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EFFECT OF PLASTIC CRUSHING OF THE CAR BODY ON OPTIMIZATION OF RHEOLOGICAL PROPERTIES OF SAFETY BELTS (*)

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The highly idealized problem of optimization of rheological properties of safety belts was formulated and solved by W. NACHBAR and J. B. SCHIPMÖLDER [2]. In the present paper optimization is considered taking a more realistic descriptio n of an impact. The effect of plastic crushing of the front end of the car body is studied. The perturbation method is applied to obtain an effective solution, with the small parameter proportional to the length of the deformed front end of the vehicle. Two cases of total and partial crushing of the elements in the front of the passenger are distinguished under the assumption of constant average plastic resistance of the car body. Optimum viscoelastic properties of the safety belt are determined which maximize the initial "safe" speed of the vehicle before an impact.

1. INTRODUCTION

Most problems of optimization in rheology consist in evaluating optimal shapes of the structures, Ref. [4]. However, in some cases the rheological properties of the material may also be subject to optimization — problems of this type belong partly to optimal structural design and partly to materials science. Such a situation is encountered mainly in dynamic problems: for example, the problem of optimal rheological properties of safety belts was formulated by W. NACHBAR and J. B. SCHIPMÖLDER [2]. In the present paper we extend their considerations assuming a more realistic description of the crushing of the car body.

W. Nachbar and J. B. Schipmölder assumed the maximal permissible speed of the vehicle before an impact, V_0, as the objective function. The following constraints were assumed: 1) the maximum displacement $x(t)$ of the human body after an impact is equal or less than a prescribed critical displacement D, 2) the force $P(t)$ exerted by the belt on the body during the motion following an impact does not exceed a prescribed maximum value P_{max}. Simple, two-elements Maxwell and Kelvin–Voigt viscoelastic models for the seat belt material were studied with the primary intent to establish qualitative effects of introducing viscosity in a simple but optimal way. It was found that a certain optimal value of the viscosity coefficient existed for which the initial speed V_0 reached a maximum.

In that highly idealized problem the vehicle was assumed to stop abruptly after an impact. The effect of plastic crushing of the front part of the car body was neglec-

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ted. Under real crash conditions the attachments of the belt are subjected to a retarded motion in the first period and then in the second period they are suddenly brought to rest.

In the present paper the equations of motion for both these periods are derived. Considerations of plastic deformation of the car body make the basic equations rather complex so it is necessary to use an approximate method to obtain an effective solution. The perturbation method is applied with the small parameter proportional to the length of the deformed front part of the car body.

2. Perfectly rigid crash

W. Nachbar and J. B. Schipmölder considered the problem under the following assumptions which are introduced also in the present paper:

a) The passenger and the belt are represented by a point mass m attached to two strips of length l, which are attached to the vehicle at the other ends.

b) Effects of gravity and of energy dissipation other than material viscosity are neglected.

c) The change of the initial angle β is small (the small strain assumption).

d) The vehicle and mass m undergo straight-line motions along the same straight line. For time $t<0$ the vehicle moves with the constant speed V_0.

However the fifth assumption—standstill of the vehicle starting abruptly at $t=0$—will be replaced by the assumption of plastic crushing deformations during the first period.

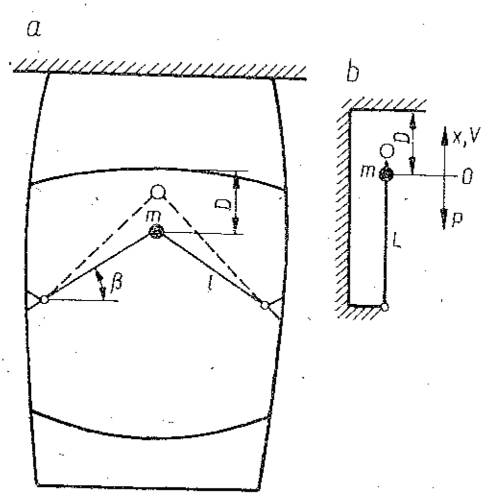

Fig. 1

The geometry of the belt is shown in Fig. 1a, and the linearization of the geometry in Fig. 1b. The criterion of the same displacements of belts a) and b) under the same loadings is used in Ref. [2] to determine the auxiliary length $L=l/2\sin^2\beta$, with the assumptions of small strain, a linear constitutive relation and a shallowness condition. Viscoelastic properties of the seat-belt material are introduced by means of the viscosity coefficient C and of the spring constant E. The basic equations are placed in a convenient nondimensional form for each model by a particular choice of the characteristic time $T_{M,K}=\sqrt{mL/E_{M,K}A}$, where A is a cross-sectional area of the seat belt, and the dimensionless parameters

$$\gamma_M=\frac{T_M E_M}{2C_M}, \qquad \gamma_K=\frac{C_K}{2T_K E_K}. \tag{2.1}$$

The indices M or K refer either to the Maxwell model or to the Kelvin–Voigt model, respectively. The parameters $\gamma_{M,K}$ are introduced in such a manner that $\gamma_M=0$ and $\gamma_K=0$ lead to ideal elasticity. A nondimensional time τ, load p, velocity v and strain ε are defined:

$$\tau=\frac{t}{T_{M,K}}, \quad p=\frac{P}{E_{M,K}A}, \quad v=\frac{VT_{M,K}}{L}, \quad \varepsilon=\frac{x}{L}. \tag{2.2}$$

The material constants C and E should maximize V_0 under the constraints

$$\sup x(t)=D, \qquad 0\leqslant t\leqslant\infty; \tag{2.3}$$

$$\sup P(t)=P_{\max}, \qquad 0\leqslant t\leqslant\infty. \tag{2.4}$$

After some calculations and elimination of E in Ref. [2] the relations for the „criterion quantity" (dimensionless objective function) defined

$$K=\frac{DP_{\max}}{mV_0^2}, \tag{2.5}$$

were eventually obtained. As we can see from the definition of criterion quantity K (Eq. (2.5)), the maximization of V_0 is equivalent in this case to the minimization of D under fixed $P_{\max}$ and V_0 or to the minimization of $P_{\max}$ under fixed D and V_0.

3. Crash analysis allowing for plastic deformation of the car body

The possibility of plastic deformation of the front part of the car body during a crash imposes a change of the fifth assumption to the problem. In the present paper we shall assume that in the first period when plastic crushing occurs, the attachments of the belt are subjected to a retarded motion. In the second period after an impact the attachments are instantaneously stopped. For simplicity we shall also assume perfectly plastic idealization of the car body and a constant deceleration of the attachments

$$a=\frac{\bar{\bar{P}}}{m_v}, \tag{3.1}$$

where $\bar{P}$=const is an average plastic carrying capacity of the deformed front part of the car body and m_v denotes the mass of the vehicle. The motion of the belt attachments in both periods is described by the relations

$$(3.2)\qquad \text{1 st period} \quad X_b(t)=V_0\, t-\frac{at^2}{2}, \quad 0\leqslant t\leqslant t^*;$$

$$(3.3)\qquad \text{2 nd period} \quad X_b(t)=S=\text{const}, \quad t\geqslant t^*,$$

where t^* is the end of the first crash period and S is a full displacement of the attachments in the 1st period, equal to the total length of plastic deformation of the car body. In Fig. 2 the speed of the belt attachments

$$(3.4)\qquad V_b(X_b)=\sqrt{V_0^2-2a\,X_b}$$

is plotted against the displacement X_b. In comparison with the present more adequate conditions of a crash including plastic deformation of the car body, W. Nachbar and J. B. Schipmölder considered the problem with the 1st period reduced to zero

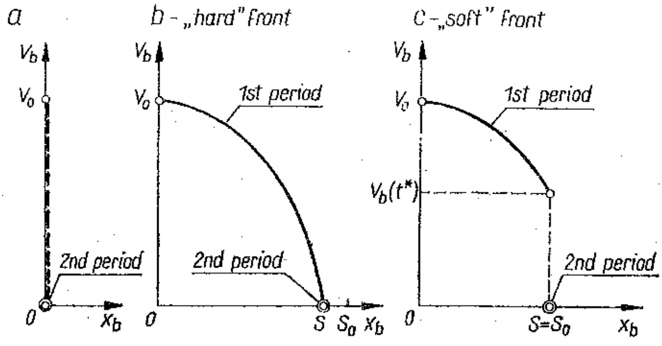

Fig. 2

(Fig. 2a). The maximum difference in curves in Figs. 2b and 2c is in the values of the boundary speed V_b^* at time t^*. This difference is related to the construction parameters of the front part of the car body, such as plastic carrying capacity $\bar{P}$ and total length S_0. For a "hard" and long front the velocity of attachments decreases from V_0 to zero (Fig. 2b) even though not the entire front part of the car body of length S_0 is deformed. In the opposite case for a "soft" and short front the plastic crushing ends before the speed of attachments reaches zero (Fig. 2c) and at the boundary time we have $V_b^*(t^*)>0$. Of course, a sharp distinction between "hard" and "soft" car bodies is impossible because it depends on the initial velocity V_0 of the vehicle, too. In the present paper we shall consider the latter of these two cases — a "soft" front — as a more general case.

An appropriate model of the seat belt is shown in Fig. 3a and the linearization of the geometry in Fig. 3b. A change of the assumption with regard to Ref. [2] involves a new expression for strain in the auxiliary belt of length L.

(3.5)
$$\varepsilon(t)=\frac{x(t)-X_b(t)}{L}.$$

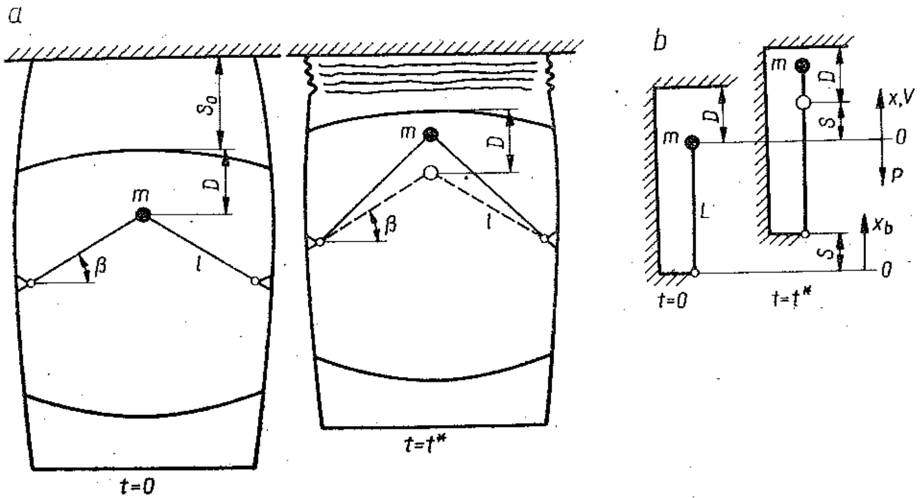

Fig. 3

The assumptions a)–d) and other relations remain the same as before.

3.1. *First crash period* $(0\leq t\leq t^*)$

Let us introduce a new nondimensional parameter

(3.6)
$$u=\frac{aT^2_{M,K}}{L},$$

connected with a constant deceleration a of the attachments due to plastic crushing of the car body.

The motion of mass m is described by a differential equation

(3.7)
$$m\frac{d^2x}{dt^2}=-P(t),\quad 0\leq t\leq t^*,$$

which together with the constitutive laws between uniaxial tensile stress σ and linear strain ε of the seat belt for each model $\Omega_{M,K}\{\sigma(t),\varepsilon(t)\}=0$ the strain—displacement relation (3.5) and the definition of stress $\sigma(t)=P(t)/A$, constitute the following two systems of dimensionless algebraic and differential equations:

Maxwell $(0 \leqslant \tau \leqslant \tau^*)$

$$\frac{dp_1(\gamma_M, \tau)}{d\tau} = v_1(\gamma_M, \tau) - 2\gamma_M p_1(\gamma_M, \tau) - v_0 + u\tau,$$

(3.8)
$$\frac{dv_1(\gamma_M, \tau)}{d\tau} = -p_1(\gamma_M, \tau),$$

$$\varepsilon_1(\gamma_M, \tau) = p_1(\gamma_M, \tau) + 2\gamma_M [v_0 - v_1(\gamma_M, \tau)],$$

with the initial conditions: $p_1(\gamma_M, 0) = 0$, $v_1(\gamma_M, 0) = v_0$,

Kelvin-Voigt $(0 \leqslant \tau \leqslant \tau^*)$

$$p_1(\gamma_K, \tau) = \varepsilon_1(\gamma_K, \tau) + 2\gamma_K [v_1(\gamma_K, \tau) - v_0 + u\tau],$$

(3.9)
$$\frac{dv_1(\gamma_K, \tau)}{d\tau} = -\varepsilon_1(\gamma_K, \tau) - 2\gamma_K [v_1(\gamma_K, \tau) - v_0 + u\tau],$$

$$\frac{d\varepsilon_1(\gamma_K, \tau)}{d\tau} = v_1(\gamma_K, \tau) - v_0 + u\tau,$$

with the initial conditions: $v_1(\gamma_K, 0) = v_0$, $\varepsilon_1(\gamma_K, 0) = 0$. After integrating Eqs. (3.8) and (3.9) with the appropriate initial conditions, the resulting equations for stress, velocity and strain in the 1st period are given in the Appendix.

3.2. *Second crash period* $(t \geqslant t^*)$

The attachments of the belt are brought to rest and therefore the same equations as in Ref. [2] can be used to determine the motion of the mass m. The initial conditions result from Eqs. (I) and (II) for the 1st period in order to obtain the continuity of speed v and strain ε at the boundary time τ^*. Solution of the appropriate equations is given in the Appendix for the 2nd period.

3.3. *Optimization*

Consideration of plastic crushing of the car body complicates the resulting equations (I)–(IV). It is no longer possible to derive the exact algebraic solution such as in Ref. [2]. The perturbation method is applied to obtain an effective solution. The small parameter is defined as

(3.10)
$$\psi = \frac{S}{D}$$

and is proportional to the length of the deformed front part S of the car body.

The smallest positive τ for which $p_1(\gamma_{M,K}, \tau)$ and $\varepsilon_1(\gamma_{M,K}, \tau)$ would reach a maximum in the 1st period are, respectively

Maxwell

(3.11a)
$$\tau_{p1} = \frac{\pi}{\sqrt{1 - \gamma_M^2}},$$

(3.12a)
$$\tau_{\varepsilon 1} = \sqrt{6},$$

Kelvin-Voigt

(3.13a) $$\tau_{p1} = \frac{1}{\sqrt{1-\gamma_K^2}} [\pi - \arccos (1-2\gamma_K^2)],$$

3.14a) $$\tau_{\varepsilon_1} = \frac{\pi}{\sqrt{1-\gamma_K^2}}.$$

The functions $p_2 (\gamma_{M,K}, \tau)$ and $\varepsilon_2 (\gamma_{M,K}, \tau)$, valid in the 2nd period, reach the (maximum at times

Maxwell

(3.11b) $$\tau_{p2} = \frac{1}{\sqrt{1-\gamma_M^2}} \arccos \gamma_M + \frac{\delta}{v_0} \psi + \ldots,$$

(3.12b) $$\tau_{\varepsilon 2} = \frac{1}{\sqrt{1-\gamma_M^2}} (\pi - \arccos \gamma_M) + \frac{\delta}{v_0} \psi + \quad ,$$

Kelvin-Voigt

(3.13b) $$\tau_{p2} = \frac{1}{\sqrt{1-\gamma_K^2}} \arccos (3\gamma_K - 4\gamma_K^3) + \frac{\delta}{v_0} \psi + \ldots,$$

(3.14b) $$\tau_{\varepsilon 2} = \frac{1}{\sqrt{1-\gamma_K^2}} \arccos \gamma_K + \frac{\delta}{v_0} \psi + \ldots,$$

where $\delta = D/L$.

Now we have to compare the values of $\tau_{p1}, \tau_{\varepsilon 1}, \tau_{p2}, \tau_{\varepsilon 2}$ for both models with the value of the boundary time $\tau^* = v_0/u \left(1 - \sqrt{1 - 2\delta u/v_0^2 \psi}\right)$ in order to evaluate the true maxima and to select the proper relations from Eqs. (3.11a, b)-(3.14a, b) to the further analysis. At the present stage of considerations such comparison cannot be made generally because the times for which stress and strain reach a maximum are functions of γ_M, γ_K and v_0 which are to be determind in the process of optimization. However, it turns out that in most practical cases stress and displacement reach their maximum in the 2nd period which involves the inequalities for both (models:

(3.15) $$\tau_{p1} \geqslant \tau^*, \quad \tau_{\varepsilon 1} \geqslant \tau^*;$$

(3.16) $$\tau_{p2} \geqslant \tau^*, \quad \tau_{\varepsilon 2} \geqslant \tau^*.$$

The inequalities (3.15) shall be proved in further analysis while the inequalities (3.16) are always true. The latter conclusion results from the comparison of the right-hand sides of Eqs. (3.11b)-(3.14b) with the expression for the boundary time τ^* expanded into a power series with respect to ψ

(3.17) $$\tau^* = \frac{\delta}{v_0} \psi + \ldots.$$

Further considerations will be presented for the Maxwell model only. The maximum of stress and strain in the 2nd period may be calculated as

$$p_{\max}=v_0 \exp\left(-\frac{\gamma_M}{\sqrt{1-\gamma_M^2}} \arc\cos \gamma_M\right)\left(1-\frac{\delta^3 u}{6v_0^4}\psi^3+\ldots\right), \tag{3.18}$$

$$\varepsilon_{\max}=v_0 \exp\left[-\frac{\gamma_M}{\sqrt{1-\gamma_M^2}}(\pi-\arc\cos \gamma_M)\right]\left(1-\frac{\delta^3 u}{6v_0^4}\psi^3+\ldots\right)+2\gamma_M v_0. \tag{3.19}$$

In the expansions (3.18) and (3.19) only the first term at ψ^3 is retained. The appropriate accuracy requires retaining terms at ψ only, Ref. [5], in Eqs. (3.11b)–(3.14b). With a partial transformation back to dimensional quantities the constraints (2.3) and (2.4) become

$$\frac{P_{\max}}{m}=\frac{V_0}{T_M}\exp\left(-\frac{\gamma_M}{\sqrt{1-\gamma_M^2}} \arc\cos \gamma_M\right)\left(1-\frac{aD^3}{6V_0^4}\cdot\frac{1}{T_M^2}\psi^3+\ldots\right), \tag{3.20}$$

$$\frac{D}{T_M}=V_0 \exp\left[-\frac{\gamma_M}{\sqrt{1-\gamma_M^2}}(\pi-\arc\cos \gamma_M)\right]\times \left(1-\frac{aD^3}{6V_0^4}\cdot\frac{1}{T_M^2}\psi^3+\ldots\right)+2\gamma_M V_0. \tag{3.21}$$

The expansion of the characteristic time T_M into a generalized power series is applied to eliminate E (hidden in T_M) between Eqs. (3.20) and (3.21)

$$\frac{1}{T_M}=T_0+T_1\psi+T_2\psi^2+T_3\psi^3+\ldots. \tag{3.22}$$

Substituting Eq. (3.22) into Eq. (3.20) we obtain a set of equations determining T_i. For the few first terms we have

$$T_0=\frac{P_{\max}}{mV_0\exp\left(-\frac{\gamma_M}{\sqrt{1-\gamma_M^2}}\arc\cos\gamma_M\right)},\qquad T_1=0,\qquad T_2=0,\qquad T_3=\frac{aD^3}{6V_0^4}T_0^3.$$

Substitution of the series (3.22) into Eq. (3.21) results in

$$K+\frac{\alpha}{6}\,\frac{K+\exp\left(-\frac{\pi\gamma_M}{\sqrt{1-\gamma_M^2}}\right)}{\left[\exp\left(-\frac{\gamma_M}{\sqrt{1-\gamma_M^2}}\arc\cos\gamma_M\right)\right]^2}K^3\psi^3+\ldots=K_0, \tag{3.23}$$

where $\alpha=a/a_{\max}$ is a ratio of the deceleration of the attachments and a maximum prescribed deceleration $a_{\max}=P_{\max}/m$ which can be exerted on a human body, and K_0 stands for the first approximation obtained in Ref. [2], namely

Maxwell

$$K_0=2\gamma_M\exp\left(-\frac{\gamma_M}{\sqrt{1-\gamma_M^2}}\arc\cos\gamma_M\right)+\exp\left(-\frac{\pi\gamma_M}{\sqrt{1-\gamma_M^2}}\right), \tag{3.24}$$

Kelvin–Voigt

$$K_0=\exp\left\{-\frac{\gamma_K}{\sqrt{1-\gamma_K^2}}\left[\arccos\gamma_K+\arccos\left(3\gamma_K-4\gamma_K^3\right)\right]\right\}. \tag{3.25}$$

The perturbation method ($\psi=S/D$) is used again to calculate K from Eq. (3.23). Eventually, the criterion quantities K for both models are

Maxwell

$$K=K_0-\frac{\alpha}{3}\,\frac{\gamma_M+\exp\left[-\dfrac{\gamma_M}{\sqrt{1-\gamma_M^2}}(\pi-\arccos\gamma_M)\right]}{\exp\left\{-\dfrac{\gamma_M}{\sqrt{1-\gamma_M^2}}\arccos\gamma_M\right\}}K_0^3\,\psi^3+\ldots, \tag{3.26}$$

Kelvin-Voigt

$$K=K_0-\frac{\alpha}{3}\exp\left\{-\frac{\gamma_K}{\sqrt{1-\gamma_K^2}}\left[\arccos\gamma_K-\arccos\left(3\gamma_K-4\gamma_K^3\right)\right]\right\}K_0^3\,\psi^3+\ldots. \tag{3.27}$$

Applying the optimization condition to Eqs. (3.26) and (3.27) we obtain

$$\gamma_{M\,\mathrm{opt}}=F_M(\psi), \tag{3.28}$$

$$\gamma_{K\,\mathrm{opt}}=F_K(\psi), \tag{3.29}$$

where the functions F_M and F_K may be defined and realized numerically only. For several values of α the relations (3.28) and (3.29) are plotted in Fig. 4. The criterion

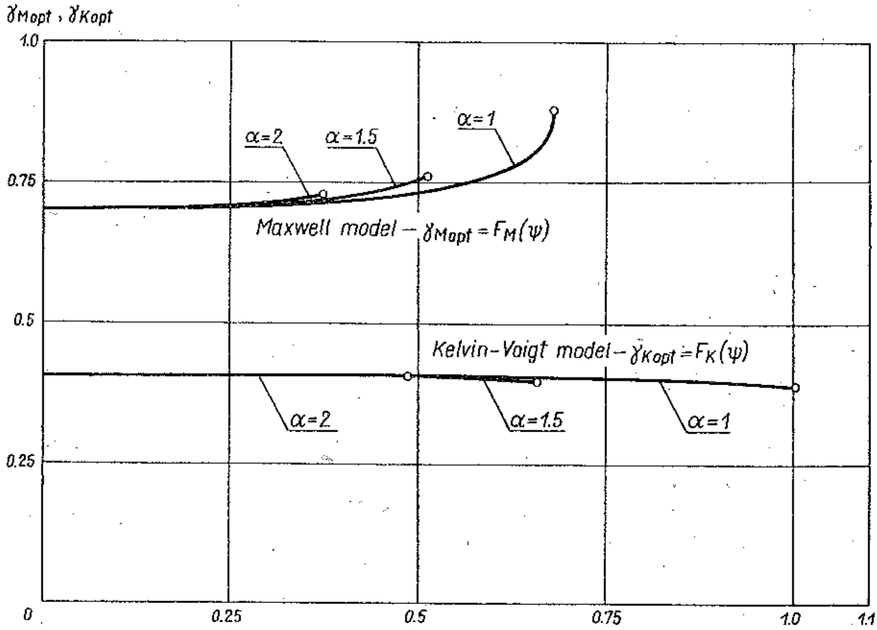

Fig 4.

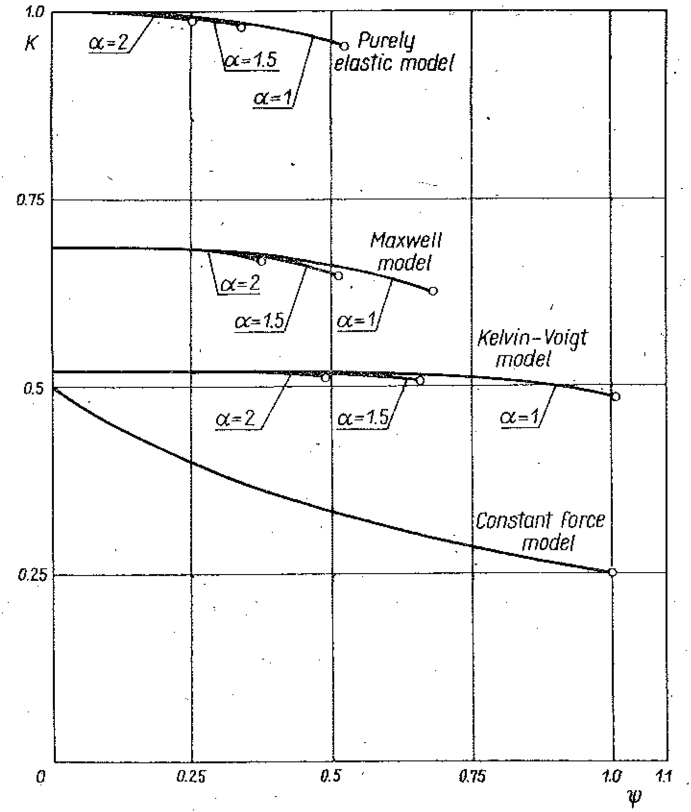

Fig. 5

quantities K versus the small parameter ψ (including the purely elastic model and constant force model) are shown in Fig. 5.

The resulting diagram in Fig. 4 may be used for detailed calculations of $\gamma_{M\,\mathrm{opt}}$ and $\gamma_{K\mathrm{opt}}$ for particular values of α and ψ. Optimum values of spring constants $E_{M\,\mathrm{opt}}$, $E_{K\mathrm{opt}}$ can be obtained using Eqs. (3.18) or (3.19) (for the Maxwell model) whereas optimum values of the viscosity coefficients $C_{M\,\mathrm{opt}}$, $C_{K\,\mathrm{opt}}$ from the substituting relations (2.1).

4. Purely elastic material

This model is considered in order to determine the effects of introducing viscosity into the seat belt material in connection with the more adequate conditions of an impact. The optimization in this case can be found by applying $\gamma_{M,K}=0$ to the solution for either the Maxwell or the Kelvin-Voigt models, Eqs. (3.26) or (3.27). The result is

$$K=K_0-\frac{\alpha}{3}K_0^3\psi^3+\dots, \tag{4.1}$$

where, according to Ref. [2], $K_0=1$.

5. Constant force model ($P(t)=P_{max}$)

After some simple rearrangements we obtain

$$K=K_0\frac{1}{1+\psi}, \tag{5.1}$$

where $K_0=0.5$. In this case the inequality $\alpha\geqslant 1$ must be satisfied; it results from the strain relation (3.5) for the 1st period $\varepsilon_1=(\alpha-1)\,a_{max}\,t^2/2L$. If $\alpha<1$ the mass m is decelerated with the force $P<P_{max}$, which does not coincide with the assumption.

6. Limitation of the validity of solution

The 1st crash period when the attachments of the belt move may continue at most to the time $t^*_{max}=V_0/a$ at which the velocity $V_b=0$. The distance covered during that time equals $S_{max}=X_b(t^*_{max})=V_0^2/2a$ and for a particular value of a depends on the initial speed V_0 of the vehicle. As a conclusion we have a limitation for the small parameter which is connected with S

$$\psi\leqslant\psi_v, \tag{6.1}$$

where $\psi_v=V_0^2/2aD=1/2\alpha K$.

The inequalities (3.15) involve the conditions

$$\psi\leqslant\psi_p, \qquad \psi\leqslant\psi_\varepsilon, \tag{6.2}$$

where $\psi_p=V_0\,t_{p1}/D$ and $\psi_\varepsilon=V_0\,t_{\varepsilon 1}/D$.

The inequalities (6.1) and (6.2) may be verified numerically only at the very end of the optimization process after determining γ_{Mopt} or γ_{Kopt} and V_{0max}.

Collecting Eqs. (6.1) and (6.2) gives finally the general condition of validity of the solution

$$\psi\leqslant\psi_{max}=\min\{\psi_v,\psi_p,\psi_\varepsilon\}. \tag{6.3}$$

7. Numerical examples

We assume the following data: $D=0.6$ m, $A=0.0001$ m^2, $l=0.5$ m, $m=80$ kg, $m_v=1000$ kg, $\beta=30°$, two values of $P_{max}=5000$ N and 10 000 N (1), three values of $\alpha=1$, 1.5 and 2. We have six different sets of data; the difference is in the values of the most controversial data: the prescribed force P_{max} which can be exerted on a human body and (indirectly) the average plastic capacity $\overline{\overline{P}}$ of the front part of the car body. The numerical results are gathered in Table 1, including a perfectly rigid crash ($\psi=0$). All the results (except the constant force model) are obtained for ψ_{max} in order to determine extremal effects due to the plastic crushing of the car body. The values of ψ_{max} are given in Table 2. As we could expect, the condition (6.1) is the sharpest so that $\psi_{max}=\psi_V$; the values of ψ_p and ψ_ε are greater than that of ψ_V at about a whole range of magnitude. For the constant force model

(1) Taken after Ref. [1]

it is necessary to limit ψ with respect to real dimensions of the car body and $\psi=1$ is taken as the representative value in this case. The diagrams of force $P(t)$, velocity $V(t)$ and dispalcement $\varepsilon(t)L$ of m towards the attachments versus time t are shown in Figs. 6, 7, 8 and 9 for all models under consideration for the following data: $P_{max}=10\,000$ N, $\alpha=2$ ($\bar{\bar{P}}=250\,000$ N) and $\psi=0.17$ ($S=0.1$ m).

Table 1.

	$P_{max}=5000$ N				$P_{max}=10\,000$ N			
	$\psi=0$	$\psi=\psi_{max}$			$\psi=0$	$\psi=\psi_{max}$		
		$\alpha=2$	$\alpha=1.5$	$\alpha=1$		$\alpha=2$	$\alpha=1.5$	$\alpha=1$
	Maxwell model							
γ_{Mopt}	0.700	0.724	0.756	0.876	0.700	0.724	0.756	0.876
E_{Mopt} (MN/m^2)	273	291	317	414	546	582	633	828
C_{Mopt} (MNs/m^2)	10.55	10.55	10.53	10.39	14.92	14.91	14.89	14.69
V_{0max} (km/h)	26.6	27.0	27.4	27.9	37.6	38.1	38.7	39.5
	Kelvin-Voigt model							
γ_{Kopt}	0.404	0.401	0.398	0.385	0.404	0.401	0.398	0.385
E_{Kopt} (MN/m^2)	58	58	59	60	115	116	117	121
C_{Kopt} (MNs/m^2)	5.49	5.46	5.44	5.35	7.76	7.73	7.70	7.57
V_{0max} (km/h)	30.6	30.8	31.0	31.7	43.2	43.5	43.8	44.8
	purely elastic model							
E_{opt} (MN/m^2)	83	83	83	84	167	167	167	167
V_{0max} (km/h)	22.0	22.2	22.3	22.6	31.2	31.3	31.5	32.0
	constant force model $P(t)=P_{max}$ (*)							
V_{0max} (km/h)	31.2	44.1	44.1	44.1	44.1	62.4	62.4	62.4

(*) Results obtained for $\psi=1$

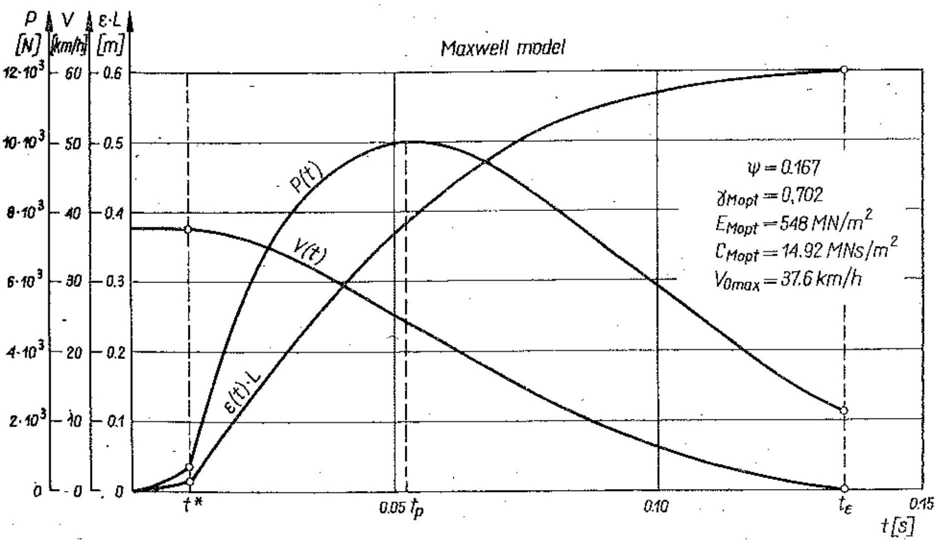

Fig. 4

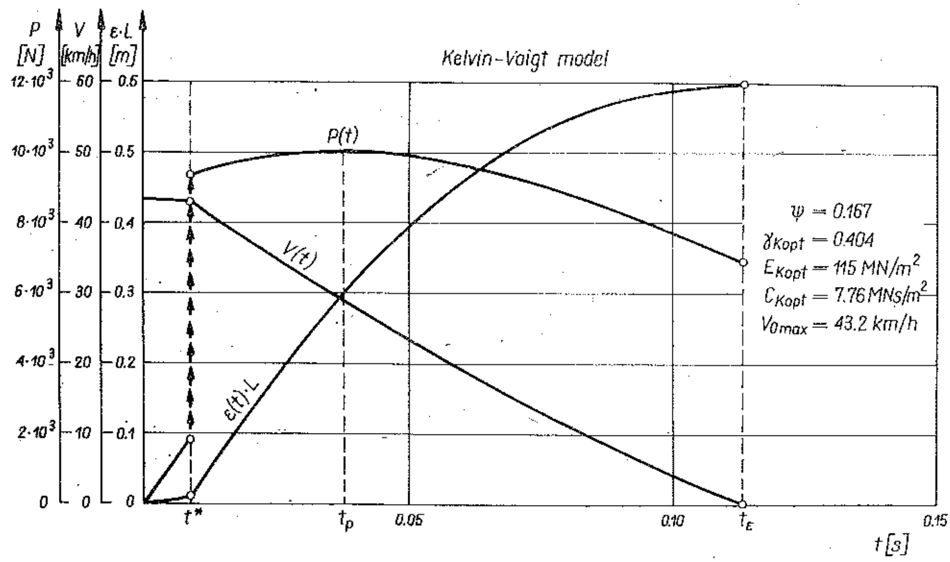

Fig. 7

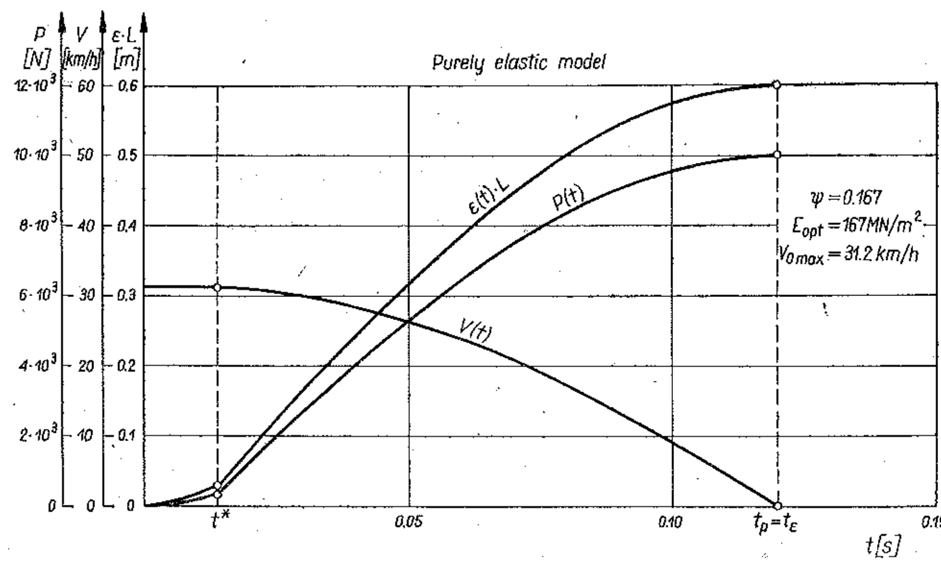

Fig. 8

Table 2.

α	ψ_{max}			
	Maxwell model	Kelvin-Voigt model	purely elastic model	constant force model
2	0.374	0.487	0.253	1
1.5	0.514	0.658	0.340	2
1	0.684	1.033	0.525	∞

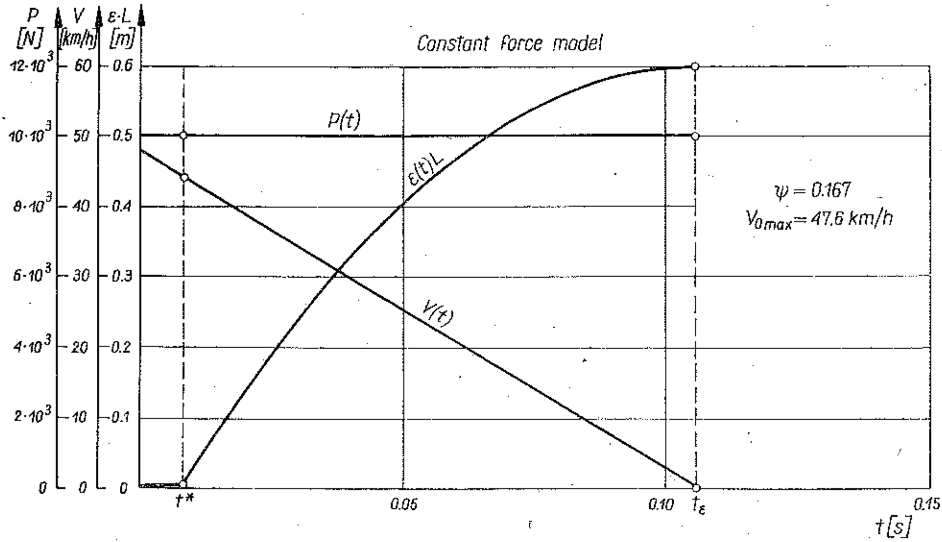

Fig. 9

8. Final remarks

It appears that the influence of plastic deformation of the car body during a crash does not produce an appreciable difference in initial speed V_0 and in optimum values of material constants for all considered models. A rise in the initial speed V_0 is neglected. Nevertheless we can observe the tendency of changes of C and E if ψ increases and the conditions of an impact become closer to reality. For all models the optimum values of C decrease and optimum values of E increase with increasing ψ. The changes are greater for the Maxwell model, since the Kelvin–Voigt model is closer to absolute optimality $(P(t)=\text{const})$ and less can be gained.

Appendix

The resulting equations for stress p, velocity v and strain ε in both crash periods:
1st period $(0\leqslant\tau\leqslant\tau^*)$
Maxwell

(I)
$$p_1=u(E_1\varphi_1+1),$$
$$v_1=u(E_1\varphi_2+2A_1+B_1),$$
$$\varepsilon_1=u(E_1\varphi_3+A_4+2A_1\tau);$$

Kelvin–Voigt

(II)
$$p_1=u(E_1\varphi_4+1),$$
$$v_1=u(E_1\varphi_5+B_1),$$
$$\varepsilon_1=u(E_1\varphi_1+1).$$

2nd period $(\tau\geqslant\tau^*)$.

Maxwell

$$\text{(III)}\quad \begin{aligned} p_2 &= uE_2\left(\frac{A_1+B_1^*}{A_2}S+C+E_1^*\varphi_1\right),\\ v_2 &= uE_2\left[\frac{A_1B_1^*-A_3}{A_2}S+(2A_1+B_1^*)C+E_1^*\varphi_2\right],\\ \varepsilon_2 &= uE_2\left[\frac{A_1(A_4+2)A_3B_1^*}{A_2}S+(A_4-2A_1B_1^*)C+E_1^*\varphi_3\right]+2A_1v_0; \end{aligned}$$

Kelvin-Voigt

$$\text{(IV)}\quad \begin{aligned} p_2 &= uE_2\left[-\frac{A_1-A_3B_1^*}{A_2}S+(1+2A_1B_1^*)C+E_1^*\varphi_4\right],\\ v_2 &= uE_2\left(-\frac{1+A_1B_1^*}{A_2}S+B_1^*C+E_1^*\varphi_5\right),\\ \varepsilon_2 &= uE_2\left(\frac{A_1+B_1^*}{A_2}S+C+E_1^*\varphi_1\right). \end{aligned}$$

The following notations are introduced in the above relations:

$$A_1=\gamma,\quad A_2=\sqrt{1-\gamma^2},\quad A_3=1-2\gamma^2,\quad A_4=1-4\gamma^2,$$

$$B_1=\frac{v_0}{u}-\tau,\quad B_1^*=\frac{v_0}{u}-\tau^*,$$

$$E_1(\gamma,\tau)=\exp(-\gamma\tau),\quad E_1^*(\gamma)=\exp(-\gamma\tau^*),\quad E_2(\gamma,\tau)=\exp[-\gamma(\tau-\tau^*)],$$

$$S(\gamma,\tau)=\sin[A_2(\tau-\tau^*)],\quad C(\gamma,\tau)=\cos[A_2(\tau-\tau^*)],$$

$$\varphi_1(\gamma,\tau)=-\frac{A_1}{A_2}\sin(A_2\tau)-\cos(A_2\tau),$$

$$\varphi_2(\gamma,\tau)=\frac{A_3}{A_2}\sin(A_2\tau)-2A_1\cos(A_2\tau),$$

$$\varphi_3(\gamma,\tau)=-\frac{A_1(A_4+2)}{A_2}\sin(A_2\tau)-A_4\cos(A_2\tau),$$

$$\varphi_4(\gamma,\tau)=\frac{A_1}{A_2}\sin(A_2\tau)-\cos(A_2\tau),$$

$$\varphi_5(\gamma,\tau)=\frac{1}{A_2}\sin(A_2\tau).$$

The indices M and K are not used in the Appendix. Therefore the parameter γ has to be completed with M in Eqs. (I) and (III) or with K in Eqs. (II) and (IV).

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Streszczenie

WPŁYW PLASTYCZNEGO ZGNIOTU NADWOZIA NA OPTYMALIZACJĘ REOLOGICZNYCH WŁASNOŚCI SAMOCHODOWYCH PASÓW BEZPIECZEŃSTWA

Zagadnienie optymalizacji reologicznych własności samochodowych pasów bezpieczeństwa zostało sformułowane i rozwiązane przez W. Nachbara i J. B. Schipmöldera [2] przy silnie upraszczających założeniach. W obecnej pracy problem optymalizacji przedstawiono w ujęciu bardziej odpowiadającym realnym warunkom zderzenia samochodu. W szczególności uwzględniony został efekt plastycznego zgniotu przedniej części nadwozia. Do rozwiązania zastosowano metodę małego parametru, za który przyjęto wielkość proporcjonalną do długości zgniatanej części nadwozia. Przy założeniu stałej uśrednionej nośności plastycznej nadwozia, rozróżniono przypadki całkowitego lub częściowego zgniotu elementów, znajdujących się przed pasażerem. Podobnie jak w pracy [2] określone zostały optymalne własności lepkosprężyste pasa bezpieczeństwa, maksymalizujące dopuszczalną, bezpieczną dla pasażera prędkość pojazdu przed zderzeniem.

Резюме

ВЛИЯНИЕ ПЛАСТИЧЕСКОГО СДАВЛЕНИЯ КУЗОВА НА ОПТИМАЛИЗАЦИЮ РЕОЛОГИЧЕСКИХ СВОЙСТВ АВТОМОБИЛЬНЫХ ПРЕДОХРАНИТЕЛЬНЫХ ПОЯСОВ

Проблема оптимализации реологических свойств автомобильных предохранительных поясов сформулирована и решена В. Нахбаром и Дж. Б. Шипмельдером [2] при сильно упрощающих предположениях. В настоящей работе проблема оптимализации представлена в подходе более отвечающим реальным условиям столкновения автомобилей. В частности, учтен эффект пластического сдавления передней части кузова. Для решения применен метод малого параметра, за который принята величина пропорциональная длине сдавленной части кузова. При предположении постоянной усредненной пластической несущей способности кузова, различаются случаи полного или частичного сдавления элементов, находящихся перед пассажиром. Аналогично как в работе [2], определены оптимальные вязкоупругие свойства предохранительного пояса, максимизирующие допустимую, безопасную для пассажира, скорость автомобиля перед столкновением.

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