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TRANSIENT THERMAL STRESS IN BIMODULUS SPHERE WITH ORTHOTOROPIC TEMPERATURE-DEPENDENT PROPERTIES

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Transient thermal stress problems for a bimodulus elastic material are analysed in this paper. Temperature-dependent mechanical and thermal properties of the material are considered. Fundamental governing equations for an orthotropic spherically symmetric bimodulus model are derived. Numerical illustrations of transient thermal stresses in a hollow sphere are presented.

1. Introduction

As an idealized material model, AMBARTSUMYAN and co-workers, e.g. [1, 2], formulated a so-called "bimodulus or different modulus elastic material", whose uniaxial stress-strain relation is represented by two respective straight lines emerging from the origin with different slopes under tension and compression. Systematic study on different response materials was initiated by these authors and has attracted wide interest among several investigators. WESOŁOWSKI [3–5] treated with a piecewise linear material, which involves the bimodulus material, as a special case. Some stress and deformation analyses of fundamental structural elements such as plates or shells have partially been performed, but investigation as a whole is now at an initial stage. Constitutional microscopic mechanisms yielding the above-mentioned difference in tension and compression have not been sufficiently interpreted.

In the present paper we intend to analyse transient stress or deformation of idealised bimodulus elastic bodies due to thermal loading. Some introductory problems on stationary and transient thermal stresses have been considered by the present author [6–8]. It has been known for a long time that thermal and mechanical properties of solids are not constant for a wide range of temperature; i.e. they are in general, temperature-dependent. Therefore we must take into account these influences in designing structural elements which are subjected to high temperature gradients. Several investigations on transient thermal stress problems considering temperature-dependent properties for classical elastic, plastic, viscoelastic and visco-elastic-plastic materials, except for the bimodulus case, are found in Refs. [9–22]. Some of these papers treated with anisotropic temperature-dependent elastic materials such as artificial graphites [10, 13, 14]. The constitutive equations for a general anisotropic bimodulus material are extremely complicated, so the present consideration is bounded to an orthotropic bimodulus material. First, the fundamental equations of a temperature-dependent bimodulus material are derived and then an example concerning hollow sphere is discussed.

Nomenclature

- a, b inner and outer radii of hollow sphere,
- c specific heat,
- E_i Young's modulus,
- K heat conductivity,
- r, θ, φ spherical coordinate system,
- s reduced time,
- s_{ij} compliance matrix,
- T temperature (measured from a reference temperature, $T=0$),
- $\bar{T}=T/T_a$ dimensionless temperature,
- T_a temperature on the inner surface,
- t time,
- u radial displacement,
- α_i thermal expansion coefficient,
- γ mass density,
- ε_i strain,
- ξ_i thermal strain,
- σ_i stress,
- ν_{ij} Poisson's ratio.

Subscripts

- 0 value at a reference temperature,
- $1, 2$ correction coefficients due to temperature change,
- $i, j=r, \theta, \varphi$,

Superscripts

- t tension value,
- c compression value,
- $*$ dimensionless correction function due to temperature change.

2. Bimodulus material model

One of the essential difficulties we encounter in analysing bimodulus stress/deformation problems is that the entries of elastic compliance and stiffness matrices have direct correlation to the sign of the corresponding stresses and are not known in advance. In order to describe properties of a bimodulus model, let us consider a polar-symmetric problem. Mechanical as well as thermal properties of the material are thought to be orthotropic (presently, polar-anisotropic).

According to the Ambartsumyan-type formulation [1, 2], the relations between stress and strain components are written in reference to the spherical coordinate system as follows:

$$
\begin{aligned}
\varepsilon_r &= s_{rr}\,\sigma_r + s_{r\theta}\,\sigma_\theta + s_{r\phi}\,\sigma_\phi + \xi_r, \\
\varepsilon_\theta &= s_{r\theta}\,\sigma_r + s_{\theta\theta}\,\sigma_\theta + s_{\theta\phi}\,\sigma_\phi + \xi_\theta, \\
\varepsilon_\phi &= s_{r\phi}\,\sigma_r + s_{\theta\phi}\,\sigma_\theta + s_{\phi\phi}\,\sigma_\phi + \xi_\phi .
\end{aligned}
\tag{2.1}
$$

The directions θ and φ are reckoned identically on account of the spherical symmetry.

The diagonal elements of the compliance appearing in Eqs. (2.1) are connected to the sign of the principal stresses as follows:

$$
s_{rr} = \begin{cases} 1/E_r^t(T) & (\sigma_r > 0), \\ 1/E_r^c(T) & (\sigma_r < 0), \end{cases} \qquad s_{\theta\theta} = s_{\phi\phi} = \begin{cases} 1/E_\theta^t(T) & (\sigma_\theta = \sigma_\phi > 0), \\ 1/E_\theta^c(T) & (\sigma_\theta = \sigma_\phi < 0). \end{cases}
\tag{2.2}
$$

The off-diagonal compliances are expressible if we suppose the symmetry of the compliance, $s_{ij} = s_{ji}$ $(i \neq j)$, as follows:

$$
\begin{aligned}
s_{r\theta} &= -\frac{\nu_{r\theta}^t(T)}{E_r^t(T)} = -\frac{\nu_{r\theta}^c(T)}{E_r^c(T)} = -\frac{\nu_{\theta r}^t(T)}{E_\theta^t(T)} = -\frac{\nu_{\theta r}^c(T)}{E_\theta^c(T)}, \\
s_{\theta\phi} &= -\frac{\nu_{\theta\varphi}^t(T)}{E_\theta^t(T)} = \ldots .
\end{aligned}
\tag{2.3}
$$

In deriving these formulae, we employed the reciprocity relations from classical orthotropic elasticity

$$
\frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j} \qquad (i \neq j).
\tag{2.4}
$$

For the bimodulus material we postulate the similar identities

$$
\frac{\nu_{ij}^t}{E_i^t} = \frac{\nu_{ij}^c}{E_i^c} \qquad (i \neq j).
\tag{2.5}
$$

Consequently, $s_{r\theta}$ and $s_{\theta\phi}$ are determined independently of the sign of stresses as opposed to the diagonal elements of the compliance. Furthermore, for an isotropic bimodulus material $s_{r\theta} = s_{\theta\phi}$.

3. Temperature-dependent material properties

The Young's moduli and Poisson's ratios which are the basic engineering parameters of a linear elastic material are known to vary with a change of temperature. They are not constant for a wide range of temperature. Let us define these parameters as follows:

$$
E_i^t(T) = E_{i0}^t\, E_i^{t*}(\bar{T}), \qquad E_i^c(T) = E_{i0}^c\, E_i^{c*}(\bar{T}),
\tag{3.1}
$$

$$
\nu_{ij}^t(T) = \nu_{ij0}^t\, \nu_{ij}^{t*}(\bar{T}), \qquad \nu_{ij}^c(T) = \nu_{ij0}^c\, \nu_{ij}^{c*}(\bar{T}),
\tag{3.2}
$$

where the values at a reference temperature are denoted by a suffix 0 and the correction functions in terms of the dimensionless temperature are marked by an asterisk.

The generalised reciprocity identities, Eqs. (2.4) and (2.5), must hold not only at the reference temperature but at any temperature, therefore

$$\frac{\nu^t_{ij0}}{E^t_{i0}}=\frac{\nu^c_{ij0}}{E^c_{i0}}=\frac{\nu^t_{ji0}}{E^t_{j0}}=\frac{\nu^c_{ji0}}{E^c_{j0}} \tag{3.3}$$

and

$$\frac{\nu^{t*}_{ij}(\bar{T})}{E^{t*}_i(\bar{T})}=\frac{\nu^{c*}_{ij}(\bar{T})}{E^{c*}_i(\bar{T})}=\frac{\nu^{t*}_{ji}(\bar{T})}{E^{t*}_j(\bar{T})}=\frac{\nu^{c*}_{ji}(\bar{T})}{E^{c*}_j(\bar{T})}. \tag{3.4}$$

Familiar empirical temperature-dependent formulae for E and ν are linear, quadratic and exponential. If we use the linear formulae

$$\begin{aligned} &E^{t*}_i(\bar{T})=1+E^t_{i1}\bar{T}, \quad E^{c*}_i(\bar{T})=1+E^c_{i1}\bar{T}, \\ &\nu^{t*}_{ij}(\bar{T})=1+\nu^t_{ij1}\bar{T}, \quad \nu^{c*}_{ij}(\bar{T})=1+\nu^c_{ij1}\bar{T} \end{aligned} \tag{3.5}$$

by virtue of Eqs. (3.4), we derive the following identities: either

$$\frac{E^t_{j1}}{E^*_{i1}}=\frac{\nu^t_{ji1}}{\nu^t_{ij1}}=1, \quad \frac{E^c_{j1}}{E^c_{i1}}=\frac{\nu^c_{ji1}}{\nu^c_{ij1}}=1, \quad \frac{E^c_{i1}}{E^t_{i1}}=\frac{\nu^c_{ij1}}{\nu^t_{ij1}}=1, \tag{3.6}$$

or

$$\frac{\nu^t_{ij1}}{E^t_{i1}}=\frac{\nu^t_{ji1}}{E^t_{ij}}=1, \quad \frac{\nu^c_{ij1}}{E^c_{i1}}=\frac{\nu^c_{ji1}}{E^c_{i1}}=1, \quad \frac{\nu^t_{ij1}}{E^t_{i1}}=\frac{\nu^c_{ij1}}{E^c_{i1}}=1. \tag{3.7}$$

For the quadratic functions for E

$$E^{t*}_i(\bar{T})=1+E^t_{i2}\bar{T}^2, \quad E^{c*}_i(\bar{T})=1+E^c_{i2}\bar{T}^2, \quad \text{(linear for } \nu) \tag{3.8}$$

we obtain Eqs. (3.6) [suffixes 1 of E_i must be converted into 2].

As shown in the above illustrations, the coefficients of correction terms due to temperature change, E_{i1}, E_{i2} and ν_{ij1}, are not independent.

4. Governing equations for spherically symmetric problems

Consider a transient stress field due to a polar-symmetric thermal load. According to the classical uncoupled thermoelasticity and the heat conduction theory, the temperature field is determined independently of the stress field; in other words, indepently of the mechanical properties of material. This fact is invariable even when the thermal properties of a material may be influenced by temperature change. In the present polar-symmetric problem, the temperature field $T=T(r, t)$ is obtained by solving the heat conduction equation subjected to given thermal conditions.

By considering identical conditions in the θ and φ directions, Eqs. 2.1 are rewritten as

$$\begin{aligned}\varepsilon_r &= s_{rr}\,\sigma_r + 2s_{r\theta}\,\sigma_\theta + \xi_r,\\ \varepsilon_\theta &= s_{r\theta}\,\sigma_r + (s_{\theta\theta}+s_{\theta\phi})\,\sigma_\theta + \xi_\theta,\end{aligned} \tag{4.1}$$

where the thermal strains are defined as follows:

$$\xi_r = \int_0^T \alpha_r\, dT, \qquad \xi_\theta = \int_0^T \alpha_\theta\, dT. \tag{4.2}$$

By solving Eqs. (4.1) with respect to stress components, we obtain

$$\begin{aligned}\sigma_r &= \frac{1}{s_{rr}(s_{\theta\theta}+s_{\theta\phi})-2s_{r\theta}^2}\,[(s_{\theta\theta}+s_{\theta\phi})(\varepsilon_r-\xi_r)-2s_{r\theta}(\varepsilon_\theta-\xi_\theta)],\\ \sigma_\theta &= \frac{1}{s_{rr}(s_{\theta\theta}+s_{\theta\phi})-2s_{r\theta}^2}\,[s_{rr}(\varepsilon_\theta-\xi_\theta)-s_{r\theta}(\varepsilon_r-\xi_r)].\end{aligned} \tag{4.3}$$

The equilibrium equation and the kinematic relations for polar-symmetric stress and deformation fields are respectively

$$\frac{\partial\sigma_r}{\partial r}+\frac{2}{r}(\sigma_r-\sigma_\theta)=0 \tag{4.4}$$

and

$$\varepsilon_r=\frac{\partial u}{\partial r}, \qquad \varepsilon_\theta=\frac{u}{r}. \tag{4.5}$$

Since the elastic compliances are determined distinctly in relation to the sign of the principal stresses, a body under consideration is supposedly divided into a finite number of domains by combinations of their signs. On the domain boundaries, the stress and deformation components normal to the boundary must be continuous. In the present polar-symmetric problem, a finite number of concentric hollow spheres correspond to such a partition and σ_r and u are continuous on the spherical boundaries. σ_θ may suffer some discontinuities. Thus it will be convenient to take σ_r and u as the basic variables in the following analysis.

Substitution of Eqs. (4.3) into Eqs. (4.5) and (4.6) yields the following derivatives:

$$\frac{\partial\sigma_r}{\partial r}=-\frac{2}{r\,[s_{rr}(s_{\theta\theta}+s_{\theta\phi})-2s_{r\theta}^2]}\left[(s_{\theta\theta}+s_{\theta\phi}+s_{r\theta})\left(\frac{\partial u}{\partial r}-\xi_r\right)-(2s_{r\theta}+s_{\theta\theta})\left(\frac{u}{r}-\xi_\theta\right)\right], \tag{4.6}$$

$$\frac{\partial u}{\partial r}=\frac{1}{s_{\theta\theta}+s_{\theta\phi}}\left\{\sigma_r\,[s_{rr}(s_{\theta\theta}+s_{\theta\phi})-2s_{r\theta}^2]+2s_{r\theta}\frac{u}{r}+(s_{\theta\theta}+s_{\theta\phi})\,\xi_r-2s_{r\theta}\,\xi_\theta\right\}. \tag{4.7}$$

Also, σ_θ becomes

$$\sigma_\theta=\frac{1}{s_{\theta\theta}+s_{\theta\phi}}\left[\frac{u}{r}-s_{r\theta}\,\sigma_r-\xi_\theta\right]. \tag{4.8}$$

5. Solution procedure

Inside each domain, the equations corresponding to Eqs. (4.6) to (4.8) should be satisfied. Usually, we analyse solid bodies, which are supposedly divided into a finite number of domains, by arranging unknown constants involved formally in the solutions for each domain, so as to satisfy the prescribed boundary conditions and continuous requirements. The problem of bimodulus material, however, belongs to a category of so-called "unknown boundary problem," and therefore the conventional method seems inadequate for our purpose. In our transient stress problem, we first separate two independent variables r and t, the latter being only contained in the temperature field T, and next apply an integration scheme, such as the Runge--Kutta and Milne methods, to the differential equations (4.6) and (4.7).

At each r, by starting from appropriate magnitudes of s_{rr} and $s_{\theta\theta}$ based on the initially supposed signs of σ_r and σ_θ, we repeat the trial-and-error procedure until we gain perfect correspondence between the magnitudes of the compliances and the stresses. This procedure determines the right-hand sides of the derivatives (4.6) and (4.7) at any r, and makes it possible to determine the values of σ_r, u and also σ_θ at $r+\Delta r$, where Δr is a finite small value of r.

6. Illustration

In what follows we consider as an example a hollow bimodulus sphere which is affected only thermally on the surfaces and is free from mechanical loading.

6.1. Temperature distribution

The following temperature condition is considered: the temperature at the inner surface of a hollow sphere which was initially in the reference temperature $T=0$ is elevated by $T_a\,(>0)$ at the time $t=0$ and held thereafter. It suffices to consider only the thermal properties in the radial direction because of polar-symmetry of the temperature field.

The temperature-dependencies of thermal conductivity, mass density and specific heat are expressed as follows:

$$K(T)=K_0\,K^*(\bar{T}), \quad \text{where} \quad K^*(\bar{T})=1+K_1\,\bar{T}, \tag{6.1}$$

$$\gamma(T)=\gamma_0\,\gamma^*(\bar{T}), \tag{6.2}$$

$$c(T)=c_0\,c^*(\bar{T}). \tag{6.3}$$

Substituting these functions into the heat conduction equation and assuming that the temperature correction term $K^*/\gamma^*\,c^*$ is negligibly small [23], we obtain the following solution:

$$\bar{T}=\frac{1}{K_1}\,[(1+2K_1\,\psi)^{\frac{1}{2}}-1], \tag{6.4}$$

where

$$(6.5)\quad \psi=\frac{a}{r}\left(1+\frac{K_1}{2}\right)\left\{\frac{b/a-r/a}{b/a-1}-\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left[\frac{n\pi\,(r/a-1)}{b/a-1}\right]\exp\left[-\left(\frac{n\pi}{b/a-1}\right)^2 s\right]\right\}$$

and the time parameter s is defined as follows:

$$(6.6)\qquad s=\frac{K}{a^2\,\gamma c}\,t\,.$$

If $K_1=0$, i.e. for the temperature-independent thermal conductivity, Eq. (6.4) becomes

$$(6.7)\qquad \bar{T}=\psi|_{K_1=7}\,.$$

6.2. Field equations

A system of the field equations which must be solved consists of Eqs. (4.6) to (4.8) and the temperature field (6.4) in terms of the independent variables r and t. In addition, thermal expansion coefficients are assumed to depend on the temperature as follows:

$$(6.8)\qquad \begin{aligned} \alpha_r(T)&=\alpha_{r0}\,\alpha_r^*(\bar{T}), \quad \text{where} \quad \alpha_r^*(\bar{T})=1+\alpha_{r1}\,\bar{T},\\ \alpha_\theta(T)&=\alpha_{\theta 0}\,\alpha_\theta^*(\bar{T}), \quad \text{where} \quad \alpha_\theta^*(\bar{T})=1+\alpha_{\theta 1}\,\bar{T}. \end{aligned}$$

Then, the dimensionless thermal strains become

$$(6.9)\qquad \xi_r/\alpha_{r0}\,Ta=\bar{T}\left(1+\frac{1}{2}\,\alpha_{r1}\,\bar{T}\right), \qquad \xi_\theta/\alpha_{r0}\,T=\frac{\alpha_{\theta 0}}{\alpha_{r0}}\,\bar{T}\left(1+\frac{1}{2}\,\alpha_{\theta 1}\,\bar{T}\right).$$

Since the sphere is not loaded mechanically but thermally on the inner surface, the boundary conditions are

$$(6.10)\qquad \sigma_r=0 \quad \text{at} \quad r=a \text{ and } b\,.$$

The problem is formally a "two-point boundary value problem" for each prescribed time s, and thus the trial-and-error method starting with an unknown value of u at the inner surface is employed in order to satisfy the above boundary conditions (6.10). Accordingly, we pursue the two kinds of trial-and-error procedure in the present analysis; i) to obtain proper correspondence between the compliances and the sign of stresses at each r, and ii) to satisfy the boundary conditions on the surfaces.

6.3. Numerical examples

In the following formulation, the material anisotropy, the bimodulus characteristics and the temperature-dependencies of mechanics and thermal properties are taken into account. With reference to the basic physical quantities E^c_{r0}, $\nu^c_{r\theta 0}$ and α_{r0}, several parameters are introduced, such as

$$(6.11)^{(1)}\qquad E^t_{\theta 0}/E^c_{r0}, \quad E^c_{\theta 0}/E^c_{r0}, \quad \nu^c_{\theta\varphi 0}/\nu^c_{r\theta 0}, \quad \alpha_{\theta 0}/\alpha_{r0}\,.$$

(1) In the following example, $\sigma_r \leqslant 0$ and so E^t_r is absent.

For the sake of clarity, the correction coefficients of the several parameters are assumed as follows:

$$E^c_{r2}=E^t_{\theta 2}=E^c_{\theta 2}\ (\equiv E_2),\qquad \alpha_{r1}=\alpha_{\theta 1},$$
$$\nu^t_{r\theta 1}=\nu^c_{r\theta 1}=\nu^t_{\theta r1}=\nu^c_{\theta r1}\ \nu^t_{\theta\varphi 1}=\nu^c_{\theta\varphi 1}=\nu^t_{\varphi\theta 1}=\nu^c_{\varphi\theta 1}. \tag{6.12}$$

The quadratic functions, Eqs. (3.8), are adopted as the temperature-dependent property of E.

For several values of the ratios listed in Eq. (6.11), $\nu^c_{r\theta 0}$ and the time parameter s, numerical calculations are carried out. The bimodulus property in the θ direction at the reference temperature is represented by the ratio

$$E^t_{\theta 0}/E^c_{\theta 0}. \tag{6.13}$$

The anisotropy between the r and θ directions is not described in an ordinary way because of the bimodulus nature. For this purpose, a ratio of the averages of Young's moduli is tentatively employed:

$$\frac{1}{2}(E^t_{\theta 0}+E^c_{\theta 0})/E^c_{r0}. \tag{6.14}$$

Results are obtained for $E^t_{\theta 0}/E^c_{\theta 0}=0.5$, 1 and 2 corresponding respectively to the tensile-weak, conventional elastic and compressive-weak materials when the above-defined anisotropy parameter $\frac{1}{2}(E^t_{\theta 0}+E^c_{\theta 0})/E^c_{r0}=0.5$, 1 and 2. The correction coefficients of E, α and K of temperature-dependent material are specified as

$$E_2=-0.5,\qquad \alpha_1=0.5,\qquad K_1=-0.5,$$

as well as the temperature-independent material (vanishing of these coefficients). However, the correction terms of Poisson's ratios are taken as zero because these are shown to affect stress fields extrinsically when compared to the others. The following values are also used:

$$\nu^c_{\theta\varphi 0}/\nu^c_{r\theta 0}=1,\qquad \alpha_{\theta 0}/\alpha_{r0}=1,\qquad \nu^c_{r\theta 0}=0.2,$$

$$s=0.1,\ 0.2 \text{ and } 1 \text{ (near-stationary)}.$$

The results for a hollow sphere with the ratio of outer to inner radius $b/a=2$ are shown in Figs. 1 to 6 together with the results of the temperature-independent material. The values of the several parameters employed in the calculations are shown in Table 1.

Figures 1(a) and 1(b) show the temperature-independent conventional orthotropic solutions of σ_r and σ_θ as the reference for comparison. The results when one of the three correction coefficients E_2, α_1 and K_1 is non-vanishing are depicted in Figs. 2 to 4; for example, Figs. 2(a) and 2(b) correspond to the case when $E_2\neq 0$, $\alpha_1=K_1=0$. Figures 5(a), 5(b) and 6(a) to 6(d) are results with non-vanishing correction coefficients for ordinary orthotropic and bimoduls orthotropic materials respectively.

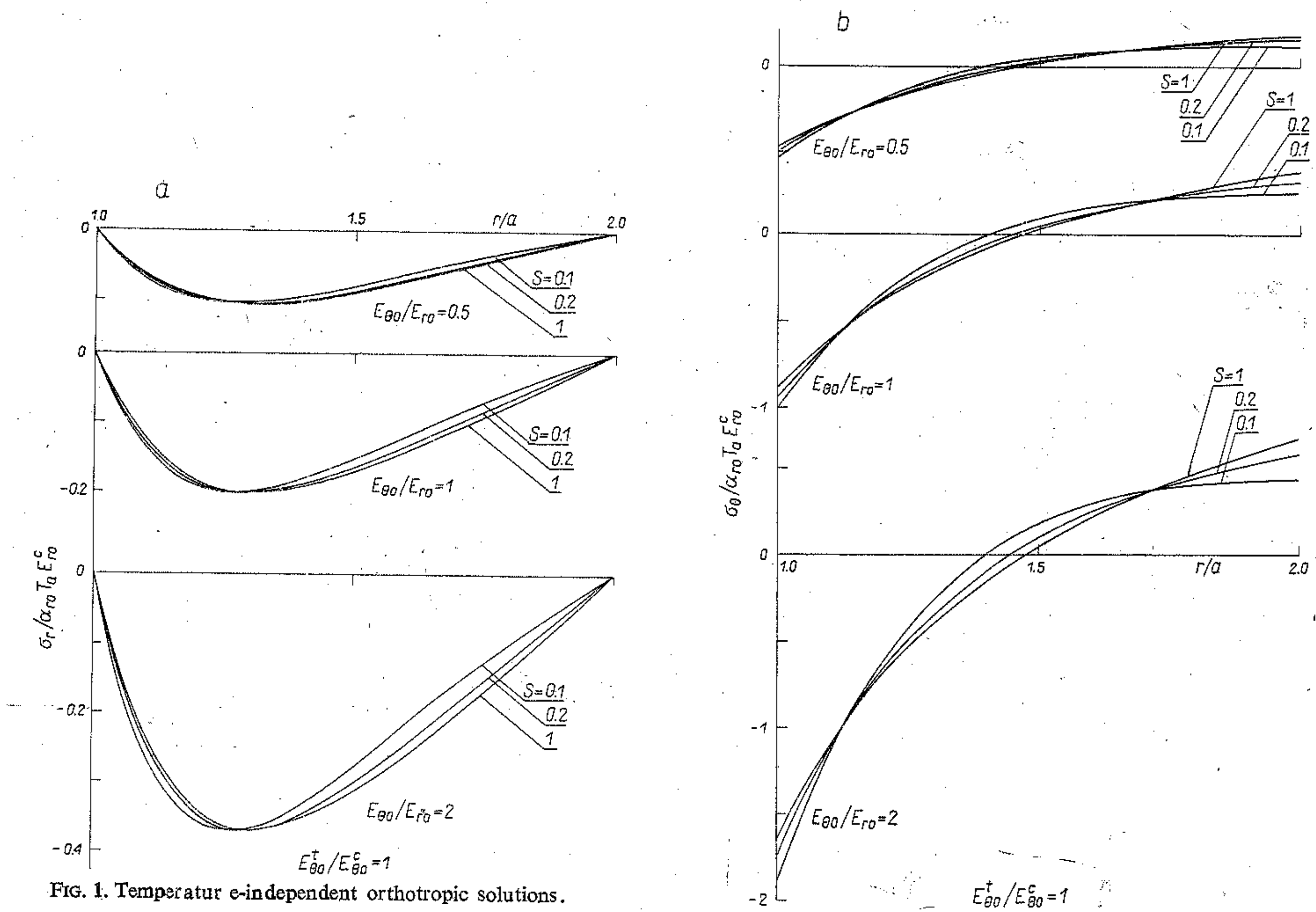

FIG. 1. Temperature-independent orthotropic solutions.

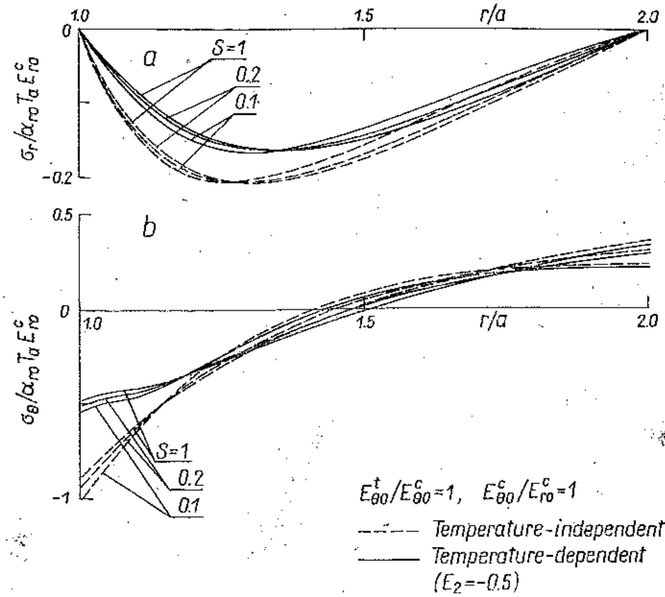

FIG. 2. Effects of temperature-dependent Young's modulus

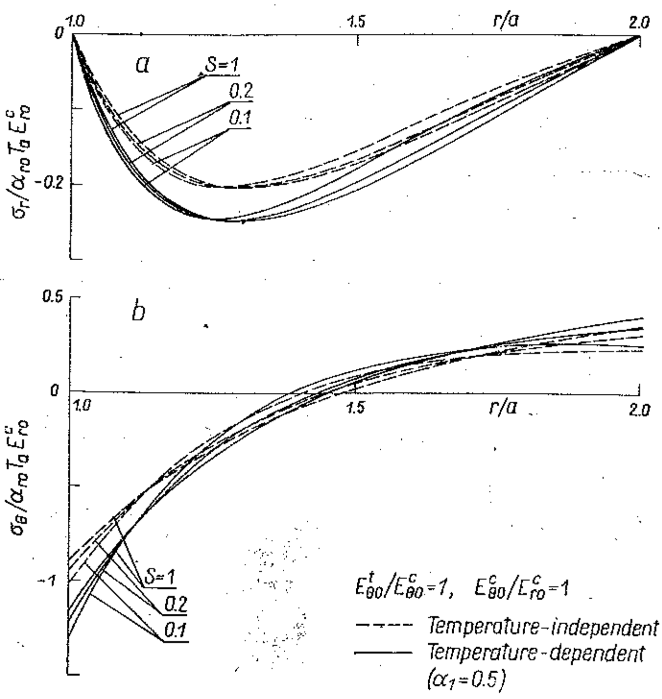

FIG. 3. Effects of temperature-dependent thermal expansion coefficient

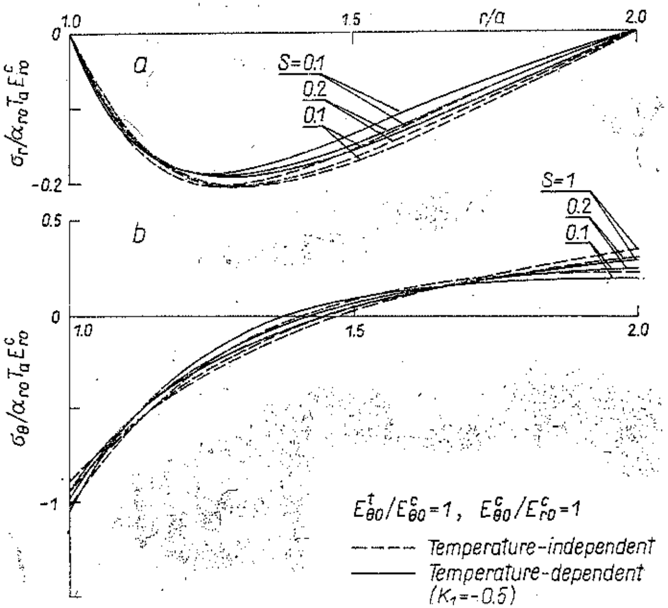

FIG. 4. Effects of temperature-dependent heat conductivity.

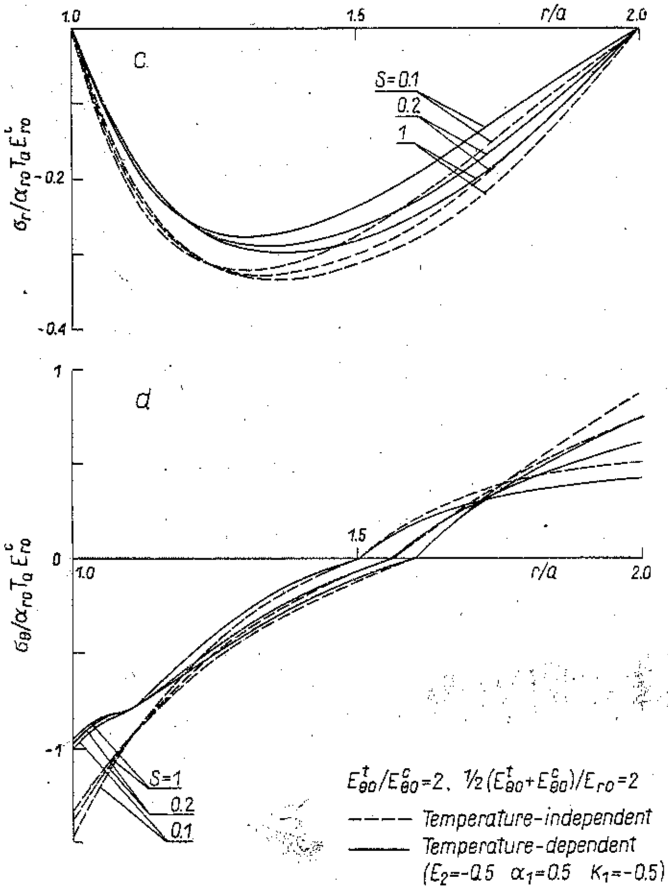

FIG. 5. Temperature-dependent orthotropic solutions.

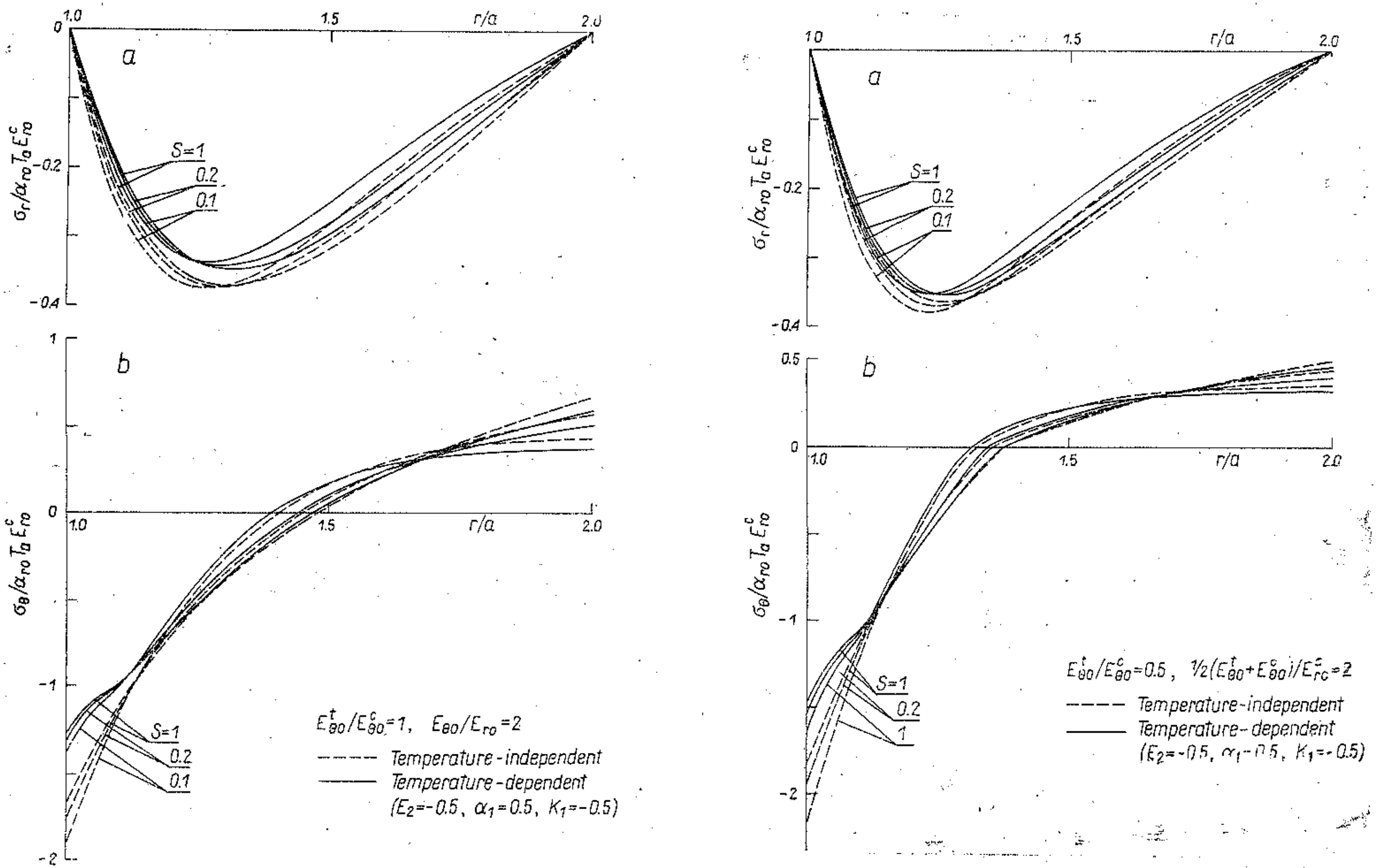

FIG. 6. Temperature-dependent bimodulus solutions

Table 1. Parameters employed in calculations.

	Figure No.	$E^t_{\theta 0}/E^c_{\theta 0}$	$\frac{1}{2}(E^t_{\theta 0}+E^c_{\theta 0})/E^c_{r0}$	E_2	α_1	K_1	Remarks
1	T-ID	1	0.5, 1, 2	0	0	0	orthotropic
2	T-ID	1	1	0	0	0	
	T-D	1	1	—0.5	0	0	varying E, isotropic
3	T-ID	1	1	0	0	0	
	T-D	1	1	0	—0.5	0	varying α, isotropic
4	T-ID	1	1	0	0	0	
	T-D	1	1	0	0	—0.5	varying K, isotropic
5	T-ID	1	2	0	0	0	orthotropic
	T-D	1	2	—0.5	—0.5	—0.5	varying E, α and K, orthotropic
6	T-ID	0.5, 2	2	0	0	0	bimodulus orthotropic
	T-D	0.5, 2	2	—0.5	—0.5	—0.5	varying E, α and K, bimodulus orthotropic

Note: $b/a=2$, $\nu^c_{\theta\varphi 0}/\nu^c_{r\theta 0}=1$, $\alpha_{\theta 0}/\alpha_{r0}=1$, $\nu^c_{r\theta 0}=0.2$ for all results. T-D: Temperature-dependent, T-ID: Temperature-independent.

In previous studies [6–8] the author showed that thermal stresses were affected greatly by the bimodulus property of a temperature-independent material. Similar results are also found in the present examples for a temperature-dependent material. Furthermore, the present results show that stress distribution in a hollow sphere are also influenced by the temperature-dependent mechanical and thermal properties of a material. The respective influence due to E_2, α_1 and K_1 seems different from each other. In general, stresses increase with increasing α_1 but decrease with increasing E_2 and K_1 which are negative. Since K_1 is accounted for, while E_2 and α_1 are not considered in calculating the temperature field, one may notice from Figs. (4)a and 4(b) that the results due to K_1 and those due to E_2 and α_1 are rather distinct. The situation is further complicated as shown in Figs. 5 and 6 when the three parameters E_2, α_1 and K_1 are considered simultaneously. In any case, the results suggest that the temperature-dependent properties of material parameters cannot be ignored.

References

1. S. A. Ambartsumyan and A. A. Khachatryan, Inzh. Zhur. MTT, 2, 44–53 1966 [in Russian].
2. S. A. Ambartsumyan, Inzh. Zhur. MTT. 3, 51–61, 1969 [in Russian].
3. Z. Wesołowski, Arch. Mech. Stos., **21**, 449–468, 1969.
4. Z. Wesołowski, Arch. Mech. Stos., **22**, 253–365, 1970.
5. Z. Wesołowski, *Trends in elasticity and thermoelasticity*, W. Nowacki Anni. Vol., M. Sokołowski *et al.* ed., Wolters-Noordhoff Pub., 280–292, 1971.
6. N. Kamiya, Bull. l'Acad. Polon. Sci., Série. Sci. Tech., **26**, 581–588, 1976.
7. N. Kamiya, Nuc. Eng. Des., **40**, 383–391, 1977.

8. N. KAMIYA, Mech. Res. Comm., **4**, 51–56, 1977.
9. J. NOWINSKI, J. Appl. Mech., Trans. ASME, **84**, 399–410, 1962.
10. TU-LUNG WENG, J. Basic Eng., Trans. ASME, **87**, 391–397, 1965.
11. T. KOIZUMI and T. TANIWAKI, Trans. Japan Soc. Mech. Eng., **40**, 9–15, 1965 [in Japanese].
12. H. KISHIGAMI, Trans. Japan Soç. Mech. Eng., **40**, 1–8 1965 [in Japanese].
13. T. HATA and A. ATSUMI, Bull. Japan Soc. Mech. Eng., **11**, 404–412, 1968.
14. T. HATA and A. ATSUMI, Bull. Japan Soc. Mech. Eng., **12**, 445–452, 1969.
15. R. MUKI and E. STERNBERG, J. Appl. Mech., Trans. ASME, **83**, 193–207, 1961.
16. D. G. DASTIDAR and P. GHOSH, Int. J. Mech. Sci., **16**, 359–371, 1974.
17. H. ISHIKAWA, Trans. Japan. Soc. Mech. Eng., **43**, 3238–3245, 1977 [in Japanese].
18. H. ISHIKAWA, Trans. Japan Soc. Mech. Eng., **43**, 4019–4028, 1977 [in Japanese].
19. H. ISHIKAWA, Trans. Japan Soc. Mech. Eng., **43**, 4454–4462, 1977 [in Japanese].
20. A. MENDELSON and S. W. SPERO, J. Appl. Mech., Trans. ASME, **84**, 151–158, 1962.
21. H. G. LANDAU and E. E. TWICKY, Jr., J. Appl. Mech., ASME, **82**, 481–488, 1960.
22. H. G. LANDAU, J. H. WEINER and E. E. ZWICKY, Jr., J. Appl. Mech., Trans. ASME, **82**, 297–302, 1960.
23. H. S. CARSLAW and J. C. JAEGER, *Conduction of heat in solids*, 2nd Ed., Oxford Clarendon Press, 1959.

Streszczenie

NIEUSTALONE NAPRĘŻENIA CIEPLNE W ORTOTROPOWEJ KULI DWUSKŁADNIKOWEJ O WŁASNOŚCIACH ZALEŻNYCH OD TEMPERATURY

Rozpatrzono zagadnienia nieustalonych naprężeń cieplnych w dwuskładnikowym materiale sprężystym. Uwzględniono zależność mechanicznych i termicznych własności materiału of temperatury. Wyprowadzono podstawowe równania dla ortotropowego, kuliście symetrycznego modelu ciała bimodualnego. Przedstawiono numeryczne rozwiązania problemów nieustalonych stanów naprężenia dla kuli wydrążonej.

Резюме

НЕУСТАНОВИВШИЕСЯ ТЕРМИЧЕСКИЕ НАПРЯЖЕНИЯ В ОРТОТРОПНОМ ДВУХКОМПОНЕНТНОМ ШАРЕ СО СВОЙСТВАМИ ЗАВИСЯЩИМИ ОТ ТЕМПЕРАТУРЫ

Рассмотрена задача неустановившихся термических напряжений в двухкомпонентном упругом материале. Учтена зависимость механических и термических свойств материала от температуры. Выведены основные уравнения для ортотропной, сферически симметричной модели бимодулярного тела. Представлены численные решения задач неустановившихся напряженных состояний для полого шара.

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