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ON THE THREE-DIMENSIONAL CONTACT PROBLEM OF A RIGID INCLUSION PRESSED BETWEEN TWO ELASTIC HALF-SPACES

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An approximation solution is found for the three-dimensional contact problem of two elastic half-spaces pressed against each other with a rigid inclusion between them. We confine ourselves to identical spaces nad will examine an inclusion of a such a nature that rotational symmetry is achieved.

We apply asymptotics to a small parameter which is the quotient of the radius of the contact region with the inclusion and of the radius of the region where the spaces touch each other. These quantities can be compouted if the pressure at infinity and the geometry of the inclusion are given. After solving this part of the problem the surface pressure may be obtained.

Numerical results are given for the small parameter and the surface pressure.

INTRODUCTION

This paper may be considered as a generalization and a further continuation of [1, 2] by J. B. ALBLAS, dealing with the corresponding two-dimensional problem.

Here we solve the three-dimensional problem of two elastic half-spaces which we press against each other with a rotationally symmetrical rigid inclusion between them. The spaces have equal constants of elasticity.

The half-spaces not only make contact with the inclusion but will also thouch each other at some finite distance after any arbitrarily small pressure has been applied.

We call the contact ratio k. This is the quotient of the contact parameters c and ρ, where c is the radius of the contact region with the inclusion and ρ is the distance at which the spaces meet.

We will represent the pressure at the surface in the form of a series expansion in powers of the ratio k. We are able to generate step by step the coefficients of that series, using for each term the results of the preceding ones. The proof of the convergence of this series is not included, however a rough estimation gives a radius of convergence of 0.2. Postulating limitation of the pressure, we get two equations for the contact parameters, the diameter of the inclusion and the pressure at infinity being given. Knowing these quantities we can determine displacements, deformations and stresses all over the half-spaces.

Cutting down the series mentioned before, we get our various approximations.

For several values of the pressure at infinity and of the diameter, numerical results are given for the contact ratio and the pressure at the surface.

1. Statement of the problem

We consider two isotropic, homogeneous elastic half-spaces of the same material, occuping the regions $r \geqslant 0$, $0 \leqslant \theta < 2\pi$, $z > 0$ and $z < 0$, (r, θ, z) being a system of cylindrical coordinates with $r \geqslant 0$, $0 \leqslant \theta < 2\pi$, $-\infty < z < \infty$ (cf. Fig. 1).

To each of these half-spaces we apply a rigid body displacement d, so that the distance between them becomes $2d$.

Next, we put a smooth rigid inclusion of 2d width between the space, the surface consiting of two paraboloids which we obtain by evaluating two parabolic arcs around the z axis.

In this way we get a rotationally symmetrical geometry, the z axis being the axis of symmetry.

By pressing at infinity (pressure p°) the spaces move towards each other and make contact at some distance from the origin, the situation of which is illustrated in Fig. 1.

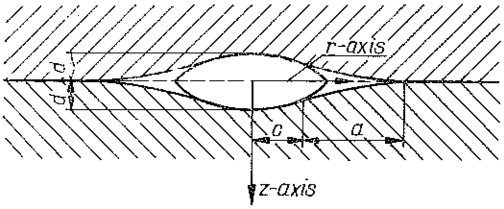

Fig. 1.

In linear elastostatics the equations of equilibrium for the displacements read

$$2\frac{1-\nu}{1-2\nu}\,\text{grad div}\,\mathbf{u} - \text{rot rot}\,\mathbf{u} = \mathbf{0}, \tag{1.1}$$

in which $\mathbf{u}$ is the vectorial displacement, Δ is the Laplacian Operator and ν is the Poisson ratio.

In the case of rotational symmetry

$$u_\theta = \frac{\partial}{\partial\theta} = 0, \tag{1.2}$$

in which u_θ is the displacement in the θ-direction.

With the aid of (1.2), (1.1) can be simplified to

$$\begin{aligned} &\beta^2\left\{\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right\} + (\beta^2 - 1)\frac{\partial^2 w}{\partial z\,\partial r} + \frac{\partial^2 u}{\partial z^2} = 0, \\ &\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + (\beta^2 - 1)\frac{\partial}{\partial z}\left\{\frac{\partial u}{\partial r} + \frac{u}{r}\right\} + \beta^2\frac{\partial^2 w}{\partial z^2} = 0, \end{aligned} \tag{1.3}$$

in which

$$\beta^2 = 2\frac{1-\nu}{1-2\nu}, \tag{1.4}$$

and u and w are the displacements in the r- and z-direction, respectively.

On account of symmetry we may confine ourselves to one half-space, for which we take $z<0$. The boundary conditions for the region mentioned before

$$r \geqslant 0, \quad 0 \leqslant \theta < 2\pi, \quad z<0; \tag{1.5}$$

are

$$\begin{aligned} z=0, \quad & 0\leqslant r<\infty, & t_{rz}=0, \\ & 0\leqslant r<c, & w=d-\frac{r^2}{2R}, \\ & c\leqslant r<\rho, & t_{zz}=0, \\ & \rho\leqslant r<\infty, & w=0, \end{aligned} \tag{1.6}$$

in which t_{ij} are the components of the stress tensor, $1/R$ is the curvature at the vertex of the parabola and c is the radius of the contact region with the conclusion. The half-spaces make contact with each other if $r>\rho$ at which

$$\rho=c+a. \tag{1.7}$$

The pressure at infinity is p^0, so

$$t_{zz}=-p^0, \quad (\sqrt{r^2+z^2}\to\infty). \tag{1.8}$$

2. Decomposition of the problem

Subtracting from the problem stated in Eqs. (1.3) and (1.6) the displacements belonging to the fundamental problem of two half-spaces without inclusion,

$$u=\frac{\nu p^0}{2\mu(1+\nu)}r, \quad w=-\frac{p^0}{2\mu(1+\nu)}z, \tag{2.1}$$

in which μ is the modulus of shear, the boundary conditions for the residual problem change to

$$\begin{aligned} z=0, \quad & 0\leqslant r<\infty, & t_{rz}=0, \\ & 0\leqslant r<c, & w=d-\frac{r^2}{2R}, \\ & c\leqslant r<\rho, & t_{zz}=p^0, \\ & \rho\leqslant r<\infty, & w=0, \end{aligned} \tag{2.2}$$

in which t_{zz} vanish at infinity.

3. Hankeltransform

By definition the Hankeltransforms of u and w are

$$\begin{aligned} \bar{u}(\xi,z) &= \int_0^\infty r u(r,z) J_1(\xi r)\,dr, \\ \bar{w}(\xi,z) &= \int_0^\infty r w(r,z) J_0(\xi r)\,dr. \end{aligned} \tag{3.1}$$

From Eqs. (1.3) and (2.2)′ we find

$$\bar{w}(\xi, z) = -\frac{\bar{t}_{zz}(\xi, 0)\, e^{-\xi z}}{2\xi\mu(\beta^2-1)}\{\beta^2+(\beta^2-1)\xi z\}, \tag{3.2}$$

in which

$$\bar{t}_{zz}(\xi, 0) = \int_0^\infty r t_{zz}(r, 0) J_0(\xi r)\, dr. \tag{3.3}$$

4. Triple integral equation

By putting $z=0$ in Eq. (3.2) we have the relation

$$\bar{w}(\xi, 0) = -K\frac{\bar{t}_{zz}(\xi, 0)}{\xi}, \tag{4.1}$$

K being defined by

$$K=(1-\nu)/\mu. \tag{4.2}$$

Together with the boundary conditions in Eqs. $(2.2)_{2,3,4}$ and (4.1) the inverse transform of Eqs. $(3.1)_2$ and (3.3) gives the triple integral equation

$$\begin{aligned} &\int_0^\infty \xi\bar{w}(\xi, 0) J_0(r\xi)\, d\xi = d-\frac{r^2}{2R}, && r\in I_1, \\ &\int_0^\infty \xi^2\bar{w}(\xi, 0) J_0(r\xi)\, d\xi = -p^0 K, && r\in I_2, \\ &\int_0^\infty \xi\bar{w}(\xi, 0) J_0(r\xi)\, d\xi = 0, && r\in I_3, \end{aligned} \tag{4.3}$$

in which

$$I_1=\{r\,|\,0\leqslant r<c\}, \quad I_2=\{r\,|\,c\leqslant r<p\}, \quad I_3=\{r\,|\,p<r<\infty\}. \tag{4.4}$$

5. Integral equations

The part on the right hand side of Eq. (4.3) is known. So we can, in principle, determine $\bar{w}(\xi, 0)$ and also $\bar{t}_{zz}(\xi, 0)$ according to Eq. (4.1). The inverse transforms of these two functions give the displacement w and stress t_{zz} at the plane $z=0$.

In Eq. (4.3) all boundary conditions meet the requirements. Solving this system with the aid of [3] and [4] we get two related integral equations:

$$\begin{aligned} f_3(u) &= m(u)+\int_0^1 M(u, t) f_1(t)\, dt, \\ f_1(v) &= n(v)+\int_0^1 M(v, s) f_3(s)\, ds. \end{aligned} \qquad 0\leqslant u,\ v\leqslant 1, \tag{5.1}$$

The functions are defined by

$$f_3(u)=\frac{\sqrt{1-u^2}}{u^2}k^{-\frac{1}{2}}K\hat{p}_3\left(\frac{\rho}{u}\right),\qquad m(u)=\frac{\sqrt{1-u^2}}{u^2}k^{-\frac{1}{2}}l_1\left(\frac{\rho}{u}\right),$$

(5.2)
$$f_1(v)=v\sqrt{1-v^2}\,kK\hat{p}_1(cv),\qquad n(v)=v\sqrt{1-v^2}\,kl_2(cv),$$

$$M(u,t)=-\frac{2}{\pi}k^{\frac{1}{2}}u\frac{\sqrt{1-k^2t^2}}{\sqrt{1-t^2}\,(1-k^2t^2u^2)},$$

in which the functions $\hat{p}_i(r)$ are the unknown pressure at the surface

$$\hat{p}_i(r)=-t_{zz}(r,0),\qquad r\in I_i$$

and the functions l_i read

(5.3)
$$l_1\left(\frac{\rho}{u}\right)=\frac{2u}{\pi}\frac{p^0K}{\sqrt{1-u^2}}\left\{\sqrt{1-k^2}-\frac{\sqrt{1-u^2}}{u}\arctan\frac{u\sqrt{1-k^2}}{\sqrt{1-u^2}}\right\},$$

$$l_2(cv)=\frac{2}{\pi}\frac{c}{R}\sqrt{1-v^2}+\frac{2}{\pi c}\frac{d-\dfrac{c^2v^2}{R}}{\sqrt{1-v^2}}+$$
$$+\frac{2}{\pi c}\frac{p^0K}{\sqrt{1-v^2}}\left\{\rho\sqrt{1-k^2}-c\sqrt{1-v^2}\arctan\frac{\sqrt{1-k^2}}{k\sqrt{1-v^2}}\right\}.$$

The important constant k is defined by

(5.4)
$$k=\frac{c}{\rho},$$

while

(5.5)
$$0\leqslant u,\quad v\leqslant 1.$$

We can reduce the system in Eqs. (5.1) to one integral equation of Fredholm's type:

(5.6)
$$f_1(v)=h(v)+\int_0^1 H(v,t)f_1(t)\,dt,\qquad 0\leqslant v\leqslant 1,$$

with

$$h(v)=n(v)+\int_0^1 M(v,s)m(s)\,ds,$$

(5.7)
$$H(v,t)=\int_0^1 M(v,s)M(s,t)\,ds.$$

The following expressions hold:

$$h(v)=v(1-v^2)\frac{2}{\pi}\frac{c}{R}k+\frac{2}{\pi}\frac{d}{c}vk-\frac{2}{\pi}\frac{c}{R}v^3k+$$

$$+\frac{2}{\pi}p^0Kv\sqrt{1-k^2}-\frac{2}{\pi}p^0Kv\sqrt{1-v^2}\,k\arctan\frac{\sqrt{1-k^2}}{k\sqrt{1-v^2}}+$$

$$-\frac{4}{\pi^2}vp^0K\int_0^1\frac{\sqrt{1-k^2}\,s^2}{s\sqrt{1-s^2}\,(1-k^2s^2v^2)}\times \tag{5.8}$$

$$\times\left[\sqrt{1-k^2}-\frac{\sqrt{1-s^2}}{s}\arctan\frac{s\sqrt{1-k^2}}{\sqrt{1-s^2}}\right]ds,$$

$$H(v,t)=\frac{4}{\pi^2}k\frac{v\sqrt{1-k^2t^2}}{\sqrt{1-t^2}}\int_0^1\frac{s\sqrt{1-k^2s^2}}{\sqrt{1-s^2}\,(1-k^2v^2s^2)(1-k^2t^2s^2)}ds$$

and

$$m(u)=\frac{2}{\pi u}k^{-\frac{1}{2}}p^0K\sqrt{1-k^2}-\frac{2}{\pi}\frac{\sqrt{1-u^2}}{u^2}k^{-\frac{1}{2}}p^0K\arctan\frac{u\sqrt{1-k^2}}{\sqrt{1-u^2}}.$$

6. Power series substitution

We substitute

$$h(v)=\sum_{l=0}^{\infty}h_l(v)k^l,$$

$$H(v,t)=\sum_{l=0}^{\infty}H_{2l+1}(v,t)k^{2l+1}, \tag{6.1}$$

$$f_1(v)=\sum_{l=0}^{\infty}f_{1,l}(v,t)k^l,$$

in Eq. (5.6) and equate the coefficients of equal powers of k:

$$f_{1,0}(v)=h_0(v),$$

$$f_{1,l}(v)=h_l(v)+\sum_{i=0}^{\left[\frac{l-1}{2}\right]}\int_0^1 H_{2i+1}(v,t)f_{1,l-(2i+1)}(t)\,dt,\qquad l\geq 1, \tag{6.2}$$

which provides a recursive relation to calculate $f_{1,l}(v)$ $(l\geq 1)$.

Next we put

$$m(u)=k^{-\frac{1}{2}}\sum_{l=0}^{\infty}d_{2l}(u)k^{2l}, \tag{6.3}$$

(6.3) [cont.]
$$M(u,t)=k^{-\frac{1}{2}}\sum_{l=0}^{\infty} M_{2l+1}(u,t)\,k^{2l+1},$$
$$f_3(u)=k^{-\frac{1}{2}}\sum_{l=0}^{\infty} f_{3,l}(u)\,k^{l}$$

in Eq. $(5.1)_1$ which yields the recurrence relation

(6.4)
$$f_{3,0}(u)=d_0(u),$$
$$f_{3,2l}(u)=d_{2l}(u)+\sum_{i=0}^{l-1}\int_0^1 M_{2i+1}(u,t)f_{1,2l-(2i+1)}(t)\,dt,\quad l\geqslant 1,$$
$$f_{3,2l+1}(u)=\sum_{i=0}^{l}\int_0^1 M_{2l+i}(u,t)f_{1,2l-2i}(t)\,dt,\quad l\geqslant 0.$$

In this way we have determined both f_1 and f_3.

7. The parameters of the contact regions

Assuming that the pressure p^0 and the diameter d/R are given, the contact parameters c/R and ρ/R are unknown quantities. We will calculate them in the following way.

The pressure functions $\hat{p}_1(cv)$ and $\hat{p}_3(\rho/u)$ belong to the functions $f_1(v)$ and $f_3(u)$ according to $(5.2)_{1,3}$. Assuming that these pressures are limited if $v, u=1$, we find two equations for the contact parameters.

We shall illustrate this in the next sections.

8. First approximation

We approximate $f_1(v)$ by cutting down the series in Eq. $(6.1)_3$. If we do so for $l=1$, we will get the first approximation for $f_1(v)\colon f_1^{(1)}(v)$. We find

$$f_1^{(1)}(v)=\frac{4}{\pi^2}p^0Kv\left(1+\frac{4}{\pi^2}k\right)+\frac{2}{\pi}kv\left[(1-2v^2)\frac{c}{R}+\frac{d}{c}\right]-p^0Kvk\sqrt{1-v^2},\tag{8.1}$$

from which

$$K\hat{p}_1^{(1)}(cv)=\frac{\frac{4}{\pi^2}p^0K\left(1+\frac{4}{\pi^2}k\right)+\frac{2}{\pi}k\left[(1-2v^2)\frac{c}{R}+\frac{d}{c}\right]}{k\sqrt{1-v^2}}-p^0K.\tag{8.2}$$

We postulate that $\hat{p}_1(cv)$ and so $\hat{p}_1^{(1)}(cv)$ is limited in $v=1$. This gives the first of the two equations mentioned in Sect. 7:

$$\frac{4}{\pi^2}p^0K\left(1+\frac{4}{\pi^2}k\right)+\frac{2}{\pi}k\left[\frac{d}{c}-\frac{c}{R}\right]=0.\tag{8.3}$$

Because of this Eq. (8.2) leads to

$$K\hat{p}_1^{(1)}(cv) = -p^0 K + \frac{4}{\pi}\frac{c}{R}\sqrt{1-v^2}. \tag{8.4}$$

Next we calculate the first approximation of $f_3(u)$ by cutting down Eq. $(6.3)_3$ at $l=2$:

$$f_3^{(1)}(u) = k^{-\frac{1}{2}}\left\{\frac{2p^0 K}{\pi u} - \frac{2p^0 K}{\pi}\frac{\sqrt{1-u^2}}{u^2}\arctan\frac{u}{\sqrt{1-u^2}} - \frac{8}{3}\frac{c}{R}\frac{k^2}{\pi^2}u\right\}, \tag{8.5}$$

from which

$$K\hat{p}_3^{(1)}\left(\frac{\rho}{u}\right) = \frac{1}{\sqrt{1-u^2}}\left[\frac{2p^0 K}{\pi}u - \frac{8}{3}\frac{c}{R}\frac{k^2}{\pi^2}u^3\right] - \frac{2p^0 K}{\pi}\arctan\frac{u}{\sqrt{1-u^2}}. \tag{8.6}$$

Again we postulate that the pressure is limited in this case in $u=1$. This condition imposed on $\hat{p}_3^{(1)}\left(\frac{\rho}{u}\right)$ gives

$$p^0 K - \frac{4}{3}\frac{c}{R}\frac{k^2}{\pi} = 0. \tag{8.7}$$

This is the second equation we mentioned in Sect. 7. We obtain the representation of the pressure:

$$K\hat{p}_3^{(1)}\left(\frac{\rho}{u}\right) - \frac{2p^0 K}{\pi}\left[u\sqrt{1-u^2} - \arctan\frac{u}{\sqrt{1-u^2}}\right]. \tag{8.8}$$

We first solve the equations for the contact parameters. By Eq. (8.7)

$$\frac{c}{R} = \frac{3}{4}\pi\frac{p^0 K}{k}. \tag{8.9}$$

This, substituted in Eq. (8.3) gives

$$2\frac{d}{R}k^4 + \frac{12}{\pi^2}(p^0 K)^2 k^2 + 3(p^0 K)^2 k - \frac{9}{8}\pi^2 (p^0 K)^2 = 0. \tag{8.10}$$

This gives the first approximation of $k: k^{(1)}$. From Eq. (8.9) we get the first approximation of c/R: $c^{(1)}/R$ and by

$$\frac{\rho}{R} = \frac{1}{k}\frac{c}{R} \tag{8.11}$$

that of ρ/R: $\rho^{(1)}/R$.

9. Second approximation

Analogously we compute the second approximation.

$$K\hat{p}_1^{(2)}(cv) = \left(\frac{8}{\pi^2}p^0 K + \frac{4}{\pi}\frac{c}{R}\right)\sqrt{1-v^2} - p^0 K, \tag{9.1}$$

$$K\hat{p}_3^{(2)}\left(\frac{\rho}{u}\right) = \frac{2p^0 K}{\pi}\left[u\sqrt{1-u^2} - \arctan\frac{u}{\sqrt{1-u^2}}\right]. \tag{9.2}$$

and the equation for $k^{(2)}$:

(9.3)
$$k^5\left\{-\frac{1664}{3\pi^4}+\frac{32}{\pi^2}+\frac{32}{\pi^2}\frac{d}{R}\frac{1}{(p^0K)^2}\right\}+k^4\left\{-\frac{128}{\pi^4}-\frac{32}{\pi^2}+8\frac{d}{R}\frac{1}{(p^0K)^2}\right\}+$$
$$+k^3\,12\left(\frac{16}{\pi^4}-1\right)+k^2\left(\frac{48}{\pi^2}+24\right)+k\cdot 6-\frac{9}{2}\pi^2=0$$

while $c^{(2)}/R$ follows from

(9.4)
$$\frac{c}{R}=\frac{\pi}{4}p^0K\left(\frac{3}{k^2}-\frac{8}{\pi^2}\right).$$

10. Discussion and results

To simply the numerical implementation we go over the following variables:

(10.1)
$$\tilde{p}=p^0K\sqrt{\frac{R}{d}},\qquad \tilde{c}=\frac{c}{\sqrt{Rd}},\qquad \tilde{\rho}=\frac{\rho}{\sqrt{Rd}}.$$

We arrive at the following equations:

1st approximation:

(10.2)
$$2k^4+\frac{12}{\pi^4}\tilde{p}^2k^2+3\tilde{p}^2k-\frac{9}{8}\pi^2\tilde{p}^2=0,$$
$$\tilde{c}^{(1)}=\frac{3}{\pi}\frac{\pi\tilde{p}}{[k^{(1)}]^2},\qquad \tilde{\rho}^{(1)}=\frac{\tilde{c}^{(1)}}{k^{(1)}};$$

2nd approximation:

(10.3)
$$k^5\left\{\tilde{p}^2\left(\frac{32}{\pi^2}-\frac{1664}{3\pi^4}\right)+\frac{32}{\pi^2}\right\}-k^4\left\{\tilde{p}^2\left(\frac{128}{\pi^4}+\frac{32}{\pi^2}\right)-8\right\}+$$
$$+\tilde{p}^2k^3\cdot 12\left(\frac{16}{\pi^4}-1\right)+\tilde{p}^2k^2\left(\frac{48}{\pi^2}+24\right)+\tilde{p}^2k\cdot 6-\frac{9}{2}\pi^2\tilde{p}^2=0,$$
$$\tilde{c}^{(2)}=\frac{\pi}{4}\tilde{p}\frac{3\pi^2-8[k^{(2)}]^2}{\pi^2[k^{(2)}]^2},\qquad \tilde{\rho}^{(2)}=\frac{\tilde{c}^{(2)}}{k^{(2)}}.$$

We notice that the approximations of k are not identical for the same $\tilde{p}$, neither are the parametes $\tilde{c}$ and $\tilde{\rho}$.

We know that $0<k<1$. It apperas that $k^{(1)}=1$ when $\tilde{p}^2=0.29$ and $k^2=1$ when $\tilde{p}^2=0.42$. Although intuition tells us that $k\to 1$ when $\tilde{p}\to\infty$ the remarks made previously are not in the contradiction with this. Here, we cut down the power series in k and compute the contact parameters with the aid of that expression, the approximation of which is reasonable when k is small. An increase in k does not require that the equation should yield a right estimation.

In the following prints we give a graphic representation of the relation between $\tilde{p}$ and $\tilde{k}^{(i)}$ if $0<p\leqslant 0.1$, both for the first and the second approximation. We see that there is a small difference between the approximations.

Considering the pressure $\hat{p}_3$ according to Eqs. (8.8) and (9.2), we establish that $K\hat{p}_3^{(1)}$ qua forma, is equal to $K\hat{p}_3^{(2)}(\rho/u)$ but the parametres are different.

We sketch the dimensionless pressure at the surface. The pressure is $p^0+\hat{p}(r)$. We do so in the case $\hat{p}=0.1$ for both approximations. The differences are significant for this high value of the parameter $\tilde{p}$.

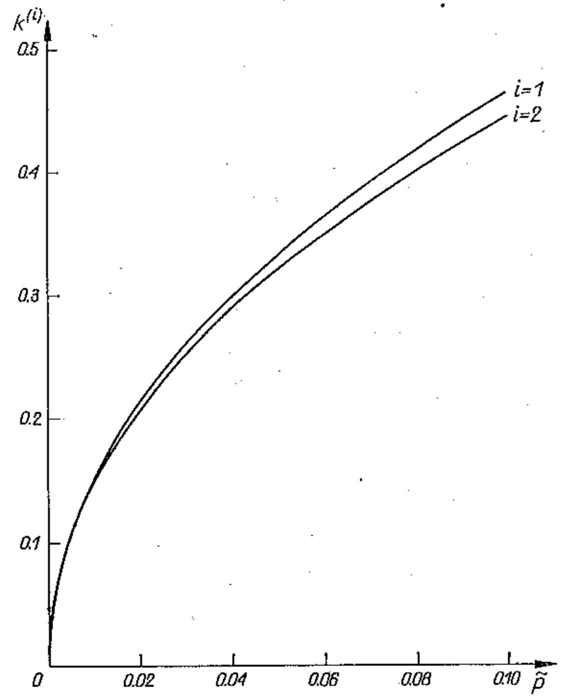

FIG. 2.

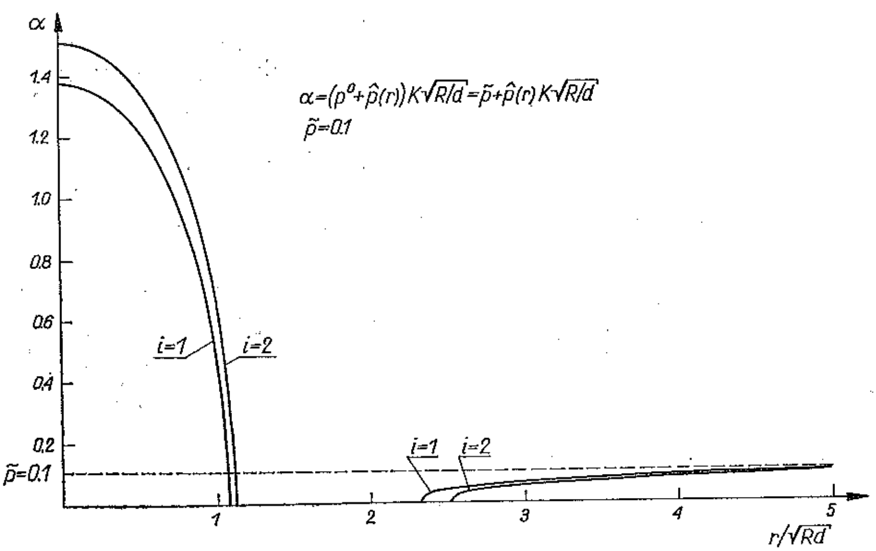

FIG. 3.

We conclude that the corrections of the 2nd to the 1st approximations are small. We took $\tilde{p}$ between 0 and 0.1. The values of $\tilde{p}$ between 0 and 0.01 are of more practical interest. In this case the derivations will be very small.

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Streszczenie

O TRÓJWYMIAROWYM PROBLEMIE KONTAKTOWYM DLA SZTYWNEJ INKLUZJI ŚCISKANEJ POMIĘDZY DWOMA SPRĘŻYSTYMI PÓŁPRZESTRZENIAMI

Znaleziono przybliżone rozwiązanie trójwymiarowego problemu kontaktowego, w którym dwie sprężyste półprzestrzenie rozdzielone sztywną inkluzją są wzajemnie ściskane. Zakłada się identyczne własności obydwu półprzestrzeni oraz obrotową symetrię inkluzji.

Otrzymuje się rozwiązanie asymptotyczne, gdzie jako mały parametr przyjmuje się stosunek promienia obszaru kontaktu z inkluzją do promienia obszaru wzajemnego kontaktu półprzestrzeni. Wielkości te mogą być obliczone gdy znane są ciśnienia w nieskończoności geometrii inkluzji. Rozwiązując tę część problemu określić można ciśnienie na powierzchni.

Otrzymano liczbowe wyniki dla małego parametru oraz ciśnienia powierzchniowego.

Резюме

О ТРЕХМЕРНОЙ КОНТАКТНОЙ ЗАДАЧЕ ДЛЯ ЖЕСТКОГО ВКЛЮЧЕНИЯ СЖИМАЕМОГО МЕЖДУ ДВУМЯ УПРУГИМИ ПОЛУПРОСТРАНСТВАМИ

Найдено приближенное решение трехмерной контактной задачи, в которой два упругих полупространства, разделенные жестким включением, взаимно сжимаются. Предполагаются идентичные свойства обоих полупространств и вращательную симметрию включения. Получается асимптотичекое решение, где как малый параметр принимается отношение радиуса области контакта с включением к радиусу области взаимного контакта популпространств. Эти величины могут быть вычислены зная давление в бесконечности для включения. Решая эту часть задачи можно определить давление на поверхьости. Получены числовые результаты для малого параметра, а также для поверхностного давления.

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