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MODELLING OF CONTINUUM DAMAGE FOR APPLICATION ELASTO-VISCOPLASTIC BODNER-PARTOM CONSTITUTIVE EQUATIONS

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The aim of the paper is to propose an application of the continuum damage model proposed by Lemaitre to the elasto-viscoplastic constitutive equations of the Bodner–Partom model. The proposed approach has been implemented into subroutines of the FE code MSC.Marc as the user's viscoplastic subroutine UVSCPL and has been used to perform the FE numerical simulations. Comparison is given of the following two variants: 1) uniaxial creep test results for a nickel-based B1900+Hf superalloy at high temperatures and 2) calculation based on the constitutive equations with the inclusion of isotropic damage models.

Key words: elasto-viscoplastic, Bodner–Partom, damage, FEM.

1. Introduction

The Bodner–Partom constitutive model belongs to the group of unified theories, proposed by Bodner and Partom [7] at the beginning of the 1970s. These constitutive equations have been frequently utilized in modelling of the elasto-viscoplastic hardening of a number of materials, with a great many practical engineering applications [2]. Its application in the elasto-viscoplastic static and dynamic analysis of plates and shells is shown in many examples: Kłosowski *et al.* [21] and [23], Sansour and Kollmann [30], Woźnica [39], Kłosowski [20], Stoffel [34] and [35], Sansour and Wagner [31] and [32], Steck [33], Kłosowski and Woźnica [22]. Description of the behaviour of glassy polymers (see e.g.: Frank and Brockman [16], Zaïri *et al.* [42]) and technical coated fabrics (see e.g.: Kłosowski *et al.* [24]) is also shown with reference to the presented model. On the other hand, Chełmiński and Gwiazda [14] studied application to the model of Bodner–Partom monotonicity of operators of the viscoplastic response.

In the present paper the author makes a detailed investigation of modelling of continuum damage for application in the elasto-viscoplastic Bodner–Partom model. In the second part of the paper, which is preceded by an introduction, the detailed description of the model is given. In the third part the author proposes an application of the continuum damage model proposed by Lemaitre [25] to the elasto-viscoplastic constitutive equations of the Bodner–Partom model. Next section concerns the description of the finite element procedure which was used for the open commercial FE program implementation. The last part gives numerical simulation examples of the creep tests for B1900+Hf alloy.

2. Bodner–Partom equations

At the beginning it is necessary to assume the isotropic material and strain additivity, where the total strain rate $\dot{\boldsymbol{\varepsilon}}$ is decomposed into the elastic part $\dot{\boldsymbol{\varepsilon}}^E$ and the inelastic part $\dot{\boldsymbol{\varepsilon}}^I$ according to the formula

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^E + \dot{\boldsymbol{\varepsilon}}^I. \tag{2.1}$$

Therefore, the relation between the stress rate $\dot{\boldsymbol{\sigma}}$ and strain rate $\dot{\boldsymbol{\varepsilon}}^E$, for the assumed isotropic material, is defined as:

$$\dot{\boldsymbol{\sigma}} = \mathbf{B}^* : \dot{\boldsymbol{\varepsilon}}^E = (1 - D) \cdot \mathbf{B} : \left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^I\right), \tag{2.2}$$

where $D \in \langle 0, 1\rangle$ is the scalar parameter of the isotropic damage and $\mathbf{B}^*$ is the effective tensor of elasticity for the damaged material, which is expressed by the standard elasticity tensor $\mathbf{B}$, reduced by the damage parameter.

Since Kachanov [19] and Rabotnov [29] have proposed the concept of effective stress, numerous damage models have been developed (see e.g.: Kachanov [18] or Lemaitre [26]). Most of the investigations on continuum damage use a power law for the damage equation evolution. Bodner and Chan [6] proposed an alternative functional form of the evaluation equation which leads to an exponential equation for damage development:

$$\dot{D} = \frac{h}{H} \cdot \left[\ln\left(\frac{1}{D}\right)^{(h+1)/h}\right] \cdot D \cdot \dot{Q}, \tag{2.3}$$

where h and H are the damage material parameters and Q is the multiaxial stress function obtained from the equation included in [17]:

$$\dot{Q} = \left(\alpha_1 \cdot \sigma^+_{\max} + \alpha_2 \cdot J(\boldsymbol{\sigma}) + \alpha_3 \cdot \operatorname{tr}(\boldsymbol{\sigma})^+\right)^z, \tag{2.4}$$

where $\sigma^+_{\max}$ and $\operatorname{tr}(\boldsymbol{\sigma})^+$ are the maximum principal tensile stress and the first stress invariant, respectively. Next α_1, α_2, α_3, z are the material constants. It should be noted that the parameters α_1, α_2 and α_3 satisfy the condition $\alpha_1 + \alpha_2 + \alpha_3 = 1.0$.

It is necessary to point out that BODNER and CHAN [6], besides of the isotropic damage model, described the procedure considering the directional damage by the load-history dependent softening variables. In this procedure the directional damage is expressed by the second-order symmetric tensor with a scalar effective value.

The inelastic strain rate $\dot{\boldsymbol{\varepsilon}}^I$ in the Bodner–Partom model is calculated according to the equation

$$\dot{\boldsymbol{\varepsilon}}^I = \frac{3}{2} \cdot \dot{p} \cdot \frac{\boldsymbol{\sigma}'}{J(\boldsymbol{\sigma}')}, \tag{2.5}$$

where $\dot{p}$ is the equivalent plastic strain, $\boldsymbol{\sigma}'$ and $J(\boldsymbol{\sigma}') = \sqrt{\frac{3}{2}\left(\boldsymbol{\sigma}' : \boldsymbol{\sigma}'\right)}$ are the deviatoric parts of stress and the stress invariant. It should be noted that in the literature it is possible to find two ways of calculating $\dot{p}$ – the rate of the equivalent plastic strain, which includes the isotropic damage evolution (see e.g. [6] and [7] for details)

$$\dot{p} = \frac{2}{\sqrt{3}} \cdot D_0 \cdot \exp\left[-\frac{1}{2} \cdot \left(\frac{\left(R + \left(\mathbf{X} : \frac{\boldsymbol{\sigma}}{J(\boldsymbol{\sigma})}\right)\right) \cdot (1-D)}{J(\boldsymbol{\sigma}')}\right)^{2\cdot n} \cdot \frac{n+1}{n}\right], \tag{2.6}$$

or

$$\dot{p} = \frac{2}{\sqrt{3}} \cdot D_0 \cdot \exp\left[-\frac{1}{2} \cdot \left(\frac{\left(R + \left(\mathbf{X} : \frac{\boldsymbol{\sigma}}{J(\boldsymbol{\sigma})}\right)\right) \cdot (1-D)}{J(\boldsymbol{\sigma}')}\right)^{2\cdot n}\right], \tag{2.7}$$

where the material parameters D_0 and n represent the limiting plastic strain rate and the strain rate sensitivity parameter, respectively. The isotropic hardening R is given as

$$\dot{R} = m_1 \cdot (Z_1 - R) \cdot \left(\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^I\right) - A_1 \cdot Z_1 \cdot \left(\frac{R - Z_2}{Z_1}\right)^{r_1}, \tag{2.8}$$

where m_1, A_1, r_1, Z_1 and Z_2 are the material parameters. The material constants m_1, A_1, r_1 are the hardening rate coefficient, recovery coefficient, and recovery exponent for isotropic hardening, respectively. The values Z_1 and Z_2 are the limiting value of isotropic hardening and the fully recovered value of isotropic hardening, respectively.

Subsequently, the kinematic hardening $\mathbf{X}$ is defined as

$$\dot{\mathbf{X}} = m_2 \cdot \left(\frac{3}{2} \cdot Z_3 \cdot \frac{\boldsymbol{\sigma}}{J(\boldsymbol{\sigma})} - \mathbf{X}\right) \cdot \left(\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^I\right) - A_2 \cdot Z_1 \cdot \frac{3}{2} \cdot \left[\frac{\frac{2}{3} J(\mathbf{X})}{Z_1}\right]^{r_2} \cdot \frac{\mathbf{X}}{J(\mathbf{X})}, \tag{2.9}$$

where m_2, A_2, r_2, Z_3 are material parameters. The material constants m_2, A_2, r_2 are the hardening rate coefficient, recovery coefficient, recovery exponent for kinematic hardening, respectively, and Z_3 is the limiting value of the kinematic hardening. Additionally, at the beginning of calculations the initial value of the isotropic hardening is assumed as $R\,(t=0)=Z_0$. It should be noted that the relation $\left(\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^I\right)$, used in Eq. (2.8) and Eq. (2.9), is called the plastic work rate.

In this described model the 20 parameters have to be determined; 14 parameters of the base model: E, ν, D_0, n, Z_0, Z_1, Z_2, Z_3, m_1, m_2, A_1, A_2, r_1, r_2, and 6 parameters of damage evolution: h, H, α_1, α_2, α_3, z. In the work [8] Bodner proposed the concept of reduction of the number of parameters, where $Z_0 = Z_2 = Z$, $A_1 = A_2 = A$ and $r_1 = r_2 = r$. According to this assumption 11 parameters of base model have to be determined: E, ν, D_0, n, $Z_0 = Z_2 = Z$, Z_1, Z_3, m_1, m_2, $A_1 = A_2 = A$, $r_1 = r_2 = r$. Detailed description of the identification procedure for the material parameters is described by Chan *et al.* [11] and Woźnica and Kłosowski [40].

In the work [6], Bodner and Chan investigated a nickel based super alloy B1900+Hf. For this material the uniaxial creep test results at various temperatures are given. The material constants for Bodner–Partom constitutive equations have been established, see Table 1. It should be noted that the damage evolution, under constant stress conditions (creep tests), was described by the equation:

$$D = \exp\left[-\frac{H}{Q}\right]^h, \tag{2.10}$$

where the stress function Q has the simplified form:

$$Q = t \cdot \sigma^z. \tag{2.11}$$

Table 1. Parameters for Bodner–Partom model for B1900+Hf [6].

	base parameters of the model								
temp. T [°C]	E [MPa]	ν [–]	D_0 $[\mathrm{s}^{-1}]$	n [–]	Z_0 [MPa]	m_1 $[\mathrm{MPa}^{-1}]$	Z_1 [MPa]	A_1 $[\mathrm{s}^{-1}]$	temp. T [°C]
871	141525	0.3	10^4	1.03	2400	0.270	3000	0.0055	871
982	125391	0.3	10^4	0.85	1900	0.270	3000	0.02	982
1093	107539	0.3	10^4	0.70	1200	0.270	3000	0.25	1093
	base parameters of the model						damage parameters		
temp. T [°C]	Z_2 [MPa]	r_1 [–]	m_2 $[\mathrm{MPa}^{-1}]$	Z_3 [MPa]	A_2 $[\mathrm{s}^{-1}]$	r_2 [–]	H $[(\mathrm{MPa})^z \cdot \mathrm{s}]$	h [–]	z [–]
871	2400	2.0	1.52	1150	0.0055	2.0	$2 \cdot 10^{27}$	1.0	8.3
982	1900	2.0	1.52	1150	0.02	2.0	$4 \cdot 10^{24}$	1.0	8.3
1093	1200	2.0	1.52	1150	0.25	2.0	$5 \cdot 10^{20}$	1.0	8.3

3. Proposed approach of damage evolution in Bodner–Partom model

The author of the present paper proposed the application of the damage concept described by Lemaitre [25] in the Bodner–Partom model. The damage evolution is specified by the following expression:

$$\dot{D} = \left(\frac{Y}{S}\right)^{s} \cdot \dot{p}, \tag{3.1}$$

where the variables s, S are the damage material parameters and $\dot{p}$ is the rate of the equivalent plastic strain assumed according to Eqs. (2.6) or (2.7). The function Y, used in Eq. (3.1), is specified by the Young's modulus E, Poisson's ratio ν, the current values of damage D, the Huber–Mises equivalent stress σ_{eq} and the hydrostatic stress σ_H, and is called the damage strain energy release rate. It is expressed by the equation

$$Y = \frac{\sigma_{eq}^2}{2 \cdot (1-D)^2 \cdot E} \cdot \left(\frac{2}{3} \cdot (1+\nu) + 3 \cdot (1 - 2 \cdot \nu) \cdot \left(\frac{\sigma_H}{\sigma_{eq}}\right)^2\right). \tag{3.2}$$

The function of the energy density Y, in above equation, in the case of uniaxial stress state can be rewritten as

$$Y = \frac{\sigma^2}{2 \cdot (1-D)^2 \cdot E}. \tag{3.3}$$

The proposal of Lemaitre damage evolution is not the only one which can be found in the literature (see e.g.: Tai [36], Wang [38], Chandrakanth [12] and [13], Dhar [15], Bonora [9], Xiao [41], Życzkowski [43]), where the authors propose alternative versions of the Eq. (3.1).

The damage model proposed by Lemaitre [25] is successfully applied to the Chaboche model [10], see e.g.: Amar and Dufailly [1]. It should be noted that the Chaboche model is an extension of the Perzyna law [28], based on the orthogonal condition in the plastic law, which requires an established yield criterion. On the other hand in the Bodner–Partom model the existence is assumed of the inelastic deformation from the beginning of the deformation process, without references to the yield limit.

Identification of the values of damage material parameters s and S is based on the concept proposed by Amar and Dufailly [1]. The present author also investigates the identification and validation of the damage parameters for the Lemaitre model, described in the paper [3]. Referring to paper [1], the value of the parameter s is chosen arbitrarily; only the factor S has to be determined. It should be noted that only the parameter S is accepted as temperature-dependent.

At the beginning of the identification process, the numerical simulation of the uniaxial tensile tests were performed on the basis on the known material parameters (see Table 1) for the Bodner–Partom model with damage. From the numerical simulation the rupture time t_r was defined. Then we can make the following transformation of the Eq. (3.1):

$$\dot{D} \cdot (1 - D)^{2\cdot s} = \left(\frac{\sigma^2}{2 \cdot E \cdot S}\right)^s \cdot \dot{p},$$

$$\int_0^1 (1 - D)^{2\cdot s}\, dD = \frac{1}{2 \cdot s + 1} = \int_0^{t_r} \left(\frac{\sigma^2}{2 \cdot E \cdot S}\right)^s \cdot \dot{p}\, dt \tag{3.4}$$

which leads to the specified value of the parameter S:

$$S = \frac{(2 \cdot s + 1)^{1/s}}{2 \cdot E} \cdot \left(\int_0^{t_r} \left(\sigma^{2\cdot s} \cdot \dot{p}\right)\, dt\right)^{1/s}. \tag{3.5}$$

In this variant of identification, the value of the parameter $s = 3.0\,[-]$ is predetermined. According to the assumed procedure, the following damage parameters for the investigated superalloy B1900+Hf are specified and given in Table 2. The basic parameters for Bodner–Partom model are known and have been collected in Table 1. The detailed identification procedure of the damage parameters s and S, for the Lemaitre model of damage evolution, is described by the author in the paper [3].

Table 2. Damage parameters of the Bodner–Partom model for B1900+Hf.

temp. T [°C]	S [MPa]	s [–]
871	0.900	3.0
982	0.275	3.0
1093	0.023	3.0

In practical applications it is necessary to specify the value of the critical damage D_c, which indicates the limit of the theory. It should be noted that this factor must be lower than 1.0. It usually lies between 0.2 and 0.8, depending on the type of material [25].

It is worth pointing out that in the proposed approach, the damage parameter has the additive character. It is dependent on the rate of the equivalent plastic strain, see Eq. (3.1), when in the concept proposed by Bodner and Chan the damage is directly calculated as a function of time and stresses, see Eq. (2.10). However, both concepts belong to the group of the isotropic continuum damage models.

4. Description of applied program and procedure

In the numerical analysis the MSC.Marc system was used. To apply the Bodner-Partom model to the MSC.Marc system the user-defined subroutines UVSCPL [37] are applied. The fundamental part of the algorithm used in the implementation of UVSCPL subroutines is presented in Figs. 1 and 2 in the form of flow charts. Early this subroutine was used successfully by the author for implementation of the Chaboche model with damage [4] and for the introduced Bodner–Partom model without damage (see e.g.: [2] and [5]).

$$[\text{step 1}] \rightarrow \begin{bmatrix} \Delta\mathbf{X} = \dfrac{\Delta t}{2}\cdot\left(\dot{\mathbf{X}}_{j-1}+\dot{\mathbf{X}}_j\right), & \mathbf{X}_j = \mathbf{X}_{j-1}+\Delta\mathbf{X} \\ \Delta R = \dfrac{\Delta t}{2}\cdot\left(\dot{R}_{j-1}+\dot{R}_j\right), & R_j = R_{j-1}+\Delta R \end{bmatrix}$$

$$[\text{step 2}] \rightarrow \left[\boldsymbol{\sigma}'_j j,\ J\left(\boldsymbol{\sigma}'_j j\right),\ J\left(\boldsymbol{\sigma}'_j\right),\ J\left(\mathbf{X}_j\right)\right]$$

$$[\text{step 3}] \rightarrow D_j = \exp\cdot\left[-\frac{H}{(t\cdot\sigma^z)}\right]^h$$

$$[\text{step 4}] \rightarrow \left[\dot{p}_j = \frac{2}{\sqrt{3}}\cdot D_0\cdot\exp\left[-\frac{1}{2}\cdot\left(\frac{\left(R+\left(\mathbf{X}_j : \dfrac{\boldsymbol{\sigma}_j}{J\left(\boldsymbol{\sigma}_j\right)}\right)\right)\cdot(1-D_j)}{J\left(\boldsymbol{\sigma}'_j\right)}\right)^{2\cdot n}\cdot\frac{n+1}{n}\right]\right]$$

$$[\text{step 5}] \rightarrow \left[\dot{\boldsymbol{\varepsilon}}^I_j = \frac{3}{2}\cdot\dot{p}_j\cdot\frac{\boldsymbol{\sigma}'_j}{J\left(\boldsymbol{\sigma}'_j\right)}\right]$$

$$[\text{step 6}] \rightarrow \left[\dot{W}^I_j = \boldsymbol{\sigma}_j : \dot{\boldsymbol{\varepsilon}}^I_j\right]$$

$$[\text{step 7}] \rightarrow \left[\dot{\mathbf{X}}_j = m_2\cdot\left(\frac{3}{2}\cdot Z_3\cdot\frac{\boldsymbol{\sigma}_j}{J\left(\boldsymbol{\sigma}_j\right)}-\mathbf{X}_j\right)\cdot\dot{W}^I_j - A_2\cdot Z_1\cdot\frac{3}{2}\cdot\left(\frac{\frac{2}{3}\cdot J\left(\mathbf{X}_j\right)}{Z_1}\right)^{r_2}\cdot\frac{\mathbf{X}_j}{J\left(\mathbf{X}_j\right)}\right]$$

$$[\text{step 8}] \rightarrow \left[\dot{R}_j = m_1\cdot(Z_1-R_j)\cdot\dot{W}^I_j - A_1\cdot Z_1\cdot\left(\frac{R_j-Z_2}{Z_1}\right)^{r_1}\right]$$

$$[\text{step 9}] \rightarrow \left[\Delta\boldsymbol{\varepsilon}^I_j = \dot{\boldsymbol{\varepsilon}}^I_j\cdot\Delta t_j\right]$$

$$[\text{step 10}] \rightarrow \left[\Delta\boldsymbol{\sigma}_j = (1-D_j)\cdot\mathbf{B}\cdot\left(\Delta\boldsymbol{\varepsilon}_j-\Delta\boldsymbol{\varepsilon}^I_j\right)\right]$$

Fig. 1. Flow chart of the UVSCPL subroutine with the damage model proposed by Bodner and Chan.

$$\text{[step 1]} \rightarrow \begin{bmatrix} \Delta\mathbf{X} = \dfrac{\Delta t}{2} \cdot \left(\dot{\mathbf{X}}_{j-1} + \dot{\mathbf{X}}_j\right), & \mathbf{X}_j = \mathbf{X}_{j-1} + \Delta\mathbf{X} \\ \Delta R = \dfrac{\Delta t}{2} \cdot \left(\dot{R}_{j-1} + \dot{R}_j\right), & R_j = R_{j-1} + \Delta R \\ \Delta D = \dfrac{\Delta t}{2} \cdot \left(\dot{D}_{j-1} + \dot{D}_j\right), & D_j = D_{j-1} + \Delta D \end{bmatrix}$$

$$\text{[step 2]} \rightarrow \left[\boldsymbol{\sigma}'_j,\ J\left(\boldsymbol{\sigma}'_j\right),\ J\left(\boldsymbol{\sigma}_j\right),\ J\left(\mathbf{X}_j\right)\right]$$

$$\text{[step 3]} \rightarrow \left[\dot{p}_j = \frac{2}{\sqrt{3}} \cdot D_0 \cdot \exp\left[-\frac{1}{2} \cdot \left(\frac{\left(R_j + \left(\mathbf{X}_j : \dfrac{\boldsymbol{\sigma}_j}{J\left(\boldsymbol{\sigma}_j\right)}\right)\right) \cdot (1 - D_j)}{J\left(\boldsymbol{\sigma}'_j\right)}\right)^{2\cdot n} \cdot \frac{n+1}{n}\right]\right]$$

$$\text{[step 4]} \rightarrow \left[\dot{\boldsymbol{\varepsilon}}_j^I = \frac{3}{2} \cdot \dot{p}_j \cdot \frac{\boldsymbol{\sigma}'_j}{J\left(\boldsymbol{\sigma}'_j\right)}\right]$$

$$\text{[step 5]} \rightarrow \left[\dot{W}_j^I = \boldsymbol{\sigma}_j : \dot{\boldsymbol{\varepsilon}}_j^I\right]$$

$$\text{[step 6]} \rightarrow \left[\dot{\mathbf{X}}_j = m_2 \cdot \left(\frac{3}{2} \cdot Z_3 \cdot \frac{\boldsymbol{\sigma}_j}{J\left(\boldsymbol{\sigma}_j\right)} - \mathbf{X}_j\right) \cdot \dot{W}_j^I - A_2 \cdot Z_1 \cdot \frac{3}{2} \cdot \left(\frac{\frac{2}{3} \cdot J\left(\mathbf{X}_j\right)}{Z_1}\right)^{r_2} \cdot \frac{\mathbf{X}_j}{J\left(\mathbf{X}_j\right)}\right]$$

$$\text{[step 7]} \rightarrow \left[\dot{R}_j = m_1 \cdot (Z_1 - R_j) \cdot \dot{W}_j^I - A_1 \cdot Z_1 \cdot \left(\frac{R_j - Z_2}{Z_1}\right)^{r_1}\right]$$

$$\text{[step 8]} \rightarrow Y_j = \frac{\sigma_{eq}^2}{2 \cdot (1 - D_j)^2 \cdot E} \cdot \left(\frac{2}{3} \cdot (1 + \nu) + 3 \cdot (1 - 2 \cdot \nu) \cdot \left(\frac{\sigma_H}{\sigma_{eq}}\right)^2\right)$$

$$\text{[step 9]} \rightarrow \dot{D}_j = \left(\frac{Y}{S}\right)^s \cdot \dot{p}_j$$

$$\text{[step 10]} \rightarrow \left[\Delta\boldsymbol{\varepsilon}_j^I = \dot{\boldsymbol{\varepsilon}}_j^I \cdot \Delta t_j\right]$$

$$\text{[step 11]} \rightarrow \left[\Delta\boldsymbol{\sigma}_j = (1 - D_j) \cdot \mathbf{B} \cdot \left(\Delta\boldsymbol{\varepsilon}_j - \Delta\boldsymbol{\varepsilon}_j^I\right)\right]$$

FIG. 2. Flow chart of the UVSCPL subroutine with the proposed damage evolution in the Bodner–Partom model.

5. Numerical simulation of creep tests for B1900+Hf alloy

In the paper two concepts of the damage evolution in the Bodner–Partom model are described. Numerical simulations of strain-time relations based on the constitutive equations with the damage models are performed for the nickel-based superalloy B1900+Hf. The results of numerical simulations obtained from the procedure described by Bodner and Chan (see Fig. 1, B–P v1) are compared with the proposed procedure (see Fig. 2, B–P v2).

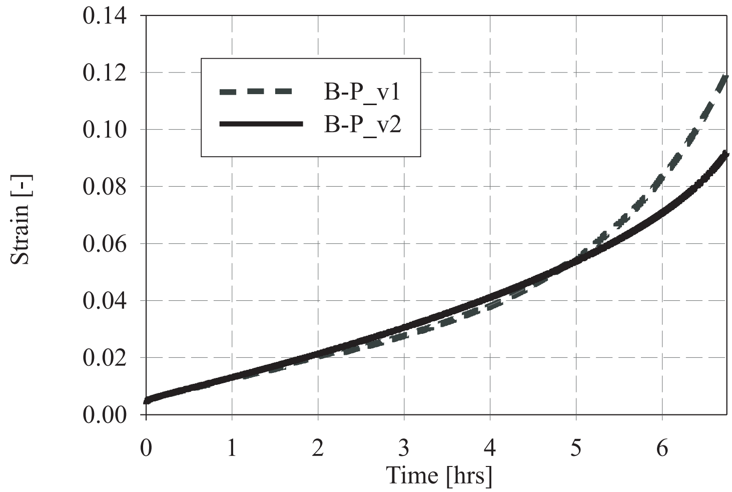

Fig. 3. Creep curves for B1900+Hf at 871° C, $\sigma = 517$ [MPa] = const.

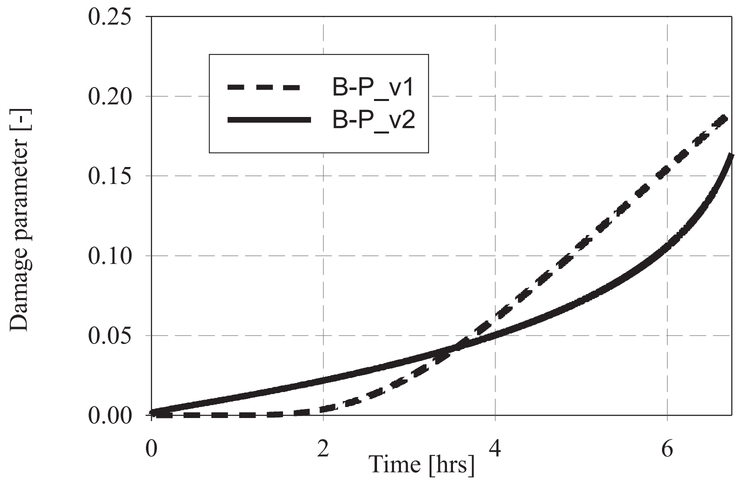

Fig. 4. Damage parameter evolution for B1900+Hf at 871° C, $\sigma = 517$ [MPa] = const.

Like in the paper [6], the numerical simulations of the uniaxial creep tests at various temperatures (see Fig. 3, Fig. 5, Fig. 7) were carried out. Additionally the damage evolution parameters for these variants of simulation are given in Fig. 4, Fig. 6 and Fig. 8. The strain-time functions obtained from the two investigated damage models gave similar results (very small differences can be observed only) in the considered range of time. The functions of the damage evolution (see Fig. 4, Fig. 8 and Fig. 6) also gave small differences.

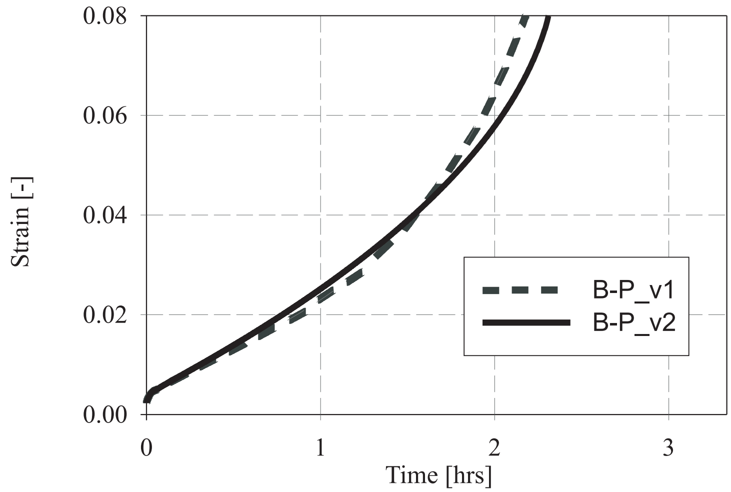

Fig. 5. Creep curves for B1900+Hf at 982° C, $\sigma = 283$ [MPa] = const.

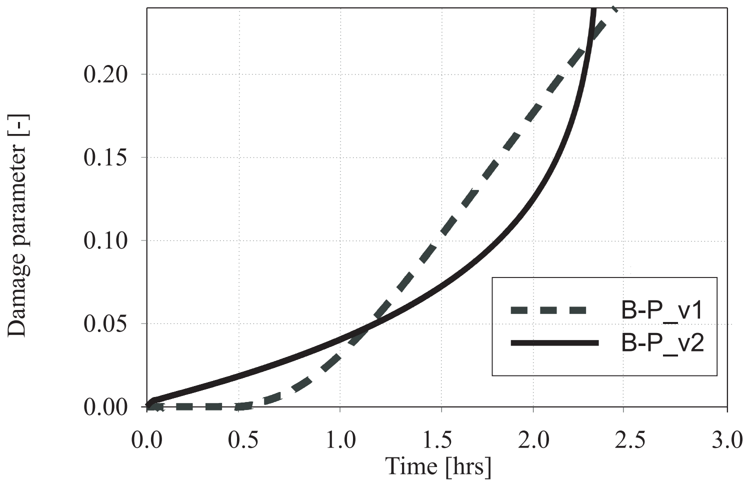

Fig. 6. Damage parameter evolution for B1900+Hf at 982° C, $\sigma = 283$ [MPa] = const.

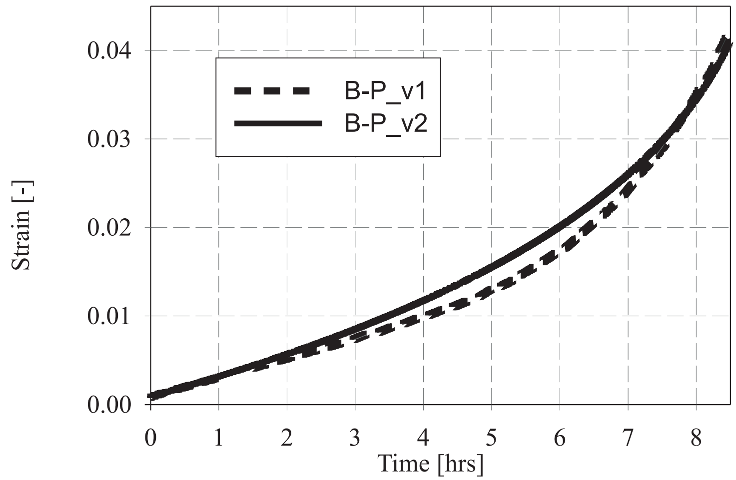

Fig. 7. Creep curves for B1900+Hf at 1093° C, $\sigma = 79.5$ [MPa] = const.

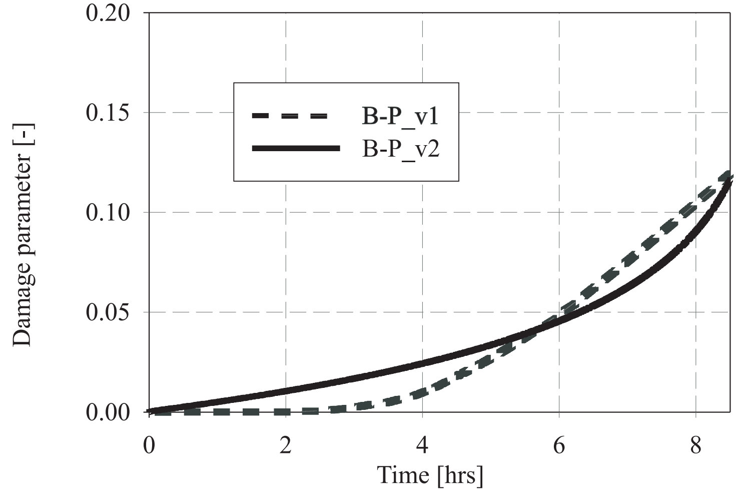

Fig. 8. Damage parameter evolution for B1900+Hf at 1093° C, $\sigma = 79.3$ [MPa] = const.

6. Concluding remarks

A new approach is proposed to the problem of continuum damage modelling for application in elasto-viscoplastic Bodner–Partom constitutive equations. The proposed method combines the damage model developed by Lemaitre and the Bodner–Partom model. The results obtained in numerical simulations of creep tests for B1900+Hf confirms the validity of the approach. Moreover, the obtained results encourage the author to continue the outlined research based on broader experimental data. Such experiment can provide a perspective of application of other types of damage evolution equation to various elasto-viscoplastic constitutive models.

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