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INTERACTION HYPERSURFACES APPLIED TO ESTIMATE THE LOAD-CARRYING CAPACITY OF SPACE TRUSSES

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When determining the load-carrying capacity of a space truss, one has assumed, usually that the result depends exclusively on properties of elements (struts) i. e. that the strengh of joints is sufficiently high. Since joints constitute up to 30 per cent of the total truss weight such an approach seems non-economical. The paper presents another approach to the joints design based on analysis of their strength by means of interaction hypersurfaces of forces acting upon them. First, the general concept is outlined and, then, possible simplifications are shown in the case of joint and load symmetries. The theoretical considerations are illustrated by a numerical example of a truss plate. Some general conclusions considering further development of the proposed approach are given.

1. INTRODUCTION

When anylysing space trusses, one usually assumes that the ultimate load of a given structure is determined by properties of its members exclusively, i. e. the strength of nodes is assumed as unlimited. To validate such an assumption, the nodes must be sufficiently strong.

Let us notice, however, that the „strength" of a truss node cannot be fully described by giving a single number or an interval of safe magnitudes of a single variable. Namely, since a node of a space truss is acted upon by a system of forces transmitted from members interconnected at that node (as well as from the loads applied directly to the truss), the stress state of the node and, thus, its safety or failure are decided by the full system of forces acting upon the node. Every particular set of the force magnitudes may be considered as a point in a certain space of appropriately many dimensions. The boundary between safe and unsafe force states, i. e. between corresponding domains in that space, is a certain hypersurface called interaction hypersurface.

Such a representation is analogous to that used in the plastic limit analysis, cf. [1], where such interaction hypersurfaces are defined for a point of continuum (yield condition), for a truss member [2] see Fig. 1a (here the hypersurface reduces to two ends of an interval), for a cross-section exposed to bending and shear (the interaction curve of Fig. 1b) etc.

Determination of the interaction hypersurface for a given node as well as the inversed problem: determine node dimensions so as to contain a pres-

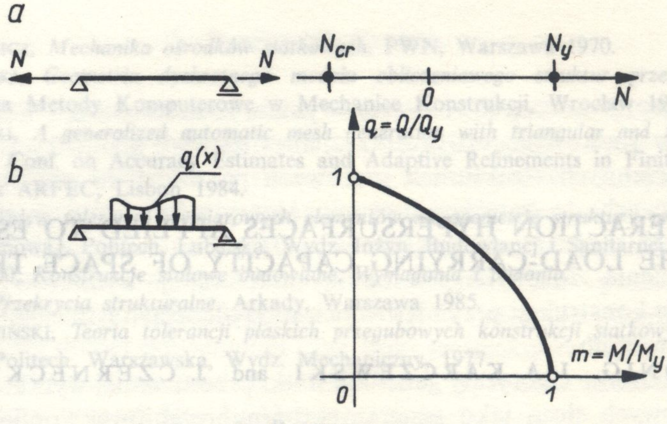


FIG. 1.

cribed force set within its interaction hypersurface are not easy tasks. However, when we know (approximately, at least) such hypersurfaces for a catalogue of nodes, then we can design the truss simply by selecting sufficiently „strong” nodes from the catalogue so as to ensure that all the force states which may happen at a given node are contained within its interaction hypersurface. This approach holds both when the axial forces in the truss members are determined by means of the classical linear (or nonlinear) elastic analysis as well as when buckling and yielding of the members are accounted for, cf. [2].

Obviously, such a procedure neglects the possibility of interaction between strut and node failures, i. e. neglects failure modes combining simultaneous failure of some nodes and members. However, in our opinion, it could be too risky to account for such and interaction.

2. GENERAL REMARKS ON CONSTRUCTING INTERACTION HYPERSURFACES FOR SPACE TRUSS NODES

First, let us notice that, contrary to the case of, e. g., a beam cross-section under bending and shear, the elements of a force set acting upon a given truss node are not independent of each other. Namely, they must satisfy three equlibrium equations. This means that, in fact, the interaction hypersurface is to be drawn out not in the space of all the forces acting upon a given node but in a subspace the dimensionality of which is lower by three.

A further space-dimensionality reduction may take place due to symmetries of the node and of the forces acting upon it, as for the node depicted in Fig. 2. If the forces acting upon this node were not symmetric, its interaction hypersurface should be constructed in a five-dimensional space of indpendent quantities ($5 = 8-3$). Symmetries reduce this number to three. Finally, the only

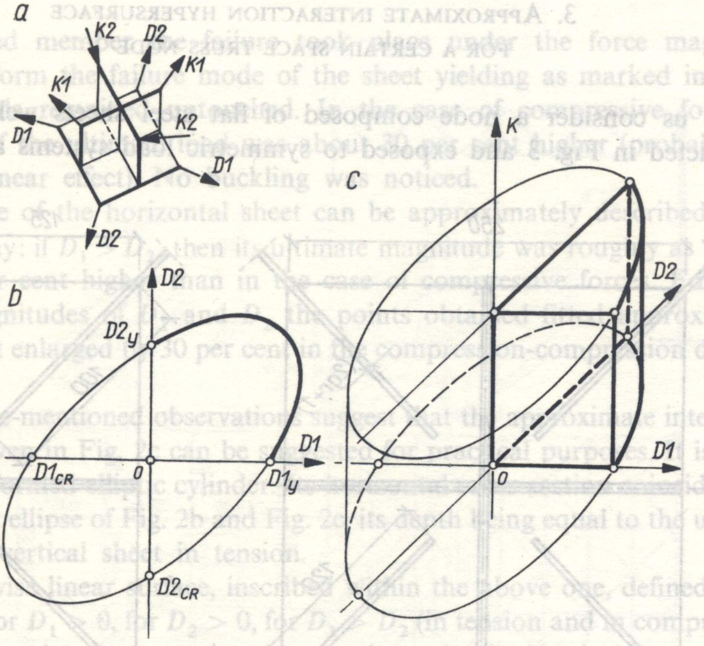


FIG. 2.

independent variables are: the forces D_1 and D_2 in the horizontal plane and the magnitude K of forces in the cross-bars (two of them are subjected to compression, the remaining two — to tension).

The interaction hypersurface for a given node can be constructed in various ways:

1. Experimentally, via determination of a certain critical state of the node for various ratios of the forces acting upon it. In this way, different points of the hypersurface will be determined. By completing sufficiently many tests, one obtains an approximation of the actual interaction hypersurface.

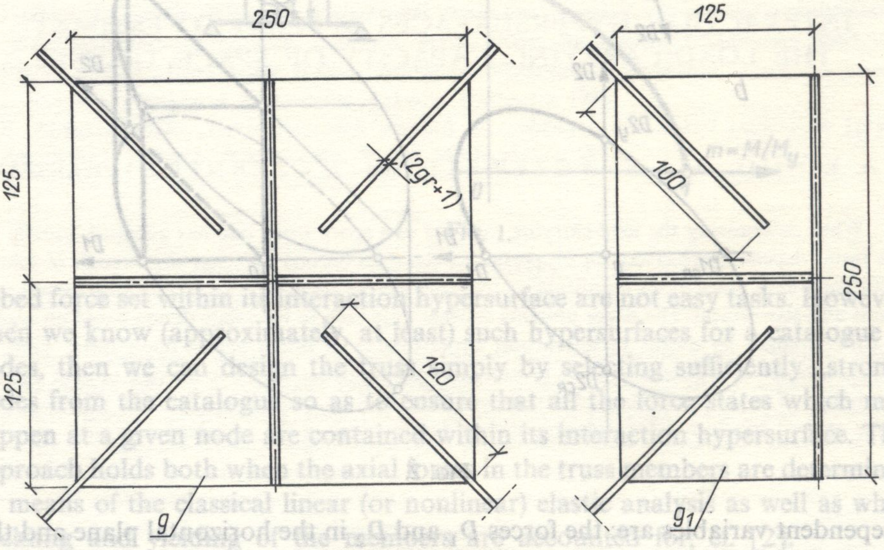
Naturally, the test results depend on the definition of the node critical state adopted. It may be first yielding, unconstrained plastic flow of the node or something intermediate as, e. g., a certain prescribed amount of local yielding.

2. Analytically or numerically by carrying out the same procedure for an appropriate computational model of the node.

Let us notice that if a node critical state is not preceded by any form of stability loss, then the interaction hypersurface defined by first yielding bounds a convex domain. The same holds also true for the interaction hypersurface defined by the unconstrained plastic flow of a node made of ductile material. Such a convexity allows to construct a safe approximation to the exact interaction hypersurface on the basis of a small number of points determined in an analytic or experimental way.

3. APPROXIMATE INTERACTION HYPERSURFACE FOR A CERTAIN SPACE TRUSS NODE

Let us consider a node composed of flat steel sheets welded together as depicted in Fig. 3 and exposed to symmetric load systems as in Fig. 2a.



Type of mode	g	g_1	Mark
I	6	4	○
II	12	5	□
III	20	5	■

gr- wall thickness of the bar

FIG. 3.

The loads are transmitted by appropriate end elements of the truss members welded to the node sheets. Such nodes were investigated experimentally, cf. [3, 4]. These investigations did show that:

a. Only a negligibly small (and diminishing with the load increase) bending of sheets was noticed.

b. Stress state of vertical sheets was, practically, independent of the state of the horizontal one. The small influence noticed was probably due to some imperfections.

c. The node failure due to failure of a single sheet can be approximately described in the following way: under positive (tension) force transmitted from the connected member the failure took place under the force magnitude determined form the failure mode of the sheet yielding as marked in Fig. 3 whereas welds remained untouched. In the case of compressive force the magnitude of the ultimate load was about 30 per cent higher (probably due to the nonlinear effect). No buckling was noticed.

d. Failure of the horizontal sheet can be approximately described in the following way: if $D_1 > D_2$, then its ultimate magnitude was roughly as in c) i.e. about 30 per cent higher than in the case of compressive forces. For intermediate magnitudes of D_1 and D_2 the points obtained fitted approximately an ellipse but enlarged by 30 per cent in the compression-compression domain, Fig. 2b.

The above-mentioned observations suggest that the approximate interaction surface as given in Fig. 2c can be suggested for practical purposes. It is of the form of a deformed elliptic cylinder. Its horizontal cross-section coincides with the deformed ellipse of Fig. 2b and Fig. 2c, its depth being equal to the ultimate load of the vertical sheet in tension.

A piece-wise linear surface, inscribed within the above one, defined by six test results: for $D_1 > 0$, for $D_2 > 0$, for $D_1 > D_2$ (in tension and in compression) provides a good safe approximation to the exact interaction hypersurface.

4. EMPLOYING INTERACTION HYPERSURFACES TO THE DESIGN OF A SPACE TRUSS

The truss roof of Fig. 4 was designed in the following way. First forces in its members were determined by means of the programme JC12 (for elastic analysis) from the SPAK system (for elastic-plastic analysis of space trusses) developed at the Warsaw Technical University, [5]. Magnitudes of those forces, for the case of a uniformly distributed load, are given in Fig. 4.

The nodes of Fig. 3 were used. The interaction hypersurfaces for the three types of nodes (i.e. in the design for various thicknesses of the steel sheets) of Fig. 3 are depicted in Fig. 5. Since the nodes of the designed truss (with exception of the central node) were loaded asymmetrically, a certain procedure was applied allowing to translate, approximately, an asymmetric nodal load system into a comparable symmetric one and, then, the requirement that forces acting upon a given node do not exceed its interaction surface was applied. The procedure consisted of adding projections of horizontal components of cross-bar forces to the forces within the horizontal sheet. It was also applied for the boundary and corner nodes.

The resulting distribution of nodes within the truss is presented in Fig. 6.

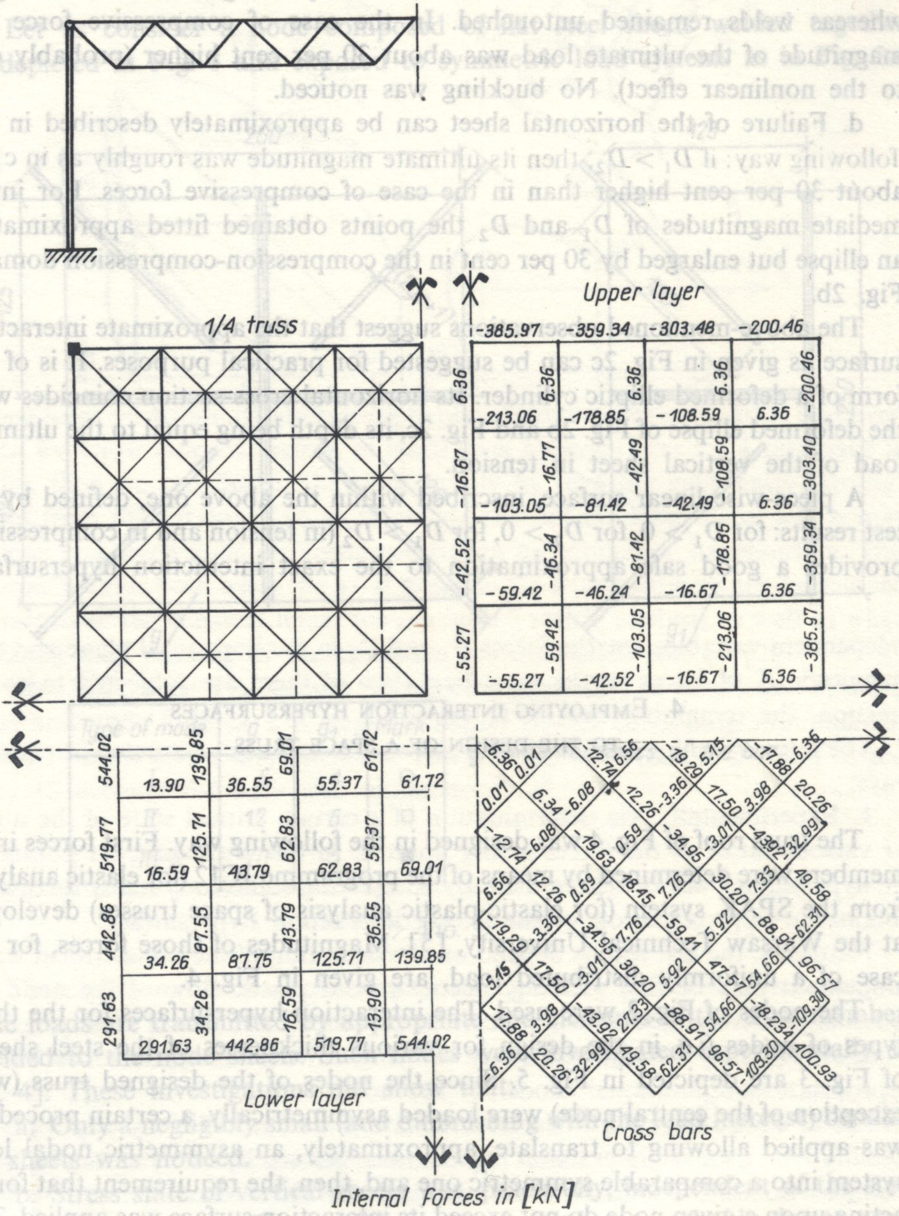
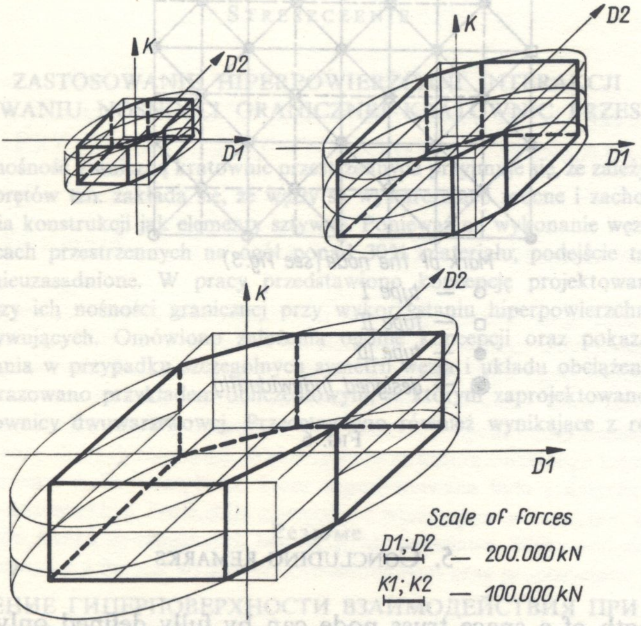

1/4 truss
Upper layer
Lower layer
Cross bars
Internal forces in [kN]

FIG. 4.



Type of node	D1	D2	D1, D2 for +D	for −D	K
I	180.00	234.00	200.19	260.45	100.00
II	360.00	468.00	400.38	520.49	125.00
III	600.00	780.00	667.28	867.46	125.00

FIG. 5.

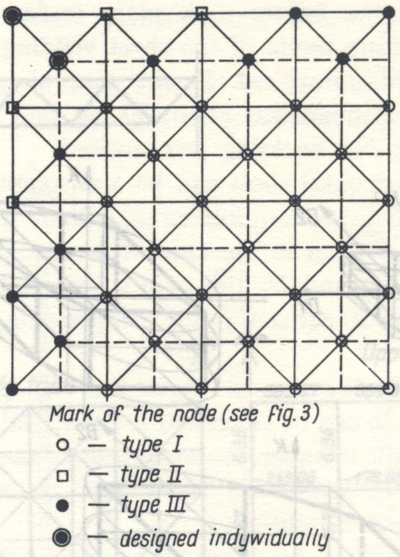


FIG. 6.

5. CONCLUDING REMARKS

1. Strength of a space truss node can by fully defined only by means of the interaction hypersurface notion as given in Sect. 1.

2. Effective determination (analytically or numerically) of the interaction hypersurface is a difficult task due to the lack of ready-to-use solutions of partial problems such as determination of the load-carrying capacity of a sheet loaded as in Sect. 3. Therefore, one should try to obtain some reasonable approximations, as shown in the example of Sect. 3.

3. The interaction hypersurface may be determined experimentally or computationally. Experimental investigations are costly and their results are of a limited range of applicability. Therefore, there is a need for possibly general computational methods.

4. It seems promising to combine some experimental observations with the analytic and numerical approaches as shown in the presented examples.

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STRESZCZENIE

ZASTOSOWANIE HIPERPOWIERZCHNI INTERAKCJI PRZY SZACOWANIU NOŚNOŚCI GRANICZNEJ KRATOWNIC PRZESTRZENNYCH

Obliczając nośność graniczną kratownic przestrzennych przyjmuje się, że zależy ona wyłącznie od podatności prętów tzn. zakłada się, że węzły są wystarczająco mocne i zachowują się aż do chwili zniszczenia konstrukcji jak elementy sztywne. Ponieważ na wykonanie węzłów przeznacza się w kratownicach przestrzennych na ogół ponad 30% materiału, podejście takie wydaje się ekonomicznie nieuzasadnione. W pracy przedstawiono koncepcję projektowania węzłów na podstawie analizy ich nośności granicznej przy wykorzystaniu hiperpowierzchni interakcji sił na nie oddziaływujących. Omówiono założenia ogólne koncepcji oraz pokazano możliwość uproszczeń zadania w przypadku szczególnych symetrii węzła i układu obciążenia. Rozważania teoretyczne zobrazowano przykładem obliczeniowym, w którym zaprojektowano węzły w bezsłupkowej kratownicy dwuwarstwowej. Przedstawiono również wynikające z rozważań uwagi i wnioski.

Резюме

ПРИМЕНЕНИЕ ГИПЕРПОВЕРХНОСТИ ВЗАИМОДЕЙСТВИЯ ПРИ ОЦЕНКЕ ПРЕДЕЛЬНОЙ НЕСУЩЕЙ СПОСОБНОСТИ ПРОСТРАНСТВЕННЫХ ФЕРМ

Вычисляя предельную несущую способность пространственных ферм, принимается, что зависит она исключительно от податливости стержней, т.зн. предполагается, что узлы достаточно крепкие и ведутся, вплоть к моменту разрушения конструкции, как жесткие элементы. Т.к. на изготовление узлов предназначается в пространственных фермах в общем свыше 30% материала, такой подход кажется экономически необоснованным. В работе представлена концепция проектирования узлов на основе анализа их предельной несущей способности, при использовании гиперповерхности взаимодействия сил воздействующих на узлы. Обсуждены общие предположения концепции и показана возможность упрощения задачи в частных случаях симметрии узла и системы нагружения. Теоретические рассуждения иллюстрированы расчетным примером, в котором запроектированы узлы в безстолбовой двухслоистой ферме. Представлены тоже замечания и следствия, вытекающие из рассуждений.

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