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NORMAL REFLECTION AND TRANSMISSION OF PLANE SHOCK WAVES IN NONLINEAR ELASTIC MATERIAL

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The normal reflection (reflection and transmission) of a finite elastic plane shock wave at a plane boundary (plane interface of two rigidly coupled materials), propagating through an elastic Murnaghan's material is examined. It is assumed that the material region in front of the shock wave is homogeneously strained and at rest. The linearized boundary value problem is considered for analysing small but finite amplitude waves. Three types of boundary conditions are considered. The obtained reflection solution is one shock wave, simple wave or progressive wave, when the only transmitted wave is always the shock wave.

1. Introduction

The oblique reflection of a finite plane shock wave at a plane boundary in a nonlinear, homogeneous, isotropic elastic material was investigated by Wright [1, 2]. Using a strictly mechanical theory, Wright presented a semi-inverse method for finding the reflected (transmitted) waves. In this method it is assumed that the incident shock wave is given a priori and that the medium ahead of the shock is homogeneously strained and at rest. It is also assumed that the state behind the incident shock wave and the state at the boundary are connected by means of a sequence of reflected simple waves and constant state regions. The most general reflection (transmission) solution gives three reflected (transmitted) simple waves or shock waves. In the case of three reflected simple waves the problem reduces to the determination of the distribution of wavelets by means of oridnary differential equations which describe the variations of the deformation gradient and particle velocity [1]. The boundary conditions on the surface of reflection (the continuity conditions on the interface) determine the intervals for the parameter corresponding to each type of waves.

In some cases the assumed reflection pattern may fail the admissibility test, the pattern must then be modified to include shock waves.

In this paper the problem of reflection at a plane boundary, or reflection and transmission at a plane interface of two rigidly coupled materials, of the normal plane shock wave propagating through an elastic

Murnaghan's material is examined. The material region ahead of the incident shock is assumed to be homogeneously strained and at rest.

According to Wright's suggestion [1 p. 180], the case of normal reflection (transmission) can be treated in a way virtually identical to the oblique reflection, with the additional simplification that the fronts of all reflected (transmitted) waves and the front of the incident wave are always parallel to the boundary plane (interface). The speeds of propagation along the normal, as a function of the deformation gradient, can be computed directly.

The general normal reflection (transmission) solution can be obtained from the system of nonlinear differential equations. This problem can be solved only numerically. For this reason, the linearized boundary value problem is considered by using the Toupin–Bernstein's equations for analysing small but finite amplitude waves.

The basic assumption is made that the reflection solution is in the form of simple waves. Two types of boundary conditions and the corresponding initial-boundary value problems are considered in Sect. 4.

The linearized problem regarded below restricts the general reflection solution to one reflected simple wave, shock wave or progressive wave, depending on the boundary conditions and the initial deformations only. In the case of the transmission at the interface, the transmitted wave is always the shock wave. Section 6 contains a numerical analysis of the reflection (transmission) solution for three types of boundary conditions. The results are illustrated graphically. The notation used here is similar to that of [1]. For convenience, some relevant formulas and results from [1] are included in this paper. The repeated index convention for summation should be systematically used in any formula where the same index appears twice unless the formula is followed by the indication "no summation".

2. Reflected shocks and simple waves

The motion of the continuum is given by $x^i = x^i(X^\alpha, t)$ where x^i, X^α are the Cartesian coordinates of the material particle in the present configuration B and the reference configuration B_R, respectively. The deformation gradient x^i_α, the velocity u^i and the Piola–Kirchhoff stress tensor for an isotropic elastic material are defined by

$$x^i_{\ \alpha} = \frac{\partial x^i}{\partial X^\alpha}, \qquad u^i = \dot{x}^i = \frac{\partial x}{\partial t}, \qquad T_{Ri}^{\ \alpha} = \varrho_R \frac{\partial \sigma}{\partial x^i_{\ \alpha}}, \tag{2.1}$$

where σ denotes internal energy per unit mass in the reference configuration B_R, ϱ_R is the density in B_R. Here and henceforth the dot above a function denotes the derivative with respect to time t, two dots denote the second derivative.

If the functions $x^i(X^\alpha, t)$ are continuous everywhere but have discontinuous first derivatives on some propagating surface $\mathscr{I}$, then the jump conditions across a surface $\mathscr{I}$ are given by

$$(2.2)\qquad \begin{gathered} [\![T_{Ri}{}^{\alpha}]\!]\, N_\alpha = -\varrho_R\, U_v\, [\![\dot{x}^i]\!], \\ [\![x^i{}_\alpha]\!] = H^i N_\alpha, \quad [\![u^i]\!] = -H^i U_v, \end{gathered}$$

$$(2.3)\qquad m = (H_i H^i)^{\frac{1}{2}} = \left([\![x^i{}_\alpha]\!]\,[\![x_i{}^\alpha]\!]\right)^{\frac{1}{2}} \geqslant 0.$$

The material normal in the direction of propagation is denoted by $\mathbf{N}$, the normal velocity relative to the material is U_v and the amplitude vector of the shock is $\mathbf{H}$. The parameter m is a measure of the intensity of the wave. The double brackets indicate the jump of arbitrary field $(\cdot)$ on $\mathscr{I}$

$$[\![\cdot]\!] = (\cdot)^B - (\cdot)^F.$$

The letters F and B indicate the limit values taken on the front and rear sides of $\mathscr{I}$. Such propagating surfaces across which the velocity and deformation gradient are discontinuous are called the shock waves.

According to [4], the propagation speed U_v and amplitude vector $\mathbf{H}$ are assumed to be expandable into power series of the parameter m.

$$(2.4)\qquad \begin{gathered} \frac{H^i(m)}{m} = \overset{0}{H}{}^i + m\overset{1}{H}{}^i + m^2 \overset{2}{H}{}^i + \ldots, \\ U_v(m) = \overset{0}{U}_v + m\overset{1}{U}_v + m^2 \overset{2}{U}_v + \ldots, \end{gathered}$$

where $\overset{0}{H}{}^i, \overset{1}{H}{}^i, \ldots, \overset{0}{U}_v, \overset{1}{U}_v, \ldots$ are constants for a fixed initial deformation in front of $\mathscr{I}$. For small but finite deformations, m is a small parameter $m \ll 1$ and it is necessary to determine the first two terms in the series only, in order to obtain a good approximation to the solution [7]. In the limit case $m \to 0$ the shock wave propagates with the same speed as the acceleration wave. In this case the amplitudes of the shock wave and the acceleration wave are colinear [4].

The Lax criterion [10] can be used to verify the stability of such shock waves. According to this criterion, a shock wave is stable if it propagates with supersonic speed with respect to the medium in front of it, and with subsonic speed with respect to the medium behind it. Analysing the connections between the shock waves with the intensity m (2.4) and acceleration waves $(m \to 0)$, Wesołowski [4] has proved that such a shock wave is stable in the sense of Lax.

If the stress and velocity fields are differentiable, then the equations expressing balance of momentum and compatibility conditions are

$$T_{Ri}{}^{\alpha}{}_{,\alpha} = \varrho_R \dot{u}^i, \qquad \frac{\partial u^i}{\partial X^\alpha} = \frac{\partial^2 x^i}{\partial X^\alpha \partial t} = \frac{\partial x^i{}_\alpha}{\partial t}. \tag{2.5}$$

Simple waves [8], [1, p. 163] are defined to be regions of space-time in which the deformation gradient and velocity fields are continuous and depend on a single parameter say $\gamma = G(X^\alpha, t)$. Regions of constant γ are propagating surfaces called wavelets with unit normal and normal velocity given in B_R by

$$N_\alpha(\gamma) = G_{,\alpha}(\nabla G)^{-1}, \qquad U(\gamma) = -\dot{G}(\nabla G)^{-1}. \tag{2.6}$$

If $\dot{G} \neq 0$, Eqs. (2.5) in the region of a simple wave may be written as

$$\begin{aligned} (Q_{ij} - \varrho_R U^2 \delta_{ij}) u'^j &= 0, \\ U x'^j{}_\beta + u'^j N_\beta &= 0, \end{aligned} \tag{2.7}$$

where the prime indicates differentiation with respect to the wave parameter γ and

$$Q_{ij} = \frac{\partial T_{Ri}{}^{\alpha}}{\partial x^j{}_\beta} N_\alpha N_\beta = \varrho_R \sigma_i{}^\alpha{}_j{}^\beta N_\alpha N_\beta,$$

is the acoustic tensor.

We denote here

$$\sigma_i{}^\alpha{}_j{}^\beta = \frac{\partial^2 \sigma}{\partial x^i{}_\alpha \partial x^j{}_\beta}.$$

For Eq. $(2.7)_1$ to have nonzero solutions in u'^i it is necessary that

$$\begin{aligned} &\det(Q_{ij} - \varrho_R U^2 \delta_{ij}) = 0, \\ &u'^j = k \overset{(\alpha)}{r}{}^j, \qquad \alpha, i, j = 1, 2, 3, \end{aligned} \tag{2.8}$$

where $\overset{(\alpha)}{r}{}^j$ is the right proper vector of Q_{ij} corresponding to a particular proper value $\varrho_R U_\alpha^2$, $(\alpha = 1, 2, 3)$, k is an arbitrary parameter. Assuming $k = U_\alpha$, it follows from Eqs. $(2.7)_2$ and $(2.8)_2$ that

$$\frac{dx^j{}_\beta}{d\gamma} = -\overset{(\alpha)}{r}{}^j N_\beta, \qquad \frac{du^j}{d\gamma} = U_\alpha \overset{(\alpha)}{r}{}^j, \qquad \alpha, \beta, j = 1, 2, 3, \tag{2.9}$$

α — no summation.

The right proper vectors can be determined, exact to an arbitrary scalar function of the deformation gradient x^i_α, $f(x^i_\alpha)$, which can be chosen for convenience since it represents only a relabeling of the wavelets. Thus the above equations may be rewritten as

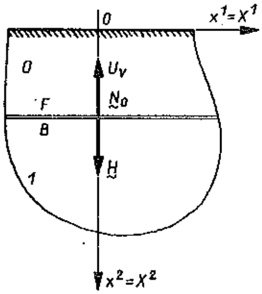

FIG. 1. Incident shock wave.

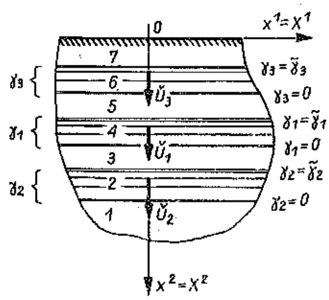

FIG. 2. Assumed shock reflection pattern.

$$
\begin{aligned}
&u'^j = U_\alpha f(x^i_\alpha) \overset{(\alpha)}{r}{}^j (x^i_\alpha), \\
&x'^j_\beta = -f(x^i_\alpha) \overset{(\alpha)}{r}{}^j (x^i_\alpha) N_\beta, \qquad \alpha, \beta, j = 1, 2, 3, \qquad \alpha \text{ — no summation.}
\end{aligned} \tag{2.10}
$$

These ordinary differential equations must be satisfied in the region of the simple wave. They can be solved with the initial conditions taken from the regions of the constant deformation gradient and particle velocity 1, 3, 5 (Fig. 2). The intervals for the parameter corresponding to the simple waves in the regions 2, 4, 6 can be found from the boundary conditions in the region 7 (Fig. 2).

3. Reflection of normal plane shock wave in hyperelastic material

We consider the halfspace $X^2 \geqslant 0$ (Fig. 1, 2) composed of a homogeneous elastic Murnaghan's material. Such a material is defined by the constitutive equation

$$
\varrho_R \sigma = \frac{l+2m}{24}(I_1-3)^3 + \frac{\lambda+2\mu+4m}{8}(I_1-3)^2 + \frac{8\mu+n}{8}(I_1-3) - \frac{m}{4}(I_1-3)(I_2-3) - \frac{4\mu+n}{8}(I_2-3) + \frac{n}{8}(I_3-1), \tag{3.1}
$$

where λ, μ — Lamé's constants, l, m, n — second order elastic constants and

$$I_1 = B^i_{\ i}, \quad I_2 = \frac{1}{2}[I_1^{\ 2} - B^j_{\ i} B^i_{\ j}], \quad I_3 = \det(B),$$

are the invariants of the left Cauchy–Green strain tensor B.

Suppose that a plane shock wave propagates through this region, in the direction normal to the boundary plane $X^2 = 0$ at which it is reflected. The characteristic parameters for such a wave can be computed by [7]. Various types of boundary conditions on $X^2 = 0$, can be considered. We assume that both $\{X^\alpha\}$ and $\{x^i\}$ are coinciding Cartesian coordinates.

With regard to second order effects, the longitudinal shock wave speed U_v is greater than the acoustic wave speed $\overset{0}{U}_v$ when the transversal shock waves propagated with the acoustic wave speed [7].

The geometry of an incident shock wave is shown in Fig. 1. The material region 0 in front of the wave is homogeneously strained and at rest. The strength of this wave $m = |H^i| > 0$, the unit material normal and the amplitude vector $\mathbf{H}$ [7] are

$$\mathbf{N}_0 = (0, -1, 0), \quad \mathbf{H}(0, m, 0). \tag{3.2}$$

It is assumed ([1]) that the reflected waves are three simple waves in regions 2, 4, 6 (Fig. 2), with unit normal

$$\mathbf{N}(0, 1, 0). \tag{3.3}$$

Regions 1, 3, 5, 7 are regions of the constant deformation gradient and constant particle velocity.

The deformation gradient in front of the incident shock wave $(\mathbf{F})^F$ and behind the wave $(\mathbf{F})^B$ are (Fig. 1)

$$(\mathbf{F})^F = (\overset{0}{\mathbf{F}}) = \begin{bmatrix} \overset{0}{x}{}^1{}_1 & 0 & 0 \\ 0 & \overset{0}{x}{}^2{}_2 & 0 \\ 0 & 0 & \overset{0}{x}{}^3{}_3 \end{bmatrix}, \quad (\mathbf{F})^B = \begin{bmatrix} \overset{0}{x}{}^1{}_1 & 0 & 0 \\ 0 & \overset{0}{x}{}^2{}_2 - m & 0 \\ 0 & 0 & \overset{0}{x}{}^3{}_3 \end{bmatrix}. \tag{3.4}$$

From the relation (3.2) it is clear that only the jumps of one component of the deformation gradient and one component of the velocity are not equal to zero:

$$[\![x^2{}_2]\!] = -m, \quad [\![u^2]\!] = -mU_v. \tag{3.5}$$

Substitution of the relation (3.3) into Eq. $(2.7)_1$ gives three homogeneous equations for u'^i $(i = 1, 2, 3)$:

$$\begin{bmatrix} \sigma_1{}^2{}_1{}^2 - U^2 & \sigma_1{}^2{}_2{}^2 & \sigma_1{}^2{}_3{}^2 \\ \sigma_2{}^2{}_1{}^2 & \sigma_2{}^2{}_2{}^2 - U^2 & \sigma_2{}^2{}_3{}^2 \\ \sigma_3{}^2{}_1{}^2 & \sigma_3{}^2{}_2{}^2 & \sigma_3{}^2{}_3{}^2 - U^2 \end{bmatrix} \begin{Bmatrix} u'^1 \\ u'^2 \\ u'^3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}. \tag{3.6}$$

It is seen from Eqs. (3.3) and $(2.9)_1$ that in the most general case three derivatives of the components of the deformation gradient in the region of the simple wave are not equal to zero:

(3.7) $$x'^1{}_2 \neq 0, \quad x'^2{}_2 \neq 0, \quad x'^3{}_2 \neq 0.$$

In front of the incident shock wave and behind it the components of the deformation gradient $x^1{}_2$, $x^3{}_2$ are equal to zero. It follows from the relations (3.7) that these components can change in the region of the reflected simple wave. For this reason we suppose that the representation of the deformation gradient in each reflected simple wave and in adjacent regions of constant state has the form

(3.8) $$\mathbf{F}_1 = \begin{bmatrix} x^1{}_1 & x^1{}_2 & 0 \\ 0 & x^2{}_2 & 0 \\ 0 & x^3{}_2 & x^3{}_3 \end{bmatrix}.$$

The elasticities required in Eqs. (3.6) for the material (3.1) and the deformation gradient (3.8) are

(3.9)
$$\begin{aligned}
\sigma_2{}^2{}_2{}^2 &= 2\sigma_1 + 2\sigma_2\left((x^1{}_1)^2 + (x^3{}_3)^2\right) + 2\sigma_3 (x^1{}_1)^2 (x^3{}_3)^2 + 4\sigma_{11} (x^2{}_2)^2 + \\
&\qquad + 8\sigma_{12} (x^2{}_2)^2 \left((x^1{}_1)^2 + (x^3{}_3)^2\right), \\
\sigma_1{}^2{}_1{}^2 &= 2\sigma_1 + 2\sigma_2 (x^3{}_3)^2 + 4\sigma_{11} (x^1{}_2)^2 + 8\sigma_{12} (x^3{}_3)^2 (x^1{}_2)^2, \\
\sigma_3{}^2{}_3{}^2 &= 2\sigma_1 + 2\sigma_2 (x^1{}_1)^2 + 4\sigma_{11} (x^3{}_2)^2 + 8\sigma_{12} (x^1{}_1)^2 (x^3{}_2)^2, \\
\sigma_1{}^2{}_2{}^2 &= x^1{}_2\, x^2{}_2 \left(4\sigma_{11} + 4\sigma_{12}\left((x^1{}_1)^2 + 2 (x^3{}_3)^2\right)\right), \\
\sigma_1{}^2{}_3{}^2 &= 4x^1{}_2\, x^3{}_2 \left(\sigma_{11} + \sigma_{12}\left((x^3{}_3)^2 + (x^1{}_1)^2\right)\right), \\
\sigma_2{}^2{}_3{}^2 &= 4x^3{}_2\, x^2{}_2 \left(\sigma_{11} + \sigma_{12}\left((x^3{}_3)^2 + 2 (x^1{}_1)^2\right)\right),
\end{aligned}$$

where

$$\sigma_K = \frac{\partial \sigma}{\partial I_k}, \quad \sigma_{KL} = \frac{\partial^2 \sigma}{\partial I_k\, \partial I_L}, \quad K, L = 1, 2, 3,$$

and

(3.10)
$$\begin{aligned}
I_1 &= (x^1{}_1)^2 + (x^2{}_2)^2 + (x^3{}_3)^2 + (x^1{}_2)^2 + (x^3{}_2)^2, \\
I_2 &= (x^2{}_2)^2 (x^3{}_3)^2 + (x^2{}_2)^2 (x^1{}_1)^2 + (x^3{}_3)^2 (x^1{}_1)^2 + (x^1{}_2)^2 (x^3{}_3)^2 + \\
&\qquad + (x^3{}_2)^3 (x^1{}_1)^2.
\end{aligned}$$

The general propagation condition $(2.8)_1$ gives three reflected waves. For the problem of normal reflection in Murnaghan's material with initial deformation gives by the relation $(3.4)_1$, the deformations are functions of space variable x^2 only. It is natural to expect that in this case, for the longitudinal incident shock wave $(3.2)_2$, the deformation possesses symmetry with respect to every plane which contains the X^2-axis. For this reason we can assume that only one lognitudinal reflected wave is possible. On this assumption the deformation gradient in the reflected simple wave region has the form $(3.4)_2$. With this additional simplification the equations for $x^1{}_2$, $x^3{}_2$ and equations for boundary conditions are still nonlinear. For this reason, for small but finite amplitude waves, the linearization of the initial-boundary problem is reasonable.

4. Linearized initial-boundary value problem

Considering the problem of propagation of a plane shock wave in an isotropic elastic material described with second order elasticity coefficients, it has been found that in the static case the principal stretches in the region of the elastic shock wave are not greater than 1 ± 0.0085 for steel and 1 ± 0.0030 for aluminium [7]. The deformations for other metals have the same order of magnitude [6]. For this reason, the linearization of the problem is justifiable. We have used the linearized Toupin–Bernstein equations of motion [9] instead of Eqs. $(2.5)_1$.

Consider a motion

$$x^i = \breve{x}^i(X^\alpha, t) + w^i(X^\alpha, t), \tag{4.1}$$

where $\breve{x}^i$ — mean the initial deformations behind, the incident shock wave front and w^i — small deformation about the initial deformation in the reflected simple wave region.

According to the definition in the simpe wave region, the deformation gradient $w^i{}_\alpha$ and particle velocity $\dot{w}^i$ depend on a single parameter γ.

From Eq. (4.1) it follows that

$$\begin{aligned} x^i{}_\alpha &= \breve{x}^i{}_\alpha(0) + w^i{}_\alpha(\gamma), \\ \dot{x}^i &= \breve{u}^i(0) + v^i(\gamma), \qquad v^i = \frac{\partial w^i}{\partial t}, \qquad i, \alpha = 1, 2, 3, \end{aligned} \tag{4.2}$$

where $\breve{x}^i{}_\alpha(0)$ — are the homogeneous components of the deformation gradient behind the incident shock wave front $(3.4)_2$, $\breve{u}^i(0)$ — are the components of the particle velocity in the same region.

It is assumed that the deformation gradient $(3.4)_{1,2}$ and particle velocity in front of the incident shock wave and behind it are constant in space and time.

For the motion (4.1), the Toupin–Bernstein equations of motion are ([9]):

$$\sigma_i{}^\alpha{}_k{}^\beta(\breve{x}^m{}_\gamma(0))\, w^i{}_{\alpha\beta} = \ddot{w}^i. \tag{4.3}$$

First, we calculate the equations of motion and compatibility condition in the simple wave region by substituting Eqs. (4.2) into Eqs. $(2.7)_{1,2}$

$$\begin{aligned} (\breve{Q}_{ij} - \varrho_R \breve{U}^2 \delta_{ij})\, v'^j &= 0, \\ \breve{U} w'^j{}_\beta + v'^j N_\beta &= 0, \end{aligned} \tag{4.4}$$

where $\breve{Q}_{ij} = \varrho_R \, \breve{\sigma}_i{}^2{}_j{}^2$ are the components of the acoustic tensor, calculated for the basic motion $\breve{x}^i{}_\alpha(0)$.

The condition that there exists nontrivial solutions for v'^j gives three solutions for the characteristic speeds:

$$(4.5)\quad \breve{U}^2{}_\alpha = \breve{\sigma}_\alpha{}^2{}_\alpha{}^2, \qquad \overset{(\alpha)}{r}{}_j = \delta_{j\alpha}, \qquad \alpha, j = 1, 2, 3, \qquad \alpha \text{ — no summation.}$$

Combining the differential equations (2.10) and (4.2) and choosing $f(x^i_\alpha) = \breve{U}_\alpha^{-1}$, we obtain the differential equations

$$(4.6)\quad \delta^\alpha{}_j \frac{dw^j{}_2}{d\gamma_\alpha} = -\breve{U}_\alpha^{-1}, \qquad \delta^\alpha{}_j \frac{d\dot{w}^j}{d\gamma_\alpha} = 1, \qquad j, \alpha = 1, 2, 3, \qquad \alpha \text{ — no summation,}$$

where γ_α — are the parameters of the reflected simple wave. The remaining derivatives of the components of the deformation gradient and particle velocities, with respect to wave parameters, γ_α are equal to zero. The above equations can be solved with the initial conditions (4.2):

$$(4.7)\quad w^\alpha{}_2(0) = v^\alpha(0) = 0, \qquad \alpha = 1, 2, 3.$$

Substituting the solutions of the initial value problem (4.6), (4.7) in Eqs. (4.2), we obtain

$$(4.8)\quad \begin{aligned} &x^\alpha{}_2 = \breve{x}^\alpha{}_2(0) - \breve{U}_\alpha^{-1}\gamma_\alpha, \\ &x^\alpha{}_\beta = x^\alpha{}_\beta(0) = \overset{0}{x}{}^\alpha{}_\beta, \\ &\dot{x}^\alpha = u^\alpha = \breve{u}^\alpha(0) + \gamma_\alpha, \qquad \beta = 1, 3, \qquad \alpha = 1, 2, 3, \qquad \alpha \text{ — no summation.} \end{aligned}$$

The above expressions are the linear functions of the wave parameters γ_α. The intervals for the parameters γ_α corresponding to the simple wave can be found from the boundary conditions in the region 7 (Fig. 1).

4.1. Clamped boundary

Let us consider the reflection problem of a plane shock wave, incident on a plane clamped boundary; this means that

$$(4.9)\quad \dot{x}^\alpha = u^\alpha = 0 \quad \text{on} \quad X^2 = 0, \qquad \alpha = 1, 2, 3.$$

The functions in the conditions (4.9) are evaluated for the final values of the wave parameters $\tilde{\gamma}_\alpha$. Hence from Eq. $(4.8)_3$

$$(4.10)\quad \tilde{\gamma}_\alpha = -\breve{u}^\alpha(0), \qquad \alpha = 1, 2, 3.$$

According to the relation $(3.5)_2$, only one component of the particle velocity $\breve{u}^2 \neq 0$, when $\breve{u}^1 = \breve{u}^3 = 0$. For this reason

$$(4.11)\quad \tilde{\gamma}_2 = -\breve{u}^2(0) \neq 0, \qquad \tilde{\gamma}_1 = \tilde{\gamma}_3 = 0.$$

The functions ${x^1}_2$, ${x^3}_2$ are equal to zero in all regions of the reflected simple waves. The parameters $\tilde{\gamma}_1$, $\tilde{\gamma}_3$ are not equal to zero in the case when the material region in front of the incident shock wave is not at rest and the components of the particle velocity: $\breve{u}^1 \neq 0$, $\breve{u}^3 \neq 0$. According to the conditions (4.9), such an assumption is fully unrealistic.

From Eqs. $(4.8)_1$ and (4.10) it follows for the fastest reflected simple wave (γ_2) that

$$
\text{(4.11)} \qquad \begin{aligned} {x^2}_2 &= {\breve{x}^2}_2(0) - \breve{U}_2^{-1}\gamma_2, & 0 \leqslant \gamma_2 \leqslant \tilde{\gamma}_2,\\ \tilde{\gamma}_2 &= -\breve{u}^2(0) = -(-mU_v) = mU_v, & m > 0. \end{aligned}
$$

This means that ${x^2}_2(\gamma_2)$ is decreasing with γ_2 changing from 0 to $\tilde{\gamma}_2 > 0$. As it was pointed out in [7], the simple wave speed $U_2({x^2}_2)$ is a decreasing function of ${x^2}_2$. Then the function U_2 is an increasing function of the wave parameter γ_2 when γ_2 ranges from 0 to its extreme value $\tilde{\gamma}_2$. The solution in the form of the simple wave fails. As a consequence of the boundary conditions assumed here, the reflected simple wave should be replaced by a reflected shock wave ([1]) with the velocity U_{v_1} and strength m_1.

The velocity jump across the reflected shock wave (Fig. 3) is the relation $(2.2)_3$

$$
\text{(4.12)} \qquad [\![\dot{x}^2]\!] = (\dot{x}^2)^B - \breve{u}^2(0) = -m_1 U_{v_1}.
$$

According to the relation $(3.5)_2$

$$
\text{(4.13)} \qquad mU_v = -m_1 U_{v_1} = -m_1 (\overset{\breve{0}}{U}_{v_1} + m_1 \overset{\breve{1}}{U}_{v_1} + \ldots).
$$

where the reflected shock wave speed U_{v_1} is represented by the power series of the parameter m_1 and $\overset{\breve{0}}{U}_v$ is the speed of the acoustic wave.

Hence

$$
\text{(4.14)} \qquad m_1 = -m\alpha_0 + 0(m_1^2),
$$

where

$$
\alpha_0 = U_v \overset{\breve{0}}{U}{}_{v_1}^{-1}.
$$

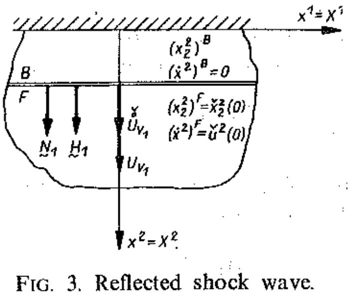

FIG. 3. Reflected shock wave.

For small deformations the terms $O(m_1^2)$ can be neglected. The speed $\overset{\delta}{U}_{v_1}$ of an acoustic wave propagating in the direction of X^2, ($\mathbf{N} = (0, \pm 1, 0)$) behind the incident shock wave front, is greater than the incident shock wave speed U_v, $\overset{\delta}{U}_{v_1} > U_v$ [4]; this implies that $\alpha_0 < 1$.

The amplitude vector of the reflected shock has an opposite direction to the amplitude vector $(3.2)_1$ of the incident shock wave:

$$\mathbf{H}_{(1)} = (0, -m\alpha_0, 0), \quad \alpha_0 < 1. \tag{4.15}$$

The jumps of the deformation gradient define the deformation in the region behind the reflected wave front:

$$(x^2{}_2)^B = \breve{x}^2{}_2(0) + H^2_{(1)} N_2 = \breve{x}^2{}_2(0) - m\alpha_0. \tag{4.16}$$

The entropy conditions admit the propagation of compressive shocks only [7]. The reflected shock wave with the amplitude (4.15) can also propagate. The velocity of the reflected simple wave $\breve{U}_2(\gamma_2)$ must be a decreasing function of the wave parameter. The propagation of the reflected simple wave for a clamped boundary is possible for $\tilde{\gamma}_2 < 0$ only. It will be possible if the material region in front of the incident shock is not at rest. For a medium with a boundary plane this requirement is fully unrealistic. The general solution reduces to a single shock wave only.

4.2. *Free boundary*

The reflection problem for the case in which the stress vector $t_i = T_{Ri}{}^\alpha K_\alpha$, $K_\alpha = (0, -1, 0)$ vanishes on the plane $X^2 = 0$ is examined. This means that

$$T_{R1}{}^2 = T_{R2}{}^2 = T_{R3}{}^2 \quad \text{on} \quad X^2 = 0. \tag{4.17}$$

For the deformation gradient $\mathbf{F}_1$ (3.8), the components of the first Piola–Kirchhoff stress tensor are

$$\begin{aligned} T_{R2}{}^2 &= 2\varrho_R\, x^2{}_2 \left[\sigma_1 + \sigma_2\left((x^1{}_1)^2 + (x^3{}_3)^2\right) + \sigma_3 (x^3{}_3)^2 (x^1{}_1)^2\right], \\ T_{R1}{}^2 &= 2\varrho_R\, x_1{}^2 (\sigma_1 + \sigma_2 I_1 + \sigma_3 I_2), \\ T_{R3}{}^2 &= 2\varrho_R\, x_3{}^2 (\sigma_1 + \sigma_2 I_1 + \sigma_3 I_2). \end{aligned} \tag{4.18}$$

To meet free boundary conditions assumed here, it is convenient to choose (in Eqs. (2.10)), $f(x^i{}_\alpha) = 1$. Combining Eqs. (2.10) and (4.2), (4.5) and making use of the initial conditions (4.7), we obtain

$$\frac{dw^\alpha{}_2}{d\gamma_\alpha} = -1, \quad \frac{d\dot{w}^\alpha}{d\gamma_\alpha} = \breve{U}_\alpha; \tag{4.19}$$

$$x^{\alpha}{}_{2} = \breve{x}^{\alpha}{}_{2}(0) - \gamma_{\alpha},$$

(4.20) $$\dot{x}^{\alpha} = \breve{u}^{\alpha}(0) + \breve{U}_{\alpha}\gamma_{\alpha},$$

$$x^{\alpha}{}_{\beta} = x^{\alpha}{}_{\beta}(0) = \mathring{x}^{\alpha}{}_{\beta}, \quad \alpha = 1, 2, 3, \quad \alpha \text{ — no summation}, \quad \beta = 1, 3.$$

The boundary conditions $(4.17)_{2,3}$ are satisfied when

(4.21) $$x^{1}{}_{2} = x^{3}{}_{2} = 0 \quad \text{on} \quad X^{2} = 0.$$

The functions $x^{1}{}_{2}$, $x^{3}{}_{2}$ are linear functions of the wave parameters γ_{α}. Assuming $\breve{x}^{1}{}_{2}(0) = \breve{x}^{3}{}_{2}(0) = 0$ we obtain $\tilde{\gamma}_1 = \tilde{\gamma}_3 = 0$. We can choose the initial deformation $(3.4)_1$ such that $\breve{x}^{1}{}_{2}(0) \neq 0$, $\breve{x}^{3}{}_{2}(0) \neq 0$; in this case the parameters $\tilde{\gamma}_1$, $\tilde{\gamma}_3$ are not equal to zero.

The components of the Piola–Kirchhoff stress tensor (2.2) for $\mathbf{F} = \mathbf{1}$ are equal to zero. The function $T_{Ri}{}^{\alpha}(\mathbf{F})$ may be expanded in the convergent Taylor series about $\mathbf{F} = \mathbf{1}$. For small but finite deformation, only the first order terms are retained:

(4.22) $$T_{Ri}{}^{\alpha}(\mathbf{F}) = T_{Ri}{}^{\alpha}(\mathbf{1}) + \frac{\partial T_{Ri}{}^{\alpha}}{\partial x^{k}{}_{\beta}}(x^{k}{}_{\beta} - \delta^{k}{}_{\beta}).$$

Assuming that all components of the representation (3.8) can change, we obtain for $T_{Ri}{}^{2}$

(4.23) $$T_{Ri}{}^{2} = \delta_{i}{}^{2}{}_{1}{}^{1}(\mathbf{1})(x^{1}{}_{1} - 1) + \sigma_{i}{}^{2}{}_{2}{}^{2}(\mathbf{1})(x^{2}{}_{2} - 1) + \sigma_{i}{}^{2}{}_{3}{}^{3}(x^{3}{}_{3} - 1) + \\ + \sigma_{i}{}^{2}{}_{1}{}^{2}(\mathbf{1})\, x^{1}{}_{2} + \sigma_{i}{}^{2}{}_{3}{}^{2}(\mathbf{1})\, x^{3}{}_{2}, \quad i = 1, 2, 3.$$

The values of the elasticity coefficients $\sigma_{ik}^{\alpha\beta}(\mathbf{1})$ are

(4.24) $$\sigma_{2}{}^{2}{}_{2}{}^{2}(\mathbf{1}) = \lambda + 2\mu, \quad \sigma_{2}{}^{2}{}_{1}{}^{1}(\mathbf{1}) = \sigma_{2}{}^{2}{}_{3}{}^{3}(\mathbf{1}) = \lambda, \\ \sigma_{1}{}^{2}{}_{1}{}^{2}(\mathbf{1}) = \sigma_{3}{}^{2}{}_{3}{}^{2}(\mathbf{1}) = \mu.$$

The other components are equal to zero.

Substitution of Eq. $(4.20)_1$ for $\alpha = 2$ into Eq. (4.23) for $i = 2$ gives the value $\tilde{\gamma}_2$ of the wave parameter γ_2 on the boundary.

(4.25) $$\tilde{\gamma}_2 = \breve{x}^{2}{}_{2}(0) - 1 + \frac{1}{1 + 2\dfrac{\mu}{\lambda}}(\breve{x}^{1}{}_{1}(0) + \breve{x}^{3}{}_{3}(0) - 2).$$

The initial deformation $(3.4)_1$ in the material region with free boundary cannot be arbitrary, it should be chosen such that

(4.26) $$T_{R2}{}^{2}(\overset{0}{\mathbf{F}}) = 0 \quad \text{on} \quad X^{2} = 0,$$

or in the linearized form

(4.27) $$\overset{0}{x}{}^{2}{}_{2} - 1 + \frac{1}{1 + 2\dfrac{\mu}{\lambda}}(\overset{0}{x}{}^{1}{}_{1} + \overset{0}{x}{}^{3}{}_{3} - 2) = 0.$$

According to Eqs. (4.25), (4.27), (4.20) and (3.5), $\breve{x}^2{}_2(0) = \overset{0}{x}{}^2{}_2 - m$

$$\tilde{\gamma}_2 = -m, \quad -m \leqslant \gamma_2 \leqslant 0. \tag{4.28}$$

The function $x^2{}_2(\gamma_2)$ is an increasing function of the wave parameter γ_2 when γ_2 changes from 0 to $\tilde{\gamma}_2 = -m$; this indicates ([7]) that the function $\breve{U}_2(\gamma_2)$ decreases when γ_2 changes from 0 to $\tilde{\gamma}_2 < 0$ and, according to the criterion given in [1], that region "2" (Fig. 2) represents a reflected simple wave. The propagation of the reflected shock is not possible because $U_2(\tilde{\gamma}_2) < U_2(0)$. According to the second equivalence theorem [3], there exist for the clamped and free boundary the solutions of the relations (4.5) for $\alpha = 1, 3$ and $j = 1, 2, 3$. The boundary conditions (4.9) and (4.17) and initial deformation $(3.4)_1$ make propagation of such waves impossible and restrict the reflection solution to one shock or simple wave.

5. Reflection and transmission of the shock wave

Let two nonlinear elastic solids (described with the Murnaghan elastic potential) differing in elastic properties and having different mass densities be rigidly coupled at the interface $X^2 = 0$. When two solids are in rigid contact, then the displacement vector or particle velocity vector u^i and stress vector t_i must be continuous from one medium to the other:

$$u^i = \hat{u}^i, \quad t_i = \hat{t}_i \quad \text{on} \quad X^2 = 0. \tag{5.1}$$

In the region of the transmitted waves $X^2 < 0$, all functions and quantities have the denotation $(\hat{\cdot})$, whereas in the region $X^2 > 0$ all symbols are such as in Sect. 4.

We assume the deformation gradient and particle velocity in regions $X^2 > 0$, $X^2 < 0$ in the form (4.2) and (5.2), respectively,

$$\begin{aligned} \hat{x}^i &= \hat{x}^i(X^\alpha, t) + \hat{w}^i(X^\alpha, t), \\ \hat{x}^i{}_\alpha &= \hat{x}^i{}_\alpha(0) + \hat{w}^i{}_\alpha(\hat{\gamma}_\alpha), \\ \dot{\hat{x}}^i &= \hat{u}^i(0) + \hat{v}^i(\hat{\gamma}_\alpha), \quad \hat{v}^i = \frac{\partial \hat{w}^i}{\partial t}, \quad i, \alpha = 1, 2, 3, \end{aligned} \tag{5.2}$$

where $\hat{x}^i_\alpha(0)$ — the components of the homogeneous deformation gradient in the form analogous to the relation $(3.4)_1$ in the region $X^2 < 0$ and $\hat{u}^i(0)$ are components of the particle velocity in the same region, $\hat{w}^i$ are the components of the small deformation about the initial deformation in the region of the transmitted simple wave.

The boundary conditions for small deformation $\hat{w}^i$, analogous to the relation (4.7) are

$$\hat{w}^\alpha{}_2(0) = \hat{v}^\alpha(0) = 0, \quad \alpha = 1, 2, 3. \tag{5.3}$$

Equations (4.4) in the region of the transmitted simple wave may be rewritten as

$$(5.4)\qquad \begin{aligned} &(\hat{Q}_{ij}-\hat{\varrho}_R\,\hat{U}^2\,\delta_{ij})\,\hat{v}'^j = 0,\\ &\hat{U}\hat{w}'^j{}_\beta+\hat{v}'^j\,\hat{N}_\beta = 0. \end{aligned}$$

According to the relations (4.5) and $(5.4)_1$, the propagation of three reflected (transmitted) waves in the halfspace $X^2 > 0$ $(X^2 < 0)$ is possible. The solutions for characteristic speeds and right proper vectors for such waves are:

Reflected waves $(X^2 > 0)$, $\mathbf{N}(0, 1, 0)$

$$(5.5)\qquad \breve{U}^2_\alpha = \breve{\sigma}_\alpha{}^2{}_\alpha{}^2,\quad \overset{(\alpha)}{r}_j = \delta_{j\alpha},\quad j, \alpha = 1, 2, 3,\quad \alpha\text{ — no summation.}$$

Transmitted waves $(X^2 < 0)$, $\hat{N}(0, -1, 0)$

$$(5.6)\qquad \hat{U}^2_\alpha = \hat{\sigma}_\alpha{}^2{}_\alpha{}^2,\quad \overset{(\alpha)}{\hat{r}}_j = \delta_{j\alpha},\quad j, \alpha = 1, 2, 3,\quad \alpha\text{ — no summation.}$$

From Eqs. $(2.10)_1$ and (5.2) we obtain the equations for particle velocity in both regions $X^2 > 0$, $X^2 < 0$:

$$(5.7)\qquad \frac{dv^\alpha}{d\gamma_\alpha} = \breve{U}_\alpha\, r_\alpha\, f_\alpha,\qquad \frac{d\hat{v}^\alpha}{d\hat{\gamma}_\alpha} = \hat{U}_\alpha\,\hat{r}_\alpha\,\hat{f}_\alpha,\qquad \alpha = 1, 2, 3,$$

α — no summation.

Taking $f_\alpha(x^i{}_\beta) = \breve{U}_\alpha^{-1}$ and $\hat{f}_\alpha(x^i{}_\beta) = \hat{U}_\alpha^{-1}$ $(\alpha = 1, 2, 3)$, with the boundary conditions (5.3), we obtain

$$(5.8)\qquad v^\alpha = \gamma_\alpha,\qquad \hat{v}^\alpha = \hat{\gamma}_\alpha,\qquad \alpha = 1, 2, 3.$$

The functions u^i, $\hat{u}^i$ in (5.1) are evaluated for the final values of the parameters $\tilde{\gamma}_\alpha$ and $\tilde{\hat{\gamma}}_\alpha$. This implies Eqs. $(4.2)_2$ and $(5.2)_3$ three equations for the final values for parameters of the reflected and transmitted waves:

$$(5.9)\qquad \tilde{\gamma}_1 = \tilde{\hat{\gamma}}_1,\qquad \tilde{\gamma}_3 = \tilde{\hat{\gamma}}_3,\qquad \tilde{\gamma}_2+\breve{u}^2(0) = \tilde{\hat{\gamma}}_2.$$

The equations for the components of the deformation gradients $w^i{}_\alpha$, and $\hat{w}^i{}_\alpha$ are Eqs. ($(2.10)_2$ and $(5.4)_2$)

$$(5.10)\qquad \frac{dw^\alpha{}_2}{d\gamma_\alpha} = -r_\alpha\, N_2\, f_\alpha,\qquad \frac{d\hat{w}^\alpha{}_2}{d\hat{\gamma}_\alpha} = -\hat{r}_\alpha\,\hat{N}_2\,\hat{f}_\alpha,\qquad \alpha = 1, 2, 3$$

α — no summation.

Taking into account the boundary conditions (4.7) and (5.4), we obtain

$$(5.11)\qquad w^\alpha{}_2 = -\breve{U}_\alpha^{-1}\,\gamma_\alpha,\qquad \hat{w}^\alpha{}_2 = \hat{U}_\alpha^{-1}\,\hat{\gamma}_\alpha,\qquad \alpha = 1, 2, 3.$$

α — no summation,

Substituting the relations (5.11) in Eqs. $(4.2)_1$ and $(5.2)_2$ we obtain the expressions for the components of the deformation gradient:

$$(5.12)\qquad x^\alpha{}_2 = -\breve{U}_\alpha^{-1}\,\gamma_\alpha,\qquad \hat{x}^\alpha{}_2 = +\hat{U}_\alpha^{-1}\,\hat{\gamma}_\alpha,\qquad \alpha = 1, 3,\quad \alpha\text{ — no summation,}$$

$$(5.13)\qquad x^2{}_2 = \breve{x}^2{}_2(0)-\breve{U}_2^{-1}\,\gamma_2,\qquad \hat{x}^2{}_2 = \hat{x}^2{}_2(0)+\hat{U}_2^{-1}\,\hat{\gamma}_2,\qquad \alpha = 2.$$

Analogously to Eq. (4.22), we expand the components of the Piola–Kirchhoff stress tensor $T_{Ri}{}^2$ (**F**) in the Taylor series about $\mathbf{F} = \mathbf{1}$. Let us assume that the initial static deformations in both material regions are pure homogeneous. These deformations cannot be arbitrary, the condition $(5.2)_2$ must be satisfied in the static state, too. Substitution of the expressions (5.12), (5.13) in Eqs. (4.23) and $(5.2)_2$ gives three equations for $i = 1, 2, 3$; they together with Eqs. (5.10) may be used to find the final values of the wave parameters $\hat{\gamma}^\alpha$, $\tilde{\tilde{\gamma}}_\alpha$.

(5.14) $$\tilde{\gamma}_2 = -m\,\frac{\varkappa - \hat{U}_2^{-1}\,U_v}{\hat{U}_2^{-1} + \varkappa\check{U}_2^{-1}}, \qquad \tilde{\tilde{\gamma}}_2 = -m\varkappa\,\frac{1 + \check{U}_2^{-1}\,U_v}{\hat{U}_2^{-1} + \varkappa\check{U}_2^{-1}},$$

where $\varkappa = (\lambda + 2\mu)(\hat{\lambda} + 2\hat{\mu})^{-1}$, μ, λ; $\hat{\mu}$, $\hat{\lambda}$ — Lamé constants in regions $X^2 > 0$ and $X^2 < 0$, respectively, and

$$ux^\alpha{}_2 = \hat{\mu}\hat{x}^\alpha{}_2 \Rightarrow (\mu U_\alpha^{-1} + \hat{\mu}\hat{U}_\alpha^{-1})\,\tilde{\gamma}_\alpha = 0, \qquad \alpha = 1, 3,$$

α — no summation.

From this it follows that

(5.15) $$\tilde{\gamma}_\alpha = \tilde{\tilde{\gamma}}_\alpha = 0 \quad \text{for} \quad \alpha = 1, 3.$$

The values of the wave parameters γ_2, $\hat{\gamma}_2$ depend on the elastic properties of two rigid connected solids and on initial deformations in these two materials (the velocities $\hat{U}_2^{-1}$ $\check{U}_2^{-1}$). According to Eq. $(5.14)_2$, the final value of the parameter $\hat{\gamma}_2$, $\tilde{\gamma}_2 < 0$. The function $\hat{x}^2{}_2$ $(5.13)_2$ is a decreasing function of the wave parameter $\hat{\gamma}_2$, when $\hat{\gamma}_2$ ranges from 0 to its extreme value $\tilde{\tilde{\gamma}}_2$. This means ([7]) that $\hat{U}_2(\hat{\gamma}_2)$ is increasing with $\hat{\gamma}_2$ changing from 0 to $\tilde{\tilde{\gamma}}_2 < 0$, and that the transmitted wave is every shock wave. Therefore, from Eq. $(5.14)_1$, we obtain for (Fig. 4)

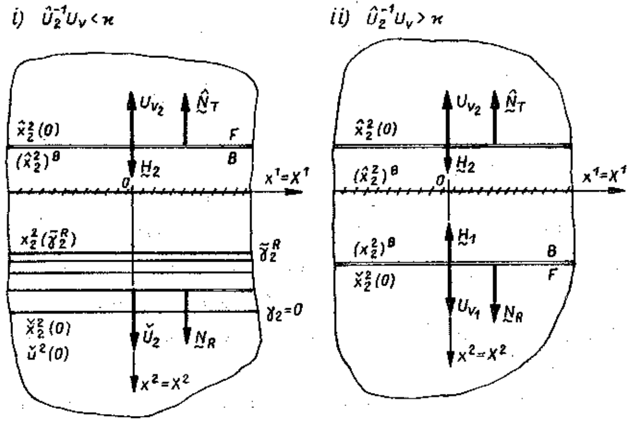

FIG. 4. Reflection-transmission patterns i) reflected simple wave and transmitted shock wave ii) reflected and transmitted shock wave.

i) $\hat{U}_2^{-1} U_v < \varkappa$ reflected simple wave,

ii) $\hat{U}_2^{-1} U_v > \varkappa$ reflected shock wave,

iii) $\hat{U}_2^{-1} U_v = \varkappa$ reflected infinitesimal progressive wave.

The unit material normals and amplitude vectors are:

$$\mathbf{N}_R = (0, 1, 0), \quad \hat{\mathbf{N}}_T = (0, -1, 0),$$
$$\mathbf{H}_1 = (0, m_1, 0), \quad \mathbf{H}_2 = (0, m_2, 0).$$

The parameters of the reflected and transmitted waves in cases i) and ii) can be computed from Eq. (2.3) in a way analogous to the expressions (4.15) and (4.16).

$$\tilde{\gamma}_2^R = -m\beta_0 + o(m_2^2), \quad m_2 = m\varkappa(1 - \check{U}_2^{-1}\beta_0), \tag{5.16}$$
$$(\hat{x}^2{}_2)^B = \hat{x}^2{}_2(0) - m_2, \quad x^2{}_2(\tilde{\gamma}_2{}^R = \check{x}^2{}_2(0) - \check{U}_2^{-1}\tilde{\gamma}_2{}^R;$$

$$mU_v = m_1 U_{v_1} + m_2 U_{v_2},$$
$$m_1 = -m\check{U}_2^{-1}\beta_0, \quad m_2 = \varkappa m(1 - \check{U}_2^{-1}\beta_0), \tag{5.17}$$
$$(\hat{x}^2{}_2)^B = \hat{x}^2{}_2(0) - m_2, \quad (x^2{}_2)^B = \check{x}^2{}_2(0) - m_1,$$

where $\beta_0 = (\varkappa - U_v \hat{U}_2^{-1})(\hat{U}_2^{-1} + \varkappa U_2^{-1})^{-1}$, m_1 and m_2 — reflected and transmitted shock wave strength, $\gamma_2{}^R$ — final value of the parameter in the reflected simple wave, U_{v_1} and U_{v_2} reflected and transmitted shock wave speed.

The second equivalence theorem [3] admits the propagation of the reflected and transmitted infinitesimal progressive waves for $\alpha = 1, 3$, Eqs. (5.5) and (5.6). The continuity conditions defined on the interface $X^2 = 0$ by the expressions (5.1) made such a propagation fully unrealistic.

6. Numerical solutions

The reflection and reflection-transmission solutions discussed in Sects. 4 and 5 are examined numerically for steel and aluminium. The elasticity constants of the first and second order were taken from [6, 7]. In the case of reflection, the results for the initial deformations in steel $x^1{}_1 = x^2{}_2 = x^3{}_3 = 1.0085$ and $x^1{}_1 = x^3{}_3 = 0.9915$, $x^2{}_2 = 1.0085$ determine in Figs. 5 and 6 the solid and broken line respectively. The initial deformation and the material region behind the wave front should remain elastic. Hence the discontinuity jumps cannot be arbitrary, and the appropriate estimate for m should be established [7].

Figure 5 refers to the case of a reflected simple wave (free boundary). The value of the incident shock strength is changing from 0 to $m = 0.017$

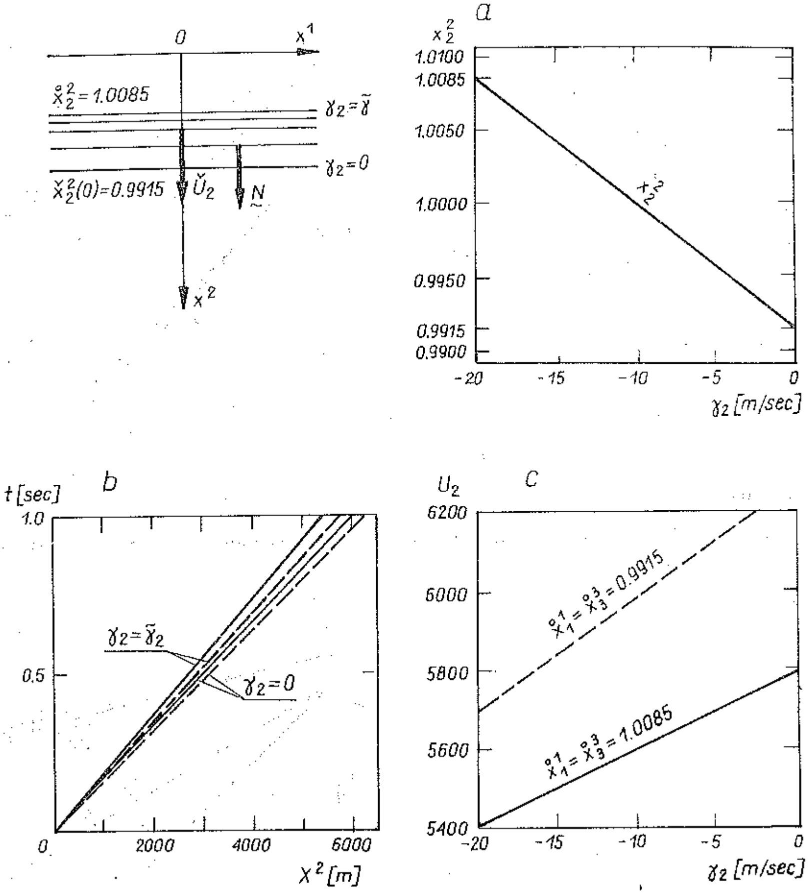

FIG. 5. Reflected simple wave: a) deformation gradient b) wave speed as functions of γ_2, c) distribution of the wavelets in reflected simple wave region.

and according to the expression (4.28) $-m \leqslant \gamma_2 \leqslant 0$. The component $x^2{}_2$ and the velocity of the reflected simple wave $U_2(\gamma_2)$ are shown as functions of the wave parameter γ_2 in Fig. 5a, c. The velocities of the wavelets in the reflected simple wave for γ_2 changing from $\gamma_2 = 0$ to $\gamma_2 = \tilde{\gamma}_2 = -m$, are shown in the $X^2 - t$ plane in Fig. 5b.

Some characteristic results in the case of a reflected shock wave (clamped boundary) are shown in Fig. 6a, b. The graphs in Fig. 6a show the incident shock wave speed U_v, the reflected shock wave speed U_{v_1} and the acoustic wave speed $\overset{0}{U}_v$ in the region behind the incident shock wave as a function of the incident shock wave strength m. The components $x^2{}_2\,(0)$ in the region behind the incident shock wave $(x^2{}_2)^B$ in the region behind the reflected shock wave and the reflection parameter α_0 are shown as functions of the

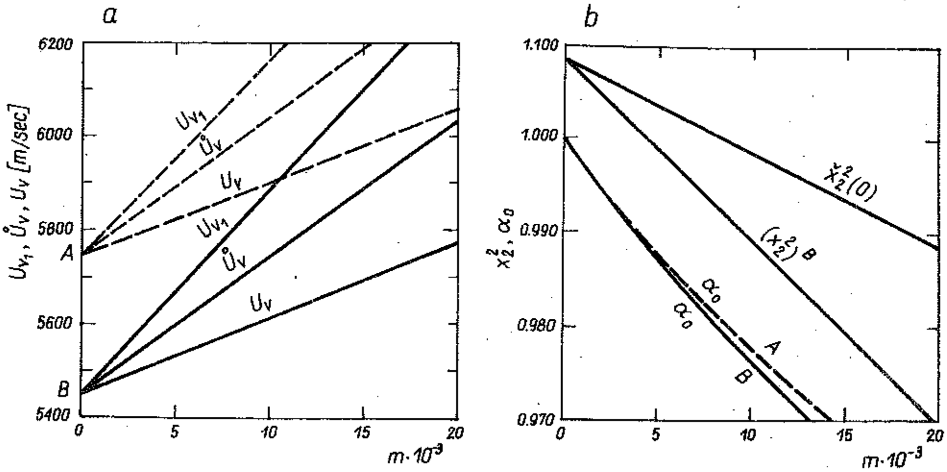

FIG. 6. Reflected shock wave: a) incident and reflected shock wave speeds U_v, U_{v_1} and acoustic wave speed $\mathring{U}_v$ b) deformation gradient and reflection parameter α_0 as functions of m.

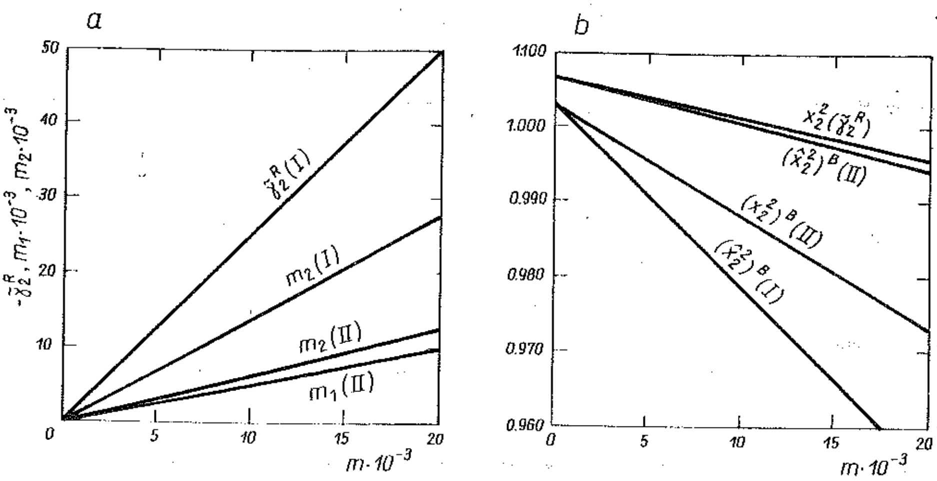

FIG. 7. Reflection-transmission case: a) final parameter $\tilde{\gamma}_2^R$ of the reflected simple wave and reflected, transmitted shock wave strength m_1, m_2, b) deformation gradient as functions of m.

incident shock strength m in Fig. 6b. Both initial deformations give nearly identical values for the components $x^2{}_2(0)$, $(x^2{}_2)^B$.

Figure 7 presents the case of reflection and transmission at a plane interface between two selected solid media: steel and aluminium, perfectly welded along the X^1-axis. Both initial deformations: in steel $x^1{}_1 = x^3{}_3 = 0.9915$, $x^2{}_2 = 1.0068$ and in aluminum $x^1{}_1 = x^2{}_2 = x^3{}_3 = 1.0030$ satisfied the conditions $(5.1)_2$ in the static case. Two cases were taken into consideration: the incident shock wave propagates through steel — case (I), $\varkappa = 2.453$ or through aluminium — case (II) $\varkappa = 0.408$, in the halfspace $X^2 > 0$ and is reflected and transmitted at the interface. The results obtained for these cases are different. The reflected wave is a simple wave (I), $(U_v \hat{U}_2^{-1} < \varkappa)$ or a shock wave (II), $(U_v \hat{U}_{v_2}^{-1} > \varkappa)$. It is characteristic that the shock reflection-transmission patterns depend first of all on the mechanical properties of both solid media ($\varkappa$). The influence of the initial deformations and incident shock wave strength is small. The final parameter $\tilde{y}_2^R$ and the reflected, and transmitted shock wave strength m_1, m_2 for both cases are shown as functions of the incident shock wave strength m in Fig. 7a, for some values of m. Figure 7b shows the graphs of the deformation gradient components as a function of m.

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Streszczenie

ODBICIE NORMALNE I TRANSMISJA PŁASKIEJ FALI UDERZENIOWEJ W NIELINIOWYM MATERIALE SPRĘŻONYM

Rozpatrzono zagadnienie odbicia (odbicia i transmisji) płaskiej fali uderzeniowej o skończonej amplitudzie od płaskiego brzegu (płaszczyzny sztywnego połączenia dwóch materiałów) propagującej się w materiale sprężystym Murnaghana. Przed frontem padającej fali uderzeniowej materiał pozostaje w spoczynku i jest jednorodnie wstępnie odkształcony. Dla analizy fal o małej lecz skończonej amplitudzie rozpatrzono zlinearyzowane zagadnienie brzegowe dla trzech różnych typów brzegów. Rozwiązanie dla fali odbitej może mieć postać fali prostej, fali uderzeniowej lub sinusoidalnej fali biegnącej, podczas gdy fala transmitowana jest zawsze w postaci fali uderzeniowej.

Резюме

НОРМАЛЬНОЕ ОТРАЖЕНИЕ И ТРАНСМИССИЯ ПЛОСКОЙ УДАРНОЙ ВОЛНЫ В НЕЛИНЕЙНОМ УПРУГОМ МАТЕРИАЛЕ

Рассмотрена проблема отражения (отражения и трансмиссии) плоской ударной волны конечной амплитуды от плоской границы (плоскости жесткого соединения двух материалов), причем волна распространяется в упругом материале Мурнагана. Перед фронтом падающей ударной волны материал остается в покое и предварительно однородным образом деформирован. Для анализа волн малой, но конечной амплитуды, рассмотрены линеаризованные краевые задачи для трех разных типов границ. Решение для отраженной волны может иметь вид простой волны, ударной волны или синусоидальной бегущей волны, в то время как переходящая волна всегда имеет вид ударной волны.

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