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THE POST-YIELD ANALYSIS OF RIGID PLASTIC BEAMS COLUMNS AND FRAMES

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In the paper the influence of second-order geometrical effects on the post-yield behaviour of plastic structures is considered. The presentation is limited to the purely mechanical (isothermal) theory and quasi-static deformation processes. The problem considered is illustrated by means of the examples of rigid-plastic beams with various kinds of partial end fixity, the rigid-plastic columns loaded by vertical and horizontal forces and multi-story portal frames. It is demonstrated that plastic deformations may result in changing the boundary conditions in such a way that the stabilizing effect of tensile axial forces is reduced. By making the proper choice of some factors unstable mechanisms and catastrophic collapse can be avoided.

1. Introduction

The post-yield behaviour of a plastic structure depends, on the one hand, on the material properties and, on the other, on the effects of geometry changes. The material is usually assumed to be strain-hardening (stable) in the range of deformations which are relevant from the engineering point of view. The influence of geometry changes, however, may be either stabilizing or destabilizing.

In this paper attention will be focused on problems concerning geometrical nonlinearities and thus, for clarity, the perfectly plastic material model will be assumed. Then the geometry changes will be the only reason for the structure to remain stable or become unstable.

The presentation is limited to the purely mechanical (isothermal) theory and quasi-static deformation processes.

The problem considered, is illustrated by means of the examples of beams with partial and fixity, the fixed column loaded by vertical and horizontal forces and multi-story portal frames.

The influence of changes of boundary conditions due to plastifications on the stability at the yield point load and on the post-yield behaviour is also considered.

In the presence of finite elastic displacement or structural imperfections, the load-carrying capacity calculated by the tools of limit analysis may never be reached if the post-yield behaviour is predicted to be unstable. Therefore the meaningful assessment of the load-carrying capacity must be accompanied by the analysis of the structural behaviour after the yield-point load has been reached.

2. Incremental analysis of post-yield behaviour of rigid-plastic beams with partial and fixity

There are numerous papers describing the post-yield behaviour of fully fixed beams. In particular, detail analysis was presented in [1]. In the papers [2]–[4] attention was focused on the problem of the changing of the collapse mechanism at advanced plastic deformation. The study of the behaviour of rigid-plastic and elastic-plastic fully fixed beams and comparison of the results obtained with the experimental data where presented in the paper [5].

There are many practical situations, however, in which it is difficult to realize the perfect end fixity of a beam. Therefore the assumption of partial end fixity represents the real condition.

The influence of elastic deformations and frictional restraints on the load-deflection relations for elastic-plastic and plastic plates were considered by M. Janas [6] and M. Janas, A. Sawczuk [7]. N. Jones [8] presented an analysis of beam which was subjected to a prescribed axial displacement at the end. The investigation of a rigid-plastic beam on elastic supports was carried by P. G. Hodge [9].

An analysis of the post-yield behaviour of rigid-plastic beams with various kinds of end fixity is presented in this chapter. The incremental formulation of the problem is proposed in contrary to the rate formulation applied in the papers [6]–[9]. The advantage of this method consists in being able to describe in the same way the whole class of beams with various boundary conditions and in applying directly the results obtained in the post-yield analysis of plastic frames (Sect. 4).

Let us consider a prismatic beam subjected, in the midspan, to monotonically increasing dead load P (Fig. 1).

The following cases of partial end fixity will be analysed:

The elastic restraints in axial direction (Fig. 1a).

Motion of the supports of beam under constant axial force (Fig. 1b).

Motion of the supports of beam under force of friction (Fig. 1c).

The material of the beam is assumed to be rigid-perfectly plastic and the beam's cross section obeys the yield condition in the form

$$F = m + n^2 - 1 = 0, \tag{2.1}$$

where m, n are, respectively, the dimensionless bending moment and axial

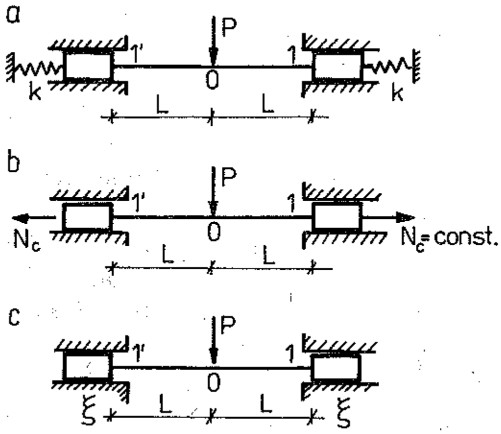

FIG. 1.

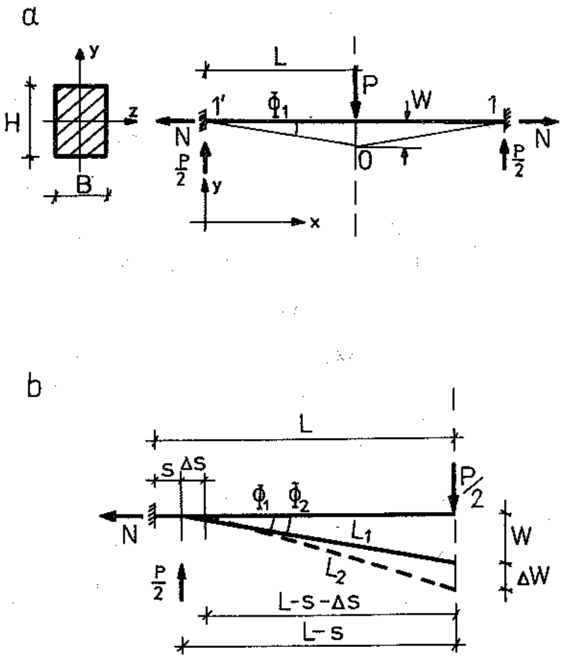

FIG. 2.

force

(2.2) $$m = \frac{M}{M_0}, \qquad n = \frac{N}{N_0},$$

(2.3) $$M_0 = \sigma_0 \frac{BH^2}{4}, \qquad N_0 = \sigma_0 BH,$$

B, H are the dimensions of the rectangular cross section of the beam (width and depth respectively), σ_0 is the yield stress at uniaxial tension.

To specify the geometrical realtions we compare the two neighbouring equilibrium states of the beam as shown in Fig. 2.

It is assumed that the plastic deformations are concentrated at the generalized plastic hinges. The length and the rotations of the rigid segments at the time t and $t+\Delta t$ are denoted, respectively, by L_1, L_2 and Φ_1, Φ_2. For the deflection W comparable with the depth of the beam H $(H \approx W)$ and for the increment of deflection ΔW much smaller than H $(\Delta W \ll H)$, the increments of elongation $\Delta \Lambda$ and rotation $\Delta \Phi$ at the generalized plastic hinges can be calculated from the relations

$$\Delta \Lambda = \frac{1}{2}(L_2 - L_1), \quad \Delta \Phi = \Phi_1 - \Phi_2, \tag{2.4}$$

where

$$(L_1)^2 = (L-s)^2 + W^2, \quad (L_2)^2 = (L-s-\Delta s)^2 + (W+\Delta W)^2, \tag{2.5}$$

$$\Phi_1 = \frac{W}{L-s}, \quad \Phi_2 = \frac{W+\Delta W}{L-s-\Delta s}. \tag{2.6}$$

Substituting Eq. (2.5) and (2.6) into the relations (2.4) and neglecting the higher order terms in the Taylor series expansion, we obtain

$$\Delta \Lambda = \frac{W \Delta W}{2(L-s-\Delta s)} - \Delta s, \tag{2.7}$$

$$\Delta \Phi = \frac{\Delta W}{L-s-\Delta s}. \tag{2.8}$$

Taking into account the flow rule (associated with the yield condition (2.1)), the dimensionless axial force n can be determined from the relation

$$n = \frac{2}{H} \frac{\dot{\Lambda}}{\dot{\Phi}} \tag{2.9}$$

or, in the incremental form,

$$n = \frac{2}{H} \frac{\Delta \Lambda}{\Delta \Phi}. \tag{2.10}$$

Next, substituting Eqs. (2.7), (2.8) into Eq. (2.10) and neglecting the higher order terms, the following relation between the deformation parameter and the axial force, can be obtained:

$$\Delta s = \frac{\Delta W}{L-s}(W - nH). \tag{2.11}$$

Finally, making use of the equilibrium equation

$$P(L-s) = 4M + 2NW \tag{2.12}$$

and of the yield condition (2.1), the external load necessary to continue the deformation process can be expressed in the form

$$p = \frac{P}{P_0} = \frac{L}{L-s}\left(1 - n^2 + 2n\frac{W}{H}\right) \tag{2.13}$$

where $P_0 = 4M_0/L$ is the limit load.

The formulas (2.11) and (2.13) describe the behaviour of the beam for any kind of considered partial end fixity. The control parameter in the incremental procedure is ΔW. Equation (2.11) determines the increment of the displacement of the ends of the beam Δs and, therefore, also the new value of s. Next, making use of the boundary (partial end fixity) condition, the load intensity p can be calculated by means of Eq. (2.13).

2.1. *Fully-fixed beam*

For a fully-fixed beam the displacements of supports are equal to zero: $s = 0, \Delta s = 0$. Then Eq. (2.11) yields

$$n = \frac{W}{H} = w, \tag{2.14}$$

where w is dimensionless deflection. Next, substituting Eq. (2.14) and $s = 0$ into Eq. (2.13), we obtain the known relation $p - w$ for a fully-fixed beam [7]

$$p = (1 + w^2). \tag{2.15}$$

2.2. *Elastic supports*

We assume that the horizontal end displacements s are proportional to the actual axial force intensity n (1)

$$n = ks, \tag{2.16}$$

where k is the stiffness of the elastic supports in the axial direction.

Substituting the relation (2.16) into Eq. (2.11) and into Eq. (2.13), we obtain

$$\Delta s = \frac{W}{L-s}(W - ksH), \tag{2.17}$$

$$p = \frac{L}{L-s}\left[1 - (ks)^2 + 2ks\frac{W}{H}\right]. \tag{2.18}$$

(1) This simplification can be accepted if the deflections are not greater than beam depth.

The results of the analysis for the different value of k are presented in Fig. 3. The post-yield behaviour of the beam is stable (the slope of $p-w$ curve is positive) for all $k>0$. The geometrical hardening is greater for the greater value of k. For $k \to 0$ the beam proves the natural stability. The analytical solution presented in [9] confirms the above results obtained by incremental procedure.

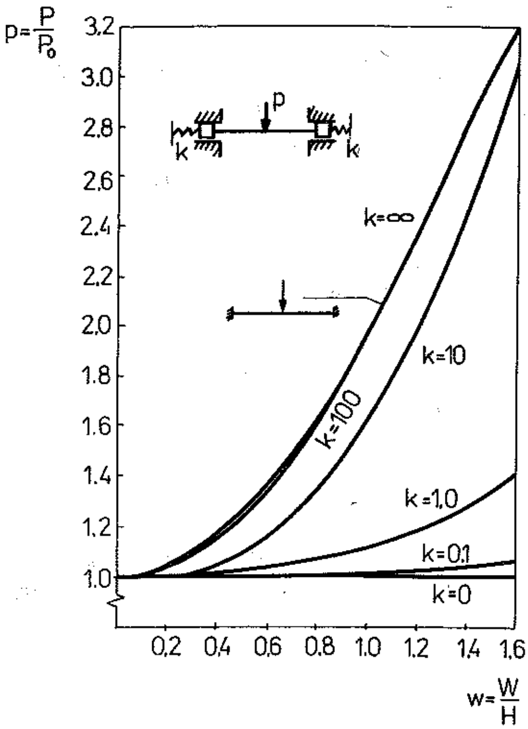

FIG. 3.

2.3. Motion of the supports of beam under the constant axial force

We assume that the horizontal displacements s start to increase when the axial force n reaches a certain critical value n_c and then remains constant.

$$n = n_c. \tag{2.19}$$

For $n < n_c$ the beam behaves as fully fixed and the motion is described by Eq. (2.15). For $n = n_c$ the $p-w$ relation given by Eqs. (2.11) and (2.13) results in

$$\Delta s = \frac{\Delta W}{L-s}(W - n_c H), \tag{2.20}$$

$$p = \frac{L}{L-s}\left(1 - n_c^2 + 2n_c \frac{W}{H}\right). \tag{2.21}$$

The post yield behaviour of the beam with supports moving at constant axial force is shown in Fig. 4 for several values of n_c. For $w < n_c$ the curves $p-w$ have the common part described by Eq. (2.15).

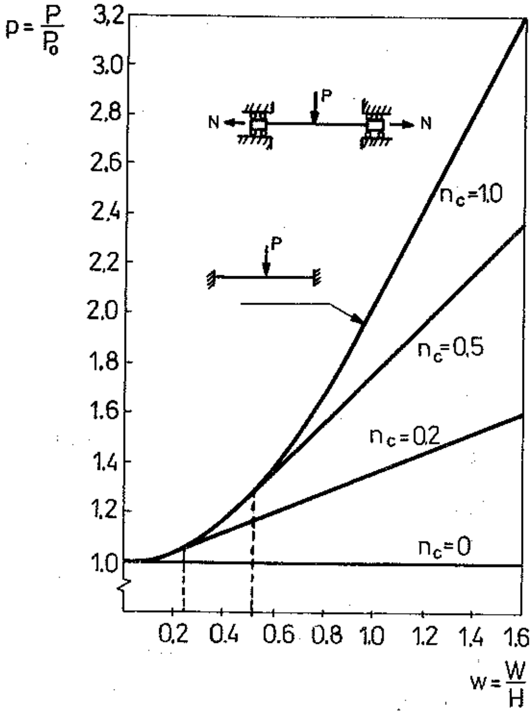

FIG. 4.

2.4. Motion of the supports of the beam under the force of friction

Now let us consider the case when the motion of the supports of the beam begins as the axial force n reaches the value of friction force determined by the relation (2)

$$n = \frac{P}{2N_0}\xi = \frac{H}{2L}p\xi. \tag{2.22}$$

The axial force n is now proportional to the load p and the friction parameter ξ. Substitution of Eq. (2.22) into Eqs. (2.11) and (2.13) yields

(2.23) $$\Delta s = \frac{\Delta W}{(L-s)}\left(W - \frac{H^2}{2L}p\xi\right),$$

(2.24) $$p = \frac{L}{(L-s)}\left[1-\left(\frac{H}{2L}p\xi\right)^2 - \frac{H}{L}p\xi\frac{W}{H}\right].$$

The set of equations (2.23) and (2.24) describes the relation between the dimensionless load p and deflection w.

The diagrams $p-w$ are plotted in Fig. 5 for several values of the friction parameter ξ. Similarly as in the previous case, for $n < \frac{H}{2L}p\xi$ the beam

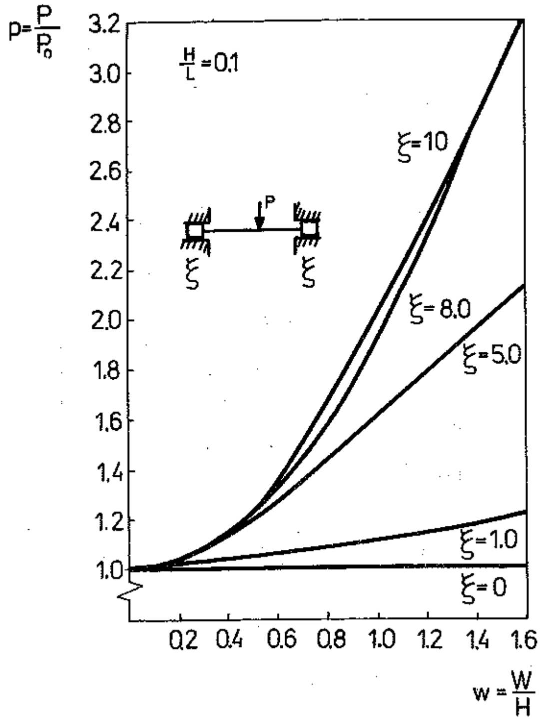

FIG. 5.

behaves as fully-fixed and the motion is described by the relation (2.15). The supports start to move horizontally at the deflection

(2.25) $$w = \frac{L}{H\xi} - \frac{L}{H\xi}\sqrt{1-\left(\frac{H\xi}{L}\right)^2}.$$

(2) This simplification (as for elastic supports) can be accepted if the deflections are not greater than beam depth.

In view of Eqs. (2.14) and (2.25) the largest value of axial force ($n = 1$ or $N = N_0$) is reached for $\xi = L/H$. Therefore for $\xi \geqslant L/H$ the friction forces are so large that the supports cannot move and the considered beam behaves like a fully-fixed one during the whole deformation process.

The plot in the $p-\delta$ diagram for the friction parameter $\xi = 8.0$ describes the situation in which the supports stop to move, reaching the plastification of the cross section only under normal force (membrane effect).

3. The post-yield behaviour of rigid plastic column

Let us consider a fixed column made of rigid perfectly plastic material and loaded by vertical and horizontal forces $P\lambda_v$ and $P\lambda_h$ increasing proportionally or independently (Fig. 6).

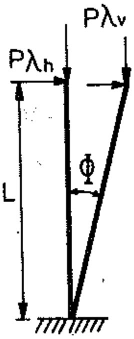

Fig. 6.

Equilibrium equations written in the deformed configuration for the base of the column have the form

$$N = -P\lambda_v \cos\Phi + P\lambda_h \sin\Phi, \tag{3.1}$$

$$M = P\lambda_h L\cos\Phi + P\lambda_v L\sin\Phi, \tag{3.2}$$

where N and M are, respectively, the axial force and bending moment at the base of the column.

Substitution of Eqs. (3.1) and (3.2) written in dimensionless form into the yield condition (2.1) leads to the following relation:

$$F = 16p_h(\cos\Phi + \eta\sin\Phi) + p_h^2\left(\frac{H}{L}\right)^2(\sin\Phi - \eta\cos\Phi)^2 - 16 = 0, \tag{3.3}$$

where

$$p_h = \frac{P\lambda_h L}{M_0}, \tag{3.4}$$

$$\eta = \frac{\lambda_v}{\lambda_h}. \tag{3.5}$$

Equation (3.3) describes the relation between the intensity of load applied p_h and the rotation angle Φ, for the determined value of the ratio of the load intensity η. Figure 7 illustrates the relation (3.3) for $\eta = -1, 0, 1, 3, 5$

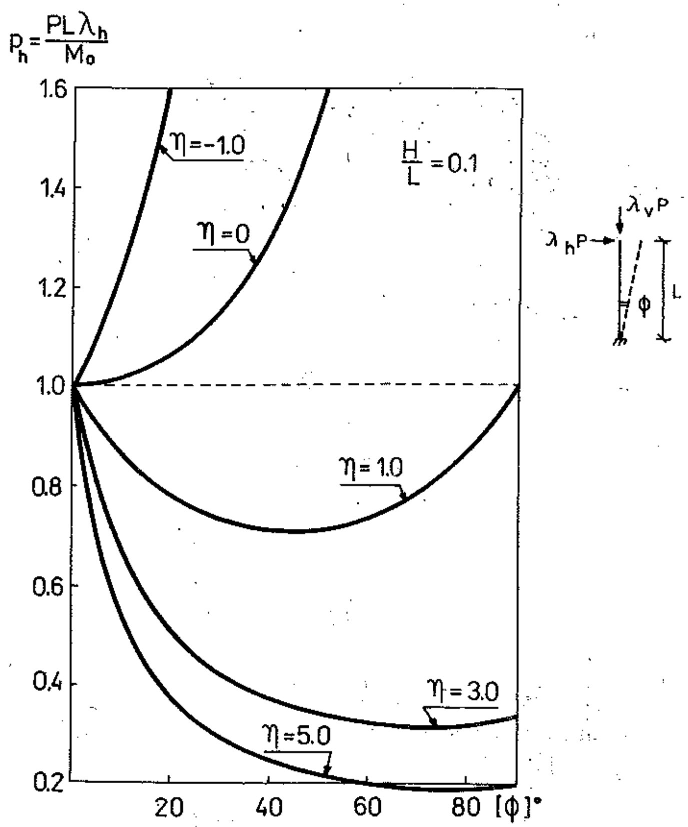

FIG. 7.

and $H/L = 0.1$. As it follows from Eq. (3.3) and Fig. 7 the post-yield behaviour of the considered column depends mostly on the parameter η. For $\eta \leqslant 0$ (the load P is directed upward) the post-yield behaviour is stable, whereas for $\eta < 0$ the deformation process is initially unstable but after the rotation Φ_1 the column becomes stable.

At the assumption that plastic deformations are concentrated in a generalized plastic hinge, the loading condition $\dot{F} = 0$ leads to the relation

$$\dot{F} = \frac{\partial F}{\partial p_h} \dot{p}_h + \frac{\partial F}{\partial \Phi} \dot{\Phi} = 0 \tag{3.6}$$

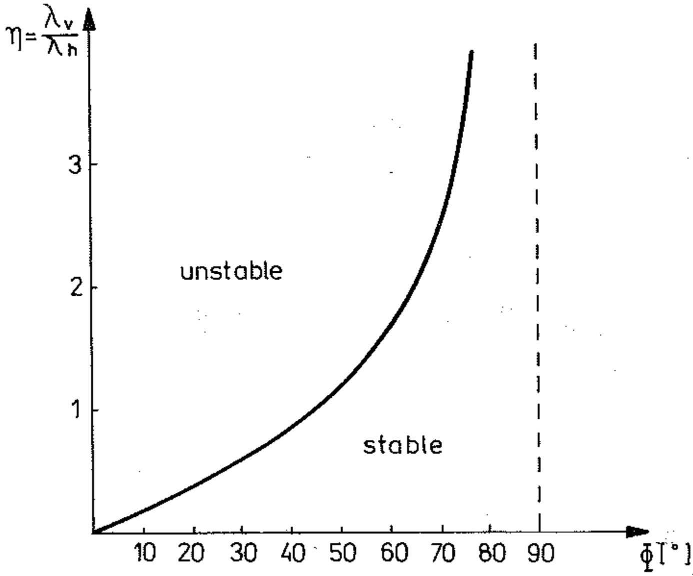
η = λv/λh
3
2
1
unstable
stable
10 20 30 40 50 60 70 80 90
Φ [°]

FIG. 8.

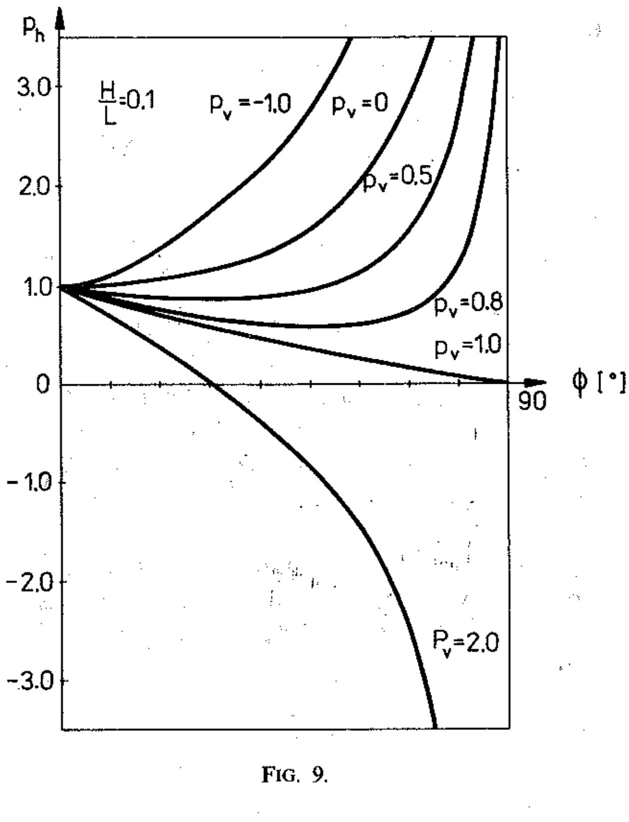
ph
3.0
2.0
1.0
0
-1.0
-2.0
-3.0
H/L=0.1
pv=-1.0
pv=0
pv=0.5
pv=0.8
pv=1.0
Pv=2.0
90
ϕ [°]

FIG. 9.

and hence

$$(3.7) \qquad \frac{dp_h}{d\Phi} = -\frac{\dfrac{\partial F}{\partial \Phi}}{\dfrac{\partial F}{\partial p_h}}.$$

Assuming $\dfrac{\partial F}{\partial p_h} \neq 0$, it follows from Eq. (3.7) that the condition $\dfrac{dp_h}{d\Phi} = 0$ is satisfied only if $\dfrac{\partial F}{\partial \Phi} = 0$. Therefore the neutral stability conditions takes eventually the form

$$(3.8) \qquad \eta = \operatorname{tg} \Phi.$$

The same result can be obtained when making use of the structural stability criterion for a perfectly plastic material in the Lagrangian sense [8]. The column is then stable if the axial force N is positive. In view of Eq. (3.1) it follows that for $\eta > \operatorname{tg} \Phi$ the column is stable,

for $\eta = \operatorname{tg} \Phi$ the column is neutral,

for $\eta < \operatorname{tg} \Phi$ the column is unstable.

Figure 8 illustrates the above relations. A similar analysis can be carried out for a constant value of one of the components of the load and an increasing value of the other. Figure 9 illustrates the relation $p_h - \Phi$ for $p_v = \dfrac{P\lambda_v L}{M_0} = -1, 0, 0.5, 0.8, 1.0, 2.0$, and $H/L = 0.1$.

4. The post-yield analysis of rigid-plastic frames

The steel portal frames used primarily in tall buildings constitute one of the most important problems in the plastic analysis of structures.

For the complex frame structures, if the yield-point load is associated with the local collapse mechanism, plastic deformations may result in such changes of the boundary conditions that the stabilizing effect of tensile forces is increased or reduced. By making the proper choice of the beam-to-column rigidity, unstable mechanisms can be avoided.

At the assumption that plastic deformations are concentrated in generalized plastic hinges and the displacements do not exceed the depths of cross sections, the structural stability criterion (for the perfectly plastic material in the Lagrangian sense) takes the form [8, 9]

$$(4.1) \qquad \dot{\mu} = \frac{\sum\limits_{i=1}^{r} N_i \dot{\Phi}_i^2 L_i}{\sum\limits_{j=1}^{m} P_{0j} \dot{W}_j} > 0,$$

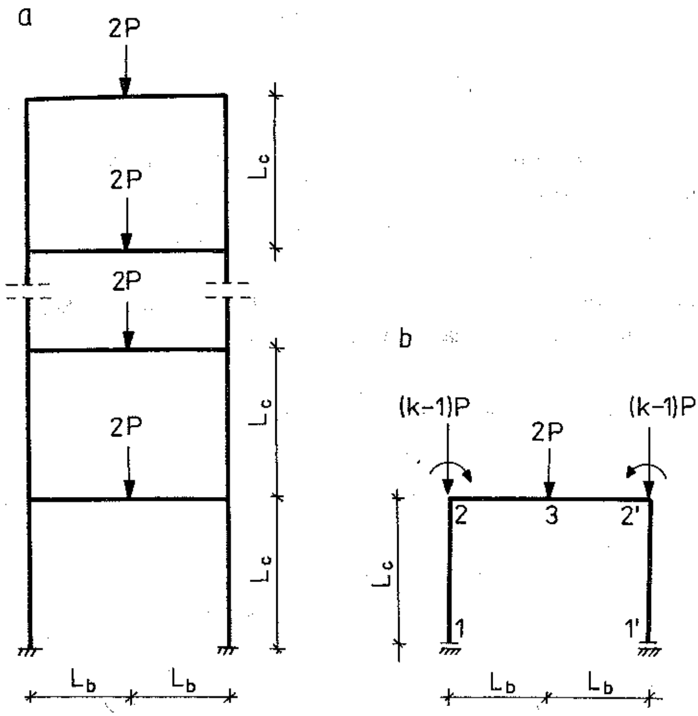

FIG. 10.

where N_i, $\dot{\Phi}_i$, L_i are, respectively, the axial force, the rate of rotation and the length of the i-th element; r is the number of elements, $\dot{W}_j$ denotes the displacement rate of the force P_j in its direction, P_{0j} is the yield point load, m is the number of points of load application.

Since the denominator in the inequality (4.1) is nonnegative, the stability of the frame structure is ensured if

$$\sum_{i=1}^{r} N_i \dot{\Phi}_i^2 L_i > 0. \tag{4.2}$$

The stabilizing effect of tensile forces and the destabilizing effect of compressive forces is, therefore, evident.

In the post-yield analysis of plastic frames at moderately large deflections, the mechanism of motion is usually assumed to be the same as at the yield point load. This assumption may result in an improper estimation of the post-yield behaviour.

To illustrate the problem let us consider an k-story portal frame subjected to concentrated vertical loads as shown in Fig. 10a. The analysis will be carried for the first floor (Fig. 10b).

The following dimensionless coefficients are introduced:

$$\alpha = \frac{H_c}{H_b}, \quad \beta = \frac{H_b}{L_b}, \quad \gamma = \frac{H_c}{L_c}, \quad w = \frac{W}{H_b}, \tag{4.3}$$

where L_c, L_b, H_c, H_b denote, respectively, the lengths of the columns and beams and the depths of the columns and beams.

All beams and columns are assumed, for simplicity, to have the same widths and to be made of the same material.

It follows from the limit analysis that the frame begins to deform according to the beam mechanism (a) presented in Fig. 11 with generalized

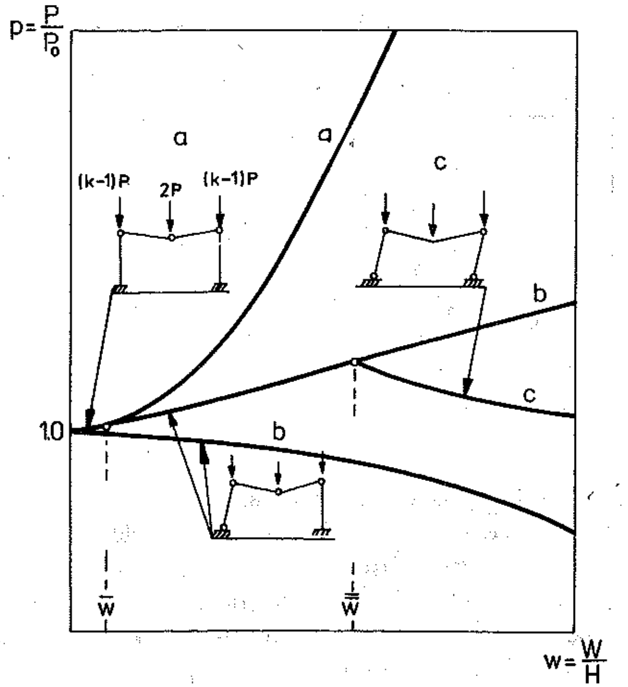

FIG. 11.

plastic hinges at the cross sections 2, 2, 3. If the columns are rigid enough, the beams of the frame work as fully-fixed beams and the relations (2.14) and (2.15) describe the behaviour of the frame. The curve (a) in Fig. 11 illustrates Eq. (2.15). The stability criterion (4.1) then yields

$$\dot{\mu} = 2w\dot{w} \geqslant 0. \tag{4.4}$$

Thus the frame is stable.

However, at the certain deflection $\bar{W}$ the increasing axial force in the beams results in the yielding of the column (or columns) and further deformation continues according to the mechanism (b) shown in Fig. 11.

The beams of the frame behave then as beams on moving supports at constant axial force. Substituting $n = W/H_b = w$ and $s = (W-\bar{W})^2/2L_b$ into Eq. (2.21), we obtain

$$p = \frac{2\,(1-\bar{w}^2+2\bar{w}w)}{2-(w-\bar{w})^2\,\beta^2}. \tag{4.5}$$

The stability criterion (4.1), after neglecting higher order terms, furnishes

$$\dot{\mu} = \frac{2\bar{w}\dot{w}}{(1+\bar{w}^2)} \geqslant 0. \tag{4.6}$$

The frame is still stable though the increase of the destabilizing effect of the compressive axial force in the columns and the reduced stabilizing effect of the tensile forces in the beams result in diminishing of the stability coefficient $\dot{u}$.

Taking into account the second-order geometrical effects as well as the influence of axial forces on the yielding of the beams and columns, the equilibrium equation, written for the mechanism (a) at the moment of plastification of the cross section 1, eventually leads to the condition

$$\alpha^2\,\gamma-2\alpha\bar{w}\,(k-1)\,k-k\gamma\,(1-\bar{w}^2)-0.25\beta^2\,\gamma k^2\,(1+2\bar{w}^2) = 0. \tag{4.7}$$

Putting $\bar{w} = 0$ into Eq. (4.7), we obtain the relation between the number of floors k and the geometry parameters for which the hinges in the cross sections 1, 2, 2′ and 3 develop simultaneously at the yield point load, namely

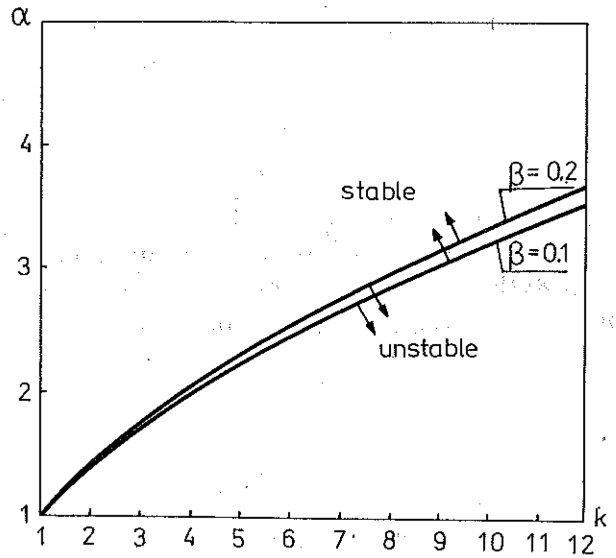

FIG. 12.

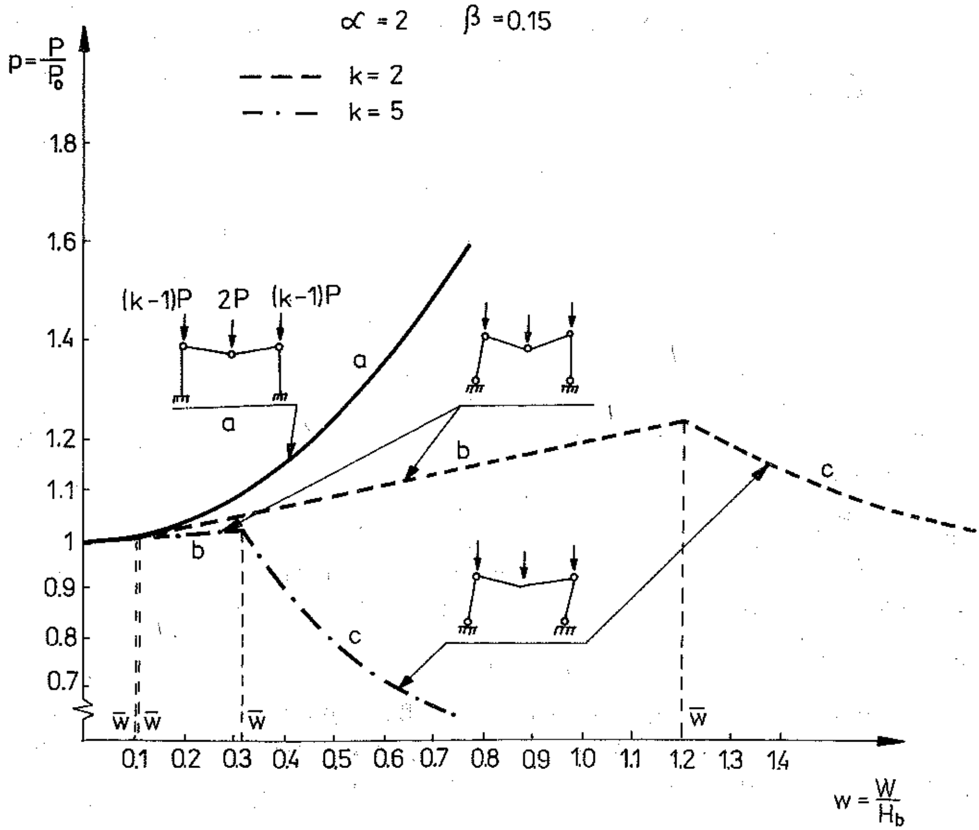

FIG. 13.

(4.8)
$$\alpha^2 = k\,(1+0.25\beta^2\,k).$$

The above relation for $\beta = 0.1$ and $\beta = 0.2$ is illustrated in Fig. 12.

If the rigidity of the columns are less than, or equal to, the critical value described by Eq. (4.8) then, from the beginning the mechanism of motion (b) occurs. The condition of equilibrium yields the relation

(4.9)
$$p = \frac{2}{n\beta}\left(\sqrt{\alpha^2-n} - w^2\,\alpha\beta^2\right),$$

therefore

(4.10)
$$\dot{\mu} = \frac{\dot{p}}{p_0} = -\frac{2\alpha\beta^2\,w\dot{w}}{\sqrt{\alpha^2-n}} < 0$$

and the frame is unstable. The foregoing relations are valid only for $\alpha^2 > n$.

Further deformation may lead to the realization of the mechanism (c) presented in Fig. 12 which proves to be unstable. However, this analysis in view of the occurrence of very large deflections would exceed the scope of this paper. Figure 13 illustrates the considered example of the frame for which the bottom of the columns at each story are connected with beams by hinges, for the following geometric parameters: $\alpha = 2, \beta = 0.15, k = 2$ and $k = 5$.

5. Conclusions

The results obtained in the previous sections describe the behaviour of some simple plastic structure at the assumptions of the lack of geometrical imperfection and the lack of elastic deformations. Those assumptions from the engineering point of view are hardly acceptable. Therefore the question arises what is the reliability of the obtained results when a real engineering structure is considered.

From a comparison of the results obtained for structures of a perfect shape, made of a rigid-plastic material and for those made of an elastic-plastic material or which exhibit initial geometric imperfections, the following conclusions can be drawn:

In the case of the stable post-yield behaviour of a rigid-plastic, structure of perfect shape the elastic-plastic structure or the structure with initial geometric imperfections is capable of sustaining a load which exceeds the load-carrying capacity assessed by the tools of limit analysis.

If the post-yield behaviour of a rigid-plastic structure of perfect shape is predicted to be unstable, then the maximum load which can be supported by the same structure but is made of an elastic-plastic material or exhibits initial geometric imperfections may near never reach the load-carrying capacity since the structure may collapse much earlier.

Therefore the full advantage of the load-carrying capacity can be taken only with an additional analysis of the behaviour of the structure after the yield-point load has been reached.

It may happen that a structure with a smaller limit load but stable is in reality capable of supporting a greater maximum load prior to collapse than an apparently stronger structure which was proved unstable.

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Streszczenie

PO-KRYTYCZNA ANALIZA SZTYWNO-PLASTYCZNYCH BELEK, SŁUPÓW I RAM

W pracy rozważany jest wpływ efektów drugiego rzędu na po-graniczne zachowanie się konstrukcji plastycznych. Analizę ograniczono do czysto mechanicznej (izotermicznej) teorii i quasi-statycznego procesu deformacji.

Rozważany problem zilustrowano przykładami sztywno-plastycznych belek z różnymi rodzajami częściowego zamocowania brzegów, sztywno-plastycznych słupów obciążonych pionowymi i poziomymi siłami oraz wielopiętrowych ram portalowych. Wykazano, że plastyczne deformacje mogą wpłynąć na zmianę warunków brzegowych w taki sposób, że stabilizujący efekt sił rozciągających jest zredukowany. Przez odpowiedni dobór pewnych parametrów można uniknąć niestatecznych mechanizmów oraz katastrofalnego zniszczenia.

Резюме

ПОСЛЕКРИТИЧЕСКИЙ АНАЛИЗ ЖЕСТКО-ПЛАСТИЧЕСКИХ БАЛОК, СТОЛБОВ И РАМ

В работе рассматривается влияние эффектов второго порядка на послепредельное поведение пластических конструкций.

Анализ ограничен к механической (изотермической) теории и квазистатическому процессу деформации.

Рассматриваемая проблема иллюстрирована примерами жестко-пластических балок с разными родами частичного закрепления краев, жестко-пластических столбов, нагруженных вертикальными и горизонтальными силами, а также многоэтажных портальных рам.

Показано, что пластические деформации могут влиять на изменение граничных условий таким образом, чтобы стабилизирующий эффект растягивающих сил был редуцированным. Путем соответствующего подбора некоторых параметров можно избежать неустойчивых механизмов и катастрофического разрушения.

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