

## DETERMINATION OF THE CRITICAL ADIABATICAL TWISTING MOMENT IN THE CASE OF THICK AND THIN-WALLED METAL TUBES

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In this paper the value of the critical adiabatical twisting moment for the case of thick- and thin-walled metal tubes is derived. The critical state means herein the maximum point of the adiabatical workhardening curve in the test of pure shearing or simple tension. The numerical results are demonstrated in a diagram form.

### 1. INTRODUCTION

The paper deals with the pure-local adiabatical twisting problem in the case of thick- and thin-walled metal tubes. The values of the critical adiabatical twisting moment are determined. Its growth can lead to the bifurcation of the equilibrium state on the outside surface of the tube (state of localization of the adiabatical shearing—see, for example, the comments in [2—4] and [19]). The critical state for which the twisting moment is determined means the state of maximum of the adiabatical workhardening curve in the test of simple tension or pure shearing (see for example [1]). In this paper some results from Chapter 3 of the work [1] are used, where the critical values of stress, strain and temperature for the pure adiabatical shearing process are determined.

The coupling problem for pure shear and twisting of thin-walled metal tubes was previously investigated, see for example [2—7]. In these papers the coupling between strain and temperature fields is taken into account. The authors use data from Manjoine's experiment [8] which was carried out for mild carbon steel. In this paper stainless steel data obtained in [12] are used. The problem of strain localization for the case of the twisting cone and for coupled strain and temperature fields is studied in the paper [9]. The influence of strain rate and heat conducting on the localization and propagation of the slip lines has been investigated in a recent work [10]. The analysis is carried out for the case of a metal ring with local wallsoftening (imperfection) as in papers [6—7].

In our work some "geometrical effect" is taken into account and its influence on the twisting moment value is studied. It is also shown how the linearization of the adiabatical workhardening law simplifies the problem and how such a result differs from the exact one. These effects are illustrated for various material constants on the diagrams, obtained by means of numerical methods.

## 2. ADIABATICAL PURE SHEARING TEST (SIMPLE TENSION) — CRITICAL STRESSES FOR SOME STAINLESS STEEL

Now some important results obtained in the paper [1] are presented. The class of metals considered in [1] can be described by the following equation for the case of simple tension [3, 5—7, 11]:

$$(2.1) \quad \sigma = Y(\varepsilon^p, \Theta) = (C - b\Theta)(b_1 + \varepsilon^p)^n,$$

where  $\sigma$  is the real stress,  $\varepsilon^p$  — the logarithmical plastic strain and  $\Theta$  — the temperature. Material constants for some stainless steels, according to experimental data — [12, 13], assume the following values:

$$\begin{aligned} C &= 45 \div 500 \text{ kG/mm}^2, & b &= 0.02 \div 0.5 \text{ kG/(mm}^2\text{C)}, \\ b_1 &= 0.016, & n &= 0.2 \div 0.5, \\ \rho_0 C_\sigma &= 0.3 \div 0.4 \text{ kG/(mm}^2\text{C)}, & E &= 2000 \div 2200 \text{ kG/mm}^2, \\ \sigma_0 &= 20 \div 60 \text{ kG/mm}^2, \end{aligned}$$

where  $E$  denotes Young's moduli and  $\sigma_0 = cb_1^n$  — the initial yield stress for  $\varepsilon^p = 0$  and  $\Theta = 0^\circ\text{C}$ .

The constitutive equation (2.1) in dimensionless form is:

$$(2.2) \quad \tau = (A - \Theta^*)(b_1 + \varepsilon^p)^n,$$

where

$$(2.3) \quad \tau = \frac{\sigma(1+\nu)}{\sqrt{3}E}, \quad \Theta^* = \frac{b(1+\nu)}{\sqrt{3}E}\Theta, \quad A = \frac{C(1+\nu)}{\sqrt{3}E}, \quad 2\mu = \frac{E}{1+\nu}.$$

The dimensionless work of the plastic strain is the following:

$$(2.4) \quad \kappa^* = \frac{1+\nu}{E} \kappa = \sqrt{3} \int_0^{\varepsilon^p} \tau(\Theta^*, \varepsilon^p) d\varepsilon^p,$$

where

$$\kappa = \int_0^{\varepsilon^p} \sigma(\Theta, \varepsilon^p) d\varepsilon^p$$

is the dimensional work of plastic strain.

By substituting Eq. (2.2) into Eq. (2.4) and after integrating one obtains

$$(2.5) \quad \kappa^* = \frac{\sqrt{3}}{n+1} (A - \Theta^*) [(b_1 + \varepsilon^p)^{n+1} - b_1^{n+1}].$$

Now let us assume that the stored energy of plastic deformation takes the form [1] —  $\rho_0 G^p = \pi \cdot \kappa$  where  $\pi = 0.1$ . Then the equation of the temperature field (see Eq. (3.1) in the work [1]) yields

$$(2.6) \quad \Theta^* = \delta \cdot \kappa^*, \quad \delta = 0.5196 \frac{b}{\rho_0 C_\sigma}$$

The parameter  $\delta$  represents the sensitivity of the yield stress point on the temperature. From Eqs. (2.2), (2.5) and (2.6) we derive the following adiabates:

$$(2.7) \quad \tau^a = \tau(\kappa^*), \quad \varepsilon^p = \varepsilon^p(\kappa^*), \quad \Theta^* = \delta \cdot \kappa^*.$$

Taking the derivative of the first equation of the set (2.7) equal to zero we obtain the critical value  $\kappa_{cr}^*$  and, corresponding to this value,

$$(2.8) \quad \varepsilon_{cr}^p = \left[ \frac{n(1+n-\sqrt{3}\delta b_1^{n+1})}{\sqrt{3}\delta} \right]^{\frac{1}{n+1}} - b_1,$$

$$\Theta_{cr}^* = A \frac{n - \sqrt{3}\delta b_1^{n+1}}{1+n-\sqrt{3}\delta b_1^{n+1}},$$

$$\tau_{cr}^a = A \left( 1 - \frac{n - \sqrt{3}\delta b_1^{n+1}}{1+n-\sqrt{3}\delta b_1^{n+1}} \right) \left( \frac{n(1+n-\sqrt{3}\delta b_1^{n+1})}{\sqrt{3}\delta} \right)^{\frac{n}{1+n}}$$

The second equation of the set (2.8) can be substituted with very high accuracy by Eq. (2.9) for  $\delta=0.028-0.730$  (from Eq. (2.6)<sub>2</sub>),  $b=0.02-0.5$  and  $\rho_0 C_\sigma=0.35$ :

$$(2.9) \quad \Theta_{cr}^* \cong A \frac{n}{1+n}.$$

Now the equation (2.8)<sub>3</sub> takes simpler form

$$(2.10) \quad \tau_{cr}^a \approx \frac{A}{n+1} \left[ \frac{n(1+n-\sqrt{3}\delta b_1^{n+1})}{\sqrt{3}\delta} \right]^{\frac{n}{1+n}}$$

### 3. TWISTING

#### 3.1. Adiabatical twisting of the thick-walled tube

We assume that:

during the twisting process there are only stiff rotations of the cross-sections; pure shear state is in the middle of the tube.

Using the results from Sect. 2 of this paper, we derive the critical value of the adiabatical twisting moment, the growth of which can lead to a bifurcation state on the outside surface of the tube (state of localization of the adiabatical shearing). For the thick-walled tube the problem is one-dimensional due to the axis symmetry (Fig. 1).

Let us introduce the following measure of plastic deformation (see [5-7]):

$$(3.1) \quad \varepsilon^p = \frac{\text{tg } \gamma^p}{\sqrt{3}},$$

where  $\gamma^p$  is the angle of plastic shearing. Decomposing the deformation into elastic and plastic parts we get

$$(3.2) \quad \gamma = \gamma^e + \gamma^p,$$

where  $\gamma^e$  is the angle of elastic shearing. From geometrical analysis (Fig. 1, [14]), one can obtain

$$(3.3) \quad \operatorname{tg} \gamma = \alpha_s r,$$

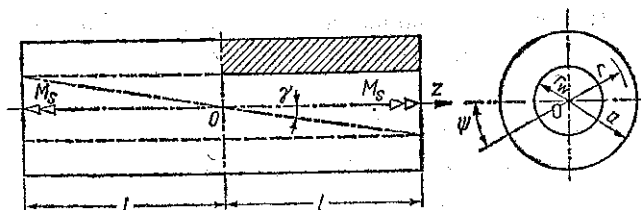


FIG. 1.

where  $\alpha_s$  is the angle of twisting corresponding to the unit of the length and  $r$  — the current radius of the tube (Fig. 1). From Eq. (3.2) and trigonometrical relations we obtain the following:

$$(3.4) \quad \operatorname{tg} \gamma^e = \frac{\operatorname{tg} \gamma - \operatorname{tg} \gamma^p}{1 + \operatorname{tg} \gamma \operatorname{tg} \gamma^p}$$

For pure twisting of the thick-walled tube, on the ground of [14], we can obtain the following expression on the correlation between the shear stress  $\tau$  and the elastic shearing angle  $\gamma^e$ :

$$(3.5) \quad \tau = \frac{1}{2} \operatorname{tg} \gamma^e.$$

Then, from Eqs. (3.1), (3.3) and (3.5) we get

$$(3.6) \quad \tau = \frac{1}{2} \left( \frac{\alpha_s r - \sqrt{3} \varepsilon^p}{1 + \sqrt{3} \alpha_s r \varepsilon^p} \right).$$

Let us introduce the following dimensionless parameters:

$$(3.7) \quad \beta_s = \alpha_s a, \quad \rho = \frac{r}{a}, \quad \rho_w = \frac{r_w}{a},$$

where  $a$  is the outside radius of the tube (Fig. 1). Substitution of the formulae (3.7) into Eq. (3.6) leads to

$$(3.8) \quad \tau = \frac{1}{2} \left( \frac{\beta_s \rho - \sqrt{3} \varepsilon^p}{1 + \sqrt{3} \beta_s \rho \varepsilon^p} \right),$$

where  $\sqrt{3} \beta_s \rho \varepsilon^p$  represents the "geometrical effect" mentioned earlier (see also denominators of Eqs. (3.4) and (3.6)). As it is known the first bifurcation states can arise on the outside surface of the tube, e.g. for  $\rho=1$ . Then the plastic strain, temperature and shear stress will have their critical values on this surface (see Eq. (2.8), or Eqs. (2.8)<sub>1</sub>, (2.9), (2.10)). Knowing this and substituting the critical values  $\varepsilon_{cr}^p$ ,  $\tau_{cr}^a$  and  $\rho=1$  into Eq. (3.8) we obtain, after some transformations,

$$(3.9) \quad \beta_s^{cr} = \frac{2\tau_{cr}^a + \sqrt{3} \varepsilon_{cr}^p}{1 - 2\sqrt{3} \tau_{cr}^a \varepsilon_{cr}^p},$$

where  $\beta_s^{cr}$  is the dimensionless critical twisting angle (see Eq. (3.7)<sub>1</sub>) for which some bifurcation states can arise on the outside surface. It is worth remarking that by setting  $2\sqrt{3}\tau_{cr}^a \epsilon_{rc}^p = 0$ , the above mentioned "geometrical effect" is neglected. The values of  $\beta_s^{cr} = \beta_s^{cr}(\delta, A, n)$ , where  $A$  and  $n$  are some parameters, are shown in Fig. 2.

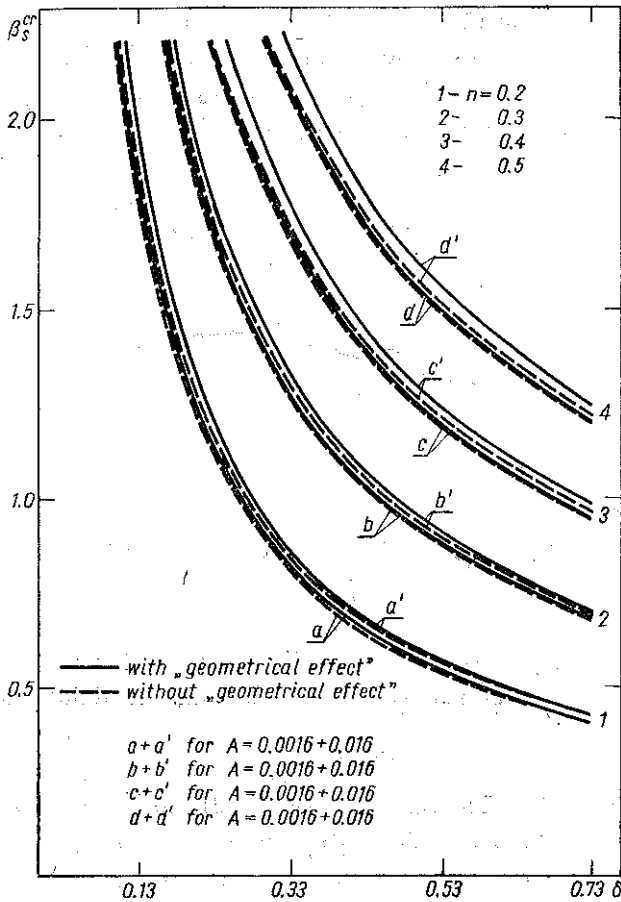


FIG. 2.

It turns out that for the critical state on the outside surface the values of  $\rho_{cr}^c$  are very low — less than  $10^{-2}$ . Therefore we can write (see Eq. (3.7)<sub>2,3</sub>) that  $\rho_w \gg \rho_{cr}^c$ . This means that the whole cross-section is in the plastic state when the critical state is attained. This remark will be useful for determining the critical adiabatical twisting moment. From Fig. 1 and Eq. (3.7)<sub>1</sub> we can observe that the angle  $\psi$  measured on the frontal surface of the tube, see for example [14], is equal to

$$(3.10) \quad \psi = \alpha_s l = \beta_s \frac{l}{a},$$

where  $l$  is the half length of the tube.

The dimensionless twisting moment  $M_s^*$  can be determined by solving the following integral [14—18]:

$$(3.11) \quad M_s^* = \frac{M \cdot (1 + \nu)}{Ea^3} = 2\pi \int_{\rho_w}^1 \tau(\rho) \rho^2 d\rho.$$

From the general equations (2.2), (2.5), (2.6) and (3.8) we cannot get an analytical solution of the expression (3.11). So in this case we have obtained a numerical solution, knowing that  $M_s^{*a} = M_s^{*a}|_{\beta_s = \beta_s^{cr}}$ . However by linearization of the function  $\tau^a(\varepsilon^p)$  — Eqs. (2.2), (2.7)<sub>1</sub>, we may obtain the analytical value of  $M_s^*$  which is

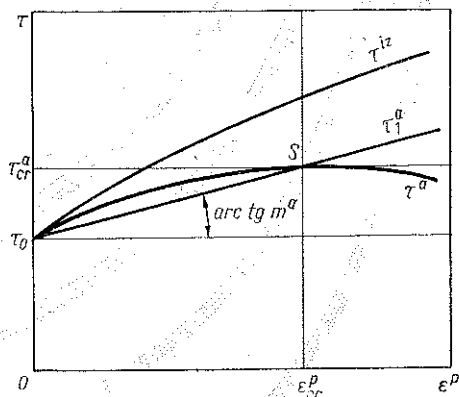


FIG. 3.

a lower estimation of the exact solution. This idea is shown in Fig. 3 where  $m^a = (\tau_{cr}^a - \tau_0) : \varepsilon_{cr}^p$  is a linear adiabatical workhardening modulus. Hence

$$(3.12) \quad \tau_1 = \tau_0 + m^a \varepsilon^p$$

is a lower estimation of  $\tau^a$ . The curve  $\tau^{i2}$  (see Fig. 3) shows the values of Eq. (2.2) for  $\theta = 0^\circ\text{C}$  (upper estimation of  $\tau^a$ ). Substituting  $\tau^a$  by  $\tau_1^a$  in Eq. (3.8) we get

$$(3.13) \quad \tau_0 + m^a \varepsilon^p = \frac{1}{2} \left( \frac{\rho \beta_s - \sqrt{3} \varepsilon^p}{1 + \sqrt{3} \varepsilon^p \beta_s \rho} \right).$$

Now from Eq. (3.13) one can determine  $\varepsilon^p(\rho)$ ,

$$(3.14) \quad \varepsilon^p(\rho) = \frac{-2m^a - \sqrt{3}}{4\sqrt{3}m^a\beta_s\rho} - \frac{\tau_0}{2m^a} + \frac{\sqrt{A(\rho)}}{4\sqrt{3}m^a\beta_s\rho},$$

where

$$(3.15) \quad \begin{aligned} A(\rho) &= a_s \rho^2 + b_s \rho + c_s, \\ a_s &= 4\beta_s^2 (3\tau_0^2 + 2\sqrt{3}m^a), \\ b_s &= 4\tau_0\beta_s (3 - 2\sqrt{3}m^a), \\ c_s &= (2m^a + \sqrt{3})^2. \end{aligned}$$

Substitution of Eq. (3.14) into Eq. (3.12) yields:

$$(3.16) \quad \tau_1^a = \frac{\tau_0}{2} - \frac{2m^a + \sqrt{3}}{4\sqrt{3}\beta_s\rho} + \frac{\sqrt{\Delta(\rho)}}{4\sqrt{3}\beta_s\rho}.$$

Finally, after placing Eq. (3.16) into Eq. (3.11), and after integrating, the lower adiabatical twisting moment takes the following form:

$$(3.17) \quad M_{1s}^{*a} = 2\pi \int_{\rho_w}^1 \tau_1^a(\rho) \rho^2 d\rho = 2\pi [A_s(\rho) + B_s(\rho) - C_s(\rho) - D_s(\rho)]|_{\rho_w}^1,$$

where

$$(3.18) \quad \begin{aligned} A_s(\rho) &= \frac{\tau_0}{6} \rho^3 - \frac{2m^a + \sqrt{3}}{8\sqrt{3}\beta_s} \rho^2, \\ B_s(\rho) &= \frac{[\Delta(\rho)]^{1.5}}{12\sqrt{3}a_s\beta_s}, \\ C_s(\rho) &= \frac{(2a_s b_s \rho + b_s^2) \sqrt{\Delta(\rho)}}{32\sqrt{3}a_s^2\beta_s}, \\ D_s(\rho) &= \frac{(4a_s b_s c_s - b_s^3) \ln(2a_s \rho + b_s + 2\sqrt{a_s \Delta(\rho)})}{64\sqrt{3}a_s^{2.5}}, \end{aligned}$$

The critical adiabatical twisting moment (see point *S*, Fig. 3), is determined from Eq. (3.17):

$$(3.19) \quad M_{1s\text{cr}}^{*a} = M_{1s}^{*a}|_{\beta_s = \beta_s^{\text{cr}}}.$$

The Figs. (4a-4d) show the critical values of the adiabatical twisting moment  $M_{1s\text{cr}}^{*a}$  (continuous line) and its estimation (dash line) given by Eq. (3.19). Each of them is the function of  $\delta$  for given values of the parameters  $A$  and  $n$ . The dimensionless internal radius (3.7)<sub>3</sub> was assumed as  $\rho_w = 0.5$ .

#### EXAMPLE 1

Let the stainless-steel of a thick-walled tube have the following material constants:

$$C = 112 \text{ kG/mm}^2, \quad b = 0.336 \text{ kG/(mm}^2 \text{ }^\circ\text{C)}, \quad b_1 = 0.016,$$

$$n = 0.3, \quad \rho_0 C_\sigma = 0.35 \text{ kG/(mm}^2 \text{ }^\circ\text{C)}, \quad \pi = 0.1,$$

$$E = 2.1 \cdot 10^4 \text{ kG/mm}^2, \quad \sigma_0 = 32.4 \text{ kG/mm}^2, \quad \nu = 0.3.$$

The following values are determined from Eqs. (2.3)<sub>3</sub>, (2.6)<sub>2</sub> and (2.8):

$$A = 0.004, \quad \delta = 0.4988, \quad \Theta_{\text{cr}}^* = 0.000923,$$

$$\varepsilon_{\text{cr}}^p = 0.52, \quad \tau_{\text{cr}}^a = 0.00252.$$

For  $\rho_w=0.5$  we obtain from the Fig. 4b:

$$M_{s\ cr}^{*a} = 0.0047, \quad M_{1s\ cr}^{*a} = 0.0042.$$

Now it is derived from Eqs. (2.3)<sub>1,2</sub> and (3.11) for  $a=20$  mm that:

$$\sigma_{cr}^a = 70.5 \text{ kG/mm}^2, \quad \Theta = 76.8 \text{ }^\circ\text{C},$$

$$M_{s\ cr}^a = 607.4 \text{ kGm}, \quad M_{1s\ cr}^a = 542.8 \text{ kGm},$$

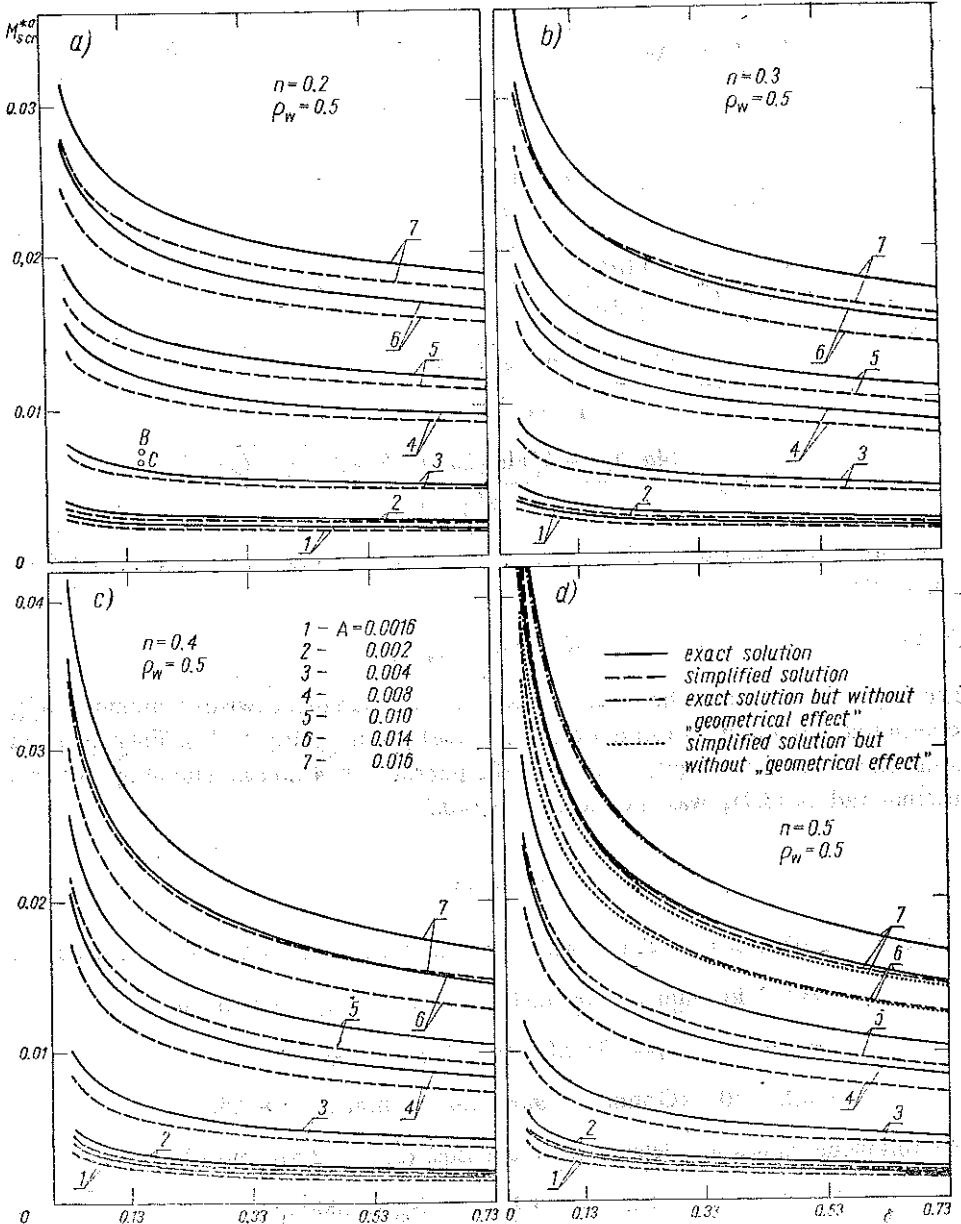


FIG. 4.



## EXAMPLE 2

In this example we take the material constants of carbon steel which suit the Manjoine's experiment [8] (see also [5-7]).

They are as follows:

$$\begin{aligned} C_0 &= 114.5 \text{ kG/mm}^2, & b &= 0.16 \text{ kG/(mm}^2 \text{ }^\circ\text{C)}, & b_1 &= 0.01, \\ n &= 0.2, & \rho_0 \cdot C_\sigma &= 0.526 \text{ kG/(mm}^2 \text{ }^\circ\text{C)}, & \pi &= 0.1, \\ E &= 1.9 \cdot 10^4 \text{ kG/mm}^2, & \nu &= 0.3. \end{aligned}$$

Hence Eqs. (2.3)<sub>3</sub>, (2.6)<sub>2</sub> and (2.8) result in:

$$A = 0.00477, \quad \delta = 0.158, \quad \theta_{cr}^* = 0.000795, \quad \tau_{cr}^a = 0.0039.$$

For  $\rho_w = 0.5$  from the Fig. 4a (points B and C) it arises:

$$M_{s\ cr}^{*a} = 0.0071 \text{ (point B)}, \quad M_{1s\ cr}^a = 0.0065 \text{ (point C)}.$$

From Eqs. (3.11) and (2.3)<sub>1</sub>, for  $a = 20$  mm it follows that:

$$\sigma_{cr}^a = 98.7 \text{ kG/mm}^2, \quad M_{s\ cr}^a = 786.5 \text{ kGm}, \quad M_{1s\ cr}^a = 720 \text{ kGm}.$$

## 3.2. Adiabatical twisting of a thin-walled tube

In this case we can also identify the stress with a pure shearing state and assume that shearing stresses are constant throughout the wall (if only the thickness of the tube is small enough). Under such assumptions the problem is one-dimensional. Hence the dimensionless critical twisting moment (3.11) takes the form

$$(3.20) \quad \bar{M}_{s\ cr}^{*a} \approx 2\pi\tau_{cr}^a \int_{\rho_w}^1 \rho^2 d\rho = \frac{2}{3} \pi \tau_{cr}^a (1 - \rho_w^3).$$

From the Fig. 3 it follows that one can get upper estimation of the moment  $\bar{M}_{s\ cr}^{*a}$  in a similar way as before, but using the isothermal curve. Then Eqs. (2.2), (2.9), (2.10) and (3.20) yield

$$(3.21) \quad \bar{M}_{s\ cr}^{*iz} \approx (1+n) \bar{M}_{s\ cr}^{*a}.$$

In this case the real critical twisting moment  $\bar{M}_{s\ cr}^{*r}$  connected with the heat transfer can be estimated:

$$(3.22) \quad \bar{M}_{s\ cr}^{*a} < \bar{M}_{s\ cr}^{*r} < (1+n) \bar{M}_{s\ cr}^{*a}.$$

The above estimation is not right in the case of thick-walled metal tubes — Sect. 3.1. It is worth remarking that by using the expression (3.20) the critical adiabatical twisting moment can be very easily obtained. It is shown in the undermentioned example.

## EXAMPLE 1

Let the material constants be the same as in the previous Example 1, and that  $\rho_w = 0.9$ . From Fig. 4 we get  $\tau_{cr}^a = 0.00252$  and Eq. (3.20) yields

$$\bar{M}_{s\ cr}^{*a} \approx 0.00143.$$

For  $a=20$  mm from Eq. (3.17) the dimensional twisting moment has the value

$$\bar{M}_{cr}^a = 184.8 \text{ kGm.}$$

The critical values of the adiabatical deformations  $\varepsilon_{cr}^p$  and temperature  $\Theta_{cr}$  are the same as in the previous Example 1.

#### 4. CONCLUDING REMARKS

The results show that taking into account the "geometrical effect" does not lead to any considerable quantitative changes in the values of the critical adiabatical twisting moment (see Fig. 4d). This refers to both linear and nonlinear physical equations (see Eqs. (2.2), (2.7)<sub>1</sub> and (3.12)). The largest differences occur for  $n=0.5$ ,  $A=0.016$ ,  $\delta=0.03$  and they are of the order of 0.1% (see Fig. 4d). So, in practice, we can neglect the "geometrical effect" because it only insignificantly increases the critical twisting moment. This effect has a slightly larger influence (about 3%) on the critical twisting angle  $B_s$ ,  $C$ , (see Fig. 3).

Simplification by describing the strain hardening curve (see Eq. (3.12)) has a considerable influence on the value of the critical adiabatical twisting moment. Differences between exact (see Eqs. (2.2) and (2.7)<sub>1</sub>) and simplified solutions (see Eq. (3.12)) are largest for  $n=0.5$ ,  $\delta=0.73$  and they are of the order of 16%. The dimensionless factor  $\delta$  which represents the sensitivity of the yield stress on the temperature has here an influence on the results while the material parameter  $A$  has not. All these differences do not appear when thin-walled tube is considered (see Sect. 3.2).

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## STRESZCZENIE

## OKREŚLENIE KRYTYCZNEJ WARTOŚCI MOMENTU PRZY ADIABATYCZNYM SKRĘCANIU GRUBOŚCIENNYCH I CIENKOŚCIENNYCH RUR METALOWYCH

W pracy wyznacza się wartość krytyczną adiabaticznego momentu skręcającego w przypadku skręcania grubo i cienkościennych metalowych rurek. Stan krytyczny, który wyznacza wartość krytycznego, adiabaticznego momentu skręcającego, jest stanem, któremu odpowiada maksimum na adiabaticznej krzywej wzmocnienia w próbie na czyste ścinanie lub proste rozciąganie. Uzyskane na drodze obliczeń numerycznych rezultaty przedstawia się w postaci odpowiednich wykresów, które mają charakter nomogramów.

## Резюме

## ОПРЕДЕЛЕНИЕ КРИТИЧЕСКОГО ЗНАЧЕНИЯ КРУТЯЩЕГО МОМЕНТА ПРИ АДИАБАТИЧЕСКОМ КРУЧЕНИИ ТОЛСТОСТЕННЫХ И ТОНКОСТЕННЫХ МЕТАЛЛИЧЕСКИХ ТРУБОК

В работе определяется критическое значение адиабатического крутящего момента при кручении толсто и тонкостенных металлических трубок. Критическое состояние, определяющее значение критического крутящего момента, является состоянием, которому соответствует максимум адиабатической кривой упрочнения при испытании на чистый сдвиг либо простое растяжение. Численно полученные результаты представлены в виде соответствующих графиков — номограмм.

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