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NORMAL PENETRATION OF THE RIGID PENETRATOR INTO ELASTIC-PLASTIC HALF-SPACE WITH VISCOSITY

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The analytical closed-form solution of the normal penetration problem of a rigid core (penetrator) of a jacket-bullet into elastic-plastic half-space (thick target) is presented in this paper. The cohesive resistance of the target, frictional effects, and acceleration of the target material in the neghbourhood of the penetrator (virtual mass effect) is taken into consideration. On the basis of the derived closed analytical formulae, influence of these parameters on the penetration depth may be investigated. Among other things, it appears from calculations that the boundary penetration depth is intensivly reduced by the force of viscous friction.

Moreover, the mean value of the coefficient of viscous friction (μ) for a given penetrator-target system may be determined. For this purpose it is necessary to define experimentally the penetration depth h_k at a given impact velocity.

Key words: terminal ballistics, normal penetration, jacket-bullet, rigid penetrator.

1. Introduction

A penetration of projectiles into various targets has been a vast field of theoretical and experimental investigations for many scientists. Wide reviews of the literature concerning this question have been presented in the papers [1, 2].

It is well known that one of the principal parameters which bounds the penetration depth of projectile into a target, is the impact velocity. The impact velocity values of small arms projectiles are contained within the interval 100÷1000 m/s. In this interval of the impact velocity, the ogival penetrator, made of a sintered carbide or of the special-purpose steel, keeps primary shape during of the penetration into the soft metal target. This fact is shown in Fig. 1. The braked undeformed core, and the pulled-off core and pressed jacket of the bullet is depicted in this figure. In these cases, the penetrator (core) can be approximated by a rigid body.

It results from the scientific literature that axial resisting force of the target, acting on a unit of the penetrator cross-section, is defined by the following formula [1, 2]:

$$\sigma = c + bv + av^2. \tag{1.1}$$

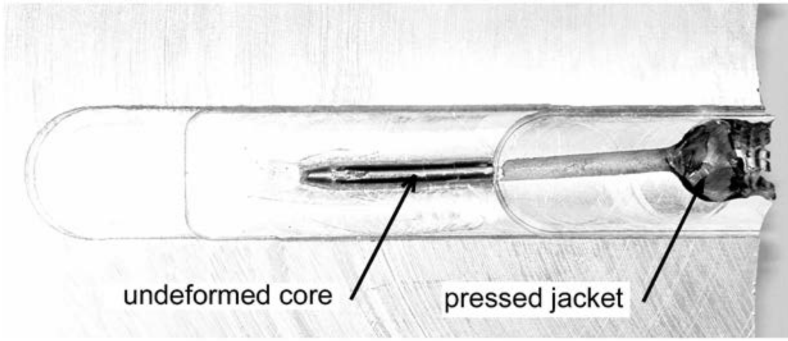

Fig. 1. The braked core (sintered carbide) of the jacket bullet (calibre: 7.62 mm) in the duralumin (PA6) target. Impact velocity $V = 812$ m/s.

The terms on the right-hand side of Eq. (1.1) are associated with the cohesive resistance of the target, frictional effects, and the acceleration of target material in the neighbourhood of the projectile (virtual mass effect), respectively.

The problem of penetration of the rigid projectile into a metal target without the second term of the formula (1.1) was considered [1–5]. Analytical solution of this problem for complete formula (1.1), and analysis of the dependence of the penetration depth on the respective terms of the formula (1.1) have been presented in this paper.

2. Formulation of the problem

We assume that a rigid core of the jacket bullet is a penetrator. The jacket is separated from the core during embedding of the projectile into a target. Subsequently the free core penetrates the target, as a rigid penetrator.

Normal penetration of the rigid penetrator into metallic semi-infinite space (thick target) is considered in this paper. The penetrator has rotational symmetry. Direction of the penetrator velocity overlaps its axis and is perpendicular to the plane of the semi-infinite space. On the penetrator acts a unit axial resisting force of the target, which is defined by the formula (1.1).

Under these assumptions, the penetrator's motion into metal target can be described by the following differential equation:

$$\rho_p L \frac{d\upsilon}{dt} = -\left(a\upsilon^2 + b\upsilon + c\right), \tag{2.1}$$

and by the initial condition

$$\upsilon(0) = V, \tag{2.2}$$

where symbols L, ρ_p and V stand for the penetrator length, density of the penetrator material and impact velocity, respectively.

According to the data given in [3–5], the coefficients a and c can be described by the following expressions:

$$a = k\rho_t, \qquad c = H_t, \tag{2.3}$$

where symbols H_t, ρ_t and k denote the dynamic hardness of the target material, density of the target material and coefficient of the nose shape of the penetrator, respectively. The value of the coefficient k can be estimated by means of the formula [6, 7]:

$$k = 1 - \frac{1}{8}\left(\frac{D}{R}\right)^2, \tag{2.4}$$

where symbols D and R stand for diameter of the greatest cross-section of the penetrator and the radius of a sphere which approximates the contact space of the penetrator nose with crater bottom, respectively.

The symbol b denotes the coefficient of the mean absolute viscosity.

We introduce the following dimensionless quantity:

$$\mu = \frac{b}{\rho_t V}. \tag{2.5}$$

The quantity μ is termed as the mean coefficient of a viscous friction.

3. Solution of the problem

On separating the variables υ and t in Eq. (2.1) and integrating, we obtain:

$$\int \frac{\mathrm{d}\upsilon}{a\upsilon^2 + b\upsilon + c} = -\frac{t}{\rho_p L} + C, \tag{3.1}$$

where an integration constant C is defined by the initial condition (2.2).

The left-hand side of Eq. (3.1) can be expressed by the following functions:

$$\frac{1}{\sqrt{\Delta}} \ln\left|\frac{2a\upsilon + b - \sqrt{\Delta}}{2a\upsilon + b + \sqrt{\Delta}}\right| \qquad \text{for} \quad b^2 > 4ac, \tag{3.2}$$

$$\frac{2}{\sqrt{-\Delta}} \operatorname{arc\,tg} \frac{2a\upsilon + b}{\sqrt{-\Delta}} \qquad \text{for} \quad b^2 < 4ac, \tag{3.3}$$

$$-\frac{2}{2a\upsilon + b} \qquad \text{for} \quad b^2 = 4ac, \tag{3.4}$$

where

$$\Delta = b^2 - 4ac. \tag{3.5}$$

It results from the above relationships that the considered problem has three different solutions, which are determined by relations between terms b^2 and $4ac$.

3.1. Solution of the problem for large absolute viscosity, i.e.:
$b^2 > 4ac$ or $\mu^2 > (4kH_t/\rho_t V^2) = \mu_0^2$

From expressions (3.1) and (3.2), as well as the initial condition (2.2), after simple transformation, we obtain:

$$\bar{\upsilon}(\eta) = \frac{\mu}{2k}\left[(1+c_1)\frac{\exp(-a_1\eta)}{b_1-\exp(-a_1\eta)} - (1-c_1)\frac{b_1}{b_1-\exp(-a_1\eta)}\right], \tag{3.6}$$

where:

$$\begin{gathered} \eta = \frac{Vt}{L}, \qquad \bar{\upsilon} = \frac{\upsilon}{V}, \qquad \mu = \frac{b}{\rho_t V}, \qquad a_1 = \mu c_1 \frac{\rho_t}{\rho_p}, \\ b_1 = \frac{2k+\mu(1+c_1)}{2k+\mu(1-c_1)}, \qquad c_1 = \sqrt{1-4k\frac{H_t}{\mu^2\rho_t V^2}} = \sqrt{1-\left(\frac{\mu_0}{\mu}\right)^2}. \end{gathered} \tag{3.7}$$

It follows from Eq. (3.6) that the penetrator is completely braked ($\upsilon = 0$) after time $t_k = (L/V)\eta_k$, where

$$\eta_k = \frac{1}{a_1}\ln\frac{1+c_1}{(1-c_1)b_1}. \tag{3.8}$$

The current penetration depth of the penetrator into the target is defined by the following integral:

$$l(t) = \int_0^t \upsilon(\tau)d\tau. \tag{3.9}$$

On substituting the relation (3.6) into Eq. (3.9) and integrating, we obtain:

$$h(\eta) = \frac{\mu}{2k}\left[\frac{2c_1}{a_1}\ln\frac{b_1-\exp(-a_1\eta)}{b_1-1} - (1-c_1)\eta\right], \tag{3.10}$$

where

$$h = \frac{l}{L}.$$

The boundary (maximal) penetration depth of the penetrator into the target is defined by the formula:

$$h_k = h(\eta_k) = \frac{\mu}{2k}\left[\frac{2c_1}{a_1}\ln\frac{b_1-\exp(-a_1\eta_k)}{b_1-1} - (1-c_1)\eta_k\right]. \tag{3.11}$$

3.2. Solution of the problem for small viscosity, i.e.: $b^2 < 4ac$ or $\mu^2 < (4kH_t/\rho_t V^2) = \mu_0^2$

In this case, from expressions (3.1) and (3.3), as well as (2.2) we get:

$$\bar{v}(\eta) = \frac{1}{2k}\left[c_2 \mathrm{tg}\left(\mathrm{arc\ tg}\frac{2k+\mu}{c_2} - \frac{c_2}{2}\frac{\rho_t}{\rho_p}\eta\right) - \mu\right], \tag{3.12}$$

$$\eta_k = \frac{2}{c_2}\frac{\rho_p}{\rho_t}\left(\mathrm{arc\ tg}\frac{2k+\mu}{c_2} - \mathrm{arc\ tg}\frac{\mu}{c_2}\right), \tag{3.13}$$

where

$$c_2 = \sqrt{4k\frac{H_t}{\rho_t V^2} - \mu^2} = \sqrt{\mu_0^2 - \mu^2}.$$

Substituting relationship (3.12) into Eq. (3.9) and integrating, we have:

$$h(\eta) = \frac{1}{k}\frac{\rho_p}{\rho_t}\left[\ln\left|\frac{\cos\alpha(\eta)}{\cos\alpha_0}\right| - \frac{\mu}{2}\frac{\rho_t}{\rho_p}\eta\right], \tag{3.14}$$

where

$$\begin{aligned} \alpha(\eta) &= \mathrm{arc\,tg}\frac{2k+\mu}{c_2} - \frac{c_2}{2}\frac{\rho_t}{\rho_p}\eta, \\ \alpha_0 &= \mathrm{arc\,tg}\frac{2k+\mu}{c_2}. \end{aligned} \tag{3.15}$$

The boundary penetration depth is defined in this case by the formula:

$$h_k = h(\eta_k) = \frac{1}{k}\frac{\rho_p}{\rho_t}\left[\ln\left|\frac{\cos\alpha(\eta_k)}{\cos\alpha_0}\right| - \frac{\mu}{2}\frac{\rho_t}{\rho_p}\eta_k\right], \tag{3.16}$$

where

$$\alpha(\eta_k) = \mathrm{arc\,tg}\frac{2k+\mu}{c_2} - \frac{c_2}{2}\frac{\rho_t}{\rho_p}\eta_k = \mathrm{arc\,tg}\frac{\mu}{c_2}. \tag{3.17}$$

It results from formulae (3.15) and (3.17) that for $\mu = 0$ [lack of the viscosity term in the expression (1.1)], there are:

$$\begin{aligned} \alpha_0 &= \mathrm{arc\,tg}\frac{2k}{\sqrt{4kH_t/\rho_t V^2}} = \sqrt{k\frac{\rho_t V^2}{H_t}}, \\ \alpha(\eta_k) &= 0. \end{aligned} \tag{3.18}$$

Substituting the formulae (3.18) into the relation (3.16) and taking into consideration the following trigonometric identity:

$$\operatorname{arc\,tg} x = \arccos\left(1/\sqrt{1+x^2}\right),$$

we get:

$$h_k = \frac{1}{2k}\frac{\rho_p}{\rho_t}\ln\left(1 + k\frac{\rho_t V^2}{H_t}\right). \tag{3.19}$$

Such expression has been derived in papers [3–5].

3.3. Singular solution: $b^2 = 4ac$ *or* $\mu^2 = \mu_0^2 = 4kH_t/\rho_t V^2$

In accordance with the formulae (3.1), (3.4) and (2.2) we obtain:

$$\bar{v}(\eta) = \frac{1}{2k}\left[\frac{2(2k+\mu_0)}{2 + (\rho_t/\rho_p)(2k+\mu_0)\eta} - \mu_0\right]. \tag{3.20}$$

In this case, the penetrator is completely braked on the lapsing of time

$$\eta_k = \frac{V t_k}{L} = \frac{4k}{\mu_0(\rho_t/\rho_p)(2k+\mu_0)}. \tag{3.21}$$

After substitution of the relationship (3.20) into integral (3.9) and integrating, we have:

$$h(\eta) = \frac{l(\eta)}{L} = \frac{1}{k}\frac{\rho_p}{\rho_t}\left[\ln\left(1 + \frac{2k+\mu_0}{2\rho_p/\rho_t}\eta\right) - \frac{\mu_0}{2\rho_p/\rho_t}\eta\right]. \tag{3.22}$$

The boundary penetration depth in this case is:

$$h_k = h(\eta_k) = \frac{1}{k}\frac{\rho_p}{\rho_t}\left[\ln\left(1 + \frac{2k}{\mu_0}\right) - \frac{2k}{2k+\mu_0}\right]. \tag{3.23}$$

Thus, we obtain the analytical solution of the examined problem for each of the mean values of the viscous friction coefficient μ.

4. Preliminary analysis of the boundary penetration depth

It follows from the above derived formulae, that normal penetration depth of the core of the jacket projectile into metal half-space depends on the following parameters: dynamic hardness and density of target metal (H_t, ρ_t), impact velocity (V), unit mass of the penetrator ($\rho_p L$), nose shape of the penetrator (k), and the mean coefficient of the viscous friction (μ).

The preliminary analysis of the boundary penetration depth h_k has been made for the following materials:

penetrator: sintered metal (WHA), $\rho_p = 17200$ kg/m^3;

target: 1. Ingot iron, $\rho = 7850$ kg/m^3, $H_t = 1638$ MPa;
2. Aluminium, $\rho = 2700$ kg/m^3, $H_t = 260$ MPa;
3. Duralumin, $\rho = 2800$ kg/m^3, $H_t = 1300$ MPa.

Two values of the coefficient k were assumed, namely:

$k = 0.5$ – hemispherical – ended projectile,

$k = 1.0$ – flat – ended projectile.

Some calculation results are depicted in Figs. 2–5. It seems that the qualitative variations of the boundary penetration depth h_k versus coefficient μ are similar for various materials of the target. On the contrary, significant quantitative differences occur among curves $h_k(\mu)$ plotted for targets made from various metals. These variations are visible in the Figs. 2–5. For example, the boundary penetration depth into aluminium target is about five times greater (at $\mu \approx 0$) than into the duralumin one (Fig. 5). This difference decreases when the coefficient μ increases.

The penetration depth is intensively reduced by increase of the coefficient μ. The highest gradient decreasing of the penetration depth occurs in the initial increase stage of the coefficient μ. The viscous friction decreases the penetration depth several times in comparison with the frictionless penetration.

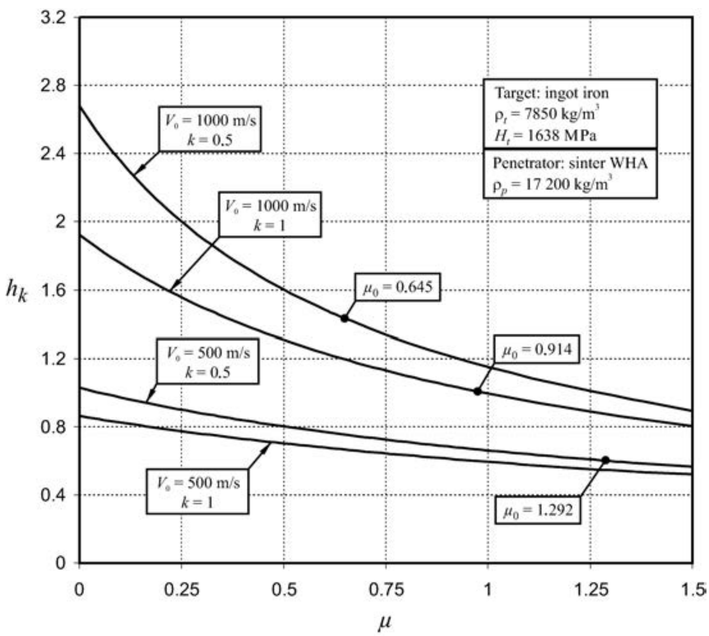

FIG. 2. Variation of quantity h_k as a function of coefficient μ.

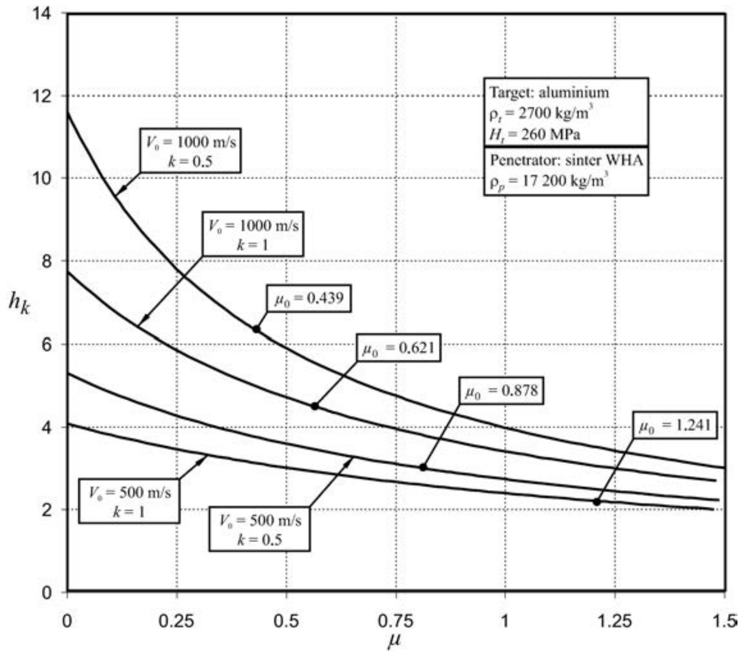

FIG. 3. Variation of quantity h_k as a function of coefficient μ.

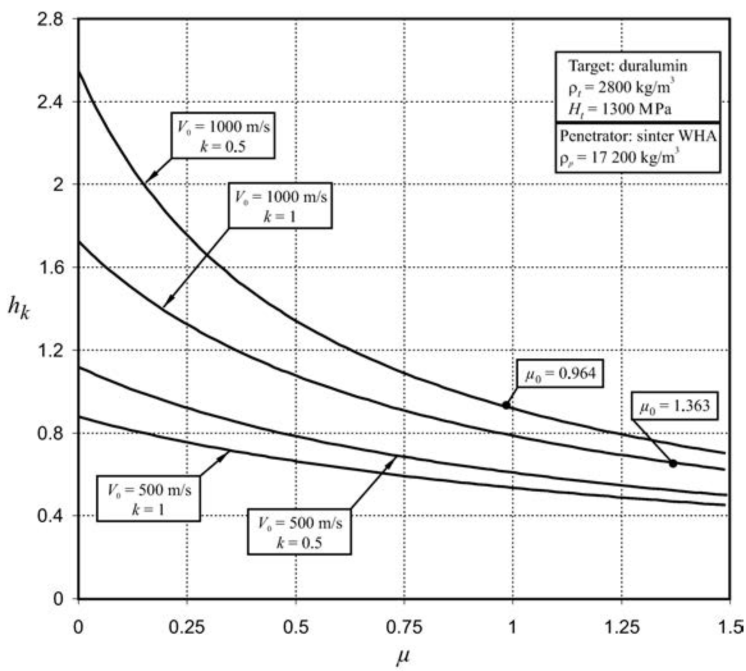

FIG. 4. Variation of quantity h_k as a function of coefficient μ.

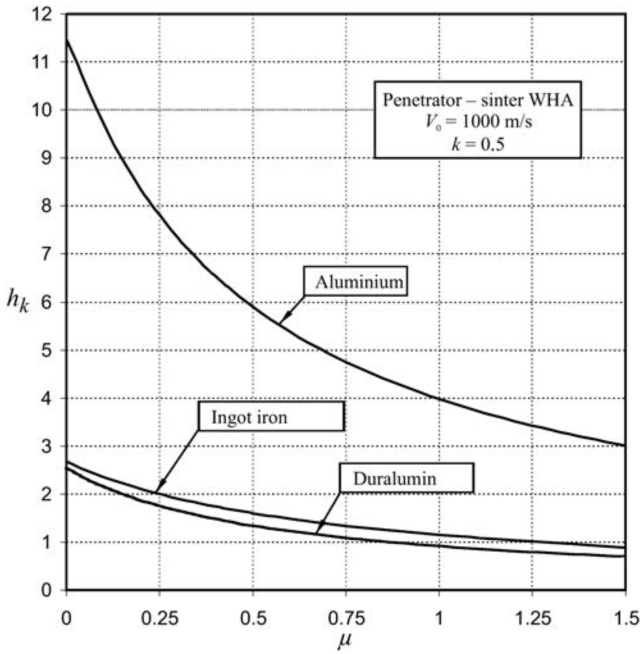

FIG. 5. Variation of quantity h_k as a function of coefficient μ for the three target-penetrator systems.

Let us observe conclusion that for a given penetrator-target system, the mean value of the coefficient μ may be defined by means of the formulae (3.11) or (3.16). For this purpose it is necessary to define experimentally the quantity h_k at the given impact velocity and then the mean value of the coefficient μ can be determined.

5. Conclusions

The analytical model of normal penetration of the rigid penetrator into the thick metal target has been presented in this paper. In this model, cohesive resistance of the target, frictional effects, and the acceleration of target material in the neighbourhood of the penetrator (virtual mass effect), have been taken into consideration. On the basis of the derived closed analytical solution, the influence of the respective terms of the formulae (1.1) on the penetration depth is investigated.

It is found that the penetration depth is intensively reduced by the forces of viscous friction. By means of this model, the mean value of the coefficient of viscous friction (μ) for a given penetrator-target system may be also determined.

Preliminary experimental investigations support the usefulness of the presented theoretical model in the calculations of the terminal ballistics.

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