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COMPRESSION EFFECTS IN STRUCTURAL DAMPING IN SANDWICH PLATES

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In the present paper, we study the thickness-stretch deformation effect in addition to other effects (bending, extentional and thickness-shear) already investigated in the previous paper [1]. An improved theory of sandwich plates is thus established. Numerical results show for small face thickness ratios, there is a rather notable discrepancy between the present theory and that presented in [1], with regard to the damping parameter.

NOTATIONS

$\vee$ sign denoting complex quantity,
$,$ differentation w.r.t. the variable standing after the comma,
t time,
$i=1, 2, 3$ subscript corresponding to upper face, core and lower face, respectively,
a, b dimensions of the sandwich plate in x- and y-directions, respectively,
u, v, w displacement components in x-, y- and z-directions, respectively,
β_x, β_y angles of rotation of the lateral side of the considered layer under deformation,
h total thickness of the sandwich structure,
t_i thickness of the i-th layer,
$k=t_2/t_1$ core thickness ratio,
$m=t_3/t_1$ face thickness ratio,
E_x, E_y storage moduli for flexure and extension in x- and y-directions, respectively,
ν_x, ν_y the Poisson's ratios in x- and y-directions, respectively,
E_{xy}, E_{xz}, E_{yz} storage shear moduli in xy-, xz-, and yz-planes, respectively,
$n=E_3/E_1$ modulus ratio of the faces,
$n_2=E_2/E_1$ modulus ratio of the core,
N_x, N_y normal forces in x- and y-directions, respectively,
S longitudinal shearing force,
H twisting moment,
G_x, G_y bending moments,
Q_x, Q_y transverse shearing forces,
σ_k normal stress,
τ_{kl} shearing stress,
e_x, e_y normal strains in x- and y-directions, respectively,
e_{xy}, e_{xz}, e_{yz} shearing strains in xy-, xz- and yz-planes, respectively,
$s_1=E_1/(1-\nu_2^2)$ stiffness coefficient of the upper face,
$B_1=E_1\, t_1/(1-\nu_1^2)$ extentional stiffness of the upper face,

$D_1=E_1 t_1^3/(1-\nu_2^2)$	bending stiffness of the upper face,
M_i	mass of the i-th layer per unit area,
$M=M_1+M_2+M_3$	mass of the sandwich structure per unit area,
γ_i	mass density of the i-th layer material,
$\theta_2=\gamma_2/\gamma_1$	mass density ratio of the core material,
$\theta_3=\gamma_3/\gamma_1$	mass density ratio of the faces material,
$d=1+\theta_2 k+\theta_3 m$	mass density ratio of the sandwich structure per unit area,
G_2	static elastic transverse modulus of the core,
μ_i	material loss factor ($i=0, 1, 2, 3, 4$),
$l^2=(i\pi/a)^2+(j\pi/b^2)$,	
$\Lambda=\sqrt{\lambda}=t_1 l$	wave number parameter,
$\varepsilon_i=\mu_i/\mu_0$	loss factor ratio,
$\psi_1=(1-\nu_1^2) G_2/\lambda E_1$	shear parameter,
ω	frequency,
$\Omega=\gamma_1 t_1^2 \omega^2/s_1$	frequency parameter,
δ	logarithmic decrement,
δ^*	damping parameter.

1. Introduction

In [1], only the bending, extensional and thickness-shear effecte are of concern in the study of damped vibrations of sandwich plates. In reality, a perfect rigidity of the core in the transversal direction cannot be ensured. Therefore, in the motion of the sandwich plate the thickness-stretch deformation vibrations should be taken into account. Yu in [2] discussed this subject but only considered the extensional motion of the sandwich plate and its corresponding frequency. In the present paper the compression effect in the structural damping in flexural motion is analysed and compared to the common theory, where a perfect transversal rigidity is assumed, and thus an improved theory can be etablished.

2. Basic assumptions

We assume the following:

(I) The material of the facings as that of the core is considered to be homogeneous and isotropic.

(II) The Kirchhof-Love's assumption holds true only for the facings.

(III) A line crossing the underformed core remains straight under deformation but not necessarily perpendicular to the midplane of the core.

(IV) No slip will occur at the contact surface between core and facings.

(V) The variation of the transverse displacement w_2 through the core is linear. As Fig. 1 illustrates, this assumption is formulated by

$$w_2^* = a_w z + w_2 , \tag{2.1}$$

where

$$a_w = \bar{w}_2/t_2 , \qquad \hat{w}_2 = w_2/2 . \tag{2.2}$$

The quantities $\bar{w}_2$ and $\hat{w}_2$ are the thickness variation of the core and the deflection at the midplane, respectively; they may be expressed in terms of the deflections w_1 and w_3 at the face midplanes as follows:

(2.3) $$\bar{w}_2 = w_1 - w_3 , \quad \hat{w}_2 = w_1 + w_3$$

or inversely:

(2.4) $$w_1 = (\hat{w}_2 + \bar{w}_2)/2 , \quad w_3 = (\hat{w}_2 - \bar{w}_2)/2 .$$

The deflection at any point in each face is assumed to be constant through its respective thickness.

Equations of motion

According to the above assumptions the displacement-strain relations are thus at the face midplane ($i=1, 3$):

(3.1) $$\begin{gathered} e_{xi} = u_{i,x} , \quad e_{yi} = v_{i,y} , \quad e_{xyi} = u_{i,y} + v_{i,x} , \\ h_{xi} = -0.5 (\hat{w}_2 \pm \bar{w}_2)_{,xx} , \quad h_{yi} = -0.5 (\hat{w}_2 \pm \bar{w}_2)_{,yy} , \\ h_{xyi} = -(\hat{w}_2 \pm \bar{w}_2)_{,xy} , \quad e_{zi} = 0 , \end{gathered}$$

at any core layer:

(3.2) $$\begin{gathered} \overset{*}{e}_{x2} = \overset{*}{u}_{2,x} , \quad \overset{*}{e}_{y2} = \overset{*}{v}_{2,y} , \quad \overset{*}{e}_{xy2} = \overset{*}{u}_{2,y} + \overset{*}{v}_{2,x} , \\ \overset{*}{e}_{xz2} = \overset{*}{u}_{2,z} + \overset{*}{w}_{2,x} , \quad \overset{*}{e}_{yz2} = \overset{*}{v}_{2,2} + \overset{*}{w}_{2,y} , \quad \overset{*}{e}_{z,2} = \overset{*}{w}_{2,z} , \\ h^*_{x2} = -\beta_{x2,x} , \quad h^*_{yx2} = -\beta_{x2,y} , \quad h^*_{yx2} = -\beta_{x2,y} + \beta_{y2,x} \end{gathered}$$

henceforth, the star indicates that the considered quantity corresponds to any layer of the core

Moreover, the longitudinal diplacements u_i, v_i ($i=1, 3$) at the midplane of each outer layer are related with those at the core midplane (Fig. 1) as follows:

(3.3) $$\begin{gathered} u_i = u_2 \mp 0.5 t_2 \beta_{x2} \mp 0.25 t_i (\hat{w}_{2,x} \pm \bar{w}_{2,x}) , \\ v_i = v_2 \mp 0.5 t_2 \beta_{y2} \mp 0.25 t_i (\hat{w}_{2,y} \pm \bar{w}_{2,y}) , \end{gathered}$$

while the longitudinal displacements at any layer of the core u_2 and v_2 are

(3.4) $$\overset{*}{u}_2 = -\beta_{x2} z + u_2 , \quad \overset{*}{v}_2 = -\beta_{y2} z + v_2 .$$

In Eqs. (3.2) and (3.3) the notation $\mp$ or $\pm$ means that the upper sign is used when $i=$ $=1$, while the lower sign corresponds to $i=3$.

The constitutive equations for the faces are assumed to be as follows:

(3.5) $$\begin{gathered} N_{xi} = \frac{E_i t_i}{1-\nu_i^2} (e_{xi} + \nu_i e_{yi}) , \quad N_{yi} = \frac{E_i t_i}{1-\nu_i^2} (e_{yi} + \nu_i e_{xi}) , \\ S_i = \frac{E_i t_i}{2(1+\nu)} e_{xyi} , \quad H_{xyi} = \frac{E_i t_i^3}{24(1+\nu_i)} h_{xyi} , \\ G_{xi} = \frac{E_i t_i^3}{12(1-\nu_i^2)} (h_{xi} + \nu_i h_{yi}) , \quad G_{yi} = \frac{E_i t_i^3}{12(1-\nu_i^3)} (h_{yi} + \nu_i h_{xi}) . \end{gathered}$$

The core is assumed to be isotropic in the plane xy and from [3] the stress-strain relations are then

(3.6)
$$\sigma_{x2}=c_1 e_{x2}+c_2 e_{y2}+c_3 e_{y2},$$
$$\sigma_{y2}=c_2 e_{x2}+c_1 e_{y2}+c_3 e_{z2},$$
$$\sigma_{z2}=c_3(e_{x2}+e_{y2})+c_4 e_{z2},$$
$$\tau_{yz2}=G_2 e_{yz2}, \quad \tau_{xz2}=G_2 e_{xz2}, \quad \tau_{xy2}=E_{xy2} e_{xy2},$$

where the coefficients c_1, c_2, c_3 are

(3.7)
$$c_1=E_2\frac{1-\nu_{z2}^2 n_z}{(1+\nu_2)(1-\nu_2-2\nu_{z2}^2 n_{z2})}, \quad c_2=E_2\frac{\nu_2+\nu_{z2}^2 n_{n2}}{(1+\nu_2)(1-\nu_2-2\nu_{z2}^2 n_{z2})},$$
$$c_3=\frac{E_2\nu_{z2}}{1-\nu_2-2\nu_{z2}^2 n_{z2}}, \quad c_4=E_2\frac{1-\nu_2}{n_{z2}(1-\nu_2-2\nu_{z2}^2 n_{z2})},$$
$$E_{xy2}=\frac{c_1-c_2}{2}=\frac{E_2}{2(1+\nu_2)}.$$

with: E_2 and ν_2—the Young's modulus and the Poisson's ratio in the plane xy, respectively; ν_{z2}—the Poisson's ratio of the deformation in the plane xy for extension

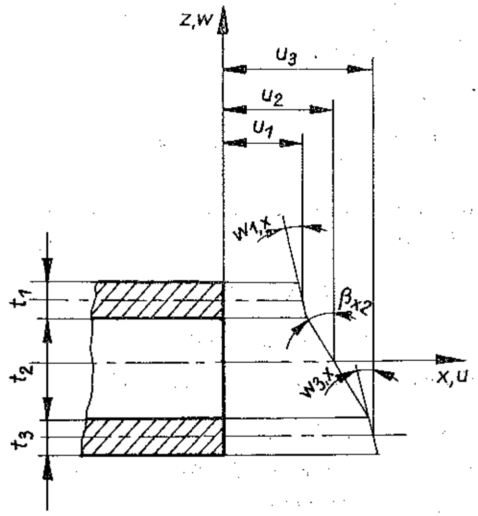

FIG. 1.

in the z-direction; $n_{z2}=E_2/E_{z2}$—ratio of the Young's modulus in the isotropy plane to that in the perpendicular direction.

The equations governing the motion of the sandwich plate are derived by using a variational procedure.

We have

(3.8)
$$\delta\int_{t_0}^{t_1}\sum_{i=1,2,3} L_i\,dt=0,$$

where

$$L_i = V_i - K_i - T_i . \tag{3.9}$$

For the faces, the variation of strain energy is

$$V_i = \int_{x_1}^{x_2}\int_{y_1}^{y_2} (N_{xi}\,\delta e_{xi} + N_{yi}\,\delta e_{yi} + S_i\,\delta e_{xyi} + G_{xi}\,\delta h_{xi} + G_{yi}\,\delta h_{yi} + H_i\,\delta h_{xyi})\,dx\,dy \qquad (i=1,3). \tag{3.10}$$

For the core, the strain energy variation is:

$$V_2 = \int_{x_1}^{x_2}\int_{y_1}^{y_2}\int_{-t_2/2}^{t_2/2} (\sigma^*_{x2}\,\delta e^*_{x2} + \sigma_{y2}\,\delta e^*_{y2} + \sigma_{z2}\,\delta e^*_{z2} + \tau^*_{xz2}\,\delta e^*_{xz2} + \tau^*_{xy2}\,\delta e^*_{xy2} + \tau^*_{xy2}\,\delta e^*_{xy2})\,dx\,dy\,dz , \tag{3.11}$$

where σ^*_i, τ^*_{ij}, e^*_i are respectively, direct, shear stresses and strain at an arbitrary point of the core thickness.

The variation of the kinetic energy of the faces are $(i=1, 3)$

$$\gamma_i t_i \int_{x_1}^{x_2}\int_{y_1}^{y_2} \Big[u_{i,t}\,\delta u_{i,t} + v_{i,t}\,\delta v_{i,t} + w_{i,t}\,\delta w_{i,t} + \frac{t_1^2}{12}(w_{i,xt}\,\delta w_{i,xt} + w_{i,yt}\,\delta w_{i,yt})\Big]\,dx\,dy . \tag{3.12}$$

For the core, the kinetic energy variation is

$$\gamma_2 t_2 \int_{x_1}^{x_2}\int_{y_1}^{y_2} \Big[u_{2,t}\,u_{2,t} + v_{2,t}\,v_{2,t} + w_{2,t}\,w_{2,t} + \frac{t_2^2}{12}(g_{x2,t}\,\delta g_{x2,t} + g_{y2,t}\,\delta y_{y2,t} + a_{w,t}\,\delta a_{w,t})\Big]\,dx\,dy . \tag{3.13}$$

Assuming that only transverse loads are acting on the sandwich plate, the variation of the external force is then

$$\delta T = \int_{x_1}^{x_2}\int_{y_1}^{y_2} \sum_{i=1,3} p_{zi}\,\delta w_i\,dx\,dy . \tag{3.14}$$

Now let use the relations (3.1) and (3.3) for the calculation of Eq. (3.10), the relations (3.2) and (3.6) together with Eqs. (2.1) and (3.4) to compute Eq. (3.11). Similar operations are performed for the expressions (3.12), (3.13) and (3.14). Next, let us perform integration by parts wherever partial derivatives are involved in the variations of the variable. Finally, from the requirement (3.8) and since δu_2, δv_2, $\delta\beta_{x2}$, $\delta\beta_{y2}$, δw_2 and $\delta\bar{w}_2$ are arbitrary values, it follows that we obtain such a set of equations:

$$\begin{aligned} &-(N_{x1}+N_{x2})_{,x} - (S_1+S_3)_{,y} - (c_1 u_{2,xx} + c_2 v_{2,xy} + c_3 w_{2,x} t_2^{-1})\,t_2 - \\ &\quad - E_{xy2}\,t_2\,(u_{2,yy} + v_{2,xy}) + M_1 u_{1,tt} + M_2 u_{2,tt} + M_3 u_{3,tt} = 0; \\ &-(N_{y1}+N_{y3})_{,y} - (S_1+S_3)_{,x} - (c_2 u_{2,xy} + c_1 u_{2,xy} + c_{1} v_{2,yy} + c_3 w_{2,y} t_2^{-1})\,t_2 - \\ &\quad - E_{xy2}\,t_2\,(u_{2,xy} + v_{2,xx}) + M_1 v_{1,tt} + M_2 v_{2,tt} + M_3 v_{3,tt} = 0 , \end{aligned} \tag{3.15}$$

(3.15) [cont.]

$$(N_{x1}-N_{x3}),_{,x}+(S_1-S_3)_{,y}-(c_1\beta_{x2,xx}+c_2\beta_{y2,xy})\frac{t_2^2}{6}-$$
$$-\frac{t_2^2}{6}E_{xy2}(\beta_{x2,yy}+\beta_{y2,xy})+2G_2(\beta_{x2}-w_{2,x})-M_1u_{1,tt}+M_3u_{3,tt}+$$
$$+\frac{t_2^2}{6}M_2\beta_{x2,tt}=0,$$

$$(N_{y1}-N_{y3})_{,y}+(S_1-S_3)_{,x}-\frac{t_2^2}{6}(c_2\beta_{x2,xy}+c_1\beta_{y2\,y})-$$
$$-\frac{t_2^2}{6}E_{xy2}(\beta_{x2,yy}+\beta_{y2,xy})+2G_2(\beta_{y2}-w_{2,y})-M_1v_{1,tt}+$$
$$+M_3v_{3\,tt}+\frac{t_2^2}{6}M_2\beta_{y2,tt}=0.$$

$$-0{,}5(t_1N_{x1}-t_3N_{x3})_{,xx}-0.5(t_1N_{y1}-t_3N_{y3})_{,yy}-(t_1S_1-t_3S_3)_{,xy}-$$
$$-(G_{x1}+G_{x3})_{,xx}-2(H_1+H_3)_{,xy}+t_2G_2(\beta_{x2,x}-w_{2,xx})+$$
$$t_2G_2(\beta_{y2,y}-w_{2,yy})-\frac{\bar{t}}{4}(u_{2,x}+v_{2,y})_{,tt}+\frac{M_2}{24}tt_2(\beta x_{2,x}+\beta_{y2,y})_{,tt}-$$
$$-\left[\frac{M_1t_1^2}{12}(1+\theta_3m^2)\nabla^2-M\right]w_{2,tt}-\left[\frac{t_1^2}{12}(1-\theta_3m^2)\nabla^2-\right.$$
$$\left.-(1-\theta_3m)\right]\frac{M_1}{2}\bar{w}_{2,tt}=(p_{z1}+p_{z3}),$$

$$-0.25(t_1N_{x1}+t_3N_{x3})_{,xx}-0.25(t_1N_{y1}+t_3N_{y3})_{yy}-0.5(t_1S_1+t_3S_3)_{,xy}-$$
$$-0.5(G_{x1}-G_{x3})_{,xx}-0.5(G_{y1}-G_{y3})_{,yy}-(H_1-H_3)_{,xy}+$$
$$+c_3(u_{2,x}+v_{2,y})c_4\bar{w}_2\bar{t}_2^1-\frac{t_2}{12}G_2\nabla^2\bar{w}_2-\frac{\bar{t}}{8}M_2(u_{2,x}+v_{2,y})_{,tt}+$$
$$+\frac{tt_2}{48}M_2(\beta_{x2,x}+\beta_{y2,y})_{,tt}-\left[\frac{t_1^2}{12}\nabla^2(1-\theta_3m^2)-(1-\theta_3m)\right]\frac{M_1}{2}w_{2,tt}-$$
$$-\left[\frac{t_1^2}{12}\nabla^2(1+\theta_3m^2)-\left(1+\frac{k}{3}\theta_2+\theta_3m\right)\right]\frac{M_1}{4}\bar{w}_{2,tt}=0.5(p_{z1}-p_{z3}),$$

where:

(3.16)
$$t=t_1+t_3,\qquad \bar{t}=t_1-t_3.$$

Equations (3.15) are equations of equilibrium of a sandwich plate where the thickness-stretch effect of the core is included in addition to the thickness-shear, extensional and bending effects.

Substituting the internal forces in the faces (N_{ij}, S_i, G_{ij}, H_i) by the constitutive equations (3.5) into Eqs. (3.15) and taking into account Eqs. (3.2) and (3.3), the governing equations are thus obtained in terms of the displacements u, v, β_x and β_y. Let us consider the particular case where only transverse inertia forces are concerned.

By using the procedure mentioned in [1], the set of six equations (3.15) is then reduced to a set of four equations dealing with the previous variables w_2, $\bar{w}_2$ and the new variables

$$u^* = u_{2,x} + v_{2,y}, \qquad u^{**} = \beta_{x2,x} + \beta_{y2,y}. \tag{3.17}$$

Thus, the equations governing the motion of the sandwich plate are as follows:

$$\begin{aligned}
&-[B_1(1+nm)+t_2c_1]\nabla^2 u^* + 0.5t_2 B_1(1-nm)\nabla^2 u^{**} + \\
&\qquad + 0.5t_1 B_1(1-nm^2)\nabla^2 w^2 + [0.25B_1(1+nm^2)\nabla^2 - c_3]\nabla^2 \bar{w}_2 = 0,\\
&-B_1(1-nm)\nabla^2 u^* + [0.5t_2 B_1(1+nm)\nabla^2 + (t_2^2/6)c_1\nabla^2 - 2G_2]u^{**} + \\
&\qquad + [0.5t_1 B_1(1+nm^2)\nabla^2 + 2G_2]\nabla^2 w_2 + 0.25t_1 B_1(1-nm^2)\nabla^4 \bar{w}_2 = 0,\\
&-0.5t_1 B_1(1-nm^2)\nabla^2 u^* + [0.25t_1 t_2 B_1(1+nm^2)\nabla^2 + t_2 G_2]u^{**} + \\
&\qquad + \left[M\frac{\partial^2}{\partial t^2} + 4D_1(1+nm^3)\nabla^4 - t_2 G_2\nabla^2\right]w_2 + \left[\frac{\bar{M}}{2}\frac{\partial^2}{\partial t^2} + \right.\\
&\qquad \left. + 2D_1(1-nm^3)\nabla^4\right]\bar{w}_2 = p_z,\\
&-[0.25B_1 t_1(1+nm^2)\nabla^2 - c_3]u^* + (t_1 t_2/8)B_1(1-nm^2)\nabla^2 u^{**} + \\
&\qquad + \left[\frac{\bar{M}}{2}\frac{\partial^2}{\partial t^2} + 2D_1(1-nm^3)\nabla^4\right]w_2 + \left[\frac{\tilde{M}}{4}\frac{\partial^2}{\partial t^2} + \right.\\
&\qquad \left. + D_1(1+nm^3)\nabla^4 + \frac{c_4}{t_2} - \frac{t_2}{12}G_2\nabla^2\right]\bar{w}_2 = \bar{p}_z,
\end{aligned} \tag{3.18}$$

where:

$$\begin{aligned}
&M = M_1 + M_2 + M_3 = \gamma_1 t_1(1+\theta_2 k + \theta_3 m),\\
&\bar{M} = M_1 - M_3 = \gamma_1 t_1(1-\theta_3 m),\\
&\tilde{M} = M_1 + M_3 = \gamma_1 t_1(1+\theta_3 m),\\
&B_1 = E_1 t_1/(1-\nu_1^2), \qquad D_1 = E_1 t_1^3/12(1-\nu_1^2),\\
&p_z = p_{z1} + p_{z3}, \qquad \bar{p}_z = 0.5(p_{z1} - p_{z3}).
\end{aligned} \tag{3.19}$$

Let us assume that the sandwich plate is rectangular and simply supported at its edges. Thus, the response functions of u^* and u^{**} are as in [1]

$$\begin{aligned}
u^* &= \sum_i^\infty \sum_j^\infty U_{ij}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b},\\
u^{**} &= t_2^{-1}\sum_i^\infty \sum_j^\infty \bar{U}_{ij}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}.
\end{aligned} \tag{3.20}$$

For the individual upper and lower faces, according to the boundary conditions, the response functions are selected as follows:

$$w_1 = \sum_i^{\infty} \sum_j^{\infty} W_{1ij} \sin\frac{i\pi x}{a} \sin\frac{j\pi y}{b},$$

(3.21)

$$w_3 = \sum_i^{\infty} \sum_j^{\infty} W_{3ij} \sin\frac{i\pi x}{a} \sin\frac{j\pi y}{b}.$$

That leads to

$$w_2 = \sum_i^{\infty} \sum_j^{\infty} \frac{1}{2}(W_{1ij}+W_{3ij}) \sin\frac{i\pi x}{a} \sin\frac{j\pi y}{b} =$$

(3.22)

$$= \sum_i^{\infty} \sum_j^{\infty} W_{0ij} \sin\frac{i\pi x}{a} \sin\frac{j\pi y}{b},$$

$$\bar{w}_2 = \sum_i^{\infty} \sum_j^{\infty} (W_{1ij}-W_{3ij}) \sin\frac{i\pi x}{a} \sin\frac{j\pi y}{b} = \sum_i^{\infty} \sum_j^{\infty} \bar{W}_{0ij} \sin\frac{i\pi x}{a} \sin\frac{j\pi y}{b}.$$

Here and henceforth, the subscripts of the unknowns U, $\bar{U}$, W and $\bar{W}$ are omitted and implied.

Let us replace u^*, u^{**}, w_2 and $\bar{w}_2$ from Eqs. (3.21) and (3.22) into the equations of motion (3.18) and next perform some transformations; this yields the following set:

$$B_{11} U + B_{12} \bar{U} + B_{13} W + B_{14} \bar{W} = 0,$$

$$B_{12} U + B_{22} \bar{U} + B_{23} W + B_{24} \bar{W} = 0,$$

(3.23)

$$B_{13} U + B_{23} \bar{U} + \left(\frac{t_1 M}{s_1 \lambda^2} \frac{\partial^2}{\partial t^2}\right) W + \left(-\frac{t_1 \bar{M}}{2 s_1 \lambda^2 \partial t^2} + B_{34}\right) \bar{W} = -p_z/s_1 \lambda,$$

$$B_{14} U + B_{24} \bar{U} + \left(-\frac{t_1 \bar{M}}{2 s_1 \lambda^2} \frac{\partial^2}{\partial t^2} + B_{43}\right) W + \left(\frac{t_1 \bar{M}}{4 s_1^2 \lambda^2} \frac{\partial^2}{\partial t^2} + B_{44}\right) \bar{W} = \frac{\bar{p}_z}{\lambda s_1}.$$

The notations used above are as follows:

$$B_{11} = 1 + nm + kn_2, \quad B_{12} = 0.5(nm-1), \quad B_{13} = 0.5(nm^2-1),$$

$$B_{14} = 0.25(1+nm^2) + \psi_3, \quad B_{22} = 0.25(1+nm) + (kn_2/12) + (\psi_1/k),$$

(3.24)

$$B_{23} = 0.25(1+nm^2) - \psi_1, \quad B_{24} = (nm^2-1)/8,$$

$$B_{33} = 0.25(1+nm^3) + k\psi_1, \quad B_{34} = (nm^3-1)/6,$$

$$B_{44} = \frac{1}{12}(1+nm^3) + \frac{\kappa_2}{k} + \frac{k}{12}\psi_1.$$

(3.25)

$$W = -\lambda W_{0ij}/t_1, \quad \bar{W} = \lambda \bar{W}_{0ij}/t_1,$$

$$s_1 = E_1/(1-\nu_1^2), \quad \lambda = (i\pi t_1/a)^2 + (j\pi t_1/b)^2,$$

$$n_2 = \frac{E_2}{E_1} \frac{(1-\nu_1^2)(1-\nu_{z2}^2 n_{z2})}{(1+\nu_2)(1-\nu_2-2\nu_{z2}^2 n_{z2})}, \qquad \psi_1 = (1-\nu_1^2) G_2/\lambda E_1 ,$$

(3.26)

$$\psi_3 = \frac{E_2}{E_1} \frac{(1-\nu_1^2)\nu_{z2}}{\lambda(1-\nu_2-2\nu_{z2}^2 n_{z2})}, \qquad \kappa_2 = \frac{E_{z2}}{E_1} \frac{(1-\nu_1^2)(1-\nu_2)}{\lambda^2(1-\nu_2-2\nu_{z2}^2 n_{z2})}.$$

In the notations (3.26), the subscript 2 refers to the core.

4. Damped free vibrations

Let us assume that the sandwich plate is executing such free vibrations so that the dependence upon time of the deflection and thickness-stretch deformation responses may be expressed as below:

$$W(t) = We^{I\omega t} e^{-\alpha t} = We^{I\check{\omega} t} ,$$

(4.1)

$$\bar{W}(t) = \bar{W}e^{I\omega t} e^{-\alpha t} = \bar{W}e^{I\check{\omega} t} ,$$

where:

(4.2) $$\check{\omega} = \omega + I\alpha , \qquad I^2 = -1 .$$

Let us denote the damped frequency parameter as

(4.3) $$\check{\Omega} = \frac{t_1^2 \gamma_1}{s_1} \check{\omega}^2 = \frac{t_1^2 \gamma_1}{s_1} \omega^2 \left(1 + I\frac{\delta}{2\pi}\right)^2 ,$$

where $\delta = 2\pi\alpha/\omega$ is the logaritmic decrement.

Moreover, let us perform the substitutions according to the introduction of complex material characteristics

$$\psi_1 = \psi_1 (1 + I\mu_0 \varepsilon_1) , \qquad n_2 = n_2 (1 + I\mu_0 \varepsilon_2) ,$$

(4.4)

$$\psi_3 = \psi_3 (1 + I\mu_0 \varepsilon_2) , \qquad \kappa_2 = \kappa_2 (1 + I\mu_0 \varepsilon_4) ,$$

with

(4.5) $$\varepsilon_1 = \mu_1/\mu_0 , \qquad \varepsilon_2 = \mu_2/\mu_0 , \qquad \varepsilon_4 = \mu_4/\mu_0 ,$$

where μ_0 is an arbitrary material loss factor, μ_1 is the loss factor associated with the shear modulus of the core, $\mu_3 = \mu_2$ is the loss factor associated with the core longitudinal Young's modulus and μ_4—that associated with the core transverse Young's modulus.

If we introduce Eq. (4.4) into Eq. (3.24), the coefficients B_{ij} take the complex forms:

$$B_{11} = \check{B}_{11} = B_{11} + I\mu_0 B_{11p} , \qquad B_{11p} = \varepsilon_2 k n_2 ,$$

(4.6)

$$B_{14} = \check{B}_{14} = B_{14} + I\mu_0 B_{14p} , \qquad B_{14p} = \varepsilon_3 \psi_3 ,$$

$$B_{22} = \check{B}_{22} = B_{22} + I\mu_0 B_{22p} , \qquad B_{22p} = \varepsilon_2 n_2 \frac{k}{12} + \varepsilon_1 \frac{\psi_1}{k} ,$$

(4.6) [cont.]

$$B_{33}=\check{B}_{33}=B_{33}+I\mu_0 B_{33p}, \quad B_{33p}=\varepsilon_1\psi_1 k,$$
$$B_{23}=\check{B}_{23}=B_{23}+I\mu_0 B_{23p}, \quad B_{23p}=-\psi_1\varepsilon_1,$$
$$B_{44}=\check{B}_{44}=B_{44}+I\mu_0 B_{44p}, \quad B_{44p}=\varepsilon_4\frac{\kappa_2}{k}+\varepsilon_1\psi_1\frac{k}{12}$$

all the remaining coefficients are not altered.

Next, let us introduce Eq. (4.1) into Eq. (3.23), taking into account Eq. (4.3). It can be seen from the set of above obtained equations that exist nontrivial solutions for U, $\bar{U}$, W and $\bar{W}$ if the determinant formed by the coefficients of these unknowns vanish. We have

$$\check{a}_0\check{\Omega}^2+\check{a}_1\check{\Omega}+\check{a}_2=0, \tag{4.7}$$

where

$$\check{a}_0=\frac{1}{4\lambda^4}(\theta_2 k+\theta_2\theta_3 km+4\theta_3 m)\begin{vmatrix}\check{B}_{11} & B_{12}\\ B_{12} & \check{B}_{22}\end{vmatrix},$$

$$\check{a}_1=-\frac{1}{\lambda^2}(1+\theta_2 k+\theta_3 m)\begin{vmatrix}\check{B}_{11} & B_{12} & \check{B}_{14}\\ B_{12} & \check{B}_{22} & B_{24}\\ \check{B}_{14} & B_{24} & \check{B}_{44}\end{vmatrix}-\frac{1}{4\lambda^2}(1+\theta_3 m)\begin{vmatrix}\check{B}_{11} & B_{12} & B_{13}\\ B_{12} & \check{B}_{22} & \check{B}_{23}\\ B_{13} & \check{B}_{23} & \check{B}_{33}\end{vmatrix}+$$

$$+\frac{1}{2\lambda^2}(1-\theta_3 m)\left\{\begin{vmatrix}\check{B}_{11} & B_{12} & \bar{B}_{13}\\ B_{12} & \check{B}_{22} & \check{B}_{23}\\ \check{B}_{14} & B_{24} & B_{34}\end{vmatrix}+\begin{vmatrix}\check{B}_{11} & B_{12} & \check{B}_{14}\\ B_{12} & \check{B}_{22} & B_{24}\\ B_{13} & \check{B}_{23} & B_{34}\end{vmatrix}\right\}, \tag{4.8}$$

$$a_2=\begin{vmatrix}B_{11} & B_{12} & B_{13} & B_{14}\\ B_{12} & B_{22} & B_{23} & B_{24}\\ B_{13} & B_{23} & B_{33} & B_{34}\\ B_{14} & B_{24} & B_{34} & B_{44}\end{vmatrix}.$$

These determinants can be expanded in real and imaginary parts. The solution of the above equation performed in the similar way is given in previous papers [1, 5] and will be not presented in detail here. The numerical results calculated for various parameter are given in the next section.

From Eq. (4.3) we have

$$\check{\Omega}=\Omega(1+I\mu_0\delta^*/2)^2\simeq\Omega(1+I\mu_0\delta^*),$$

where

$$\Omega=\frac{t_1^2\gamma_1}{s_1}\omega^2 \qquad \text{frequency parameter,}$$

$$\delta^*=\frac{\delta}{\mu_0\pi} \qquad \text{damping parameter.}$$

It should be pointed out that in the case of a sandwich plate with a symmetrical structure (e.g. $n=m=\theta_3=1$), the set of equations of motion (3.18) will degenerate into two sets of equations, uncoupled one from another. Each set consists of two equations depending upon two unknowns and describes one of the two motions: transverse vibrations or thickness-stretch vibrations.

5. Numerical results

Investigations on the various frequencies and their associated damping parameters are conducted on the basis of the above equations. However, it should be noted that the frequencies of the two types of motion, the transverse flexural type and the thickness-stretch deformation type, are generally very far apart. The former takes on values of low magnitude, while the latter — that of much higher magnitude. This feature enables us to set up from [4] simpler formulae in checking up on the frequencies of the sandwich plate.

We assume the same data as used in [1] for sandwich plates where the thickness--stretch deformation of the core is not taken into account (hereafter, for the sake of convenience, this theory will be named the „common" theory of sandwich plates), for example

$$t_1/a=1/50, \quad a/b=2.0, \quad \lambda=\pi\left[\left(\frac{it_1}{a}\right)^2+\left(\frac{jt_1}{b}\right)^2\right]=0.02,$$

$$\theta_2=\gamma_2/\gamma_1=0.1, \quad \theta_3=\gamma_3/\gamma_1=1.0, \quad n=E_3/E_1=1.0,$$

$$G_2/E_1=10^{-4}\div 0.1, \quad \psi_1=\frac{1-\nu_1^2}{\lambda}\frac{G_2}{E_1}=5.10^{-3}\div 5.0.$$

However, the additional data have to be used here. The ratio n_{z2} of the longitudinal Young's modulus of the core E_2 to the transverse one E_{z2} is assumed to cover the following range:

$$10^{-6}\leqslant n_{z2}\leqslant 1.0.$$

For simplicity, we set

$$\nu_1=\nu_2=\nu_3=\nu_{z2}=0.3.$$

Then,

$$(5.1)\qquad n_2\approx 1.22\frac{E_2}{E_1}, \quad \psi_3\approx 0.53\,\lambda^{-1}\frac{E_2}{E_1}, \quad \kappa_2\approx 1.225\,\lambda^{-2}\frac{E_{z2}}{E_1}.$$

We assume that

$$10^{-6}\leqslant E_2/E_1\leqslant 0.1 \quad \text{and} \quad 10^{-6}\leqslant E_{z2}/E_1\leqslant 1.0.$$

Accordingly, from Eqs. (5.1) it follows that

$$1.22.10^{-6}\leqslant n_2\leqslant 0.122, \quad \leqslant 2.65.10^{-6}\leqslant \psi_3\leqslant 2.65, \quad 3.1.10^{-3}\leqslant \kappa_2\leqslant 3.1.10^3.$$

Moreover, the loss factor ratios ε_i are taken to be equal to 1. Taking into account the above mentioned ranges of ψ_1, n_2, ψ_3, κ_2, the frenquency parameter and the damping parameter corresponding to each type of motion are calculated. The analysis of the numerical results allows us to draw some conclusions.

I. Fig. 2 shows the variation of the frequency parameters Ω_1 and Ω_2 with regard to the face thickness ratio m, for various values k of the core thickness. The frequency of vibrations of the flexural type and that of the thickness-stretch type are: very far apart when the core is not too thick.

Variation of the frequency parameters Ω_1 and Ω_2 vs. the face thickness ratio. With $\theta_2=0.1$, $n=1.0$, $n_2=0.01$, $\psi_1=10^{-3}$, $\psi_3=0.227$, $\kappa_2=25.0$, $\lambda=0.02$, $\varepsilon_1=\varepsilon_2=$

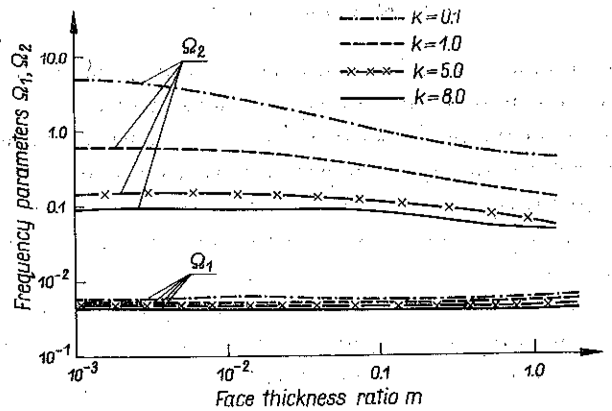

FIG. 2.

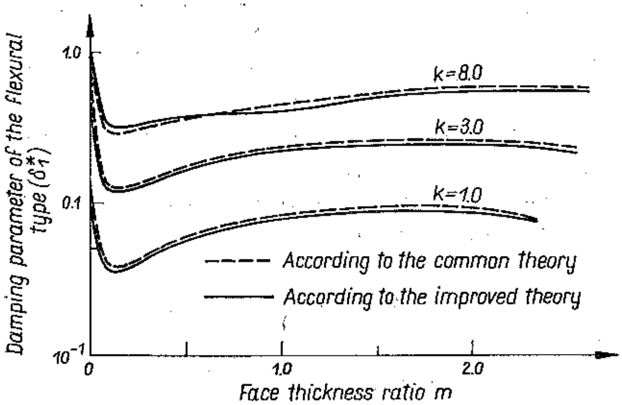

FIG. 3.

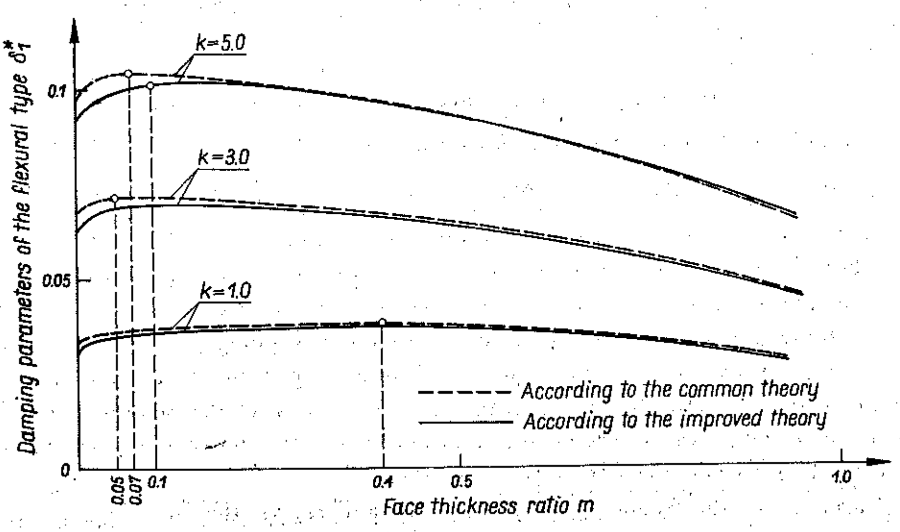

FIG. 4. Variation of the damping parameter of the fluxural type δ_1^* versus the face thickness ratio m.

$=\varepsilon_3=\varepsilon_4=1.0$, where $E_2/E_1=E_{z2}/E_1=0.82.10^{-2}$, $G_2/E_1=0.82.10^{-2}$, $G_2/E_1=2.10^{-5}$, $t_1/a=1/50$.

II. The damping caused by the transverse compressibility of the core contributes essentially to deaden the vibrations of the thickness-stretch type and the damping parameter of this type (δ_1^*) approximates to unity.

III. The frequency and the damping parameter of flexural type may be considered as totally unaffected by the inertia force produced by the thickness-stretch vibrations of the core. In general, therefore, it is not necessary to include these inertia forces in the equations of motion; hence the calculation of the vibration characteristics of flexural type becomes simpler.

IV. Figures 3 and 4 show the damping parameter of the flexural type δ_1^* plotted against the face thickness ratio m, according to the improved theory and to the common one, in the casses of sandwich plates with a thick core. Except for very small values of m, the $\delta_1^* - m$ curves checked from the two theories are all but the same, their difference in negligibly small. However, the improved theory will depart markedly from the common one when m tends to zero; the thicker the core the stronger this discrepancy. For instance, at $m=0.001$ and $k=5.0$, the divergence between these two theories with regard to the damping effectiveness may amount to 10%.

V. Although there is not a too large difference between the improved theory and the common one with regard to the magnitude of the damping parameter, it should be pointed out that there exists a rather strong discrepancy between the optimum face thicknes ratios m_{opt} checked according to these theories (Fig. 3). The optimum peak obtained from the improved theory moves to the right-hand side with respect to that obtained from the common theory.

VI. The larger is the transverse rigidity the better is the damping. The results from the improved becomes closer to that of the common theory.

VII. Although the improved theory gives a more real picture of the sandwich plate deformation, the computation of the vibration characteristics of the sandwich plates in engineering, however, may be based on the common theory which gives sufficiently good accuracy.

Variation of the damping parameter of the flexural type δ_1^* versus the face thickness ratio m; Fig. 3 with the same data as given in Fig. 2; Fig. 4 with $\psi_1=5.0$ (e.g. $G_2/E_1=0.1$) and the remaining data are as in Fig. 2.

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Streszczenie

WPŁYW ŚCISKANIA NA TŁUMIENIE STRUKTURALNE W PŁYTACH SANDWICZOWYCH

W pracy bada się wpływ odkształceń w kierunku normalnym do płaszczyzny płyty na tłumienie strukturalne, w uzupełnieniu do poprzednio zbadanych [1] efektów zginania oraz ścinania (w płaszczyźnie płyty oraz w kierunku do niej prostopadłym). Otrzymano skorygowaną teorię płyt sandwiczowych. Wyniki numeryczne wskazują, że dla małych stosunków grubości warstwy zewnętrznej do grubości płyty obserwuje się znaczną stabilność wartości parametru tłumienia pomiędzy omawianą teorią, a teorią przedstawioną w [1].

Резюме

ВЛИЯНИЕ СЖАТИЯ НА СТРУКТУРНОЕ ДЕМИФИРОВАНИЕ В ПЛАСТИНАХ ТИПА СЭНДВИЧ

В работе исследуется (в дополнение к раннее исследованному эффекту изгиба и сдвига в плоскости пластины, и в нормальном направлении) влияние деформации растяжения-сжатия в нормальном направлении на структурное демифирование. Получена улучшенная теория пластин типа сэндвич. Численные результаты указывают на то, что при малом значении отношения толщины внешних слоев к толщине пластины наблюдается существенное расхождение значений параметра демифирования полученных по данной теории, по сравнению с полученными в работе [1].

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