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A NEW METHOD CALCULATING STRESSES AND DISPLACEMENTS IN ELASTIC BARS

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A new method is presented enabling rapid numerical evaluation of stresses and displacements of orthotropic, linear-elastic thin-walled bars of polygonal cross-sections. The algorithm is based on Vlasov's shell theory. The differential equations derived are solved numerically by means of the method of splines. Examples of numerical results prove the possibility of application of the existing FORTRAN programs. The paper contains a novel formulation of the boundary conditions.

1. Introduction

One of the first steps in construction design is an overall estimation of displacements and stresses, which appear under the action of given external forces. The finite element method is the most widely used tool for such calculations. This method, however, often demands very much computer time, especially for large and complicated constructions. In the present paper we propose a new method of computation of stresses and displacements which — at least in some cases — is much faster than the finite element method. Our work concerns thin-walled prismatic bars with cross-sections consisting of arbitrary arranged polygons. We consider only orthotropic and linear-elastic materials. The Vlasov general theory of prismatic and cylindrical constructions [1, 2, 3, 4] was the starting point for our study. Under the assumptions of the Vlasov theory the general equilibrium equations of considered bars were obtained and adopted for bulkheadless ships [5, 6]. The resulting system of linear ordinary differential equations with constant coefficients was solved numerically. The algorithm was based on the spline representation of the unknown functions. The existing FORTRAN program makes possible the integration of differential systems corresponding to the constructions with 60—70 joints in their cross-section (100 equations). We have not made tests with greater systems, but permanent high stability of the method allows to predict the possibility of solving larger systems. The authors claim that such models cover all technically important types of constructions applied in ship-building. The present paper contains also a new general formulation of the boundary conditions.

2. Total energy of a bar

Consider thin-walled bars with constant profile, consisting of an arbitrary arrangement of polygons (one can approximate a hull of a ship with such a bar). The profile is completely determined by the Cartesian coordinates OXY of its joints, provided that it is known how these joints are connected (Fig. 1). Each polygon K_i has its curvilinear coordinate s. The direction

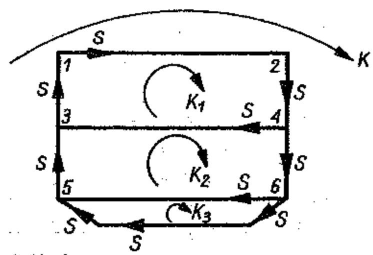

FIG. 1. An example of a thin-walled bar with multicircuit cross-section longitudinal coordinate perpendicular to the figure.

of this coordinate may be chosen arbitrarily. Let us construct for each K_i the local system of orthonormal versors $\mathbf{l}_i$, $\mathbf{n}_i$, $\mathbf{b}_i$ where $\mathbf{l}_i$ agrees with the s-direction, $\mathbf{n}_i$ is the normal external versor, $\mathbf{b}_i$ is orthogonal to both $\mathbf{l}_i$, $\mathbf{n}_i$ and points in the z-direction (longitudinal coordinate). Loadings are given as a vector function $\mathbf{p}(z, s)$, which we decompose in the local basis $\mathbf{l}$, $\mathbf{n}$, $\mathbf{b}$:

$$\mathbf{p}(z, s) = p_n(z, s)\,\mathbf{n} + p_s(z, s)\,\mathbf{l} + p_b(z, s)\,\mathbf{b}. \tag{2.1}$$

Displacements of the points of the shell are represented by a vector function $\mathbf{R}(z, s)$, which we decompose in the local basis $\mathbf{l}$, $\mathbf{n}$, $\mathbf{b}$:

$$\mathbf{R}(z, s) = u(z, s)\,\mathbf{b} + v(z, s)\,\mathbf{l} + w(z, s)\,\mathbf{n}. \tag{2.2}$$

Let us introduce such constraints that the coordinates of the displacement vector may be written as

$$\begin{aligned} u(z, s) &= \sum_{i=1}^{n} U_i(z)\,\varphi_i(s), \\ v(z, s) &= \sum_{k=1}^{m} V_k(z)\,\psi_k(s), \\ w(z, s) &= \sum_{l=1}^{r} W_l(z)\,\chi_l(s), \end{aligned} \tag{2.3}$$

where the functions $U_i(z)$, $V_k(z)$, $W_l(z)$ are to be found, and the given functions $\varphi_i(s)$, $\psi_k(s)$, $\chi_l(s)$ constitute the basis for u, v, w.

The elastic material is assumed to be orthotropic with the orthotropy axes **b**, **l**, **n**. Using the semi-momentless model of the bar [1, 5], we get the following expression for the total energy of the bar deformation:

$$(2.4)\qquad \pi=\frac{1}{2}\int\limits_0^L\left[\oint\limits_K\left\{\breve{E}_1\left(\frac{\partial u}{\partial z}\right)^2+\breve{E}_2\left(\frac{\partial v}{\partial s}\right)^2+2\breve{E}_1\nu_{21}\frac{\partial v}{\partial s}\frac{\partial u}{\partial z}+\right.\right.$$
$$\left.\left.+\breve{G}\left(\frac{\partial u}{\partial s}+\frac{\partial v}{\partial z}\right)^2+D\left(\frac{\partial^2 w}{\partial s^2}\right)^2\right\}ds\right]dz,$$

where

$$D=\frac{E_2\,\delta^3}{12\,(1-\nu_{12}\,\nu_{21})},\qquad \breve{G}=G\delta,\qquad \breve{E}_i=\frac{E_i\,\delta}{1-\nu_{12}\,\nu_{21}},\qquad i=1,2,$$

E_1, E_2 — Young's moduli, ν_{12}, ν_{21} — Poisson constants, G — Kirchhoff modulus, δ — shell thickness (constant in the interjoint segments), K — contour of the cross-section, L — the length of the bar.

The work of external forces is

$$(2.5)\qquad A=\int\limits_0^L\left[\oint\limits_K \mathbf{p}(z,s)\,\mathbf{R}(z,s)\,ds\right]dz=\int\limits_0^L\left[\oint\limits_K (up_b+vp_s+wp_n)\,ds\right]dz,$$

and the total energy of the shell

$$(2.6)\qquad \Omega=\pi-A.$$

The functions $w(z,s)$ may be expressed by the functions $v(z,s)$ [5] and the total energy is

$$(2.7)\qquad \Omega=\frac{1}{2}\int\limits_0^L\left[\oint\limits_K\left\{\breve{E}_1\left(\frac{\partial u}{\partial z}\right)^2+\breve{E}_2\left(\frac{\partial v}{\partial s}\right)^2+2\breve{E}_1\nu_{21}\frac{\partial v}{\partial s}\frac{\partial u}{\partial z}+\right.\right.$$
$$\left.\left.+\breve{G}\left(\frac{\partial u}{\partial s}+\frac{\partial v}{\partial z}\right)^2+\frac{M_F^2}{D}+\frac{2M_F M_g}{D}\right\}ds\right]dz-$$
$$-\int\limits_0^L\left[\oint\limits_K\{up_b+vp_s+(w_1+w_2)\,p_n\}\,ds\right]dz,$$

where M_F — the total bending moment arising from the displacements of the joints in the plane of the cross-section, M_g — the bending moment which originates from external loadings $p_n(z,s)$, acting on the cross-section frame with joints that are immovable, but can rotate freely about their axes, w_1 — displacement in the direction **n** that arises from displacements of the joints, w_2 — bending of the bar when the joints are immovable but can rotate.

3. Equilibrium equations

Let us accept the shape functions of the first order [5] (Fig. 2). The functions $\varphi_i(s)$ determine the displacements orthogonal to the plain of the contour, thus they must be continuous all over K. They have the value 1 in a joint and decrease linearly towards zero in the neighbouring joints.

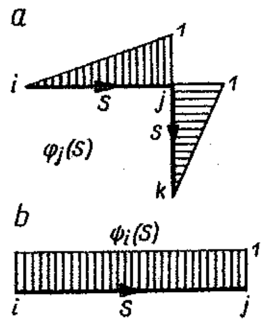

FIG. 2. Shape functions which constitute the basis for deformation. a) $\varphi(s)$ describes binormal displacement, b) $\psi(s)$ describes tangent displacement.

The functions $\psi_k(s)$ determine displacements tangent to the contour, and thus must be continuous on each interjoint segment of the cross-section of the shell.

As an example we shall consider one of the components of (2.7):

$$\oint_K \breve{E}_1 \left(\frac{\partial u}{\partial z}\right)^2 ds.$$

With the aid of Eq. (2.3) we get

$$(3.1)\qquad \oint_K \breve{E}_1 \left(\frac{\partial u}{\partial z}\right)^2 ds = \oint_K \breve{E}_1 \left[\frac{\partial}{\partial z}\Big(\sum_{i=1}^{n} U_i(z)\,\varphi_i(s)\Big)\right]^2 ds =$$

$$= \oint_K \breve{E}_1 \Big[\sum_{i=1}^{n} U_i'(z)\,\varphi_i(s)\Big]^2 ds = \{U'\}^T [M_{\varphi\varphi}]\,\{U'\},$$

where $[M_{\varphi\varphi}]$ is a matrix of integrals $\oint_K \breve{E}_1\,\varphi_i\,\varphi_j\,ds$. Treating all the components in the quadratic form (2.7) analogically, we get the following matrix representation of the total bar energy:

$$(3.2)\qquad \pi = \frac{1}{2}\int_0^L \Big(\{U'\}^T [M_{\varphi\varphi}]\,\{U'\} + \{V'\}^T [M_{\psi\psi}]\,\{V'\} +$$

(3.2) [cont.]
$$+\{V\}^T([M_{\psi\psi}]+[M_{FF}])\{V\}+\{U\}^T[M_{\varphi'\varphi'}]\{U\}+$$
$$+2\{V\}^T[M_{\varphi\psi'}]\{U'\}+2\{V'\}^T[M_{\varphi'\psi}]\{U\}\Big)dz-$$
$$-\int_0^L(\{b\}^T\{U\}+\{d\}^T\{V\})\,dz.$$

The details of the coefficient calculations are given in [5].

From the existence condition of the extremum of the total-energy functional we get

$$(3.3)\quad \begin{bmatrix}[M_{\varphi\varphi}] & 0\\ 0 & [M_{\psi\psi}]\end{bmatrix}\{T''\}+\begin{bmatrix}0 & [M_{\varphi\psi'}]^T-[M_{\varphi'\psi}]^T\\ [M_{\varphi'\psi}]-[M_{\varphi\psi'}] & 0\end{bmatrix}\{T'\}-$$
$$-\begin{bmatrix}[M_{\varphi'\varphi'}] & 0\\ 0 & [M_{\psi'\psi'}]+[M_{FF}]\end{bmatrix}\{T\}=\{q\},$$

where $\{T\}=\{U_1,\ldots,U_n,V_1,\ldots,V_m\}^T$ is the vector of the unknown functions, $\{q\}=\{b_1,\ldots,b_n,d_1,\ldots,d_n\}^T$ is the vector of the generalized forces, denotes differentiation of $U(z)$, $V(z)$, $\varphi(s)$ and $\psi(s)$ with respect to their arguments.

The derivatives of $\psi(s)$ with respect to s equal zero, and all the matrices whose elements have $\psi'(s)$ under the integration sign become zero matrices. Thus we have

$$(3.4)\quad \begin{bmatrix}[M_{\varphi\varphi}] & 0\\ 0 & [M_{\psi\psi}]\end{bmatrix}\{T\}''+\begin{bmatrix}0 & -[M_{\varphi'\psi}]^T\\ [M_{\varphi'\psi}] & 0\end{bmatrix}\{T\}'-$$
$$-\begin{bmatrix}[M_{\varphi'\varphi'}] & 0\\ 0 & [M_{FF}]\end{bmatrix}\{T\}=\{q\}.$$

This is the system of the second-order equations for U and V which is to be solved. Let us formulate the boundary conditions for Eq. (3.4).

4. Boundary conditions

Let us add the work of the external forces acting on the bar ends to the energy Ω given by Eq. (2.6). The works of the external forces acting on the bar ends are

$$(4.1)_1\qquad A_1=\oint_K[u(0,s)\,p_b(0,s)+v(0,s)\,p_s(0,s)+w(0,s)\,p_n(0,s)]\,ds,$$

for $z=0$ and

$$(4.1)_2\qquad A_2=\oint_K[u(L,s)\,p_b(L,s)+v(L,s)\,p_s(L,s)+w(L,s)\,p_n(L,s)]\,ds,$$

for $z = L$. Under the assumptions of the semi-momentless theory Eqs. (4.1) simplify to

$$A_1 = \sum_{i=1}^{n} U_i\, b_i\big|_{z=0} + \sum_{k=1}^{m} V_k\, d_k\big|_{z=0}, \tag{4.2$_1$}$$

and

$$A_2 = \sum_{i=1}^{n} U_i\, b_i\big|_{z=L} + \sum_{k=1}^{m} V_k\, d_k\big|_{z=L}. \tag{4.2$_2$}$$

The total energy of the shell is now

$$\Omega_1 = \Omega - A_1 - A_2, \tag{4.3}$$

where Ω is given by Eq. (2.7). Equation (4.3) is a special case of the functional

$$J[y] = \int_{x_1}^{x_2} F(x_i, y_i, y_i')\, dx - \sum_{i=1}^{n} \gamma_i(y_{i_1}) + \sum_{i=1}^{n} \beta_i(y_{i_2}), \tag{4.4}$$

where y_{i_2}, y_{i_2} are the values of the functions y_i in the ends of the domain, γ_i, β_i — any functions of their arguments. The variation of the functional $J[y]$ given by Eq. (4.4) is

$$\begin{aligned}\delta J = \int_{x_1}^{x_2} \left(\sum_{i=1}^{n} F_{y_i} - \frac{d}{dx} F_{y_i'}\right) h_i(x)\, dx + \sum_{i=1}^{n} \left(F_{y_i}\big|_{x=x_2} + \frac{\partial \beta}{\partial y_{i_2}}\right) \delta y_{i_2} - \\ - \sum_{i=1}^{n} \left(F_{y_i}\big|_{x=x_1} + \frac{\partial \gamma_i}{\partial y_{i_1}}\right) \delta y_{i_1} + \left(F\big|_{x=x_2} - \sum_{i=1}^{n} y_i' F_{y_i'}\big|_{x=x_2} - \right. \\ \left. - \sum_{i=1}^{n} \frac{\partial \beta_i}{\partial x_2}\right) \delta x_2 - \left(F\big|_{x=x_1} - \sum_{i=1}^{n} y_i' F_{y_i'} - \sum_{i=1}^{n} \frac{\partial \gamma_i}{\partial x_1}\right) \delta x_1 .\end{aligned} \tag{4.5}$$

Using Eq. (4.5) for finding the extremum of the functional (4.3) we get equilibrium equations which contain the dependence on the fastening of the bar. In the case of free ends, the boundary conditions are

$$\begin{aligned}\begin{bmatrix} 0 & [M_{\varphi\psi'}] & [M_{\varphi\varphi}] & 0 \\ [M_{\varphi'\psi}] & 0 & 0 & [M_{\psi\psi}] \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} T \\ T' \end{Bmatrix}\Big|_{z=0} + \\ + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & [M_{\varphi\psi'}] & [M_{\varphi\varphi}] & 0 \\ [M_{\varphi'\psi}] & 0 & 0 & [M_{\psi\psi}] \end{bmatrix} \begin{Bmatrix} T \\ T' \end{Bmatrix}\Big|_{z=L} = \begin{Bmatrix} q|_{z=0} \\ q|_{z=0} \end{Bmatrix}.\end{aligned} \tag{4.6}$$

In the case when one end is fastened and one free, the boundary conditions are

$$(4.7)\quad \begin{bmatrix} E & 0 & 0 & 0 \\ 0 & E & 0 & 0 \\ 0 & 0 & E & 0 \\ 0 & 0 & 0 & E \end{bmatrix} \begin{Bmatrix} T \\ T' \end{Bmatrix}\Big|_{z=0} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & [M_{\varphi\psi'}] & [M_{\varphi\varphi}] & 0 \\ [M_{\varphi'\psi}] & 0 & 0 & [M_{\psi\psi}] \end{bmatrix} \times$$
$$\times \begin{Bmatrix} T \\ T' \end{Bmatrix}\Big|_{z=L} = \begin{Bmatrix} 0 \\ q|_{z=L} \end{Bmatrix},$$

where E denotes a unit matrix. Similarly we can write the boundary conditions for all technically important situations.

The solution of Eq. (3.3) or Eq. (3.4) with the boundary conditions (4.6), (4.7) or others resulting from Eq. (4.3), gives us the functions $U_i(z)$ and $V_k(z)$ which, in accordance with the accepted hypothesis, allow us to find displacements in any point of the shell Eq. (2.3). The stresses may be found on the basis of the following equations:

$$(4.8)\qquad \sigma(z, s) = \breve{E}_1 \sum_{i=1}^{n} U_i'(z)\, \varphi_i(s),$$

and

$$(4.9)\qquad \tau(z, s) = \breve{G}\Big(\sum_{i=1}^{n} U_i(z)\, \varphi_i'(s) + \sum_{k=1}^{m} V_k'(z)\, \psi_k(s)\Big),$$

where $\sigma(z, s)$ — normal stresses, $\tau(z, s)$ — tangent stresses.

5. Numerical results

The equilibrium equations are the system of the second-order ordinary differential equations, linear, with constant coefficients. These equations, however, turned to be numerically troublesome. All standard numerical methods for such equations were ineffective (long computation time, low accuracy even in the double précision calculations with the aid of the ICL-4-70 computer). In order to achieve satisfactory numerical results, the resolution of the unknown functions in the Tchebyshev series was applied (matrix generalisation of the method worked out in [7] for one equation with two-point boundary conditions). The Tchebyshev series method was numerically effective for constructions described by no more than 20 equations of the second order, what corresponds to bars with 10—20 joints in their cross-section. Contemporary bulkheadless ships (e.g. RO-RO ships), however, should by approximated with a bar of 30 or more joints in their cross-section. This leads to the systems of about 50 differential equations of the second order with two-point boundary conditions. The proper algorithm — based on the spline representation — has been worked out. This algorithm makes it possible to integrate differential systems corresponding to the constructions with 60—70 joints in their cross-section (100 equations). The general idea is as follows. The equations to be solved (3.3) and (3.4) are of the form

$$\begin{aligned} \mathscr{L}y = \alpha y''(x)+\beta y'(x)+\gamma y(x) = f(x), \quad 0 \leqslant x \leqslant L, \\ a_1\, y(0)+a_2\, y'(0) = a_0, \\ b_1\, y(L)+b_2\, y'(L) = b_0, \end{aligned} \tag{5.1}$$

where α, β, γ, a_1, a_2, b_1, b_2 are matrices and y, $f(x)$, a_0, b_0 are vectors. The cardinal splines for the problem (5.1) are a set of $N+3$ independent cubic splines forming a basis for all cubic splines on the mesh $\Delta: 0 = x_0 < x_1 < \ldots < x_N = L$. These are defined by the following relations [8]:

$$\begin{aligned} A_{\Delta,k}(x_j) &= \delta_{k,j}, && j = 0, 1, \ldots, N; && k = 0, 1, \ldots, N; \\ A'_{\Delta,k}(x_i) &= 0, && i = 0, N; && k = 0, 1, \ldots, N; \\ B_{\Delta,k}(x_j) &= 0, && j = 0, 1, \ldots, N; && k = 0, N; \\ B'_{\Delta,k}(x_i) &= \delta_{k,i}, && i = 0, N; && k = 0, N, \end{aligned} \tag{5.2}$$

where $\delta_{k,i}$ is the Kronecker delta.

Let

$$S_\Delta(y; x) = \sum_{j=0}^{N} A_{\Delta,j}(x)\, y(x_j) + y'(0)\, B_{\Delta,0}(x) + y'(L)\, B_{\Delta,N}(x),$$

be the spline satisfying the proper end conditions and interpolating on the mesh Δ the solution of Eqs. (5.1). The convergence properties of cubic splines allow us to represent the solution of the differential system in the following matrix form:

$$H_\Delta\, Y_\Delta = R_\Delta,$$

where

$$H_\Delta = \begin{bmatrix} a_2 & a_1 & 0 & \ldots & 0 & 0 \\ \mathscr{L}B_{\Delta,0}(x_0) & \mathscr{L}A_{\Delta,0}(x_0) & \mathscr{L}A_{\Delta,1}(x_0) & \ldots & \mathscr{L}A_{\Delta,N}(x_0) & \mathscr{L}B_{\Delta,N}(x_0) \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ \mathscr{L}B_{\Delta,0}(x_N) & \mathscr{L}A_{\Delta,0}(x_N) & \mathscr{L}A_{\Delta,1}(x_N) & \ldots & \mathscr{L}A_{\Delta,N}(x_N) & \mathscr{L}B_{\Delta,N}(x_N) \\ 0 & 0 & 0 & \ldots & b_1 & b_2 \end{bmatrix},$$

$$Y_\Delta = (y'_0, y_0, y_1, \ldots, y_N, y'_N)^T,$$

$$R_\Delta = (a_0, f(x_0), \ldots, f(x_N), b_0)^T,$$

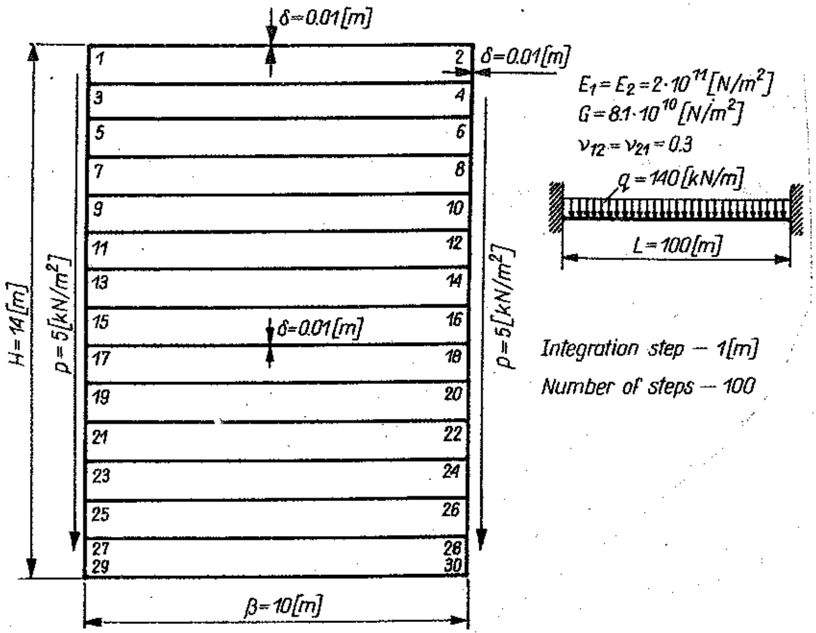

FIG. 3. Prismatic bar with multicircuit cross-section, restrained on both ends, submitted to continuous bending force. Displacements and stresses calculated with the aid of the presented theory are pictured in Figs. 4 and 5.

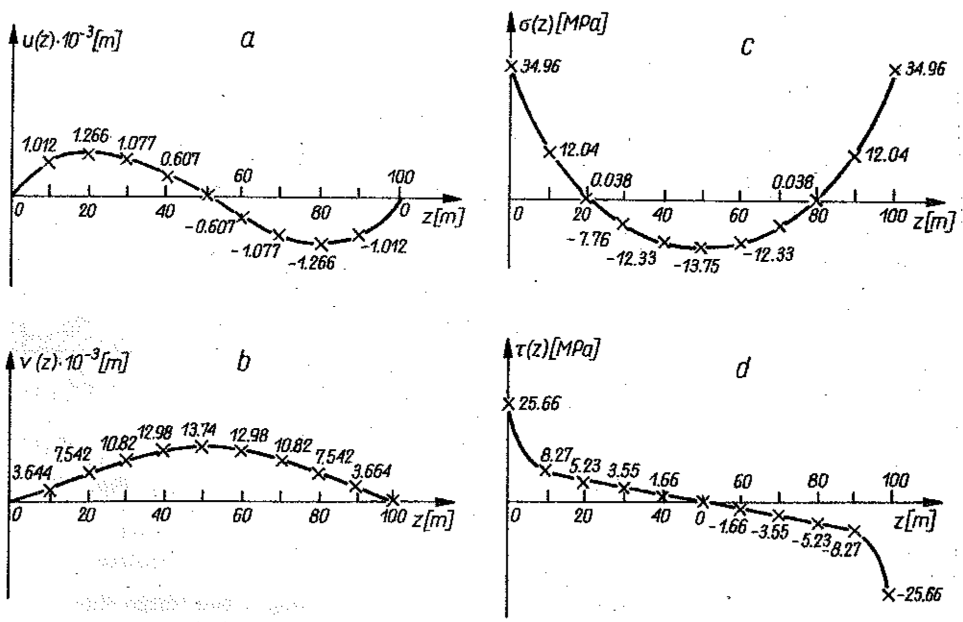

FIG. 4. a) longitudinal displacements $u(z)$, b) tangent displacements $v(z)$, c) normal stresses $\sigma(z)$, d) tangent stresses $\tau(z)$ calculated for the joint 1 and the segment 1—3 for the bar from Fig. 3.

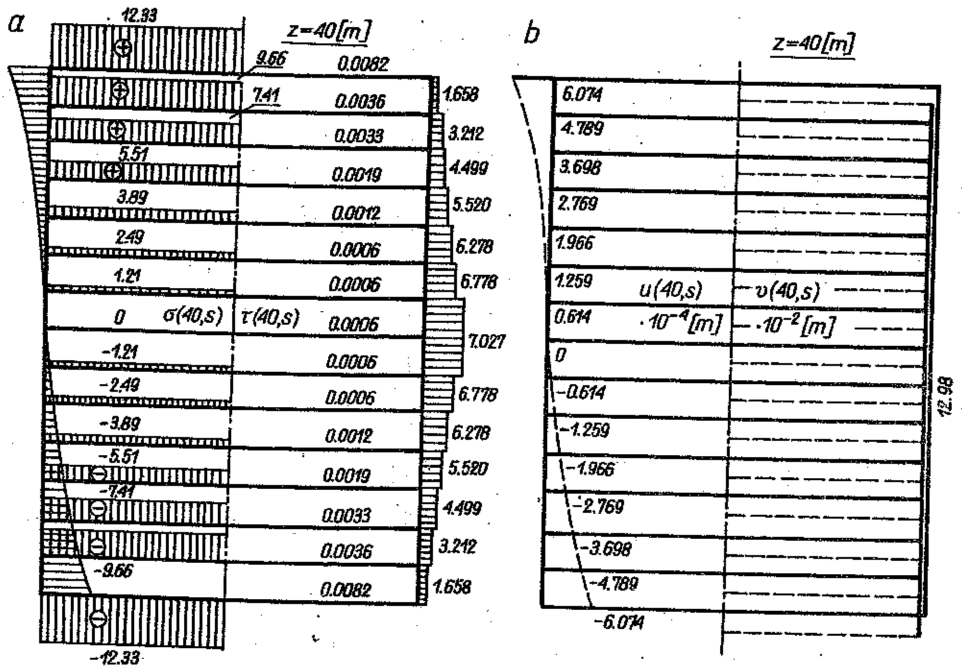

FIG. 5. a) normal stresses σ (40, s) and tangent stresses τ (40, s). b) longitudinal displacements u (40, s) and tangent displacements v (40, s) calculated in $z = 40$ m for the bar from Fig. 3.

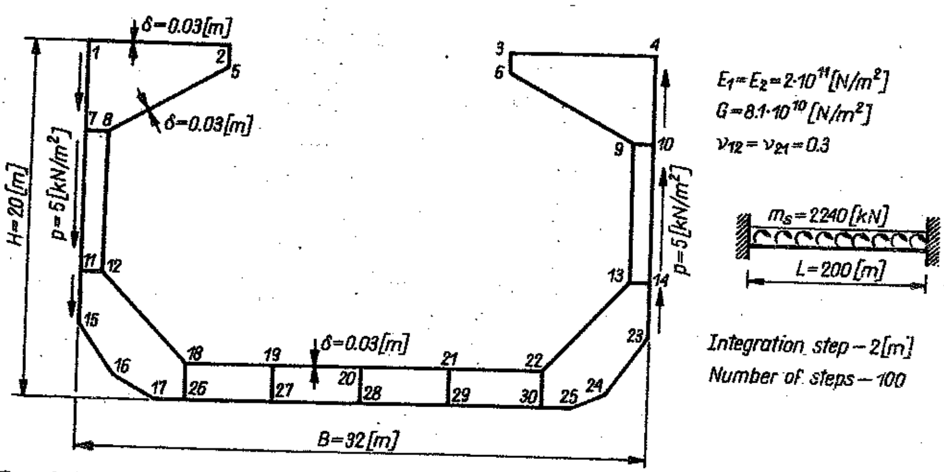

FIG. 6. Prismatic bar with multicircuit cross-section approximating a bulk cargo ship, strained on both ends and submitted to continuous torsional force. Displacements and stresses are pictured in Figs. 7 and 8.

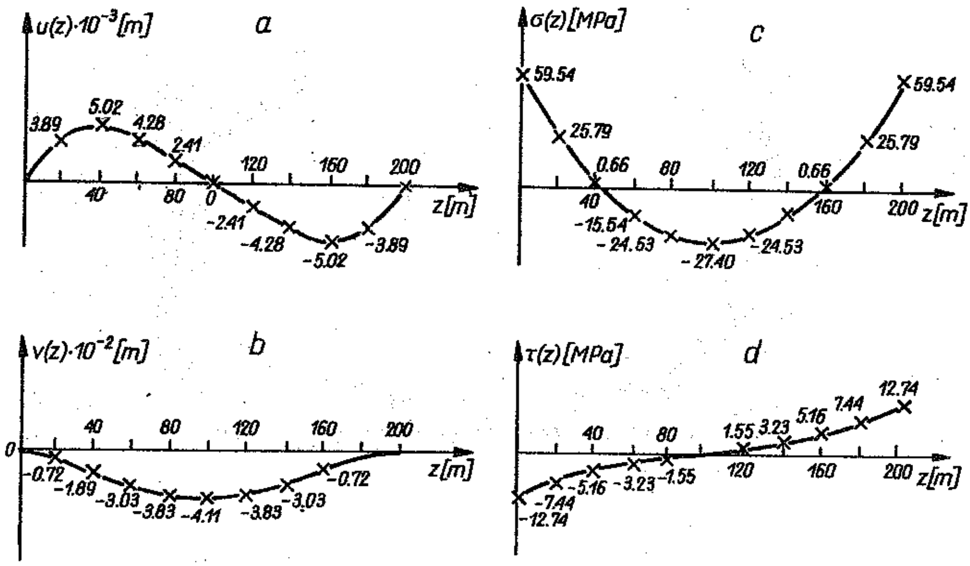

FIG. 7. a) longitudinal displacements $u(z)$, b) tangent displacements $v(z)$, c) normal stresses $\sigma(z)$, d) tangent stresses $\tau(z)$ calculated for joint 1 and on the segment 1—2 for the bar from Fig. 6.

y_i, $f(x_i)$ being n-dimensional vector of solutions and the right-hand sides of the system in the mesh point x_i, n is the number of equations in the system. The matrix H_Δ, which is to be inversed, is — in view of the definition of the cardinal splines — a band matrix.

On the basis of the above theory, the FORTRAN-program "CEZAR" has been prepared. This program can be used for calculating displacements and stresses at an arbitrary point of any prismatic thin-walled bar of a considered class. The preparation of the input data is extremely simple. One has only to give the coordinates of the joints in the cross-section in the arbitrary Cartesian system, and the "coupling matrix", containing information about connections of joints. Also the thicknessess of the plates, material constants and external loadings are necessary. It proves possible to prepare the input data for the construction with 10—12 joints in 15—20 minutes, and for the construction with 30—50 joints — in 2—3 hours (no more than 30 cards).

We enclose two examples of numerical results (Fig. 3—8). The calculations have been performed for the bars with cross-sections of 30 joints, which are sketched in Figs. 3 and 6, for material constants, dimensions of the bar, directions and values of external forces as indicated in the figures. The bar from Fig. 6 may be treated as a model of the hull of a bulk cargo ship, with sides approximated by cuboids (our theory deals only closed cross-sections). The displacements $u(z)$ and the stresses $\sigma(z)$

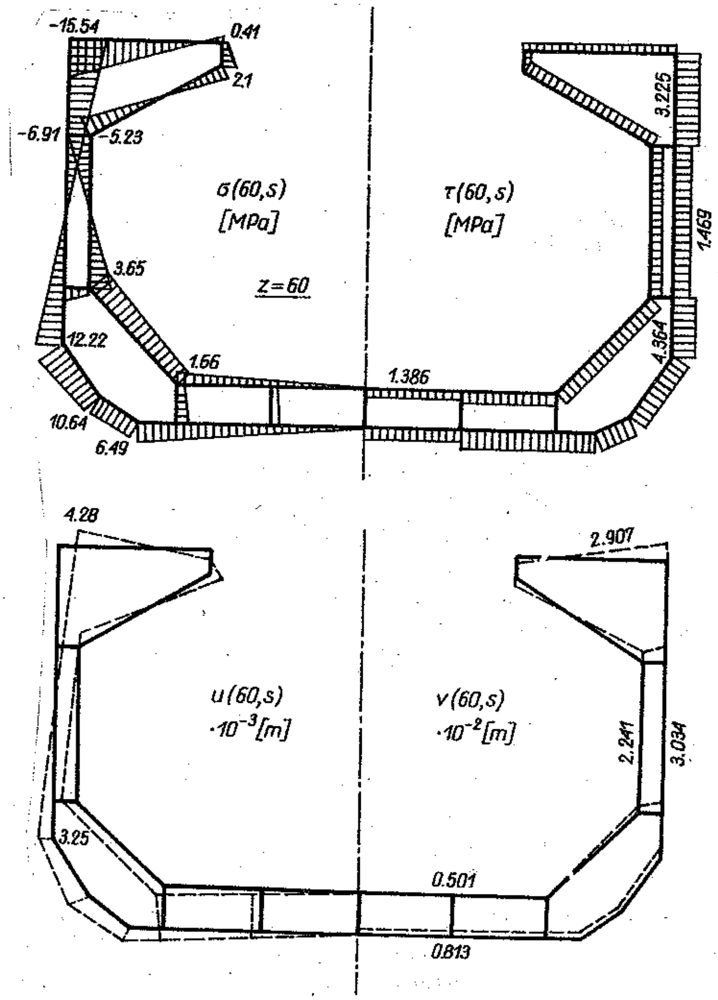

FIG. 8. a) normal stresses $\sigma(60, s)$ and tangent stresses $\tau(60, s)$ b) longitudinal displacements $u(60, s)$ and tangent displacements $v(60, s)$ calculated in $z = 60$ m for the bar from Fig. 6.

for the joint 1, and the displacements $v(z)$ and the stresses $\tau(z)$ for the inter-joint segment 1—3 are given for both constructions in Fig. 4 and in Fig. 7, all the quantities being the functions of the longitudinal coordinate z. Figures 5 and 8 illustrate the distributions of the displacements $u(z_0, s)$, $v(z_0, s)$ and stresses $\sigma(z_0, s)$, $\tau(z_0, s)$ in a chosen distance from the end of the bar (here $z_0 = 40$ meters for the construction from Fig. 5, and $z = 60$ meters for the construction from Fig. 8).

The computer calculation times for the constructions from Figs. 3 and 6 were 67 and 71 min, respectively (ICL-4-70). An overall estimation of stresses and displacement for the same constructions with accuracy less than that obtained with the aid of our program demands at least ~1000 finite elements. The computer time of such finite element calculations was ~5 hours. In order to obtain an accuracy comparable with that obtained by our method, one should use ~10000 finite elements. The computer time should be ~50 hours. The first practical applications of our program in ship designing completely support the opinion that for great constructions our program gives the distribution of stresses and displacements in the structures of the considered class, within computer times tens times shorter than the finite element method.

6. Possible extensions of the results

The authors are interested mainly in calculations concerning ship design and thus ship-building terminology appears several times in the above paper. However, the applicability of our model is of course not confined to ship-bulding. Every construction which may be approximated by a prismatic thin-walled bar with a cross-section consisting of arbitrarily arranged polygons may be investigated with the aid of the described program.

The assumption of a constant cross-section may be easily relaxed. For constructions with dimensions of the cross-section continuously changing along z we simply have to calculate the matrix coefficients of Eq. (3.4) in each point along z which appears in the integration procedure. It is the only change, and it is sufficient for including streamline constructions into the consideration.

Another possible extension is to supplement the present theory with the inclusion of prismatic bars with a cross-section constant on segments and changing in several points on the bar length (e.g. bulkhead ships). Both theoretical and numerical results concerning bars with a noncontinuous cross-section will be published soon.

References

1. В. З. Власов, *Тонкостиенные упругие стержни*, Госуд. Издат. Физ.-Мат. Лит., Москва 1959.
2. А. Ф. Феофанов, *Строительная механика авиационных конструкций*, изд. Машиностроение, Москва 1969.
3. Й. Ф. Образцов, *Методы рассчёта на прочность кесонных конструкций типа крыла*, Оборнгиз, 1960.
4. Ф. Г. Кандель, Ю. Раскин, Ю. Беляк, *Проблемы прочности судов смешанного плавления*, в: Проблемы прочности судов, В. Чувиковский, изд. Судостроение, Ленинград 1975.
5. Z. Górecki, *Zastosowanie półbezmomentowej teorii powłok w obliczeniach statycznych ortotropowych liniowo-sprężystych prętów pryzmatycznych o przekroju wieloobwodowym zamkniętym*, Mech. Teor. Stosow., **20**, 339, 1982.

6. Z. GÓRECKI, *Metoda automatycznego układania równań równowagi w obliczeniach statycznych ortotropowych liniowo-sprężystych, cienkościennych prętów pryzmatycznych o dowolnym przekroju wieloobwodowym*, Mech. Teor. Stos., 24, 519, 1986.
7. G. OLUREMI-OLAOFE, *On the Tchebyschev method of solution of ordinary differential equations*, J. Math. Anal. and Appl., **60**, 1, 1977.
8. J. H. ALBERGH, J. L. WALSH, E. N. NILSON, *Theory of splines and their applications*, Pergamon Press, 1967.

STRESZCZENIE

NOWA METODA OBLICZANIA NAPRĘŻEŃ I PRZEMIESZCZEŃ W PRĘTACH SPRĘŻYSTYCH

Opracowano nową metodę szybkiego numerycznego wyznaczania naprężeń i przemieszczeń w cienkościennych, liniowo-sprężystych prętach o przekrojach złożonych z dowolnych wielokątów. Algorytm obliczeń oparto na teorii powłok Własowa. Otrzymane równania różniczkowe rozwiązano numerycznie aproksymując niewiadome funkcje splajnami kubicznymi. Przedstawiono przykłady wyników liczbowych wykazujących możliwość praktycznego wykorzystania istniejącego programu w języku FORTRAN. Praca zawiera nowe sformułowanie warunków brzegowych.

РЕЗЮМЕ

НОВЫЙ МЕТОД РАСЧЕТА НАПРЯЖЕНИЙ И ПЕРЕМЕЩЕНИЙ В УПРУГИХ СТЕРЖНИЯХ

Разработан новый метод быстрого, численного расчёта напряжений и перемещений в тонкостенных, линейно упругих стержнях о поперечном сечении в виде произвольного многоугольника. Алгоритм расчёта исходит из теории оболочек Власова. Полученные дифференциальные уравнения были решены с использованием аппроксимации неизвестных функций сплайнами. Приведённые примеры численных расчётов доказывают практическую применность существующей программы, составленной на языке фортран. Работа содержит новую формулировку краевых условий.

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