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PARAMETRIZATION OF THE ODQVIST'S CREEP LAW IN THE CIRCULAR-SYMMETRICAL PROBLEM

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This work presents function parametrization method which describes the state of stress in case of the solution of plane axial symmetrical problems. In result of this parametrization for the simplified law in which the member describing unsteady creep is neglected, the basic equation system can be reduced to a hyperbolic form. In the general formulation of the Odqvist's creep law however for the case $n=n_0$ iteration procedure will be used. The efficiency of calculation method will be illustrated by an example of creep of the disk subjected to the action of pressure inside the hole.

1. INTRODUCTION

The theory of axial-symmetrical problems of circular stress state has an important application in solving problems of practical use; therefore this is one of the better known problems of the theory of elasticity and plasticity. Many structures having discs with a circular hole as element work in raised temperatures. Hence there is a need of considering the creep phenomena in analysis. The present paper takes into consideration, among others, the change in disc thickness as the effect of the creep process. In this analysis the creep law given by Odqvist was adopted,

$$\epsilon_{ij}=\frac{3}{2}\left\{\frac{d}{dt}\left[\left(\frac{\sigma_e}{\sigma_0}\right)^{n_0-1}\frac{S_{ij}}{\sigma_0}\right]+\left(\frac{\sigma_e}{\sigma_c}\right)^{n-1}\frac{S_{ij}}{\sigma_c}\right\}, \tag{1.1}$$

where ϵ_{ij} is a tensor of creep velocity, S_{ij} a deviator of stress $\sigma=\frac{3}{2}S_{ij}S_{ij}$ — stress intensity, whereas $\sigma_0, \sigma_c, n_0, n$ are constants describing the properties of the material. The symbol d/dt denotes the substantional derivative with respect to the time of the expression in square brackets (unsteady state creep). Parametriaztion of the described function of stress state will be the starting point in working out a method for solving axial symmetrical problems. This method will be given further on.

As a result of this parametrization, for the simplified law (1.1) in which the member describing unsteady creep is neglected, the basic equation system can be reduced to a hyperbolic form. In the general formulation of the law (1.1) however for the case $n=n_0$ iteration procedure will be used. The efficiency of the method of calculation will be illustrated by an example of creep of the disc subjected to the action of pressure inside the hole after programs have been elaborated for a digital computer.

2. Formulation of problem

It follows from the assumption of axial-symmetry in circular stress state for a polar system of coordinates $\{r, \theta\}$ that all the inquired magnitudes will be functions of the radius r and time t. Only such boundary problems in which the radial and circumferential directions are simultaneoulsy the main directions will be considered. In this particular case the constitutive equation (1.1) in Euler's description, after substituting ϵ_{ij} by a radial constituent of creep velocity $v(r, t)$ and S_{ij} by radial and circumferential stress respectively, takes the form

$$\frac{\partial v}{\partial r}=\frac{1}{2}\left\{\frac{d}{dt}\left[\left(\frac{\sigma_e}{\sigma_0}\right)^{n_0-1}\frac{2\sigma_r-\sigma_\theta}{\sigma_0}\right]+\left(\frac{\sigma_e}{\sigma_c}\right)^{n-1}\frac{2\sigma_r-\sigma_\theta}{\sigma_c}\right\}, \tag{2.1}$$

$$\frac{v}{r}=\frac{1}{2}\left\{\frac{d}{dt}\left[\left(\frac{\sigma_e}{\sigma_0}\right)^{n_0-1}\frac{2\sigma_\theta-\sigma_r}{\sigma_0}\right]+\left(\frac{\sigma_e}{\sigma_c}\right)^{n-1}\frac{2\sigma_\theta-\sigma_r}{\sigma_c}\right\}, \tag{2.2}$$

where the intensity of stress σ_e is given by

$$\sigma_e^2=\sigma_r^2-\sigma_r\sigma_\theta+\sigma_\theta^2. \tag{2.2a}$$

For a full formulation of the axial symmetrical problem the relations (2.1) and (2.2) should be completed with the equilibrium equation

$$\frac{\partial\sigma_r}{\partial r}+\frac{1}{h}\frac{\partial h}{\partial r}\sigma_r+\frac{1}{r}(\sigma_r-\sigma_\theta)=0 \tag{2.3}$$

and the non-compressibility equation

$$\frac{1}{h}\left(\frac{\partial h}{\partial t}+v\frac{\partial h}{\partial r}\right)+\frac{\partial v}{\partial r}+\frac{v}{r}=0, \tag{2.4}$$

in which $h(r, t)$ is a function of change of thickness. Finally, the solution of an initial boundary problem is reduced to the solution of a system of four nonlinear differential partial equations of the first order, Eqs. (2.1) to (2.4), with respect to the inquired functions dependent on two variables r and t.

The general idea of parametrization conists in reducing the number of equations in the system to three in result of identical satisfaction of relation (2.2).

With this aim in view the stresses σ_r and σ_θ are expressed by the parametrizing function

$$\left.\begin{matrix}\sigma_r\\ \sigma_\theta\end{matrix}\right\}=\left(\frac{4}{3}\right)^{\frac{n-1}{2n}}\sigma_c\left[\frac{4f}{r(\sqrt{3}\cos\psi-3\sin\psi)}\right]^{\frac{1}{n}}\cos\left(\psi\mp\frac{\pi}{6}\right). \tag{2.5}$$

As we shall see, the function $f(r, t)$ generally adopted in the formula (2.5) will be different for particular cases of the creep law.

The problem of steady and unsteady state creep as well as the problem of the combined effect of the two constituants for $n=n_0$ will be solved separately.

3. Steady state creep

Basing upon parametrization in Eq. (2.5) the solution of steady state creep is very simple.

Equations (2.1) and (2.2) will be in this case

$$\frac{\partial v}{\partial r}=\frac{1}{2}\left(\frac{\sigma_e}{\sigma_c}\right)^{n-1}\frac{2\sigma_r-\sigma_\theta}{\sigma_c}, \tag{3.1}$$

$$\frac{v}{r}=\frac{1}{2}\left(\frac{\sigma_e}{\sigma_c}\right)^{n-1}\frac{2\sigma_\theta-\sigma_r}{\sigma_c}. \tag{3.2}$$

The algebraic relation (3.2) is here identically satisfied by Eq. (2.5) if $f(r, t)=v(r, t)$, hence,

$$\left.\begin{matrix}\sigma_r\\ \sigma_\theta\end{matrix}\right\}=\left(\frac{4}{3}\right)^{\frac{n-1}{2n}}\sigma_c\left[\frac{4v}{r(\sqrt{3}\cos\psi-3\sin\psi)}\right]^{\frac{1}{n}}\cos\left(\psi\mp\frac{\pi}{6}\right). \tag{3.3}$$

It follows that the subsequent equations of the system describing the steady state creep with regard to the expressions σ_r and σ_θ by the relation (3.3) will take the form

the equilibrium equation

$$\frac{\partial\psi}{\partial r}\left[\frac{1}{n}\frac{\sqrt{3}\sin\psi+3\cos\psi}{\sqrt{3}\cos\psi-3\sin\psi}\cos\left(\psi-\frac{\pi}{6}\right)-\sin\left(\psi-\frac{\pi}{6}\right)\right]+ \tag{3.4}$$
$$+\cos\left(\psi-\frac{\pi}{6}\right)\left[\frac{1}{nv}\frac{\partial v}{\partial r}-\frac{1}{nr}+\frac{1}{h}\frac{\partial h}{\partial r}+\frac{1}{r}\right]-\frac{1}{r}\cos\left(\psi+\frac{\pi}{6}\right)=0,$$

the constitutive relation (3.1)

$$\frac{\partial v}{\partial r}=\frac{v}{r}\,\frac{2\cos\left(\psi-\frac{\pi}{6}\right)-\cos\left(\psi+\frac{\pi}{6}\right)}{2\cos\left(\psi+\frac{\pi}{6}\right)-\cos\left(\psi-\frac{\pi}{6}\right)}, \tag{3.5}$$

the non-compressibility condition

$$\frac{1}{h}\left(\frac{\partial h}{\partial t}+v\frac{\partial h}{\partial r}\right)=\frac{v}{r}\,\frac{\cos\left(\psi-\frac{\pi}{6}\right)+\cos\left(\psi+\frac{\pi}{6}\right)}{\cos\left(\psi-\frac{\pi}{6}\right)-2\cos\left(\psi+\frac{\pi}{6}\right)}. \tag{3.6}$$

According to Courant's classification [2] this system is not fully hyperbolic. Along three line of the characteristics: double $t=\text{const}$ and $dr-vdt=0$ two differention relations can be derived for $t=\text{const}$

$$\frac{dv}{dr}=\frac{v}{r}\,\frac{2\cos\left(\psi-\frac{\pi}{6}\right)-\cos\left(\psi+\frac{\pi}{6}\right)}{2\cos\left(\psi+\frac{\pi}{6}\right)-\cos\left(\psi-\frac{\pi}{6}\right)}, \tag{3.7}$$

and for $dr - v dt = 0$

$$\frac{dh}{dr} = \frac{h}{r}\,\frac{\cos\left(\psi - \frac{\pi}{6}\right) + \cos\left(\psi + \frac{\pi}{6}\right)}{\cos\left(\psi - \frac{\pi}{6}\right) - 2\cos\left(\psi + \frac{\pi}{6}\right)}. \tag{3.8}$$

The existence of the third missing relation along the characteristic $t = \text{const}$ is, however, indicated by the form of the equilibrium equation (3.4). The unknown functions occuring in it which are differential only with respect to the radial variable will permit to assign to this equation the character of a relation along the characteristic curve $t = \text{const}$. After eliminating the velocity $v(r, t)$ in Eq. (3.4) by substituting Eq. (3.5), it will be for $t = \text{const}$

$$\frac{d\psi}{dr} = \tag{3.9}$$

$$= \frac{\cos\left(\psi - \frac{\pi}{6}\right)\left[\frac{1}{nr}\left(\frac{2\cos\left(\psi - \frac{\pi}{6}\right) - \cos\left(\psi + \frac{\pi}{6}\right)}{2\cos\left(\psi + \frac{\pi}{6}\right) - \cos\left(\psi - \frac{\pi}{6}\right)} - 1\right) + \frac{1}{h}\frac{\partial h}{\partial r} + \frac{1}{r}\right] - \frac{1}{r}\cos\left(\psi + \frac{\pi}{6}\right)}{\sin\left(\psi - \frac{\pi}{6}\right) - \frac{1}{n}\cos\left(\psi - \frac{\pi}{6}\right)\frac{\sqrt{3}\sin\psi + 3\cos\psi}{\sqrt{3}\cos\psi - 3\sin\psi}}.$$

In this form the system of equations (3.4) to (3.6) will be considered further on as hyperbolic since the solution of the problem is reduced to the integration of the simple equations (3.7) to (3.9) along respective characteristic directions.

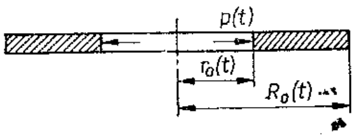

FIG. 1.

This is similar to the case of axial-symmetrical problems of the theory of plasticity [4].

The way of formulating the initial boundary conditions will be shown in the example of disc creep under the influence of pressure exerted inside the hole according to the scheme in Fig. 1. Boundary conditions for the radial stress function are established both on the external edge where $\sigma_r(R_0, t) = 0$ and the internal edge of the disc where $\sigma_r(r_0, t) = -p(t)$.

Basing upon parametrization in Eq. (3.3), the first of the conditions for the circumferential tensile stress ($\sigma_\theta > 0$) yields

$$\psi(R_0, t) = \frac{5}{3}\pi. \tag{3.10}$$

The second, on the other hand, yields the relation which, after transformation, takes the form

$$v(r_0, t) = \left(\frac{3}{4}\right)^{\frac{n-1}{2}}\left(\frac{-p(t)}{\sigma_c\cos\left(\psi_0 - \frac{\pi}{6}\right)}\right)^n \frac{r_0(\sqrt{3}\cos\psi_0 - 3\sin\psi_0)}{4}, \tag{3.11}$$

where $\psi_0 = \psi(r_0, t)$.

The conditions (3.10) and (3.11) form the so-called double—point problem for Eqs. (3.7) and (3.9). It consists in choosing such values $\psi(r_0, t)$ on the internal edge in the expression (3.11) so that after integration of Eqs. (3.7) and (3.9) the function $\psi(r, t)$ satisfies simultaneously the boundary condition (3.10).

For the purpose of choosing the values $\psi(r_0, t)$, the linear interpolation method was used in the paper. It proved to be of rapid convergence even in the case of satisfying the condition (3.10) with very good accuracy. Using the known distribution of the function values $\psi(r, t)$ for a determined moment t, we solve Eq. (3.8) along the characteristic $dr - vdt = 0$ with the initial condition

$$h(r, 0) = h_0(r). \tag{3.12}$$

A disc of const initial thickness $h_0(r) = \text{const}$ was adopted in calculation; this did not limit the generality of solution. Passing over to the next stage of the creep process $t = \text{const}$, small displacements are admitted. Depending on the adopted material constants the magnitude of the assumed displacements of particles will have an influence on the duration of the motion.

In the subsequent stage of the process the solution for the system of equations (3.7) to (3.9) with the conditions (3.10) to (3.12) is again undertaken. Performing a sufficient number of calculation steps one can finally achieve optionally big deformations.

Hence the stress state and creep velocity will be related at each time with the existing advance state of the deformation process of the disc [5].

The numerical example was solved for a disc made of soft carbon steel $\sigma_c = 17584$ kGcm$^{-2}h^{1/5}$, $n = 5$ at a temperature of 450 C with the initial dimensions $r_0 = 60$ mm, $R_0 = 80$ mm assuming the pressure inside the hole to be $p(t) = 6$ kG/mm^2. The calculated net of characteristics is presented in Fig. 2. Thick lines correspond

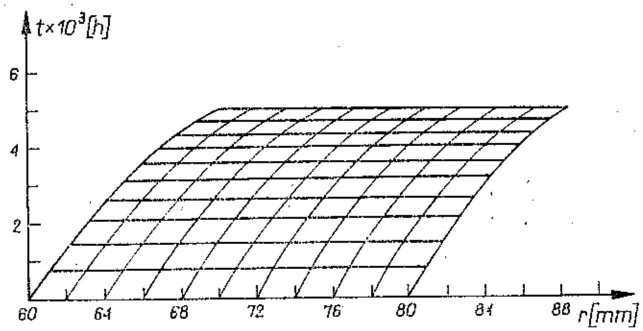

FIG. 2.

to the domain of the characteristics $t = \text{const}$. Thin lines on the other hand present the trajectories of the chosen disc particles in time of creep, describing at the same time the second domain of the characteristic $dr - vdt = 0$.

In Figs. 3, 4 and 5 areas of stress, velocities and changes of the disc thickness in various stages of a creep process are presented. Continuous lines present the

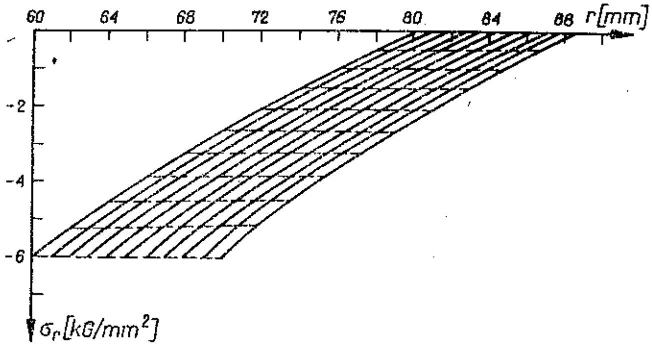

FIG. 3.

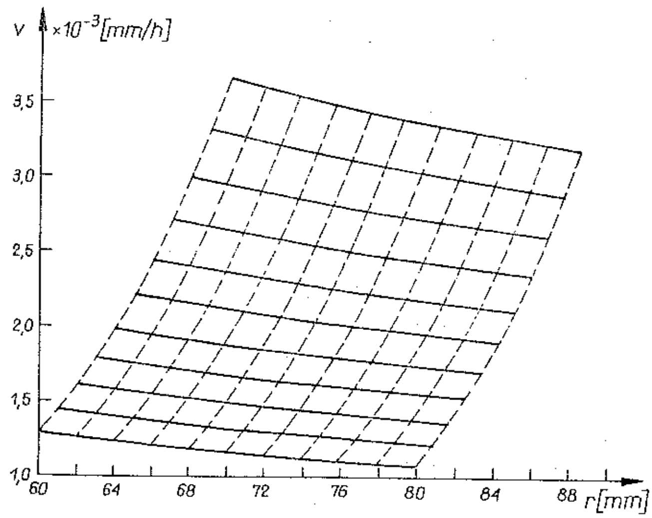

FIG. 4.

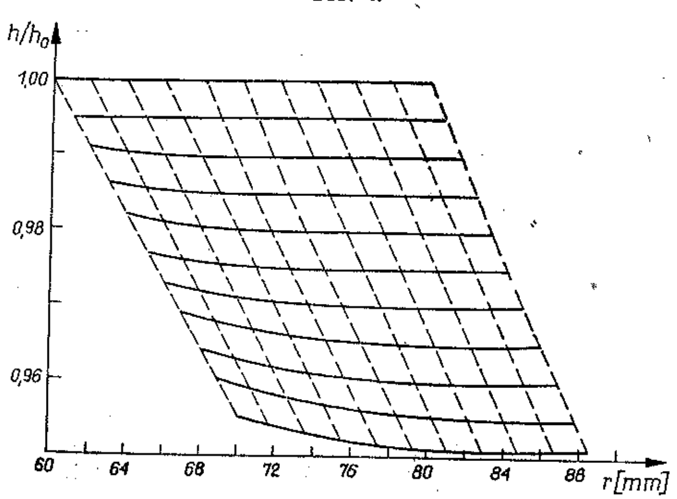

FIG. 5.

course of the function for the established moment of time $t=\text{const}$ whereas broken ones refer to the history of described magnitudes in individual particles during the deformed process.

It should be stressed that the solution can be obtained for optionally big strains; nevertheless it has a physical sense until the cohesion or stability has been disturbed. In the presented example the domain of solution was limited to the moment in which the internal radius of the disc increased its length by 10 mm.

4. Unsteady state creep

The solution of the problem, if only unsteady state creep is considered, is presented in a similar way. After a suitable simplification and bilateral integration with respect to time, Eqs. (2.1) and (2.2) take the form

$$\frac{\partial u}{\partial r}=\frac{1}{2}\left(\frac{\sigma_e}{\sigma_0}\right)^{n_0-1}\frac{2\sigma_r-\sigma_\theta}{\sigma_0}, \tag{4.1}$$

$$\frac{u}{r}=\frac{1}{2}\left(\frac{\sigma_e}{\sigma_0}\right)^{n_0-1}\frac{2\sigma_\theta-\sigma_r}{\sigma_0}, \tag{4.2}$$

where $u(r,t)$ is the radial constituent of displacement. The relation (4.2) is satisfied identically if the stresses σ_r and σ_θ are expressed by

$$\left.\begin{matrix}\sigma_r\\ \sigma_\theta\end{matrix}\right\}=\left(\frac{4}{3}\right)^{\frac{n_0-1}{n_0}}\sigma_0\left[\frac{4u}{r(\sqrt{3}\cos\psi-3\sin\psi)}\right]^{\frac{1}{n}}\cos\left(\psi\mp\frac{\pi}{6}\right). \tag{4.3}$$

The inguired magnitudes do not depend in this case evidently on time, hence the basic system will be described after parametrization in Eq. (4.3) with ordinary differential equations:

$$\frac{du}{dr}=\frac{u}{r}\,\frac{2\cos\left(\psi-\frac{\pi}{6}\right)-\cos\left(\psi+\frac{\pi}{6}\right)}{2\cos\left(\psi+\frac{\pi}{6}\right)-\cos\left(\psi-\frac{\pi}{6}\right)}, \tag{4.4}$$

$$\frac{dh}{dr}=\frac{h}{r}\,\frac{\cos\left(\psi-\frac{\pi}{6}\right)+\cos\left(\psi+\frac{\pi}{6}\right)}{\cos\left(\psi-\frac{\pi}{6}\right)-2\cos\left(\psi+\frac{\pi}{6}\right)}, \tag{4.5}$$

$$\frac{d\psi}{dr}= \tag{4.6}$$

$$=\frac{\cos\left(\psi-\frac{\pi}{6}\right)\left[\frac{1}{n_0 r}\left(\frac{2\cos\left(\psi-\frac{\pi}{6}\right)-\cos\left(\psi+\frac{\pi}{6}\right)}{2\cos\left(\psi+\frac{\pi}{6}\right)-\cos\left(\psi-\frac{\pi}{6}\right)}-1\right)+\frac{1}{h}\frac{dh}{dr}+\frac{1}{r}\right]-\frac{1}{r}\cos\left(\psi+\frac{\pi}{6}\right)}{\sin\left(\psi-\frac{\pi}{6}\right)-\frac{1}{n_0}\cos\left(\psi-\frac{\pi}{6}\right)\frac{\sqrt{3}\sin\psi+3\cos\psi}{\sqrt{3}\cos\psi-3\sin\psi}}.$$

The solution of Eqs. (4.4) to (4.6), with the boundary conditions

$$\psi(R_0, t) = \frac{5}{3}\pi, \tag{4.7}$$

$$u(r_0, t) = \left(\frac{3}{4}\right)^{\frac{n_0-1}{2}} \left(\frac{-p(t)}{\sigma_0 \cos\left(\psi_0 - \frac{\pi}{6}\right)}\right)^{n_0} \frac{r_0(\sqrt{3}\cos\psi_0 - 3\sin\psi_0)}{4}, \tag{4.8}$$

$$h(r, 0) = h_0 = \text{const} \tag{4.9}$$

for a disc as in the foregoing example (Fig. 1) under the assumption that $n_0 = 6$ and $\sigma_0 = 3380$ kG/cm², are presented in Figs. 6 and 7. In an analogous way, as in the

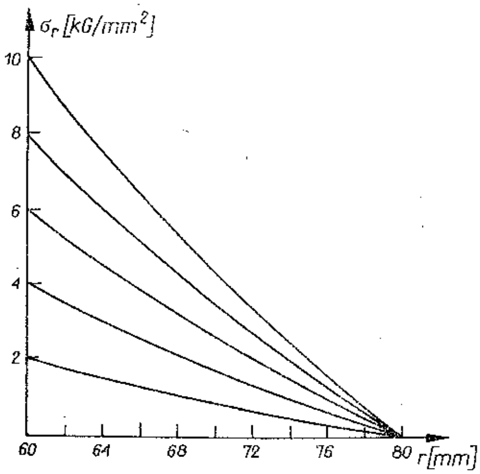

FIG. 6.

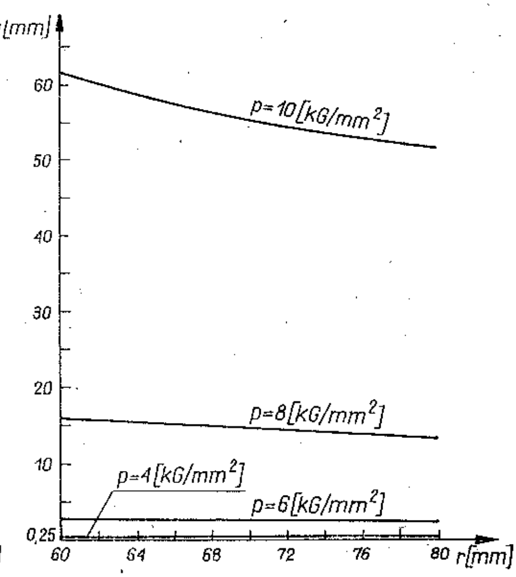

FIG. 7.

case of unsteady state creep; the conditions (4.7) and (4.8) form the double-point problem for Eqs. (4.4) and (4.6), the solution of which given the distribution of the values of the function $\psi(r, t)$ and the magnitudes of immediate displacement $u(r, t)$. In Fig. 6 particular curves present the change of radial stress along the radius for various pressures p loading the internal edge of the disc $p = 2, 4, 6, 8, 10$ kG/mm².

The values of instant displacement corresponding with curves were plotted in Fig. 7. The illustration presenting the change of the disc thickness was neglected.

5. Full Odqvist's law

In the general formulation of the Odqvist's creep law, for the case $n = n_0$ after substituting Eq. (2.5) to Eq. (2.2) the following is obtained:

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial r} + f\left(\alpha - \frac{v}{r}\right) = \alpha v, \tag{5.1}$$

where $\alpha=\left(\frac{\sigma_0}{\sigma_c}\right)^n$. The solution of Eq. (5.1) with respect to $f(r, t)$ can be reduced to the integration of the ordinary equation

$$\frac{df}{dt}+f\left(\alpha-\frac{v}{r}\right)=\alpha v, \tag{5.2}$$

along the characteristic $dr-v dt=0$ with the initial condition

$$f(r, 0)=\alpha u(r, 0). \tag{5.3}$$

The function $u(r, 0)$ describes the magnitudes of instant displacement of particles during unsteady state creep. As a result of the solution (5.2) with the initial condition (5.3) we get

$$f(r, t)=\alpha\left[\int_0^t \frac{v}{r} e^{\alpha t}\, dt+\frac{u(r, 0)}{r(0)}\right] r e^{-\alpha t}, \tag{5.4}$$

where $r(0)$ is the radius of the particle considered at the moment $t=0$. At last, the expression (2.5) satisfying identically Eq. (2.2) for $n=n_0$ will have the form

$$\left.\begin{matrix}\sigma_r\\ \sigma_\theta\end{matrix}\right\}=\left(\frac{4}{3}\right)^{\frac{n-1}{2n}} \sigma_0\left[\frac{4\left(\frac{u(r, 0)}{r(0)}+\int_0^t \frac{v}{r} e^{\alpha t}\, dt\right) e^{-\alpha t}}{\sqrt{3} \cos \psi-3 \sin \psi}\right]^{\frac{1}{n}} \cos\left(\psi \mp \frac{\pi}{6}\right). \tag{5.5}$$

The other equations of the basic system, after substituting Eq. (5.5), will be written in sequence of the calculations carried out in the iteration method suggested further on, i.e.

constitutive equation (2.2)

$$\frac{\partial v}{\partial r}=\frac{v}{r} \varphi-e^{-\alpha t}\left[\int_0^t \frac{v}{r} e^{\alpha t}\, dt+\frac{u(r, 0)}{r(0)}\right] \frac{d \varphi}{d t}, \tag{5.6}$$

where

$$\varphi(r, t)=\frac{\sqrt{3} \cos \psi+3 \sin \psi}{\sqrt{3} \cos \psi-3 \sin \psi},$$

non-compressibility condition

$$\frac{1}{h} \frac{dh}{dr}=-\frac{1}{v} \frac{\partial v}{\partial r}-\frac{1}{r}, \tag{5.7}$$

and equilibrium equation

$$\frac{\partial \psi}{\partial r}\left[\sin\left(\psi-\frac{\pi}{6}\right)-\frac{1}{n} \frac{\sqrt{3} \sin \psi+3 \cos \psi}{\sqrt{3} \cos \psi-3 \sin \psi} \cos\left(\psi-\frac{\pi}{6}\right)\right]= \tag{5.8}$$

$$=\cos\left(\psi-\frac{\pi}{6}\right)\left[\frac{\int_0^t\left(\frac{1}{r} \frac{\partial v}{\partial r}-\frac{v}{r^2}\right) e^{\alpha t}\, dt+\frac{1}{r(0)} \frac{\partial u(r, 0)}{\partial r}-\frac{u(r, 0)}{r^2(0)}}{n\left(\int_0^t \frac{v}{r} e^{\alpha t}\, dt+\frac{u(r, 0)}{r(0)}\right)}+\frac{1}{h} \frac{\partial h}{\partial r}+\frac{1}{r}\right]-\frac{1}{r} \cos\left(\psi+\frac{\pi}{6}\right).$$

The symbols d/dt and d/dr determine differentiation along the direction $dr-vdt=0$. Hence the considered problem describes the system of three differential-integral equations (5.6) to (5.8) with three unknown functions. However, there are many difficulties in solving this system directly. With regard to this the method of successive approximation based upon the observation that the inquired function should differ state creep is presented in Point 3.

Owing to this the solution of the system of equations (3.7) to (3.9) can be taken as a first approximation. The calculated value $\psi(r, t)$ will be substituted into Eq. (5.6). This results to the relation for one unknown function $v(r, t)$ which should be solved on a full area for an established moment of time $t=\text{const}$. The function $u(r, 0)$ describes the instant displacements for the moment of time $t=0$ from the state of unsteady state creep. Its value can be calculated from the solution of the initial boundary problem for the system of equations (4.4) to (4.6). The velocity function $v(r, t)$ determined through numerical integration in Eq. (5.6) is subsequently introduced into the relation (5.7) which, after integration along the direction $dr-vdt=0$, gives the next function $h(r, t)$. The calculated functions $v(r, t)$ and $h(r, t)$ are used in Eq. (5.8) which, as in the case of Eq. (5.6), is integrated in the full solution area for a given moment of time $t=\text{const}$; consequently, the second approximation of the function $\psi(r, t)$ is obtained.

This procedure should be repeated in order to obtain for the succeeding approximation sufficiently close results.

The previously established boundary conditions (3.10) and (3.12) for the disc under the effect of pressure inside the hole remain valid provided that Eq. (3.11) takes now a slightly changed form. From the condition $\sigma_r(r_0, t)=-p(t)$ we obtain for $t=0$

$$(5.9)\qquad u(r_0, 0)=\frac{1}{4\alpha}\left(\frac{4}{3}\right)^{\frac{1-n}{2}} r_0(0)\left[\frac{-p(t)}{\sigma_c\cos\left(\psi_0-\frac{\pi}{6}\right)}\right]^n(\sqrt{3}\cos\psi_0-3\sin\psi_0),$$

and for $t>0$

$$(5.10)\qquad \int_0^t \frac{v}{r_0(t)}e^{\alpha t}\,dt=\frac{1}{4\alpha}\left(\frac{4}{3}\right)^{\frac{1-n}{2}} e^{\alpha t}\left[\frac{-p(t)}{\sigma_c\cos\left(\psi_0-\frac{\pi}{6}\right)}\right]^n(\sqrt{3}\cos\psi_0-3\sin\psi_0)-\frac{u(r_0, 0)}{r_0(0)}.$$

The numerical example solved for the previously given values σ_0 and σ_c and under the assumption that $n=n_0=5$ indicates that the method is rapidly convergent. Differences between the second and third approximations lie within the limits of the error of numerical integration. Good agreement of calculation was obtained after aplying in the solution ordinary differential equations of Runge-Kutty's [5] method of higher order.

The relations of displacement of the interial edge of a disc in time for various values of pressure inside the hole are presented in Figs. 8a, b and c.

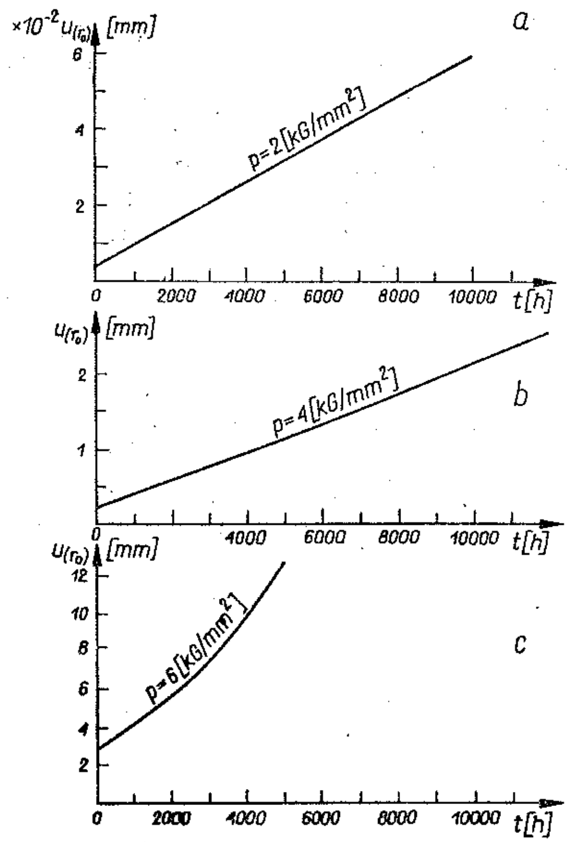

FIG. 8.

6. FINAL REMARKS

This paper presents the function parametrization method which describes the state of stress in solving plane axial symmetrical problems. In all the three considered cases we have taken as an example the pressing of a ring by pressure evenly applied on the hole surface. The solution of the case of steady state creep on the basis of Fig. 4 presenting the distribution of the velocity field has allowed us to draw conclusions on the increase of creep velocity with the progress of the strain process. This phenomena is by no means accompanied by significant changes in the radial stress distribution. We can say approximately that the radial stress distribution curves for the fixed moment of time $t=$const are parallel (Fig. 3). This is caused by a small yet at the same time steady decrease of thickness of the plate during creeping (Fig. 5). The numerical results obtained in the case of unsteady state creep are also worth noting, e.g. interdependence of the instant displacement values and the pressure applied (Fig. 7). This relation can be treated as a condition for defining the permissible pressure which limits the value instant strain. By adopting the above condition as a design condition we can considerably limit the need for the iteration procedure for the complete

Odqvist's law. This procedure could become the starting point for an exact analysis of the stress and strain states. However, in case of insignificant changes in the geometry of the plane — coured by instant strains — it will be possible to limit the solution to those describing only steady state creep.

The applicability of the analysis presented above is connected with frequent cases of the work of planes at high temperatures. The planes can function as a construction element of reactor heat exchangers or rocket hozzles. Each of the examples considered requires an exact analysis of stress and strain state. On the basis of this analysis we can establish suitable design conditions which would assure safety and proper functioning of the structure over a long period.

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Streszczenie

PARAMETRYZACJA PRAWA PEŁZANIA ODQVISTA DLA ZAGADNIEŃ OSIOWO-SYMETRYCZNYCH

Przedstawiona w pracy metoda parametryzacji funkcji opisujących stan naprężenia dotyczy rozwiązań płaskich zagadnień osiowo-symetrycznych. W wyniku tej parametryzacji dla uproszczonego prawa, w którym pomija się wyraz opisujący pełzanie nieustalone, podstawowy układ równań można sprowadzić do postaci hiperbolicznej. W ogólnym natomiast sformułowaniu prawa pełzania Odqvista dla przypadku $n=n_0$ posłużono się procedurą iteracyjną. Efektywność metod obliczeniowych zilustrowano przykładem pełzania pierścienia poddanego działaniu ciśnienia wewnątrz otworu.

Резюме

ПАРАМЕТРИЗАЦИЯ ЗАКОНА ПОЛЗУЧЕСТИ ОДКВИСТА ДЛЯ ОСЕСИММЕТРИЧНЫХ ЗАДАЧ

Представленный в работе метод параметризации функции, описывающих напряженное состояние, касается решений плоских, осесимметричных задач. В результате этой параметризации для упрощенного закона, в котором пренебрегается членом описывающим неустанивившуюся ползучесть, основную систему уравнений можно свести к гиперболическому виду. В общей же формулировке закона ползучести Одквиста для случая $n=n_0$ послуживаются итерационной процедурой. Эффективность расчетных методов иллюстрирована примером ползучести кольца подвергнутого действию давления внутри отверстия.

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