function,
l characteristic length,
ε amplitude of oscillation
A, B functions, Eqs. (2.7)
Pr Prandtl number,
m magnetic number,
ψ_j functions, Eqs. (3.1),
T_j functions, Eq. (3.2),
θ_j, ζ_j functions, Eqs. (3.8) and (3.15), (3.16)
i $\sqrt{-1}$,
E_n constants,

$+$ dimensionless variables,
$'$ derivatives with respect to y,
∞ ambient conditions,
j $0, 1, 2, \ldots,$
n $1, 2, 3, \ldots, 39.$

1. Introduction

In recent years considerable attention is being drawn to unsteady boundary layer theory with boundary layer response to imposed oscillations. The solution of this problem consists in solving a system of partial differential equations. Such type of problems have been studied, among others, by Nanda and Sharma [1], Kelleher and Yang [2], Muhuri and Maiti [3], Mital [4], Merkin [5], Zeytoumian [6] and Pop [8] who has studied a problem of oscillatory free convection flow past a semi-infinite vertical plate.

Unsteady magnetohydrodynamic free convection flow past a hot vertical plate has been studied by Chawala [9], Soundalgekar [10], Pop [11], Mishra and Mohapatra [12, 13] and many others. In this paper our aim is to study the problem of oscillatory free convection flow past a vertical plate in the presence of a magnetic field. As in most of the free convection problems, here we find that we cannot make the x-component of velocity u to be zero at a large distance from the plate. For this reason, we assume the presence of a steady outer flow. In addition, a special feature of the present paper is that it develops a formally exact solution of the time-dependent boundary layer equations. By disregarding higher power of ε amplitude of oscillation, solutions for the velocity and temperature fields in the boundary layer are obtained.

2. Formulation of the problem

We consider a semi-infinite vertical plate for which the axes x, y are taken along and transverse to the plate with the origin at the leading edge. The equations of unsteady boundary layer flow, in presence of a uniform magnetic field, are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T-T_\infty)+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma H^2}{\rho}u, \tag{2.2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}. \tag{2.3}$$

The fluid is assumed to be almost incompressible, so that changes in density are important only in producing buoyancy forces. Viscosity, specific heat and thermal conductivity are taken as constants and viscous dissipation is neglected. The induced magnetic field is also neglected.

We shall consider the case in which the plate temperature is allowed to oscillate harmonically in time with small amplitude about a certain mean distribution along the plate. The boundary conditions will, therefore, be written as:

$$u=v=0,\quad T-T_\infty=T_m(x)\cos\omega t+H(x),\quad \text{at}\quad y=0, \tag{2.4}$$

$$u=U(x),\quad T=T_\infty \quad \text{as}\quad y\to\infty, \tag{2.5}$$

where $T_m(x)$ is as yet an unspecified function of x, $H(x)$ and $U(x)$ are unknown functions to be determined. Functions H and U are interpreted as a secondary effect of free convection due to the non-linear terms which are contained in equations (2.1)–(2.3). From the continuity equation (2.1) we can define a stream function by

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}. \tag{2.6}$$

Introducing the non-dimensional quantities

$$x=lx^+,\quad y=y^+\left(\frac{\nu}{\omega}\right)^{1/2},\quad t=\frac{t^+}{\omega},\quad \varepsilon=\frac{g\beta T_\infty}{l\omega^2},\quad \psi=\frac{g\beta\nu^{1/2}T_\infty\psi^+}{\omega^{3/2}},$$

$$T=T^+T_\infty+T_\infty,\quad A(x^+)=\frac{l\omega^2H(x)}{g\beta T_\infty^2},\quad B(x^+)=\frac{l\omega^3U(x)}{(g\beta T_\infty)^2}, \tag{2.7}$$

$$\Pr=\frac{\nu}{\alpha}\quad\text{and}\quad m^2=\frac{\partial H^2}{\rho\omega},$$

equations (2.2) and (2.3), superscripts "+" being disregarded take the dimensionless form

$$\frac{\partial^2\psi}{\partial t\partial y}+\varepsilon\left(\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\right)=T+\frac{\partial^3\psi}{\partial y^3}-m^2\frac{\partial\psi}{\partial y}, \tag{2.8}$$

$$\frac{\partial T}{\partial t}+\varepsilon\left(\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}\right)=\frac{1}{\Pr}\frac{\partial^2T}{\partial y^2}, \tag{2.9}$$

where ε is a small quantity.

The corresponding boundary conditions become:

$$\begin{aligned}&\psi=\frac{\partial\psi}{\partial y}=0,\quad T=\frac{T_m(x)\cos t}{T_\infty}+\varepsilon A(x),\quad \varepsilon\ll 1,\quad \text{at}\quad y=0,\\ &\frac{\partial\psi}{\partial y}=\varepsilon B(x),\quad T=0,\quad \text{as}\quad y\to\infty.\end{aligned} \tag{2.10}$$

3. Method of solution

To develop a solution we follow the method of Schlichting. The non-linear convective terms in the boundary layer equations (2.8) and (2.9) can be assumed to be of smaller order than the linear terms when the frequency parameter is small.

Therefore equations (2.8) and (2.9) suggest the expansion of ψ and T in powers of ε,

$$\psi=\psi_0+\varepsilon\psi_1+\varepsilon^2\psi_2+\dots, \tag{3.1}$$

$$T=T_0+\varepsilon T_1+\varepsilon^2 T_2+\dots. \tag{3.2}$$

Substituting these expansions into equations (2.8) and (2.9) and equating the terms with equal powers of ε, we get the following partial differential equations:

$$\frac{\partial^2\psi_0}{\partial t\,\partial y}-\frac{\partial^3\psi_0}{\partial y^3}+m^2\frac{\partial\psi_0}{\partial y}=T_0, \tag{3.3}$$

$$\frac{\partial T_0}{\partial t}-\frac{1}{\mathrm{Pr}}\frac{\partial^2 T_0}{\partial y^2}=0, \tag{3.4}$$

$$\frac{\partial^2\psi_1}{\partial t\,\partial y}-\frac{\partial^3\psi_1}{\partial y^3}+m^2\frac{\partial\psi_1}{\partial y}=T_1+\frac{\partial\psi_0}{\partial x}\frac{\partial^2\psi_0}{\partial y^2}-\frac{\partial\psi_0}{\partial y}\frac{\partial^2\psi_0}{\partial x\,\partial y}, \tag{3.5}$$

$$\frac{\partial T_1}{\partial t}-\frac{1}{\mathrm{Pr}}\frac{\partial^2 T_1}{\partial y^2}=\frac{\partial\psi_0}{\partial x}\frac{\partial T_0}{\partial y}-\frac{\partial\psi_0}{\partial y}\frac{\partial T_0}{\partial x}, \tag{3.6}$$

subject to the boundary conditions

$$\begin{aligned}&\psi_0=\frac{\partial\psi_0}{\partial y}=\psi_1=\frac{\partial\psi_1}{\partial y}=0,\quad T_0=T_m\,\mathrm{const},\quad T_1=A(x),\quad \text{at}\quad y=0;\\ &\frac{\partial\psi_0}{\partial y}=0,\quad \frac{\partial\psi_1}{\partial y}=B(x),\quad T_0=T_1=0,\quad \text{as}\quad y\to\infty.\end{aligned} \tag{3.7}$$

In order to solve these equations it is convenient to use the complex notation for harmonic functions in which only real parts will have physical meaning.

In view of the boundary conditions (3.7) we assume the solution of (3.3) and (3.4) to be of the form

$$T_0=T_m\theta_0(y)e^{it},\qquad \psi_0=T_0\zeta_0(y)e^{it}. \tag{3.8}$$

Substituting (3.8) in (3.3) and (3.4) and using (3.7) we arrive at

$$\theta_0''-i\,\mathrm{Pr}\,\theta_0=0, \tag{3.9}$$

$$\zeta_0'''-(m^2+i)\zeta_0'=-\theta_0, \tag{3.10}$$

$$\theta_0(0)=1,\quad \zeta_0(0)=\zeta_0'(0)=0,\quad \theta_0(\infty)=\zeta_0'(\infty)=0, \tag{3.11}$$

where primes denote differentiation with respect to y.

The solutions of (3.9) and (3.10) for $\mathrm{Pr}\neq 1$ are

$$\theta_0=\exp[-\sqrt{\mathrm{Pr}/2}\,(1+i)y], \tag{3.12}$$

$$\zeta_0=\frac{3}{\{m^2+i(1-\mathrm{Pr})\}}\left[\frac{1-e^{-(1+i)\sqrt{\frac{\mathrm{Pr}}{2}}}}{(1+i)\sqrt{\frac{\mathrm{Pr}}{2}}}-\frac{1-e^{-\sqrt{(m^2+2)y}}}{(m^2+i)^{1/2}}\right]. \tag{3.13}$$

Now equations in (3.8) in real notations become:

$$T_0=\frac{1}{2}T_m(\theta_0 e^{it}+\bar{\theta}_0 e^{-it}),\qquad \psi_0=\frac{1}{2}T_m(\zeta_0 e^{it}+\bar{\zeta}_0 e^{-it}),\tag{3.14}$$

where the bar denotes the complex conjugate.

Passing to the second approximations, we see from (3.14) that T_0^2 and ψ_0^2 consist of a steady part and a time-dependent part. This suggests that the solutions of (3.5) and (3.6) may be assumed as

$$T_1=\frac{1}{2}T_m\frac{dT_m}{dx}[\theta_{11}(y)e^{2it}+\bar{\theta}_{11}(y)e^{-2it}+2\theta_{12}(y)],\tag{3.15}$$

$$\psi_1=\frac{1}{2}T_m\frac{dT_m}{dx}[\zeta_{11}(y)e^{2it}+\zeta_{11}(y)e^{-2it}+2\zeta_{12}(y)].\tag{3.16}$$

Substituting (3.15) and (3.16) in (3.5) and (3.6) and comparing the coefficients of $\exp(2it)$ and terms independent of it, we obtain

$$\theta_{11}''-2i\Pr\theta_{11}=\frac{\Pr}{2}(\zeta_0'\theta_0-\theta_0'\zeta_0),\tag{3.17}$$

$$\theta_{12}''=\frac{\Pr}{4}(\theta_0\bar{\zeta}_0'-\zeta_0\bar{\theta}_0'-\bar{\zeta}_0\theta_0+\bar{\theta}_0\zeta_0'),\tag{3.18}$$

$$\zeta_{11}'''-(m^2+2i)\zeta_{11}'=-\theta_{11}+\frac{1}{2}(\zeta_0'^2-\zeta_0\zeta_0''),\tag{3.19}$$

$$\zeta_{12}'''-m^2\zeta_{12}'=-\theta_{12}+\frac{1}{2}\zeta_0'\bar{\zeta}_0'-\frac{1}{4}(\bar{\zeta}_0\zeta_0''+\zeta_0\bar{\zeta}_0''),\tag{3.20}$$

with the boundary conditions:

$$\begin{gathered}\theta_{11}(0)=\zeta_{11}(0)=\zeta_{11}'(0)=\zeta_{12}(0)=\zeta_{12}'(0)=0,\qquad T_m\frac{dT_m}{dx}\theta_{12}(0)=A(x);\\ \theta_{11}(\infty)=\theta_{12}(\infty)=\zeta_{11}'(\infty)=0,\qquad T_m\frac{dT_m}{dx}\zeta_{12}'(\infty)=B(x).\end{gathered}\tag{3.21}$$

Now solving the equations (3.17)–(3.20) with the help of (3.21) we have

$$\theta_{11}=E_5\left(\frac{1}{E_1^2-i\Pr+2E_1E_3}-\frac{i}{\Pr}\right)e^{-\sqrt{2}E_3y}+E_5\left(\frac{i}{\Pr}-\frac{e^{-E_1y}}{E_1^2-i\Pr+2E_1E_3}\right)e^{-E_3y},$$

$$\theta_{12}=\frac{1}{4(E_2^2+i\Pr)}\left[\frac{1}{2}e^{-\sqrt{2\Pr}\,y}-\frac{\Pr}{(E_2+E_3)^2}e^{-(E_2+E_3)y}\right]+$$
$$+\frac{1}{4(E_2^2-i\Pr)}\left[e^{-E_4y}+\frac{i}{2}e^{-\sqrt{2\Pr}\,y}+\frac{E_4}{E_1}\left\{\frac{\Pr}{(E_1+E_4)^2}e^{-(E_1+E_4)y}-ie^{-E_4y}\right\}\right]+$$

$$+\frac{1}{4(E_2^2+i\,\mathrm{Pr})}\left[e^{-E_3 y}-\frac{i}{2}\,e^{-\sqrt{2}\mathrm{Pr}\,y}+\frac{E_3}{E_2}\left\{\frac{\mathrm{Pr}}{(E_2+E_3)^2}\,e^{-(E_2+E_3)y}-ie^{-E_3 y}\right\}\right]+$$

$$+\frac{1}{4(E_1^2-i\,\mathrm{Pr})}\left[\frac{1}{2}\,e^{-\sqrt{2}\mathrm{Pr}\,y}-\frac{\mathrm{Pr}}{(E_1^2+E_4)^2}\,e^{-(E_1+E_4)y}\right],$$

$$\zeta_{12}=E_{12}+E_{13}\,e^{-E_{11}y}+E_6\,e^{-\sqrt{2}E_3 y}-E_7\,e^{-E_3 y}+(E_{10}-E_8)\,e^{-(E_1+E_3)y}+E_9\,e^{-E_1 y}$$

and

$$\zeta_{12}=-E_{38}-\frac{E_{39}}{m}(1-e^{-my})+\frac{\mathrm{Pr}}{4}\,[E_{14}\,e^{-\sqrt{2}\mathrm{Pr}\,y}-E_{15}\,e^{-(E_{14}+E_3)y}+$$

$$+E_{16}\,e^{-E_4 y}+E_{17}\,e^{-(E_1+E_4)y}+E_{18}\,e^{-E_3 y}]-E_{19}\,e^{-\sqrt{2}\mathrm{Pr}\,y}+$$

$$+E_{20}\,e^{-(E_3+E_2)y}+E_{21}\,e^{-(E_1+E_4)y}-E_{22}\,e^{-(E_1+E_2)y}+E_{23}\,e^{-E_3 y}+E_{24}\,e^{-E_1 y}+E_{25}\,e^{-E_2 y},$$

where

$$E_1^2=(m^2+i),\quad E_2^2=(m^2-i),\quad E_3=(1+i)\sqrt{\frac{\mathrm{Pr}}{2}},\quad E_4=(1-i)\sqrt{\frac{\mathrm{Pr}}{2}},$$

$$E_5=\frac{\mathrm{Pr}(E_1-E_3)}{2E_1(E_1^2-iP)},\quad E_6=\frac{iE_5(E_1^2+2E_1E_3)}{\sqrt{2}E_3\,\mathrm{Pr}\,[E_1^2+(1-2\mathrm{Pr})i]\,[(E_1^2+2E_1E_3)^{-i\,\mathrm{Pr}}]},$$

$$E_7=\frac{2E_1E_5(E_1^2-i\,\mathrm{Pr})^2+i\,\mathrm{Pr}(E_1-E_3)}{2iE_1E_3\,\mathrm{Pr}(E_1^2-i\,\mathrm{Pr})^2\,[-E_1^2+i(\mathrm{Pr}-1)]},$$

$$E_8=\frac{E_5}{(E_1+E_3)(E_1^2+2E_1E_3-i\,\mathrm{Pr})\,[2E_1E_3-i(\mathrm{Pr}-1)]},$$

$$E_9=\frac{i(E_1-E_3)}{2E_1E_3(E_1^2-i\,\mathrm{Pr})^2}.$$

$$E_{10}=\frac{-(E_1-E_8)^2}{2E_1E_3(E_1^2-i\,\mathrm{Pr})^2(E_1+E_3)(E_3^2+2E_1E_3-i)},$$

$$E_{11}=(E_1^2+i)^{1/2},\quad E_{12}=E_{13}-E_6+E_7+E_8-E_9-E_{10},$$

$$E_{13}=\left[\frac{E_5}{(E_1^2+2E_1E_3-i\,\mathrm{Pr})}\left\{\frac{1}{2E_1E_3+i(\mathrm{Pr}-1)}+\frac{E_1^2+2E_1E_3}{i\,\mathrm{Pr}\,(E_1^2+i(1-2\mathrm{Pr}))}\right\}+\right.$$

$$+\frac{E_1-E_3}{2E_1\{-E_1^2+i(\mathrm{Pr}-1)\}(E_1^2-i\,\mathrm{Pr})^2}-\frac{iE_5}{\mathrm{Pr}\{-E_1^2+i(\mathrm{Pr}-1)\}}+$$

$$\left.+\frac{E_1-E_3}{2iE_3(E_1^2-i\,\mathrm{Pr})^2}+\frac{(E_1+E_3)^2}{2E_1E_3\{2E_1E_3+i(\mathrm{Pr}-1)\}(E_1^2-i\,i\mathrm{Pr})^2/(E_1^2+i)^{1/2}}\right],$$

$$E_{14}=\frac{1}{2\sqrt{2}\,\mathrm{Pr}^{3/2}(2\mathrm{Pr}-m^2)}\left(\frac{1-i}{E_2^2+i\,\mathrm{Pr}}+\frac{1+i}{E_1^2-i\mathrm{Pr}}\right),$$

$$E_{15}=\frac{E_2-E_3}{E_2(E_2^2+i\,\mathrm{Pr})(E_2+E_3)^3(i\,\mathrm{Pr}+2E_2E_3-i)},$$

$$E_{16}=\frac{(-E_1-iE_4)}{E_1E_4\Pr(E_1^2-i\Pr)(m^2-i\Pr)}-\frac{(E_4+iE_1)}{E_1E_4\Pr(i\Pr+m^2)[m^4+(\Pr-1)^2]},$$

$$E_{17}=\frac{E_4-E_1}{E_1(E_1^2-i\Pr)(E_1+E_4)^3(i-i\Pr+2E_1E_4)},$$

$$E_{18}=\frac{(-E_3+iE_2)}{iE_2E_3\Pr(E_2^2+i\Pr)(i\Pr-m^2)},$$

$$E_{19}=\frac{1}{2\sqrt{2}\Pr(2\Pr-m^2)[m^4+(\Pr-1)^2]},$$

$$E_{20}=\frac{(E_2-E_3)^2}{4E_2E_3[m^4+(\Pr-1)^2][(E_2+E_3)(i\Pr-i+2E_2E_3)]},$$

$$E_{21}=-\frac{(E_1-E_4)^2}{4E_1E_4[m^4+(P_4-1)^2][(E_1+E_4)(i-i\Pr+2E_1E_4)]},$$

$$E_{22}=-\frac{(E_1-E_2)^2}{4E_1E_2[m^4+(\Pr-1)^2][(E_1+E_2)(m^2+2E_1E_2)]},$$

$$E_{23}=\frac{(E_3+iE_2)}{4E_2E_3[m^4+(\Pr-1)^2](i\Pr-m^2)},$$

$$E_{24}=-\frac{\frac{E_1}{E_4}-\frac{E_1}{E_2}}{4E_4[m^4+(\Pr-1)^2](i\Pr+m^2)},$$

$$E_{25}==\frac{\frac{E_2}{E_3}-\frac{E_2}{E_1}}{4iE_2[m^4+(\Pr-1)^2]},$$

$$E_{26}=-\sqrt{2}\Pr E_{14},\quad E_{27}=E_{15}E_2+E_3),\quad E_{28}=E_{16}E_4,\quad E_{29}=E_{17}(E_1+E_4),$$

$$E_{30}=E_{18}E_3,\quad E_{31}=\sqrt{2}\Pr E_{19},\quad E_{32}=E_{20}(E_3+E_2),\quad E_{33}=E_{21}(E_1+E_4),$$

$$E_{34}=E_{22}(E_1+E_2),\quad E_{35}=E_{23}E_3,\quad E_{36}=E_{24}E_1,\quad E_{37}=E_{25}E_2,$$

$$E_{38}=\frac{\Pr}{4}(E_{14}-E_{15}+E_{16}+E_{17}+E_{18})-E_{19}+E_{20}+E_{21}-E_{22}+E_{23}+E_{24}+E_{25}$$

and

$$E_{39}=\frac{\Pr}{4}(E_{26}+E_{27}-E_{28}-E_{29}-E_{30})+E_{31}-E_{32}-E_{33}-E_{34}-E_{35}-E_{36}-E_{37}.$$

Substituting the values of $\theta_{11}(y)$, $\bar{\theta}_{11}(y)$ and $\theta_{12}(y)$ in (3.15) and $\zeta_{11}(y)$, $\bar{\zeta}_{11}(y)$ and $\zeta_{12}(y)$ in (3.16), we obtain the values of T_1 and ψ_1 respectively, where $\bar{\theta}_{11}(y)$ and $\bar{\zeta}_{11}(y)$ are the complex conjugates of $\theta_{11}(y)$ and $\zeta_{11}(y)$, respectively.

From (3.15) the steady part of temperature T_1 at the plate is given by

$$T_{1st}|_{y=0}=\frac{1}{25}T_m\frac{dT_m}{dx}, \tag{3.22}$$

and from (3.16) the steady part of velocity ψ_1' at latge distance from the plate is given by

$$\psi_{1st}'|_{y=\infty} = \frac{1}{500} T_m \frac{dT_m}{dx}. \tag{3.23}$$

4. Discussions

It is seen from the equations (3.15) and (3.16) that the boundary conditions for T_1 and ψ_1 are completely satisfied. This fact leads to the conclusion that a steady velocity field (3.23) is induced at latrgė distances from the plate.

Fig. 1 shows the effect of Prandtl number Pr on the temperature field. The real part of $\theta_0(y)$, what is obvious from (3.12), is independent of the effect of the magnetic field. It is seen from this figure that in a thin liquid layer in the boundary layer

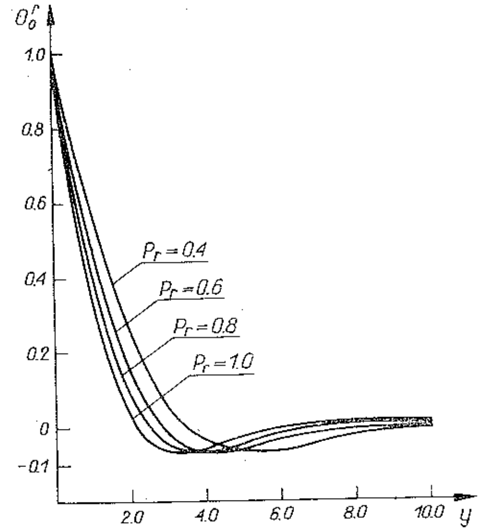

FIG. 1. Effect of Prandtl number on $\theta_0^r(y)$ with $m=4.0$.

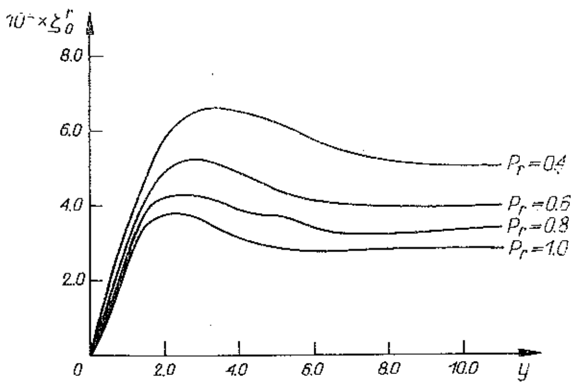

FIG. 2. Effect of Prandtl number on $\zeta_0^r(y)$ with $m=4.0$.

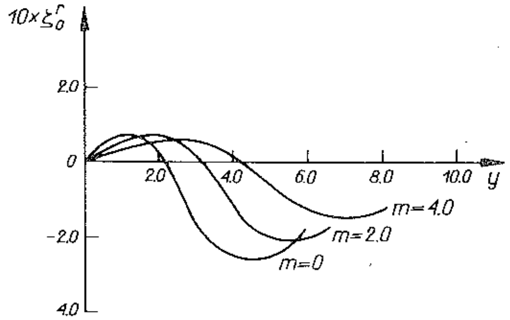

FIG. 3. Effect of magnetic number on $\zeta_0^r(y)$ with Pr=0.6.

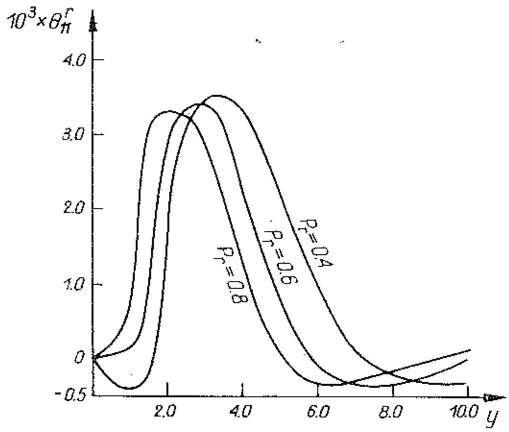

FIG. 4. Effect of Prandtl number on $\theta_{11}^r(y)$ with $m=4.0$.

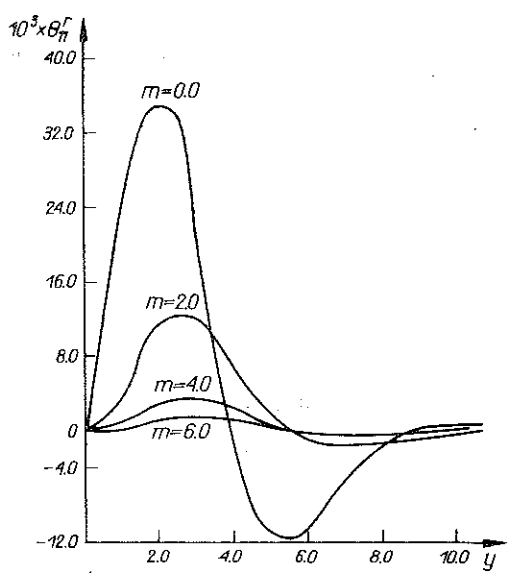

FIG. 5. Effect of magnetic number on $\theta_{11}^r(y)$ with Pr=0.6.

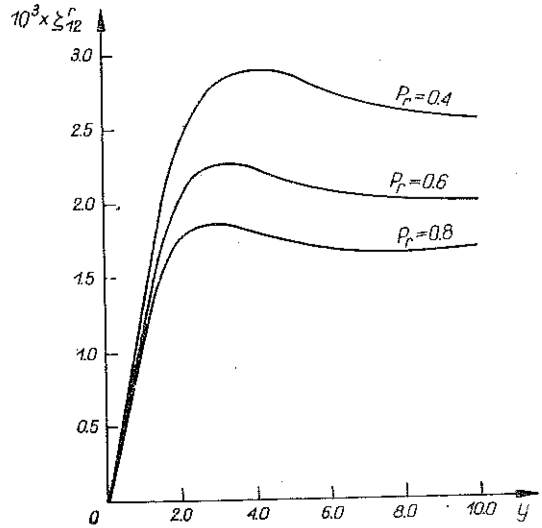

FIG. 6. Effect of Prandtl number on $\zeta^r_{11}(y)$ with $m=4.0$.

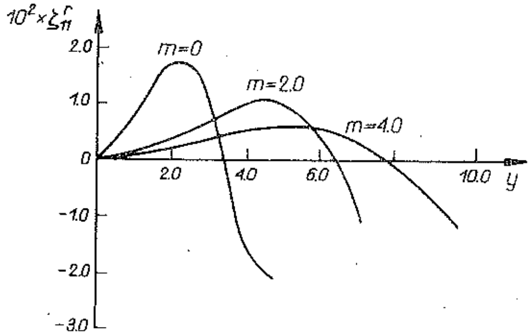

FIG. 7. Effect of magnetic number on $\zeta^r_{11}(y)$ with Pr=0.6.

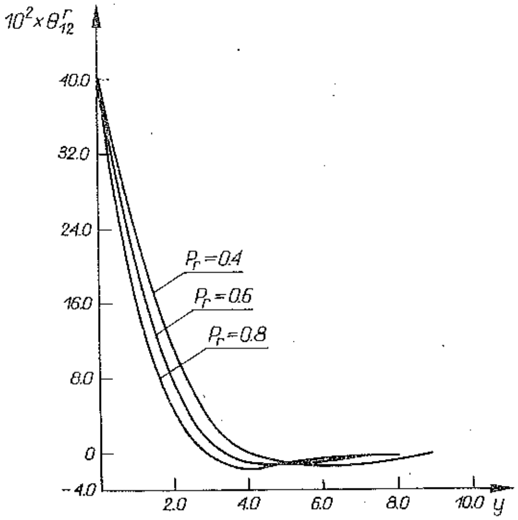

FIG. 8. Effect of Prandtl number on $\theta^r_{12}(y)$ with $m=4.0$.

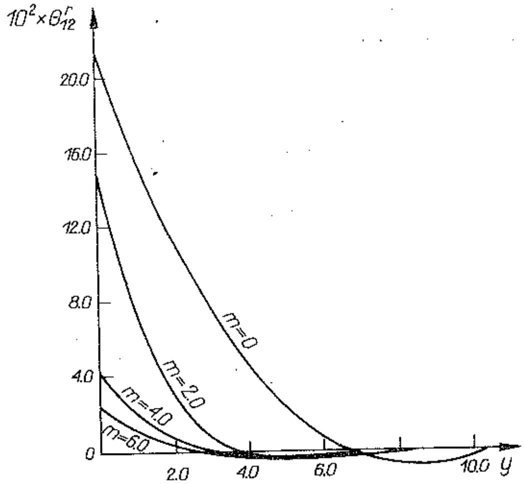

FIG. 9. Effect of magnetic number on $\theta^r_{12}(y)$ with Pr=0.6.

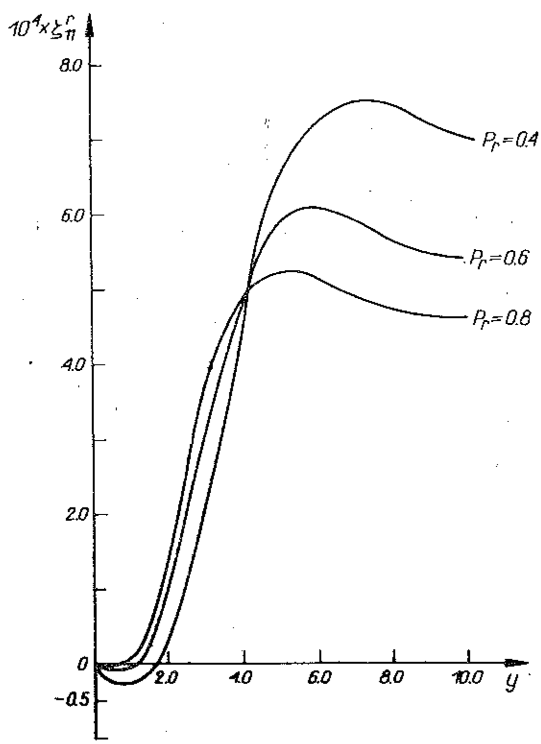

FIG. 10. Effect of Prandtl number on $\zeta^r_{12}(y)$ with m=4.0.

Table 1. Values of ζ'_{12} for different values of m and Pr=0.6

y \ m	0.0	2.0	4.0	6.0
0.0	0.00000	0.00000	0.00000	0.00000
1.0	0.34827	0.02269	0.00124	0.00029
2.0	0.79530	0.02681	0.00202	0.00044
3.0	1.15141	0.02721	0.11225	0.00049
4.0	1.33324	0.02629	0.00223	0.00048
5.0	1.37190	0.02529	0.00214	0.00046
6.0	1.33884	0.02462	0.00207	0.00046
7.0	1.29128	0.02429	0.00202	0.00043
8.0	1.25672	0.02419	0.00200	0.00043
9.0	1.24010	0.02421	0.00200	0.00043
10.0	1.23619	0.02426	0.00200	0.00043

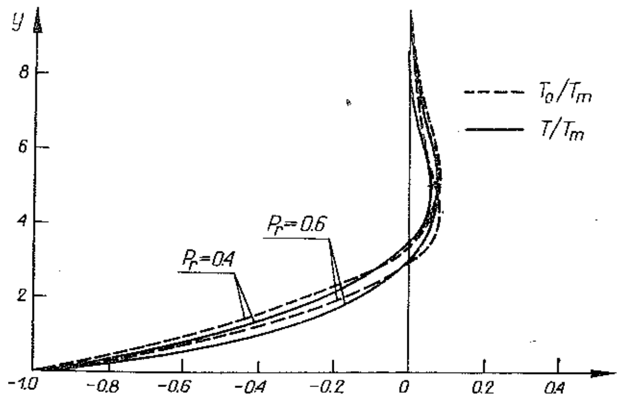

FIG. 11. Temperature profiles for different values of Pr with $\varepsilon=\frac{1}{2}$, $m=6.0$ and $t=\pi$.

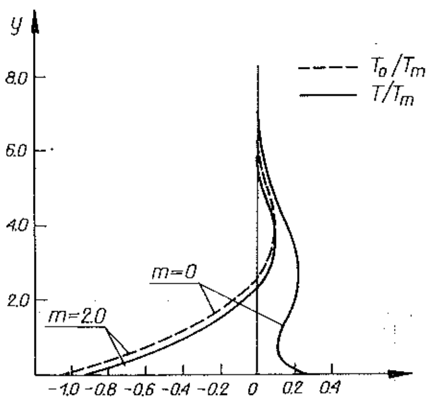

FIG. 12. Temperature profiles for different values of m with $\varepsilon=\frac{1}{2}$, Pr=0.8 and $t=\pi$.

$\theta_0^r(y)$ decreases as Pr increases. An opposite effect is observed outside this layer. Beyond a thin liquid layer near the plate, the value of $\theta_0(y)$ becomes negative and then assymptotically tends to zero as $y \to \infty$.

Fig. 2. illustrates the effect of Pr on the value of $\zeta_0^r(y)$. This figure shows that $\zeta_0^r(y)$ decreases in the boundary layer as the Prandtl number increases.

Fig. 3. shows that $\zeta_0^r(y)$ decreases as the magnetic number increases. This seems physically plausible. Beyond a thin liquid layer near the plate the value of $\zeta_0^r(y)$ becomes negative.

Fig. 4 shows that in a thin liquid layer near the plate θ_{11}^r increases as the Prandtl number increases. In this layer, for smaller valus of the Prandtl number, θ_{11}^r is negative. The maximum points of the curves for θ_{11}^r for different values of Pr move nearer to the wall as the Prandtl number Pr increases.

An examination of Fig. 5 shows that in a liquid layer near the plate the effect of m consists in decreasing θ_{11}^r, but beyond this layer an opposite effect is observed. At a large distance from the plate θ_{11}^r asymptotically tends to zero.

Fig. 6 shows that in a thin liquid layer near the plate the effect of Pr consists increasing ζ_{11}^r whereas, beyond this layer, an opposite effect is observed. For smaller values of Pr, ζ_{11}^r becomes negative near the plate.

Fig. 7 shows that $\zeta_{11}^r(y)$ decreases as the magnetic number increases.

Fig. 8 shows that $\theta_{11}^r(y)$ decreases in a thin liquid layer in the boundary layer as Prandtl number increases. Beyond this liquid layer, however, an opposite effect is observed. Furthermore, beyond a thin liquid layer, $\theta_{12}^r(y)$ is negative.

Fig. 9 shows that $\theta_{12}^r(y)$ gradually decreases as the magnetic numbre increases. For smaller values of the magnetic number, $\theta_{12}^r(y)$ becomes negative towards the edge of the boundary layer.

Fig. 10 shows that $\zeta_{12}^r(y)$ decreases as the Prandtl number increases.

From Table 1 it is seen that $\zeta_{12}^r(y)$ decreases as the magnetic number at the plate increases.

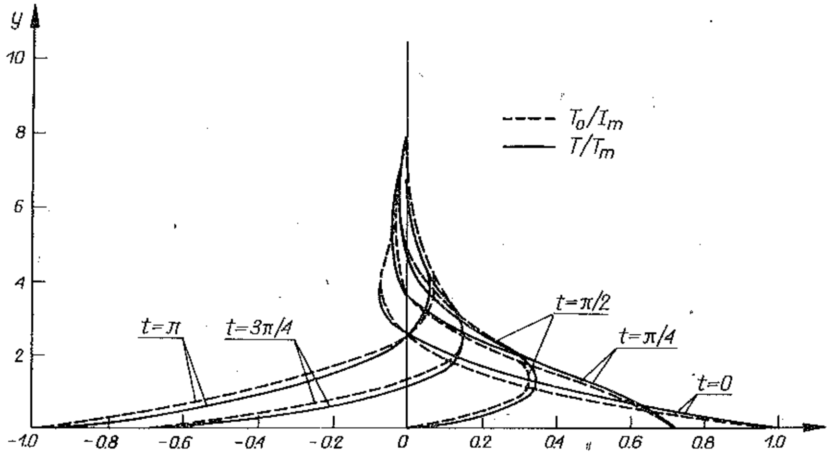

FIG. 13. Temperature profiles for different values of t with $\varepsilon=\frac{1}{2}$, Pr=0.8 and m=6.0.

Figs. 11, 12 and 13 illustrate the effect of non-linear terms on the fluid temperature distribution when $\varepsilon=1/2$ and $T_m(x)=x$. These figures show the effect of Prandtl number, magnetic number and time, respectively, on the temperature profile. It is seen that the non-linear terms significantly modify the temperature profile.

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Streszczenie

MAGNETOHYDRODYNAMICZNA OSCYLACYJNA KONWEKCJA SWOBODNA Z PIONOWEJ PŁYTY

Rozważono problem oscylacyjnego, swobodnego konwekcyjnego przepływu warstwy granicznej z półnieskończonej płyty płaskiej w położeniu pionowym przy założeniu, że średnia temperatura powierzchniowa jest funkcją odległości od krawędzi płyty, która jest ponadto poddana działaniu poprzecznego pola magnetycznego. Pominięto w rozważaniach wpływ indukowanego pola magnetycznego i członu odpowiedzialnego za lepką dysypację. Badania przepływu oscylacyjnego ograniczono do przypadku małych amplitud.

Резюме

МАГНИТОГИДРОДИНАМИЧЕСКАЯ ОСЦИЛЛЯЦИОННАЯ СВОБОДНАЯ КОНВЕКЦИЯ ИЗ ВЕРТИКАЛЬНОЙ ПЛИТЫ

Рассмотрена проблема осцилляционного, свободно конвекционного течения пограничного слоя из полубесконечной плоской плиты в вертикальном положении, при предположении, что средняя поверхностная температура является функцией расстояния от плиты, которая кроме этого подвергнута действию поперечного магнитного поля. В рассуждениях пренебрегается влиянием индуцированного магнитного поля и членом, ответственным за вязкую диссипацию. Исследования осцилляционного течения ограничены случаем малых амплитуд.

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