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THIN AEROFOIL WITH MULTIPLE SLOTTED FLAP

T. S. PATEL, T. A. DERBASI and A. N. HASSANEN (TRIPOLI)

The paper deals with the problem of thin aerofoil with multiple slotted movable hinge flaps. Two particular cases are considered: a) assuming no gaps exist at the hinges, and b) considering the existence of slots. The conventional assumptions of small angle of attack, small flap deflection and small camber are omitted.

1. Introduction

The classical thin aerofoil theory can be applied to a two-dimensional wing with a small thickness ratio, small camber under small angle of attack. In many practical cases these restrictions cannot be justified. They simplify the problem. However, the assumptions of small angle of attack and small camber are not valid for practial cases like take off, landing etc. There are numerous works available on the application of the thin aerofoil theory, for example, the aerofoil with rotating flap [1], thin aerofoil with ground effect [2], thin flapped aerofoil [3], etc.

2. Thin aerofoil with movable hinge multiple flaps (no gaps)

2.1. *Formulation of the problem*

The analysis of thin symmetric aerofoil with plain flap [3] has been extended here for the case of multiple flaps with movable hinges. The increase in chord due to movable hinges has been taken into account. The aerofoil with flap is represented by a resulting cambered thin aerofoil with distribution of vortices along its meanline as in the general case of thin aerofoil:

$$K(x) = 2V_0\left[A_0\left(\frac{l+\cos\theta}{\sin\theta}\right)+\sum_{n=1}^{\infty} A_n\sin n\theta\right], \tag{2.1}$$

where $x = (c/2)(l-\cos\theta)$; $0 \leqslant x \leqslant c$ and $0 \leqslant \theta \leqslant \pi$, c resultant chord when flaps are extended.

Let the initial chord aerofoil without deflection of flaps be c_i and the chordwise lengths of the main part of aerofoil and flaps be $c_0, c_1, c_2, \ldots, c_r$ where r is the number of multiple flaps (see Fig. 1). Let the angle of attack of the main part be α and the flap deflections be $\eta_1, \eta_2, \ldots, \eta_r$ (positive downward). The theoretical representation of this aerofoil with $(r+1)$ practically linear segments is shown in Fig. 1.

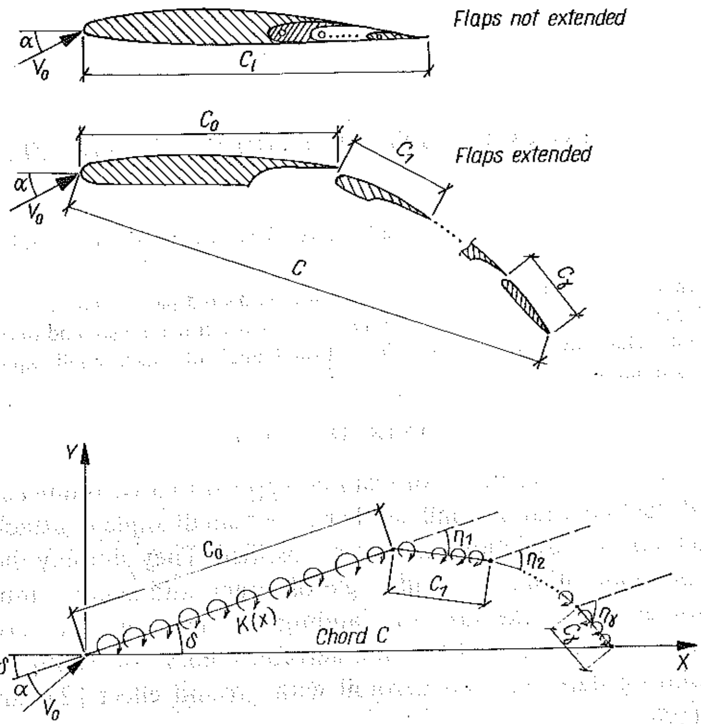


FIG. 1. Thin flapped aerofoil and its simplified vortex model. Aerofoil model consisting of $(r+1)$ linear segments.

The boundary condition over the aerofoil may be satisfied on the meanline itself as in the thin aerofoil theory. It may then be given by

$$\left(\frac{dy}{dx}\right) = \frac{V_0 \sin(\alpha+\delta)+v}{V_0 \cos(\alpha+\delta)+u}, \tag{2.2}$$

where (dy/dx) is the slope of the meanline, u and v are the induced velocities along X and Y axes, respectively.

Simplification of this condition leads to the following general solution [3] for the unknown coefficients A_0 and A_n:

$$A_0 = (\alpha+\delta) - (l/\pi)\int_0^{\pi} (dy/dx)\cdot d\theta \tag{2.3}$$

and

$$A_n = (2/\pi)\int_0^{\pi} (dy/dx)\cos n\theta \cdot d\theta \tag{2.4}$$

with δ as defined in Fig.1.

The solution of the problem then consists in evaluating A_0 and A_n using Eqs. (2.3) and (2.4) for the given geometry of the aerofoil and flap deflections. The necessary geometrical details are worked out in the following paragraph.

2.2. Geometrical details

The height h of the first hinge from the chord can be expressed by two different equations involving the unknown angle

$$c_0 \cdot \sin\delta = h \tag{2.5}$$

and

$$\sum_{p=1}^{r} c_p \cdot \sin(\eta_p - \delta) = h. \tag{2.6}$$

Equations (2.5) and (2.6) can be solved by iterations to find the value of δ. Then the actual chord after flap deflection becomes

$$c = c_0 \cdot \cos\delta + \sum_{p=1}^{r} c_p \cdot \cos(\eta_p - \delta). \tag{2.7}$$

Let the position of the hinge of the p-th flap be given by $x_p = (c/2)(1 - \Phi_p)$, then Φ_p may be expressed as below:

$$\Phi_p = \cos^{-1}\left[1 - \sum_{s=1}^{p} (c_{s-1}\cos\eta_{s-1} - \delta)/(c/2)\right]. \tag{2.8}$$

If the slope of the meanline segment for the main part is $(dy/dx)_0$ and that for the p-th flap is $(dy/dx)_p$, then their values will be

$$(dy/dx)_0 = \tan\delta \text{ and } (dy/dx)_p = -\tan(\eta_p - \delta). \tag{2.9}$$

2.3. Solution of the problem

Equations (2.3) and (2.4) for unknown A_0 and A_n can be reduced to the following form with the help of Eq. (2.9):

$$A_0 = (\alpha + \delta) - (1/\pi)\left[\Phi_1 \tan\delta - \sum_{p=1}^{r-1} (\Phi_{p+1} - \Phi_p)\tan(\eta_p - \delta) - -(\pi - \Phi_r)\tan(\eta_r - \delta)\right] \tag{2.10}$$

and

$$A_n = (2/\pi)\left[\tan\delta(\sin n\Phi_1)/n + +\tan(\eta_r - \delta)(\sin n\Phi_r)/n - \sum_{p=1}^{r-1} \tan(\eta_p - \delta)(\sin n\Phi_{p+1} - \sin n\Phi_p)/n\right], \tag{2.11}$$

where δ an Φ could be evaluated with the help of Eqs. (2.5) to (2.8).

The aerodynamic characteristics of the aerofoil with flaps may be worked out as shown in the next paragraph.

2.4. Aerodynamic characteristics

The coefficients of lift C_1 and the pitching moment C_{mLE} (nose up positive) can be obtained using Joukovsky's theorem. The final results are as below:

(2.12)
$$C_1 = 2\pi(A_0 + A_1/2)\cdot(c/c_1)$$

and

(2.13)
$$C_{mLL} = -(\pi/2)(A_0 + A_1 - A_2/2)\cdot(c/c_i),$$

where A_0, A_1, A_2 and c can be evaluated from Eqs. (2.10), (2.11) and (2.7).

The hinge moment coefficient for the q-th hinge could be reduced to the following form:

(2.14)
$$C_{Hq} = -\left(A_0 I_{0q} + \sum_{n=1}^{\infty} A_n I_{nq}\right) \Big/ \left(\sum_{p=r}^{q} c_p\right)^2,$$

where

$$I_{0q} = (\pi - \Phi_q)(2\cos\Phi_q - l)/2 + \sin\Phi_q - (\sin 2\Phi_q)/4,$$

$$I_{nq} = \frac{\cos\Phi_q}{2}\left[\frac{\sin(n+l)\Phi_q}{n+l} - \frac{\sin(n-l)\Phi_q}{n-l}\right] - \left[\frac{\sin(n+2)\Phi_q}{(n+2)} - \frac{\sin(n-2)\Phi_q}{(n-2)}\right]\Big/4,$$

$$C_{Hq} = H_q/(\tfrac{1}{2}\varrho v_0^2 c_q^2),$$

H_q hinge moment about the q-th hinge.

2.5. *Numerical illustration*

The problem of symmetrical aerofoil with double flap and movable hinges has been considered as a numerical example using the following data:

$$c_i = 1.00, \quad c_0 = 0.80, \quad c_l = 0.25, \quad c_2 = 0.15$$

and the various combinations of

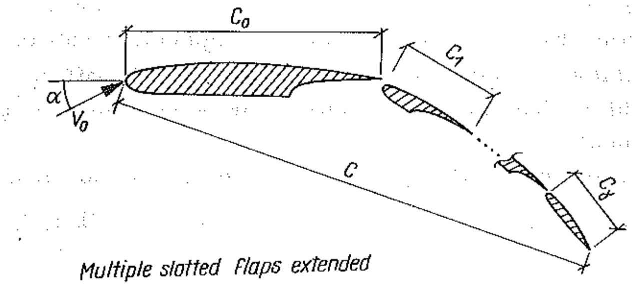
$$\alpha = 0.00°, \quad 5.73°, \quad 11.45°, \quad 17.20°,$$
$$\eta_1 = 0°, \quad 10°, \quad 20°, \quad 30° \quad \text{and} \quad \eta_2 = 0°, \quad 15°, \quad 30°, \quad 45°.$$

The computer program has been developed in FORTRAN-IV and the problem has been solved with the help of the computer centre at Al-Fateh University, Tripoli. The results are presented in the form of curves in Figs. 3, 5 and 6.

3. THIN AEROFOIL WITH MOVABLE MULTI-SLOTTED FLAP

3.1. *Formulation of the problem*

The multiple slotted flapped aerofoil has been analysed here with the help of the thin aerofoil theory. Each of the main part and multiple flaps is treated as an independent thin aerofoil having mutual interaction on each other. A typical aerofoil model with such flaps has been shown in Fig. 2.



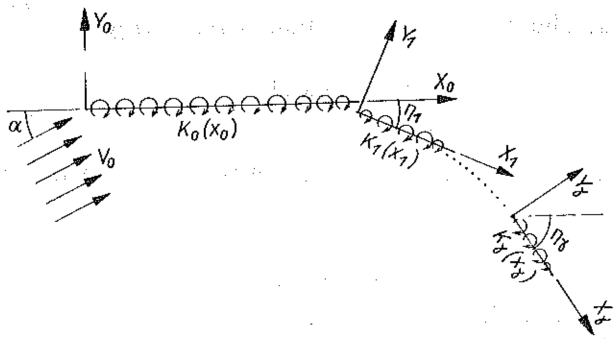


FIG. 2. Vortex model of thin aerofoil with multiple slotted movable hinge flaps.

According to the thin aerofoil theory, each part is represented by the circulation distribution over its meanline. Let K_0, K_1, $K_2, \ldots$, $K_p, \ldots$, K_r be the circulation distributions for the respective part, each satisfying the trailing edge condition. They may represented by

$$(3.1) \qquad K_p = 2V_0\left[A_{0p}(1+\cos\theta_p)/(\sin\theta_p)+\sum_{n=1}^{\infty} A_{np}\sin n\theta_p\right],$$

where the chordwise position on each segment $x_p = (c_p/2)(1-\cos\theta_p)$; $0 \leqslant x_p \leqslant c_p$ and $0 \leqslant \theta_p \leqslant \pi$; $p = 0, 1, 2, \ldots, r$ for a respective segment the aerofoil.

The boundary condition to be satisfied on the meanline of each segment the aerofoil may be given as follows:

$$(3.2) \qquad V_0[(dy/dx)_q - \sin(\alpha+\eta_p)]_{\theta_{eq}} = \sum_{p=0}^{r} v_{pq},$$

where $(dy/dx)_q$ is a slope of the meanline for the q-th segment θ_{eq} is the position on the meanline of the q-th segment where the boundary condition is satisfied at e point and v_{pq} is the normal component (to chord) on the q-th segment induced by the circulation distribution K_p of the p-th segment.

The conventional assumptions of a small angle of attack and small flap deflection are omitted in Eq. (3.2).

The problem lies in finding the values of unknown coefficients A_{0p} and A_{np} in the circulation distribution K_p of the p-th segment. It can be numerically solved by retaining N number of terms in each of the distributions K_p. The boundary condition then must be satisfied at the same number of points θ_{eq} on every q-th segment.

As suggested by MULTHOPP [5], the value of θ_{eq} may be chosen as follows:

(3.3) $$\theta_{eq} = (\pi \cdot e)/(2e+1), \quad e = 1, 2, ..., N \quad \text{and} \quad q = 0, 1, 2, ... r.$$

3.2. *Induced velocity components v_{qp}*

The induced velocity components may be obtained with the help of Biot–Savart's law. The final results are summarized below

(3.4) $$v_{qq} = v_0(-A_{0q} + \sum_{\eta=1}^{\infty} A_{nq} \cos n\theta_{eq}),$$

(3.5) $$v_{pq} = -\int_0^{c_p} K_p(Q_{pq} \sin\theta_{eq} + R_{pq} \cos\theta_{eq})(dx_p)/2\pi(Q_{pq}^2 + R_{pq}^2),$$

where

$$Q_{pq} = (x_q - x_p) + (c_q/2)(1 - \cos\theta_{eq})\cos(\eta_q - \eta_p) - $$
$$- (c_q/2)(1 - \cos\theta_p),$$
$$R_{pq} = (y_q - y_p) + (c_q/2)(1 - \cos\theta_{eq})\sin\theta_{eq},$$

v_{qq} is the induced velocity at any point θ_{eq} on the q-th segment due to circulation distribution K_q and v_{pq} is the induced velocity at any point θ_{eq} on the q-th segment due to circulation distribution K_p.

The integrals involved in Eq. (3.5) are evaluated numerically using Simpson's rule.

3.3. *Solution of the problem*

The boundary condition (3.2) become a set of linear $(r+1) \times N$ number of equations by satisfying it at $(r+1) \times N$ points given by Eq. (3.3) retaining only N terms with N unkown values of coefficients in K_p (see Eq. (3.1)). Thus, there will be $(r+1) \times N$ unknowns with $(r+1) \times N$ equations which can be solved by either matrix inversion or by the iteration method. The aerodynamic coefficients can be obtained with the help of Joukovsky's theorem. The final relations are as below:

$$C_l = \sum_{p=0}^{r} C_{lp},$$

$$C_{lp} = 2\pi(A_{op} + A_{lp}/2)(c_p/c), \qquad C_{mLE} = \sum_{p=0}^{r} C_{mLEp},$$

$$C_{mLEp} = -(\pi/2)(A_{0p} + A_{lp} - A_{2p}/2)(c_p/c)^2,$$

$$C_{Hq} = \sum_{p=q}^{r} C_{mLEp} + \sum_{p=q}^{r} c_{lp}(x_p - x_q)\left(c/\sum_{p=q}^{r} c_p\right),$$

x_p and x_q are the distances of hinges of p-th and g-th segments of the aerofoil from its leading edges.

3.4. *Numerical illustration*

The numerical problem of the aerofoil with double slotted movable hinge flaps has been worked out using data as follows:

$c_0 = 0.80$, $c_1 = 0.25$, $c_2 = 0.15$ and combinations of
$\alpha = 0.00°, 5.73°, 11.45°, 17.20°$,
$\eta_1 = 0°, 10°, 20°, 30°$ and $\eta_2 = 0°, 15°, 30°, 45°$.

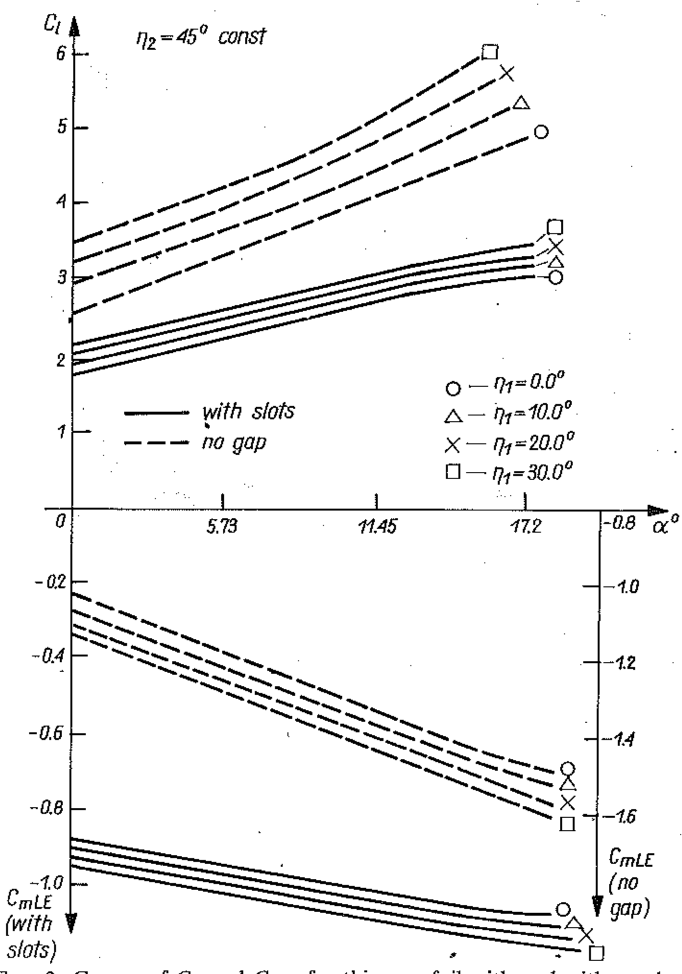


FIG. 3. Curves of C_1 and C_{mLE} for thin aerofoil with and without slots.

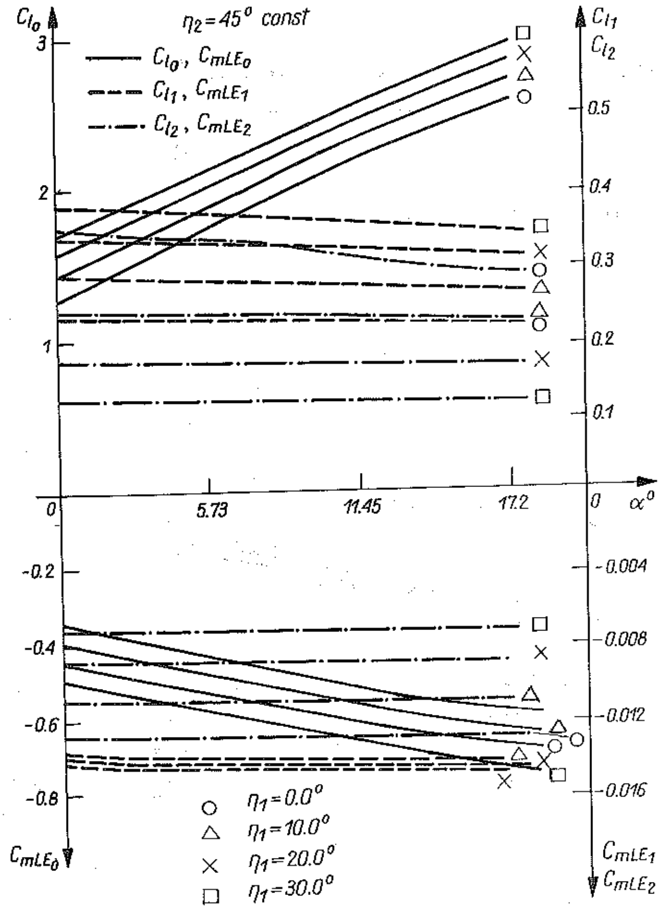


FIG. 4. Curves of components of lift and pitching moment coefficients for double slotted flaps.

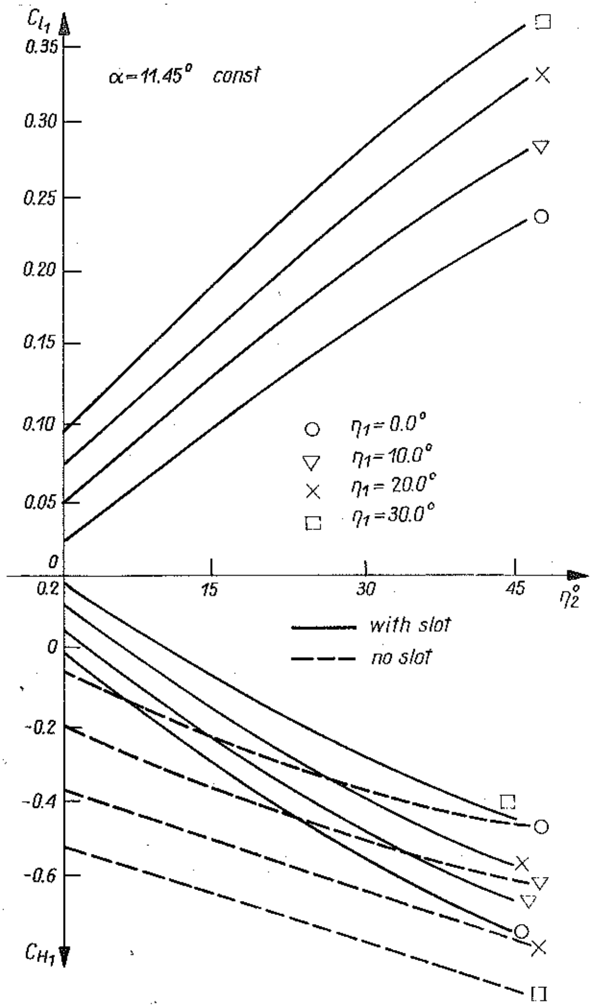


FIG. 5. Curves of C_{l1} and C_{H1} against η_2.

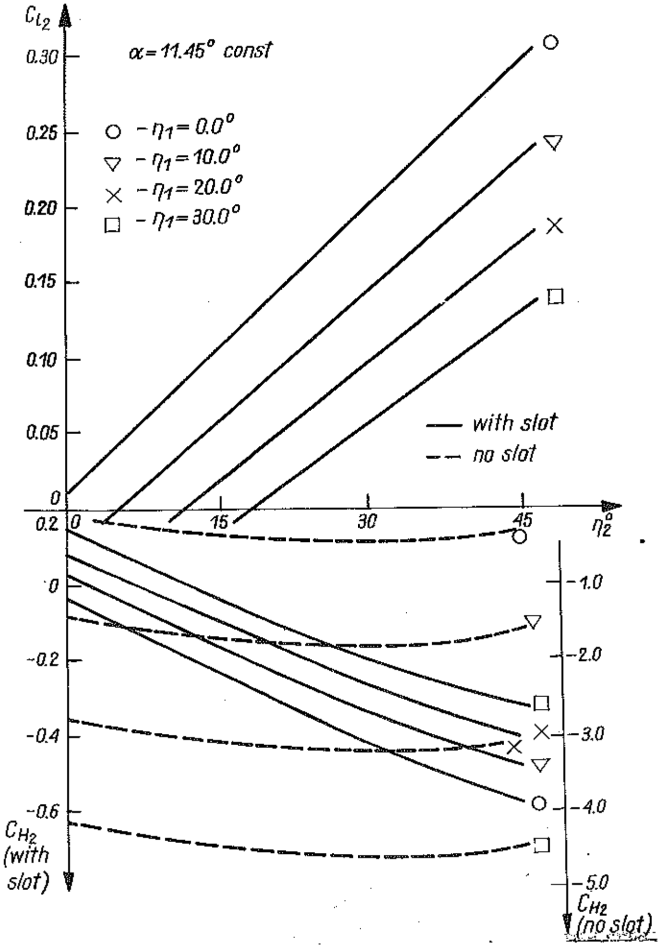


FIG. 6. Curves of C_{l2} against η_2 and C_{H2} against η_2.

The positions of the first hinge (x_1, y_1) and second hinge (x_2, y_2) are as follows:

$$x_1 = 0.80, \qquad y_1 = 0.02, \qquad x_2 = 0.80 + 0.25\cos\Phi_1,$$
$$y_1 = 0.035 + 0.25\sin\Phi_1.$$

The results of computation are presented in Figs 3, 4, 5 and 6 together with the possible results for the same problem attempted by the simplified theory without slots.

4. CONCLUDING REMARKS

Figure 3 shows that the values of C_1 and $-C_{mLE}$ for the slotted aerofoil increase with the angle of attack and flap deflection η_1 for constant flap deflection η_2. It is found that the trend is the same with variation in flap deflection η_2 also. The rate of such increment is more for the simplified model without slots than that for the slotted aerofoil. The increment rate is decreasing with an increase in α which indicates that for the slotted flapped problem the adverse effect of the front segments on the rear segments grows with an increase in α.

Figure 4 shows that the value of C_{l0} and the absolute value of C_{mLE0} increases with a rise in α and also with a rise in flap deflections η_1 when η_2 = const. This agrees with the thin aerofoil theory. The value of C_{l1} and absolute value of C_{mLE1} rise with an increase in α but decrease with an increase in η_1 when η_2 = const. The increase in the above quantities with α is slower than that for C_{l0} and C_{mLE0}. The effect of η_1 on them is opposite to that on C_{l0} and C_{mLE0}. This may be due to the adverse effect of comparatively powerful circulation over the nearby main part ahead and above the first segment. The value of C_{l2} and the absolute value of C_{mLE2} decreases with an increase in α and also with a rise in η_1 when η_2 = const. This seems to be a trend opposite to the expected one. This may be due to the combined negative induced effect of the main part and the first segment which are ahead and above the second flap.

Figure 5 represents variations in C_{l1} and C_{H1} with flap deflection η_2, with flap deflection η_1 as a parameter. The magnitudes of both of these coefficients increase with η_2 and η_1. It may be noted that at higher values of η_2, the circulation distribution over the main part becomes more powerful than that of the second flap and hence the increase in these coefficients with η_2 becomes slower. The values of C_{HL} are higher for the simplified model with no slots than for the model of slotted flapped aerofoil.

Figure 6 indicates the curves for the coefficients C_{l2} and C_{H2} against flap deflection η_2 with flap deflection η_1 as a parameter. Their values increase with a rise in η_2 but they decrease with an increase in η_1 in magnitude. This may be due to the combined adverse effect of the nearby first flap and the main part which are ahead and above the second flap. The values of hinge moments with the slotted flapped model are higher than the simplified aerofoil model without slots. This difference may be attributed to the slots which allow a flow of air from a high pressure region to a low pressure region across the slot and reduces the intensity of circulation accordingly.

The curves are not linear in most of the cases as the linearising assumption of small angles of attack, small flap deflections and small resulting camber are avoided in the present work.

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STRESZCZENIE

CIENKI PŁAT Z WIELOSZCZELINOWĄ KLAPĄ

Rozważono zagadnienie cienkiego płata zaopatrzonego w wieloszczelinowe ruchome klapy osadzone na zawiasach. Zbadano dwa przypadki szczególne: a) przy założeniu, że nie ma szczelin przy zawiasach i b) przy założeniu, że szczeliny takie istnieją. Pominięto stosowane zazwyczaj założenia dotyczące małego kąta natarcia i małego ugięcia klapy.

Резюме

ТОНКОЕ КРЫЛО С МНОГОРАЗРЕЗНЫМ ЗАКРЫЛКОМ

Рассмотрена задача тонкого крыла, снабженного в многоразрезные подвижные закрылки, осажденные на петлях. Исследованы два частных случая: а) при предположении, что нет разрезов при петлях и б) при предположении, что такие разрезы существуют. Пренебрегается применяемыми обычно предположениями, касающимися малого угла атаки и малого прогиба закрылка.

AL-FATEH UNIVERSITY, TRIPOLI, LIBYA.

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