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ACTIVE STRATEGY OF AVOIDING RESONANCE

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The problem of active control of natural frequencies by changes of local stiffness in structural elements in order to avoid resonance (due to variable frequency of excitation) is considered. The optimal strategy of play with switchable control parameters in the case of a simple discrete structure and continuous beam cantilever is discussed. It is concluded that even one structural element with a controllable stiffness coefficient gives us an efficient (in most cases) device to avoid resonance frequencies in the structure under variable excitation.

Notation

- A cross sectional area of beam,
- E modulus of elasticity,
- J modulus of inertia,
- k stiffness coefficient,
- k_d, k_u limit values of k,
- m mass of material point,
- M magnification factor,
- $W(x)$, u local deflections,
- $p(x, t)$ dynamical load,
- $P(x)$, F_0 amplitude of excitation,
- δ static deflection,
- ω frequency of forced vibration,
- ω_n frequency of free vibration,
- γ_0 density of material,
- $\gamma = A\gamma_0$,
- $C^2 = EJ/\gamma$,
- $D = kl^3/EJ$,
- $\lambda^2 = \omega/C$,
- $\beta = \lambda l$.

1. Introduction

Problems of active damping of vibration in structural design appeared in the literature in the seventies [11, 12, 15, 16, 17]. However, one can find some earlier formulations (e.g., [18]). Particularly, the following important subjects in civil engineering were considered:

a) vibration in tall structures subjected to random pressure due to turbulent wind,

b) structural vibration caused by movable load,

c) resonance forced by supported engines.

The problem of active damping in design of large space structures is also very important. Because of the requirement for low weight, such structures will lack the stiffness and damping necessary for the passive control of vibration. Therefore applying the active control idea appears to be very hopeful (e.g. [1, 10]).

Two approaches can be distinguished in the optimal design of actively damped structures. The first one (e.g., [6, 14]) starts from an initial design satisfying all sets of constraints and then an optimal control system improving the dynamic response of the structure is invented. In the second approach (e.g., [2, 3, 5, 7, 8, 13]) a simultaneous integrated design of the structure and vibration control system is achieved by improving the configuration as well as the control system. Both formulations, however, make use of some undefined external force-sources which realize the system of actuators. The idea of vibration control presented in this paper describes a closed, self-controlled system without external force-sources. It can be of some use in space structures under forced vibration but, particularly, it can be applied in civil structures.

Assuming that the geometry of the structure is fixed and only slow changes of excitation frequency are allowed, the response of the structure (following load changes) and the optimal strategy of actively avoiding resonance will be considered. Avoidance of resonance will be possible by changing natural frequencies due to the stiffness control of an element of the structure. The idea will be presented on the basis of a two-degree-of-freedom problem (the second section), on the basis of an example of beam with controllable support stiffness (the third section), and then some generalization will be made (the fourth section).

First let us discuss the problem taking the simple, one-dimensional example of the spring-mass system (shown in Fig. 1) loaded by oscillating force

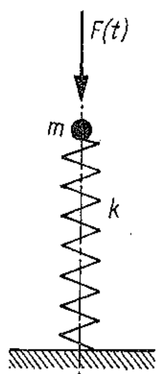

FIG. 1. One-degree-of-freedom spring-mass system.

$F(t) = F_0 \cos\omega t$. The equation of the forced vibration motion takes the well-known form:

$$m\ddot{u} + ku = F(t), \qquad \ddot{u} = d^2u/dt^2 \tag{1.1}$$

where the parameter k describes the stiffness coefficient of the structure supporting the mass m. The general solution of Eq. (1.1) is given by

$$u = C_1 \cos\alpha t + C_2 \sin\alpha t + F_0 \cos\omega t/k[1-(\omega/\alpha)^2], \tag{1.2}$$

where $\alpha^2 = k/m$, C_1, C_2 — the constant coefficients dependent on the initial conditions.

The amplitude of motion of the particular solution representing the steady-state forced vibration can be expressed in nondimensional form through the so-called magnification factor:

$$M = |1/[1-(\omega/\alpha)^2]|. \tag{1.3}$$

In engineering practice analysis of the case of variable frequency of external load $\omega = \omega(t)$ is particularly important because of the danger of resonance (when ω approaches α — cf. (1.3)). In this situation, however, the idea of active control of macrostiffness characteristic k (subject to constraints: $0 < k_d \leqslant k \leqslant k_u$) of the supporting structure can be very helpful. Assuming quasi-static changes of both: the forced frequency $\omega(t)$ and the coefficient $k = k(t)$ (which follows variations of ω), the above steady-state description can be used as the first approximation of the problem. Therefore the problem of active damping of vibration amplitude can be formulated as follows. For each forced frequency ω define such stiffness coefficient k ($k_d \leqslant k \leqslant k_u$) that the magnification factor M is minimum:

$$M = |1/[1-(\omega/\alpha)^2]| = \min. \tag{1.4}$$

Substituting the relation $\alpha^2 = k/m$ to Eq. (1.4), the following optimal solution of the above problem can be calculated (cf. Fig. 2, 3a):

$$k = \begin{cases} k_u & \text{for} \quad \omega < \omega_0, \\ k_d & \text{for} \quad \omega \geqslant \omega_0, \end{cases} \tag{1.5}$$

where

$$\omega_0 = \sqrt{2k_d k_u/m(k_d+k_u)}.$$

The solution (1.5) guarantees that the magnification factor $M \leqslant M_0$, where the value M_0

$$M_0 = (k_d+k_u)/(k_u-k_d) \tag{1.6}$$

is reached for $\omega = \omega_0$.

The step function (1.5) describing the optimal active control of the

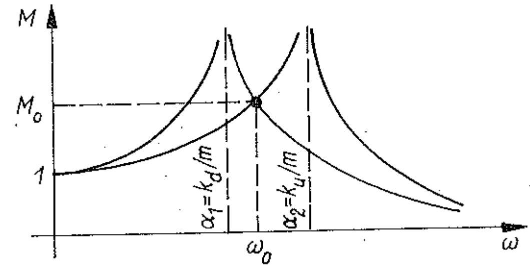

FIG. 2. Relation between magnification factor and frequency of excitation.

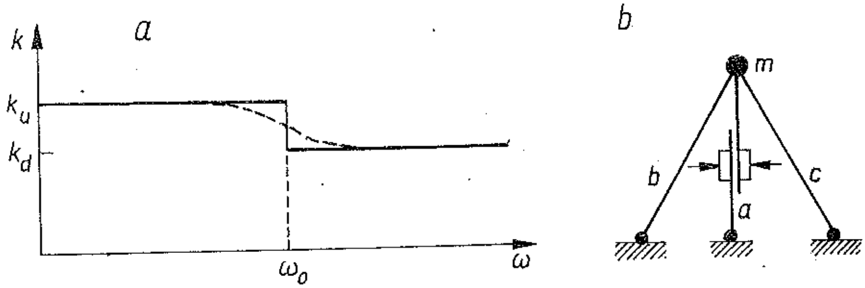

FIG. 3. Optimal control of stiffness coefficient in the one-dimensional case.

macrostiffness characteristic k can be realized, for example, by using a switchable element in the supporting structure (Fig. 3b). The discussed structure should be designed in such a way that the stiffness coefficient k_d is related to the substructure composed of elements b and c while the coefficient k_u is related to the whole structure composed of all elements a, b, c. Also the control box should switch off the element a when the forced frequency is bigger than ω_0 and switch it on when $\omega < \omega_0$. The effect of the switch-damper designed above depends on the difference $k_u - k_d$ (cf. (1.6)). Therefore the switchable element a should be as stiff as possible.

Notice that in real application, switching of the element a should be done more smoothly (Fig. 3a) in order to minimize the impact due to the sudden change of macrostiffness k. If the forced excitation is defined a priori, the switch-damping technique can be applied in the open loop control. In the opposite case the closed loop control detecting the current frequency of excitation through sensors measuring the frequency of structural vibration has to be used.

2. Optimal strategy of avoiding resonance

Generalizing the problem discussed in the previous section, let us consider forced vibration in the two-degree-of-freedom spring-mass system as shown in

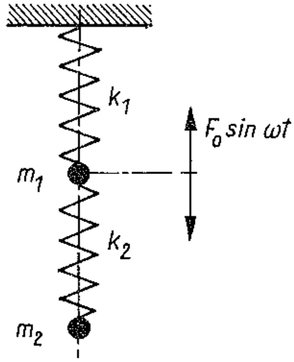

FIG. 4. Two-degree-of-freedom spring-mass system.

Fig. 4. For forced vibration without damping, the equations of motion are given by

$$(2.1)\qquad \begin{aligned} m_1\ddot{u}_1+(k_1+k_2)u_1-k_2u_2 &= F_0\sin\omega t,\\ m_2\ddot{u}_2+k_2u_2-k_2u_1 &= 0. \end{aligned}$$

Substituting $F_0e^{i\omega t}$ for $F_0\sin\omega t$, $U_1e^{i\omega t}$ for u_1 and $U_2e^{i\omega t}$ for u_2 (where U_1, U_2 do not depend on time) and dividing by $e^{i\omega t}$, the equations of motion become

$$(2.2)\qquad \begin{aligned} (k_1+k_2-m_1\omega^2)U_1-k_2U_2 &= F_0,\\ -k_2U_1+(k_2-m_2\omega^2)U_2 &= 0. \end{aligned}$$

The solution of the above set of equations takes the form

$$(2.3)\qquad \begin{aligned} U_1 &= F_0(k_2-m_2\omega^2)/\Delta,\\ U_2 &= F_0k_2/\Delta, \end{aligned}$$

where the determinant Δ is given by

$$(2.4)\qquad \Delta = m_1m_2\omega^4-[(m_1+m_2)k_2+m_2k_1]\omega^2+k_1k_2.$$

If the determinant Δ approaches zero (due to the approach of excitation frequency ω to some natural frequency ω_n of the system), resonance occurs. Therefore the formulation of the problem of optimal strategy of avoiding resonance by control of the stiffness parameters k_1, k_2 should be based on maximization of the function

$$(2.5)\qquad \max|\Delta(\omega, k_1, k_2)|.$$

for all ω from a given range and for the stiffness coefficients k_1, k_2 chosen from the following admissible range:

$$(2.6) \qquad \begin{aligned} k_d \leqslant k_1 \leqslant k_u, \\ k_d \leqslant k_2 \leqslant k_u. \end{aligned}$$

Let us note that the expression $|\Delta|$ can be treated as the measure of the distance between the actual frequency ω of the excitation and the closest natural frequency ω_n of the system $|\Delta| = \|\omega - \omega_n\|$. The goal function (2.5) and the domain of admissible control parameters (2.6) are convex. Therefore the process of optimal control of the stiffness coefficients k_1, k_2 chooses the states describing the corner points of the domain (2.6).

Computing the solution of the problem (2.5), (2.6) (for $m_1 = m_2 = 1$ kg, $k_d = 9.8$ N/cm, $k_u = 19.6$ N/cm), the optimal control process was determined (see Fig. 5a). Four functions $\Delta(\omega)$ corresponding to configurations of the control parameters k_1, k_2 which describe the corner points of the domain (2.6) are presented in Fig. 5c. After interpreting results, one can see that for low ω the maximal stiffness coefficients $k_1 = k_2 = k_u$ are required. However, in order to pass the first natural frequency of the system in the most distant way, the control parameter k_1 has to be switched for the lowest admissible value $k_1 = k_d$ when ω exceeds $\omega_1 = 0.784$. Then, when ω grows up more than $\omega' = 1.414$, the parameter k_1 can be switched back to the value $k_1 = k_u$ in order to magnify additionally the cost function $|\Delta|$. The most distant way of avoiding the second resonance frequency requires the final switching of both control parameters for the lowest possible values $k_1 = k_2 = k_d$ when ω exceeds $\omega_2 = 1.962$. The function describing the value of the determinant Δ depends on the actual frequency of excitation ω, and the corresponding optimal configuration of the control parameters (cf. Fig. 5c) demonstrates that the cost function (2.5) takes its minimal value $|\Delta| = 0.69$ for $\omega = \omega_1$.

After generalizing considerations for n-degree-of-freedom systems, one can check that the optimal control process requires the maximal possible values of parameters k_i ($i = 1, \ldots, n$) for ω below the first resonance frequency ω_1 and the minimal possible values of parameters k_i for ω above the highest resonance frequency ω_n of the system. The optimal strategy of play with parameters k_i for $\omega \in \langle \omega_1, \omega_n \rangle$ depends on the particular problem.

In general, having n control parameters available gives us the best solutions in the sense of the criterion (2.5). However, the problem of active control by m ($m < n$) parameters is more reasonable from the engineering point of view and can be successfully used. In most cases just one control stiffness parameter is enough to avoid all resonance frequencies, unless its location coincides with a nodal point of some mode. The optimal location of the controlled elements of the structure is an important problem and can be determined through

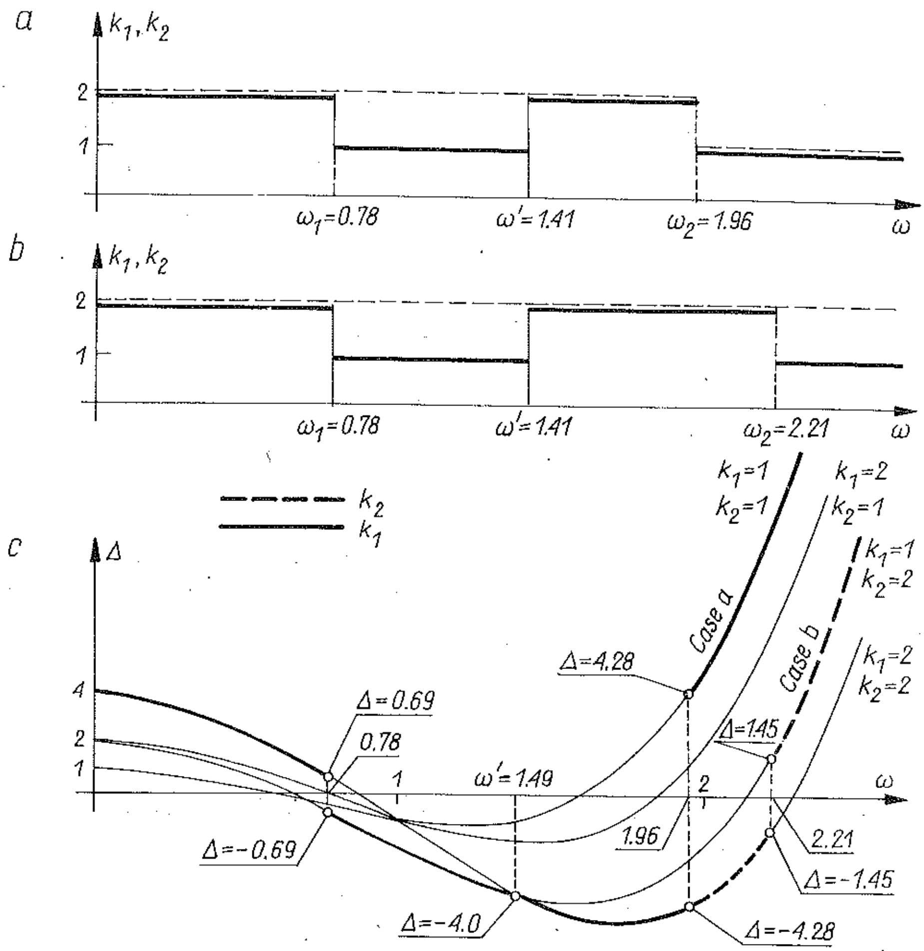

FIG. 5. Optimal control of stiffness coefficients in two dimensional case.

sensitivity analysis. Briefly speaking, the location of the controlled elements should coincide with the maximal modal deflections.

Assuming in the example considered above that only the coefficient k_1 is controllable (while $k_2 = k_u$ is constant), the solution of the active control problem is determined (see Fig. 5b). The two discussed results differ about the "distance" Δ of the structural response from the resonance for the second mode of vibration. If the forced excitation describes a combination of several forced excitations with frequencies ω_i independently (but slowly) variable, then the strategy (2.5) of active damping has to be modified. The maximization of the smallest distance $|\Delta| = \|\omega_i - \omega_n\|$ between the actual frequencies ω_i of excitation components and the closest natural frequency ω_n of the system should be required in this case.

3. Example of beam with controllable support stiffness

Finally, the idea of optimal strategy of avoiding resonance frequencies will be discussed in detail using the example of continuous beam structure with only one control parameter (the stiffness of the support B controllable — see Fig. 6).

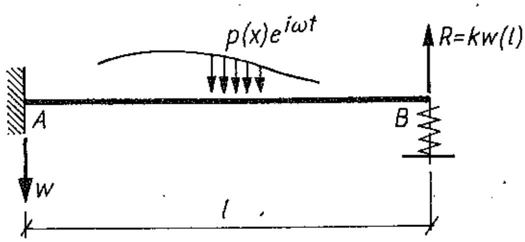

FIG. 6. Beam with the support stiffness controllable.

Let us consider the transverse harmonic vibration of the beam

$$w(x, t) = W(x)e^{i\omega t} \tag{3.1}$$

due to some external excitation

$$p(x, t) = P(x)e^{i\omega t}. \tag{3.2}$$

The forced vibration caused by the excitation (3.2) is described by the well-known nonhomogeneous equation

$$C^2(\partial^4 w/\partial x^4) + \ddot{w} = \frac{1}{\gamma}P(x)e^{i\omega t}, \tag{3.3}$$

where $\gamma = A\gamma_0$, γ_0 — the density of the material, A — the cross-sectional area, $C^2 = EJ/\gamma$, E — the modulus of elasticity, J — the moment of inertia.

Substituting the assumed form (3.1) of generated vibration to (3.3), the equation of amplitude is obtained:

$$d^4W(x)/dx^4 - \lambda^4 W(x) = P(x)/EJ, \tag{3.4}$$

where

$$\lambda^4 = \omega^2/C^2.$$

Let us develop now the functions $W(x)$ and $P(x)$ in series:

$$W(x) = \sum_{n=1}^{\infty} A_n W_n(x), \quad P(x) = \sum_{n=1}^{\infty} P_n W_n(x) \tag{3.5}$$

where the orthogonal eigen functions $W_n(x)$ of natural vibrations satisfy the homogeneous problem (Appendix (A.1)) with the boundary conditions (A.4).

The natural vibration dependent on the stiffness k of the support B is discussed in the Appendix.

The coefficients A_n, P_n can be determined by multiplying Eqs. (3.5) by the normalized eigen functions $W_n(x)$, integrating these formulae in the range $\langle 0, l\rangle$ and making use of the orthonormality $(\int_0^l W_k(x)W_n(x)dx = \delta_{kn})$:

$$A_n = \int_0^l W_n(x)W(x)dx, \quad P_n = \int_0^l W_n(x)P(x)dx. \tag{3.6}$$

Multiplying both sides of Eq. (3.4) by $W_n(x)$ and integrating this formula in the range $\langle 0, l\rangle$, the relation between the coefficients is reached [9]:

$$A_n = P_n/EJ\lambda_n^4(1-\lambda^4/\lambda_n^4). \tag{3.7}$$

Substituting Eq. $(3.6)_2$ to Eq. (3.7) and then the calculated coefficient A_n to Eq. $(3.5)_1$, the amplitude $W(x)$ is determined

$$W(x) = \frac{1}{EJ}\sum_{n=1}^{\infty}\frac{W_n(x)}{\lambda_n^4(1-\lambda^4/\lambda_n^4)}\int_0^l P(u)W_n(u)du. \tag{3.8}$$

Finally, making use of the relations

$$\lambda^4 = \omega^2/C^2, \quad \lambda_n^4 = \omega_n^4/C^2, \quad C^2 = EJ/\gamma, \tag{3.9}$$

the solution of Eq. (3.3) describing forced vibration due to excitation (3.2) is obtained:

$$w(x, t) = \frac{e^{i\omega t}}{\gamma}\sum_{n=1}^{\infty}\frac{W_n(x)}{\omega_n^2(1-\omega^2/\omega_n^2)}\int_0^l P(u)W_n(u)du. \tag{3.10}$$

The last formula was derived under the assumption that the frequency of external load ω is constant. However, in the case of quasi-static changes of the forced frequency, the accuracy of the above solution is also acceptable.

The problem of optimal strategy of avoidance of resonance can be formulated as follows. For a given frequency ω' of external excitation define the support B stiffness coefficient $k = k(\omega)$ which corresponds to the sequence of natural frequencies $\omega_n(k)$, the most distant from ω':

$$\max_k \|\omega' - \omega_n(k)\|, \tag{3.11}$$

where the distance $\|\omega' - \omega_n(k)\|$ is defined as the following distance between ω' and the closest natural frequency $\omega_n(k)$

$$\|\omega' - \omega_n(k)\| \overset{\text{df}}{=} \min_n |\omega' - \omega_n(k)|[\omega' + \omega_n(k)]. \tag{3.12}$$

Maximization of the functional (3.11) means maximization of the denominator in the formula (3.10), and so minimization of the amplitude of vibration.

The solution k, $\omega_n(k)$ of the above problem follows the changeable externally forced frequency $\omega(t)$. Therefore the idea of active control of the support B compliance guarantees avoiding all resonance frequencies in the most distant way and continuous minimizing of vibration amplitudes.

Let us notice that considering a value ω' in the range $\omega_i \leqslant \omega' \leqslant \omega_{i+1}$ (ω_i, ω_{i+1} — the closest to ω' natural frequencies) the formula (3.12) determines ω' as the geometric mean point of this interval.

Introducing the notation $\beta = \lambda l$, $D = kl^3/EJ$ (cf. (A.8)) and the relation $(3.9)_1$, the optimal criterion (3.11), (3.12) can be expressed in the equivalent from

$$\max_D \min_n \varrho(\beta, \beta_n(D)), \tag{3.13}$$

where the distance between β and $\beta_n(D)$ is measured by

$$\varrho = |\beta^4 - \beta_n^4(D)|. \tag{3.14}$$

The relation between the control parameter D and the corresponding sequences of natural frequencies β_n is described by the equation (cf. Appendix (A.9)):

$$\frac{D}{\beta^3}(\cos h\beta \sin\beta - \sin h\beta \cos\beta) + \cos h\beta \cos\beta + 1 = 0. \tag{3.15}$$

Now a computational procedure of the active control (3.13), (3.15) can be discussed in two formulations. One of them, when the control parameter D can be changed continuously between zero and infinity and the other, when only some discrete set of values can be taken by D.

In the case of continuous control of the parameter D, the optimal solution of the problem (3.13), (3.15) was computed and exposed in Fig. 7 where the transformation $D = [\xi/(1-\xi)]^2$ was applied, merely for better visualization of the result. The function ϱ of distance between β and the closest eigen value β_n takes its minimal value $\varrho = 113.4$ for $\beta = 3.345$, when the controlled parameter ξ is switched from 1 (the position describing full support of the end B) to the position 0 (describing free end B of the beam). Apart from this sore point, the control characteristic passes relatively close to the first and the second branch of lines describing the points of resonance frequencies (broken lines in Fig. 7). For the higher modes, however, the function ϱ grows up rapidly, so the resonance frequencies evade far away. The first elements of the sequence of distances ϱ_n (Fig. 7) in the switching points (from $\xi = 1$ to $\xi = 0$) are presented in Table 1a.

In the case of discrete control of the parameter D the numerical results of the problem (3.13), (3.15) are presented in Fig. 8. If only two realizations of the support conditions ξ_1, ξ_2 ($0 \leqslant \xi_i \leqslant 1$, $i = 1, 2$) are available, the optimal control characteristic is shown in Fig. 8a. If the control parameter ξ can take three values: ξ_1, ξ_2, ξ_3 ($0 \leqslant \xi_i \leqslant 1$, $i = 1, 2, 3$), the optimal solution is exposed

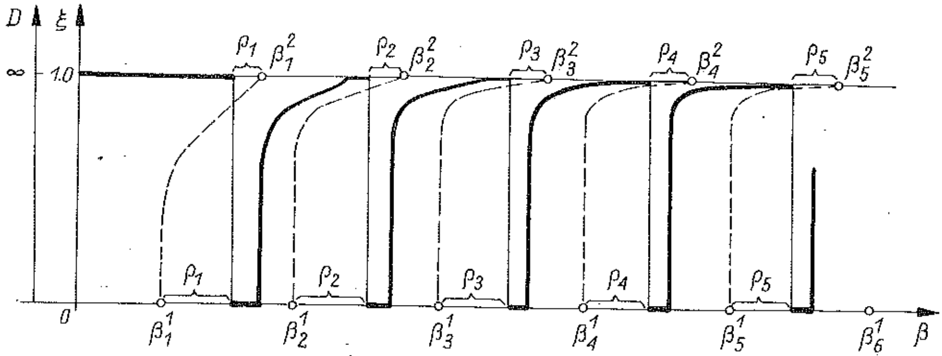

FIG. 7. Optimal control of the support stiffness — case of continuous control.

Table 1.

a)

i	1	2	3	4	5
ϱ_i	113.4	1009	3633	8826	17506
${}^a\varrho'_i$	129.3	710	2030	4420	8180

b)

i	1	2	3	4	5
${}^b\varrho'_i$	198	1634	5330	12446	24087
${}^b\varrho''_i$	198	771	2092	4479	8256

c)

i	1	2	3	4
${}^c\varrho'_i$	215	1645	5375	12594
${}^c\varrho''_i$	215	1620	5374	12594
${}^c\varrho'''_i$	215	765	2050	4325

in Fig. 8b. The intermediate control parameter ξ_2 is defined in such a way that the condition $\varrho'_1 = \varrho''_1$ (Fig. 8b and Table 1b) is satisfied. Similarly, when four values $\xi_1, \xi_2, \xi_3, \xi_4$ $(0 \leqslant \xi_i \leqslant 1,\ i = 1, \ldots, 4)$ are available, the control characteristic takes the form of step function as in Fig. 8c. Notice that optimal value ξ_2, ξ_3 should be chosen from the conditions $\varrho'_1 = \varrho''_1 = \varrho'''_1$ (cf. Table 1c). The optimal solution in the case of n available values of the control parameters ξ_i $(i = 1, \ldots, n,\ \xi' \leqslant \xi_i \leqslant \xi'')$ can be constructed analogously. Therefore the limit values should be reached: $\xi_1 = \xi'$, $\xi_n = \xi''$, and the condition $\varrho_j^{(i)} = \text{const}$ (for each j) be satisfied. Of course the effect of control is bigger if the difference $\xi'' - \xi'$ is bigger.

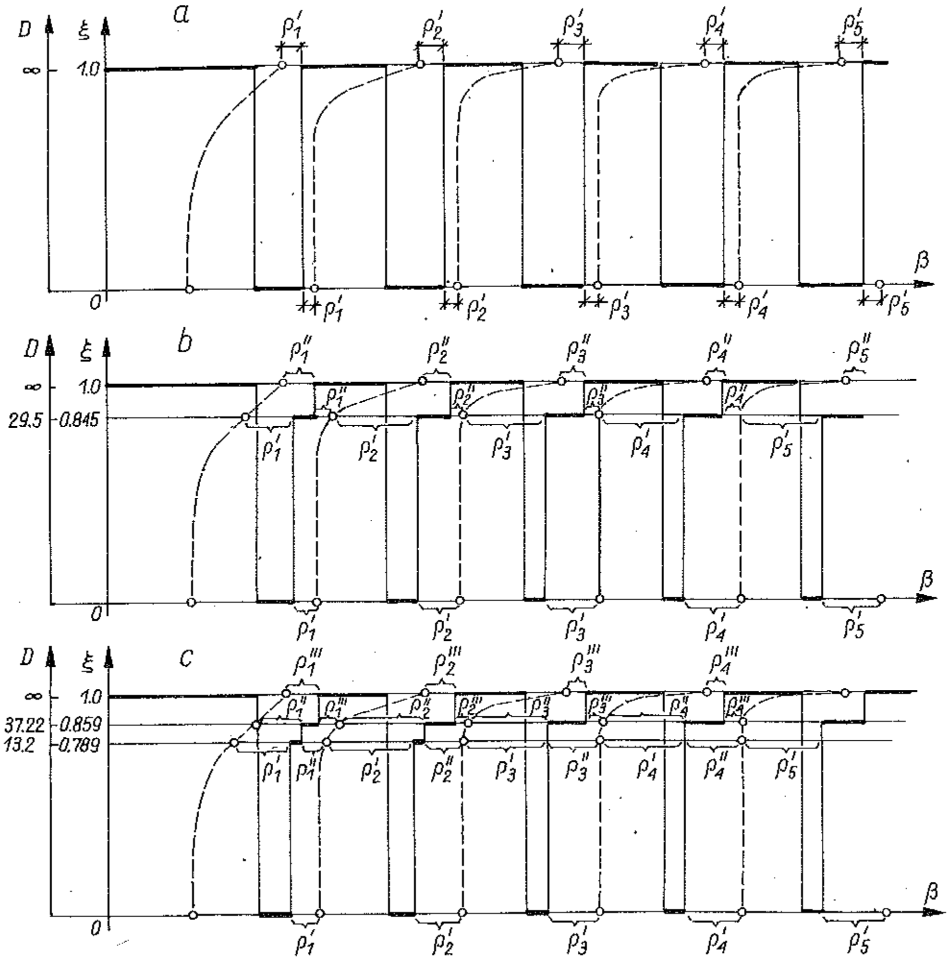

FIG. 8. Optimal control of the support stiffness — cases of discrete control.

From the results presented above one can see that the simplest case of discrete, biparameter control (Fig. 8a) is relatively effective and easy to apply. It allows us to avoid all infinite numbers of resonance frequencies. The control box installed in the support B of the beam should realize the optimal strategy of switching prescribed by the optimal solution.

4. CONCLUSIONS

As it was mentioned at the beginnig, the above analysis was presented assuming quasi-static changes of the frequency ω. If this condition is not satisfied, then the function $w(x, t)$ takes a more general form than the expression (3.1) and the whole problem has to be considered again. Also realization of switching points in active control (cf. jumps in step functions — Figs. 7 and 8) needs some comment. In real applications change of the control parameter should be made more smoothly (cf. Fig. 3a). If not, the impact due to a sudden change of the stiffness coefficient will cause some local perturbation of the optimal solution. This perturbation can be calculated and added to the solution. However, the range of its influence in the quasi-static case is localized. The direct method of active damping computation for the case of any variable load and a structure discretized will be presented in [4].

The problem of active damping of forced vibration was discussed on the basis of the example of the beam with controllable support stiffness. However, the problem is more general and no obstacles prevent the application of the presented approach to optimal control of other engineering structures like frames, trusses or continuous beams. The problem of where to locate (and how many) control devices for maximization of the control effect is still open. The sensitivity analysis should be applied for particular cases but, generally speaking, one can notice that the control devices should be located in the cross-sections where the modal deflections take maximal values and away from nodal points (especially of the first modes of vibration). Normally, the support points satisfy these conditions).

The advantage of the present method for avoiding resonance is the fact that one or two properly located control devices (e.g., dynamical clutch) can successfully damp the forced vibration in the whole structure. The control procedure realizes a new constitutive characteristic $k = k(\omega)$ for an element of the structure and therefore it can be classified as an "active-passive" method. It means that the procedure actively changes some internal, own properties of the structure but does not generate some external forces acting on the structure (the "active-active" case). Usually the active-passive method can be realized in an easier and cheaper way because the considered structure itself plays the role of the controlling device.

The method of active damping by control of constitutive characteristics will be applied for damping of natural vibrations of structures in the next paper [4].

Appendix. Free vibration of beam with elastic support

Substituting the expression (3.1) to the well-known equation of transverse free vibration of beam

$$C^2\frac{\partial^4 w}{\partial x^4}+\ddot{w} = 0, \qquad C^2 = EJ/\gamma, \tag{A.1}$$

where $\gamma = A\gamma_0$, γ_0 — the density of material, A — the cross sectional area,

E — the modulus of elasticity, J — the moment of inertia, the following ordinary differential equation is obtained:

$$\frac{d^4W}{dx^4} - \lambda^4 W = 0, \qquad \lambda^4 = \omega^2/C^2.$$

Making use of the Laplace's transformation, the solution of the equation takes the form [9]

$$\text{(A.2)}\quad W(x) = W(0)S(\lambda x) + \frac{1}{\lambda}W'(0)T(\lambda x) + \frac{1}{\lambda^2}W''(0)U(\lambda x) + \frac{1}{\lambda^3}W'''(0)V(\lambda x),$$

where

$$V(\lambda x) = \frac{1}{2}(\sin h\lambda x - \sin\lambda x),$$

$$U(\lambda x) = \frac{1}{2}(\cos h\lambda x - \cos\lambda x),$$

$$T(\lambda x) = \frac{1}{2}(\sin h\lambda x + \sin\lambda x),$$

$$S(\lambda x) = \frac{1}{2}(\cos h\lambda x + \cos\lambda x).$$

The formula (A.2) easily allows us to take into account the boundary conditions because the constant coefficients $W(0)$, $W'(0)$, $W''(0)$, $W'''(0)$ have a mechanical meaning. Determining the following derivatives of the function (A.2):

$$W'(x) = W(0)\lambda V(\lambda x) + W'(0)S(\lambda x) + \frac{1}{\lambda}W''(0)T(\lambda x) + \frac{1}{\lambda^2}W'''(0)U(\lambda x),$$

$$\text{(A.3)}\quad W''(x) = W(0)\lambda^2 U(\lambda x) + W'(0)\lambda V(\lambda x) + W''(0)S(\lambda x) + \frac{1}{\lambda}W'''(0)T(\lambda x),$$

$$W'''(x) = W(0)\lambda^3 T(\lambda x) + W'(0)\lambda^2 U(\lambda x) + W''(0)\lambda V(\lambda x) + W'''(0)S(\lambda x),$$

the boundary conditions for the studied beam can be considered

$$W(0) = W'(0) = 0,$$

$$\text{(A.4)}\quad W''(l) = 0,$$

$$W'''(l) = W(l)k/EJ.$$

The formula $(A.4)_3$ describes the equilibrium between the transverse internal force in the end B of the beam $Q(l) = -EJW'''(l)$ and the reaction force in the elastic support $R = -kW(1)$. Taking into account the boundary conditions $(A.4)_1$, the function $W(x)$ describing the deflection of the beam (A.2) can be expressed as follows:

$$W(x) = \frac{1}{\lambda^2}W''(0)U(\lambda x)+\frac{1}{\lambda^3}W'''(0)V(\lambda x), \tag{A.5}$$

where the coefficients $W''(0)$ and $W'''(0)$ should be determined from the two remaining conditions $(A.4)_{2,3}$.

Calculating $W''(l)$, $W'''(l)$ and $W(l)$ from the formulae $(A.3)_{2,3}$, (A.5) respectively, the conditions $(A.4)_{2,3}$ lead to the set of two equations:

$$\begin{bmatrix} S(\lambda l) & \frac{1}{\lambda}T(\lambda l) \\ \lambda V(\lambda l)-\frac{k}{EJ\lambda^2}U(\lambda l), & S(\lambda l)-\frac{k}{EJ\lambda^3}V(\lambda l) \end{bmatrix} \begin{bmatrix} W''(0) \\ W'''(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{A.6}$$

The solution of Eqs. (A.6) exists if the main determinant Δ vanishes:

$$\Delta = S^2(\beta)-\frac{D}{\beta^3}S(\beta)V(\beta)-T(\beta)V(\beta)+\frac{D}{\beta^3}T(\beta)U(\beta) = 0, \tag{A.7}$$

where

$$\beta = \lambda l, \qquad D = kl^3/EJ. \tag{A.8}$$

The transcendental equation (A.7) describes the infinite number of roots β_n (where $\beta_n^2 = \omega_n l^2/C$) defining the sequence of natural frequencies ω_n.

Substituting the definitions $(A.2)_{2-5}$ the frequency equation (A.7) takes the final form

$$\frac{D}{\beta^3}(\cos h\beta \sin\beta - \sin h\beta \cos\beta)+\cos h\beta\cos\beta+1 = 0. \tag{A.9}$$

In the one, limit case of a cantilever beam with a free end $B(k = 0)$, the parameter D vanishes and the frequency equation (A.9) takes the form

$$\cos h\beta\cos\beta+1 = 0. \tag{A.10}$$

The second limit case of the beam fully supported at the end $B(k = D = \infty)$ leads to another particular form of the equation (A.9):

$$\cos h\beta\sin\beta-\sin h\beta\cos\beta = 0. \tag{A.11}$$

The first five elements of the sequence β_n^1 of roots of the transcendental equation (A.10) take the following values

$$\text{(A.12)}\qquad \begin{aligned} &\beta_1^1 = 1.875, \quad \beta_2^1 = 4.694, \quad \beta_3^1 = 7.855,\\ &\beta_4^1 = 10.996, \quad \beta_5^1 = 14.137, \end{aligned}$$

while the corresponding elements of the sequence β_n^2 of roots of Eq. (A.11) are equal, respectively,

$$\text{(A.13)}\qquad \begin{aligned} &\beta_1^2 = 3.927, \quad \beta_2^2 = 7.069, \quad \beta_3^2 = 10.210,\\ &\beta_4^2 = 13.352, \quad \beta_5^2 = 16.493. \end{aligned}$$

We can estimate now the sequence $\beta_n = \beta_n(D)$ of the roots of the transcendental equation (A.9)

$$\text{(A.14)}\qquad \beta_n^1 \leqslant \beta_n \leqslant \beta_n^2.$$

When D grows up from zero to infinity, the elements $\beta_n(D)$ of root sequence grows up from β_n^1 to β_n^2. Therefore, by controlling the flexibility k of the support B, the natural frequencies ω_n of the beam can be changed taking values from the ranges $\langle \omega_n^1, \omega_n^2 \rangle$, respectively.

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Streszczenie

AKTYWNA STRATEGIA UNIKANIA REZONANSU

Rozważany jest problem aktywnego sterowania częstości drgań własnych przez wprowadzanie lokalnych zmian sztywności w elementach konstrukcyjnych. Celem sterowania, śledzącego zmienne w czasie wymuszenia, jest omijanie częstości rezonansowych. Dyskutowana jest optymalna strategia przełączania sterowalnego, lokalnego parametru sztywności w przypadku prostej struktury dyskretnej oraz ciągłego wspornika belkowego. Wykazano, że nawet jeden element konstrukcyjny ze sterowalnym współczynnikiem sztywności pozwala zazwyczaj ominąć częstości rezonansowe w konstrukcji poddanej zmiennym w czasie wymuszeniom.

Резюме

АКТИВНАЯ СТРАТЕГИЯ ИЗБЕЖАНИЯ РЕЗОНАНСА

Рассматривается проблема активного управления собственными частотами колебаний путем введения локальных изменений жесткости в конструкционных элементах. Целью управления, слежащего переменные во времени вынуждения, является избежание резонансных частот. Обсуждается оптимальная стратегия управляемого переключения локального параметра жесткости, в случае простой дискретной структуры и непрерывной балочной консоли. Показано, что даже один конструкционный элемент с управляемым коэффициентом жесткости позволяет обычно избежать резонансных частот в конструкции, подвергнутой переменным во времени вынуждениям.

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