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PLANE STRESS PROBLEM OF PLASTIC FORMING OF CYLINDRICAL SHELLS MADE FROM COATED METAL SHEETS

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The method of characteristics is applied to solve the axially symmetric problem of plastic yielding of coated metal sheets under plane stress conditions, in spite of the layered structure of the shell. The material properties are modelled by introducing different Lankford anisotropy factors and different strain-hardening curves for individual layers. The relations derived for the characteristics are then applied to solve the problem of press forming of cylindrical shells on a flat or conical die without a blankholder. The numerical examples illustrate the effect of zinc and tin coats on the forming stresses of coated steel sheets.

1. Introduction

The theory of axially symmetric plastic flow under plane stress conditions has an application in solving the problems of forming sheet metal shells. For principal processes of deep drawing, the theoretical solutions are already known [1, 2]. The present development tendency of this theory consists in the formulation of solutions accounting for real properties of sheet metal, such as strain-hardening [3], anisotropy, nonhomogeneity, superficial coats, etc.

In modern technology of plastic forming sheet metals coated by zinc, tin, varnish and films of plastic materials are used. The existence of coats influences the stress state in plastic forming processes.

The theoretical solutions can also be used to predict the behaviour of the coats during metal deformation.

Because the thickness of sheet metal is small in comparison with its other dimensions, we will be able to propose a simplified solution which fulfil the plane stress conditions, in spite of the existence of superficial coats.

The stress state is expressed by its components in each material layer. Radial components of the strain-rate tensor are equal for all the layers of sheet metal. This simplified solution will enable us to determine the variation of the stresses and thickness of each layer during the deformation process.

2. Formulation of problem

Consider a coated metal sheet composed of the principal middle layer h_1 and two thin superficial coats of equal thickness $h_2' = h_2''$, which are made of the same material different from that of the principal layer h_1. The magnitudes concerning the principal middle layer will always be denoted by subscript 1, and the superficial coats – by 2.

In the following considerations, the model of plastic anisotropic materials for all layers will be assumed. The material of each layer will satisfy the Hill yield criterion [4, 5] formulated for anisotropic material characterized by transversal anisotropy only. This anisotropy is determined by the Lankford factor R [6], and this factor is assumed to be independent of the strain ratio. Furthermore, the properties of both the materials will be described by strain-hardening curves, different for each of them.

Considering the stress state in such a material as a particular case of a three-dimensional axially symmetric problem, we have to introduce radial stress σ_r, circumferential stress σ_θ and, in addition, a distribution of stresses σ_z, τ_{rz} in each layer. Such a stress state is very complex what makes it impossible to obtain a simple solution of the problem. However, since the superficial coats of the sheet metal are usually very thin (not exceeding 0.1 of the principal layer thickness), we will assume for simplification the conditions of plane stress state (Fig. 1). Application of this model will be justified in the case of deformation processes in which the surface of the sheet metal is free (without the action of blankholder).

From the assumption of plane stress conditions it follows that: σ_z and τ_{rz} are equal to zero in each layer; stress components σ_r, σ_θ are independent of the z coordinate in each layer, but the lines of junction between the layers are lines of discontinuity for stresses; across these lines, jumps of stresses σ_r, σ_θ are admissible.

Since $h_2' = h_2''$, the symmetry of geometry and material properties is secured in each cross-section along the z-coordinate. So, the moments of

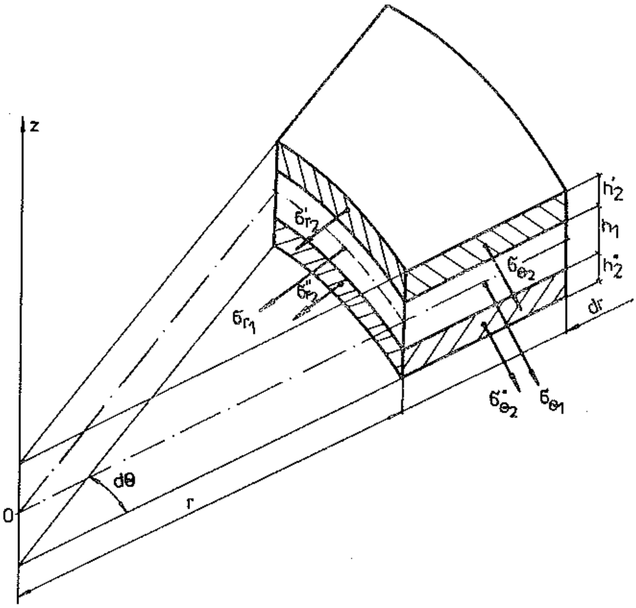

FIG. 1.

radial forces operating in superficial coats and calculated with respect to the middle plane are self-equilibrated and they can be neglected provided the radii of shells curvature are sufficiently large.

The kinematics of deformation will be described by the radial velocity v_r, and the geometry of the material – by thicknesses h_1, h_2. All the magnitudes considered will be functions of the radius r and time t.

The system of basic equations for an axially-symmetric case of plane stress contains:

the equilibrium equation

$$\frac{d}{dr}(\sigma_r r h) - \sigma_\theta h = 0; \tag{2.1}$$

the Hill yield criterion

$$\sigma_r^2 - \frac{2R}{1+R}\sigma_r\sigma_\theta + \sigma_\theta^2 = \sigma_p^2; \tag{2.2}$$

the flow law associated with the yield criterion

$$\frac{\dot{\varepsilon}_r}{(1+R)\sigma_r - R\sigma_\theta} = \frac{\dot{\varepsilon}_\theta}{(1+R)\sigma_\theta - R\sigma_r}; \tag{2.3}$$

the incompressibility condition

$$\dot{\varepsilon}_r + \dot{\varepsilon}_\theta + \dot{\varepsilon}_h = 0. \tag{2.4}$$

These equations will be adapted to our problem. To simplify the form of relations in further considerations, we will introduce an "equivalent" layer $h_2 = h_2' + h_2''$. The stress state in the middle plane is written in polar coordinates, and we take into consideration the stress components occurring in two layers: the principal middle layer h_1 and the equivalent layer of coats $h_2 = h_2' + h_2''$.

The equilibrium condition (2.1) must contain now the sum of the respective components for both the layers:

$$\frac{d\sigma_{r1}}{dr}h_1 + \frac{d\sigma_{r2}}{dr}h_2 + (\sigma_{r1} - \sigma_{\theta 1})\frac{h_1}{r} + (\sigma_{r2} - \sigma_{\theta 2})\frac{h_2}{r} + \sigma_{r1}\frac{dh_1}{dr} + \sigma_{r2}\frac{dh_2}{dr} = 0. \tag{2.5}$$

The Hill yield criterion (2.2) and flow law (2.3) must be satisfied in both materials, but we must introduce the respective parameters σ_{p_1}, R_1 and σ_{p_2}, R_2. If all the layers are perfectly joined, the radial component of flow velocity is the same in each layer, so that $v_{r1} = v_{r2} = v_r(r,t)$.

The strain-rates in the directions $\{r, \theta\}$ are given by the relations:

$$\begin{aligned} \dot{\varepsilon}_{r_1} &= \dot{\varepsilon}_{r_2} = \frac{\partial v_r}{\partial r}, \\ \dot{\varepsilon}_{\theta_1} &= \dot{\varepsilon}_{\theta_2} = \frac{v_r}{r}. \end{aligned} \tag{2.6}$$

In view of $\dot{\varepsilon}_{r_2} = \dot{\varepsilon}_{r_1}$ and $\dot{\varepsilon}_{\theta_2} = \dot{\varepsilon}_{\theta_1}$, by double application of the flow law (2.3) we obtain also the relation

$$\frac{(1+R_1)\sigma_{r_1} - R_1\sigma_{\theta_1}}{(1+R_1)\sigma_{\theta_1} - R_1\sigma_{r_1}} = \frac{(1+R_2)\sigma_{r_2} - R_2\sigma_{\theta_2}}{(1+R_2)\sigma_{\theta_2} - R_2\sigma_{r_2}}. \tag{2.7}$$

The yield condition (2.2) will be identically satisfied if the stresses σ_r and σ_θ are expressed in terms of the following parametrizing functions, introduced separately for each layer:

$$\begin{aligned} \sigma_{r_{1,2}} &= \sigma_{p_{1,2}}\sqrt{\frac{1+R_{1,2}}{2}}\left(\cos\omega_{1,2} + \frac{1}{\sqrt{1+2R_{1,2}}}\sin\omega_{1,2}\right), \\ \sigma_{\theta_{1,2}} &= \sigma_{p_{1,2}}\sqrt{\frac{1+R_{1,2}}{2}}\left(\cos\omega_{1,2} - \frac{1}{\sqrt{1+2R_{1,2}}}\sin\omega_{1,2}\right). \end{aligned} \tag{2.8}$$

The relation (2.7) implies also a dependence between ω_1 and ω_2 as follows:

$$\operatorname{tg}\omega_2 = \sqrt{\frac{1+2R_1}{1+2R_2}}\operatorname{tg}\omega_1. \tag{2.9}$$

As a result of the parametrization, the basic equation system is reduced to three equations which can be solved by the method of characteristics in the following way.

The strain-rates in the transversal direction can be expressed as relative increments of thicknesses for the time increment dt

$$\dot{\varepsilon}_{h_{1,2}} = \frac{dh_{1,2}}{h_{1,2}}dt. \tag{2.10}$$

These results follow from the non-compressibility condition (2.4) which is satisfied in each layer. Introduction of the respective rates, yields finally the result

$$\frac{1}{h_{1,2}}\left(\frac{\partial h_{1,2}}{\partial t} + v_r\frac{\partial h_{1,2}}{\partial r}\right) + \frac{\partial v_r}{\partial r} + \frac{v_r}{r} = 0. \tag{2.11}$$

Introducing the relation (2.3) into the incompressibility condition (2.11), we obtain the equations to be satisfied in each layer

$$\frac{\partial h_{1,2}}{\partial t} + v_r\frac{\partial h_{1,2}}{\partial r} = \frac{v_r h_{1,2}}{r}\,\frac{\sigma_{r_{1,2}} + \sigma_{\theta_{1,2}}}{R_{1,2}\sigma_{r_{1,2}} - (1+R_{1,2})\sigma_{\theta_{1,2}}}\,. \tag{2.12}$$

The Eq. (2.12) contains partial differentials of the functions h_1 or h_2 only, and it posseses a line of characteristics described by the equation

$$dr - v_r dt = 0, \tag{2.13$_1$}$$

with the differential relation satisfied along it

$$\frac{dh_{1,2}}{dr} = \frac{h_{1,2}}{r}\,\frac{2\cos\omega_{1,2}}{\sqrt{1+2R_{1,2}}\sin\omega_{1,2} - \cos\omega_{1,2}}\,. \tag{2.13$_2$}$$

A second line of characteristics results from the equation (2.3) for $t =$ const, and the following relation must be hold along it:

$$\frac{dv_r}{dr} = \frac{v_r}{r}\,\frac{\cos\omega_{1,2} + \sqrt{1+2R_{1,2}}\sin\omega_{1,2}}{\cos\omega_{1,2} - \sqrt{1+2R_{1,2}}\sin\omega_{1,2}}\,. \tag{2.14}$$

The third missing relation along the characteristic $t =$ const is provided by the equilibrium equation (2.5), which enables us to calculate $\omega_{1,2}$ for

$t = \text{const}$. After expressing the streses $\sigma_{r_{1,2}}$ and $\sigma_{\theta_{1,2}}$ in terms of the parametrizing functions (2.8), this equation yields the differential relation

$$\text{(2.15)} \quad \frac{d\omega_1}{dr} = \left[\left(\frac{d\sigma_{p_1}}{dr}h_1 + \sigma_{p_1}\frac{dh_1}{dr}\right)(A_1\cos\omega_1 + B_1\sin\omega_1)\right.$$
$$+2\sigma_{p_1}B_1\frac{h_1}{r}\sin\omega_1 + \left(\frac{d\sigma_{p2}}{dr}h_2 + \sigma_{p_2}\frac{dh_2}{dr}\right)(A_2\cos\omega_2 + B_2\sin\omega_2)$$
$$\left.+2\sigma_{p_2}B_2\frac{h_2}{r}\sin\omega_2\right] / \left[\sigma_{p_2}h_2(A_2\sin\omega_2 - B_2\cos\omega_2)D\cdot(1+\mathrm{tg}^2\omega_1)\right.$$
$$\left./(1+D^2\mathrm{tg}^2\omega_1) + \sigma_{p_1}h_1(A_1\sin\omega_1 - B_1\cos\omega_1)\right],$$

where

$$A_1 = \sqrt{\frac{1+R_1}{2}}, \qquad A_2 = \sqrt{\frac{1+R_2}{2}},$$
$$B_1 = A_1/\sqrt{1+2R_1}, \qquad B_2 = A_2/\sqrt{1+2R_2},$$
$$D = \sqrt{\frac{1+2R_1}{1+2R_2}}.$$

The solution of boundary-value problems consists in determining the characteristic lines in the system of coordinates $\{r,t\}$. The positions of material particles at each instant of time are calculated with the aid of Eq. $(2.13)_1$. The magnitudes $\omega_{1,2}h_{1,2}$, v_r sought for are calculated by numerical integration of Eqs. $(2.13)_2$, (2.14), (2.15) along the characteristics.

3. Deep drawing on a flat die

If the deep drawing process is effected on a flat die and the action of the blankholder is relatively weak, the theoretical problem is reduced to the case of unsteady flow of a circular flange having an initial external diameter D_0 and internal diameter d. The draw stress is exerted inside the hole on r_A (Fig. 2).

The boundary condition for the radial stress components is established at the outer rim B where $\sigma_{r_{1,2}}(r_B,t) = 0$. At the inner rim r_A the material flows into the hole with a constant radial velocity $v_r(r_A,t) = v_0$. The initial conditions for thicknesses are: $h_1(r,0) = h_{0_1}$, $h_2'(r,0) = h_2''(r,0) = h_{0_2}$ and $h_2 = h_2' + h_2''$.

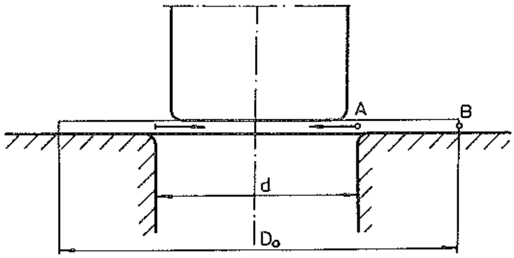

FIG. 2.

The initial and boundary conditions are sufficient to solve the problem in coordinates $\{r, t\}$. For $t = 0$, when h_1=const, h_2=const and σ_{p_1}=const, the value of ω_1 as a function of r can be calculated by numerical integration of Eq. (2.15), and the initial velocity distribution – by integration of relation (2.14). After time increment dt, new positions of the considered r-points and the new thicknesses of the layers at these points are given by the relation (2.13) written for the first line of characteristics; then the strains can be calculated.

Using the Hencky logarithmic strains φ_r, φ_θ, φ_h written for the principal directions, the strain intensity is expressed by the relation

$$\varphi_{i_{1,2}} = \sqrt{\frac{R_{1,2}+1}{2R_{1,2}+1}} \int \sqrt{d\varphi_r^2 + d\varphi_\theta^2 + R_{1,2} d\varphi_{h_{1,2}}^2}. \tag{3.1}$$

In spite of the assumption $v_{r_1} = v_{r_2}$, the strain intensity is not equal for both layers, and it depends on the stress state in the entire material. This difference is indicated by the plastic flow law (2.3) written in the complete form

$$\frac{d\varphi_r}{(1+R_{1,2})\sigma_{r_{1,2}} - R_{1,2}\sigma_{\theta_{1,2}}} = \frac{d\varphi_\theta}{(1+R_{1,2})\sigma_{\theta_{1,2}} - R_{1,2}\sigma_{r_{1,2}}} = \frac{d\varphi_{h_{1,2}}}{-\sigma_{r_{1,2}} - \sigma_{\theta_{1,2}}} = \frac{d\varphi_{i_{1,2}}}{(1+R_{1,2})\sigma_{p_{1,2}}}. \tag{3.2}$$

This remark is important for the technology if one of the two materials will have a critical admissible strain value lower than that of the other material.

When the distributions of $\sigma_{p_{1,2}}(r)$ and $h_{1,2}(r)$ are known, the derivatives $d\sigma_{p_1}/dr$, $d\sigma_{p_2}/dr$, dh_1/dr, dh_2/dr can be calculated. The stresses and velocities after time dt will then be determined by relations (2.15) and (2.14) holding along the second line of characteristics. This procedure is repeated in order to increase the accuracy of calculations and to attain a presumed difference between the successive approximations. This procedure of solution was elaborated by W. SZCZEPIŃSKI [2]. It takes into consideration the strain-hardening of metal. The assumption of linear strain-hardening indicates that the maximum force occurs just at the beginning of the deep drawing process, which is in contradiction with the experiment.

In our calculations, the strain-hardening will be described by the equation

$$\sigma_p = C(\varphi_0 + \varphi_i)^n, \tag{3.3}$$

where C, φ_0, n are the material properties.

The numerical examples will be given for the deep drawing process of cylindrical shells formed from a low carbon steel sheet coated with zinc or tin. Steel containing about 0.08% carbon has a strain hardening dependence (3.3) described by constants: C = 680 MPa, φ_0 = 0.002, n = 0.22 and Lankford factor R = 1.3. For zinc coat C = 147 MPa, φ_0 = 0.002, n = 0.185, R = 0.52, and for tin coat C = 34 MPa, φ_0 = 0.002, n = 0.444, R = 0.35. Steel sheet of 2 mm thickness has two-sided zinc or tin coats

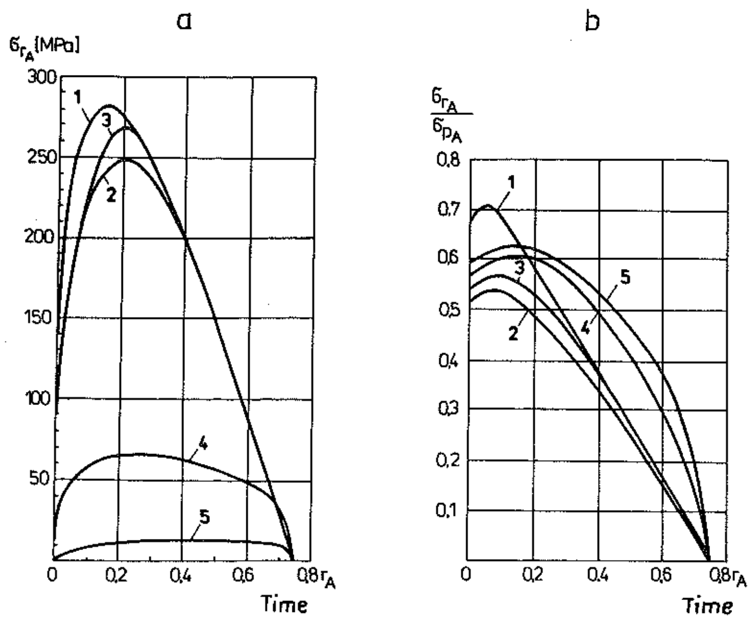

FIG. 3. 1 – coatless steel, 2 – middle steel layer of zinc coated steel, 3 – middle steel layer of tin coated steel, 4 – zinc coat, 5 – tin coat.

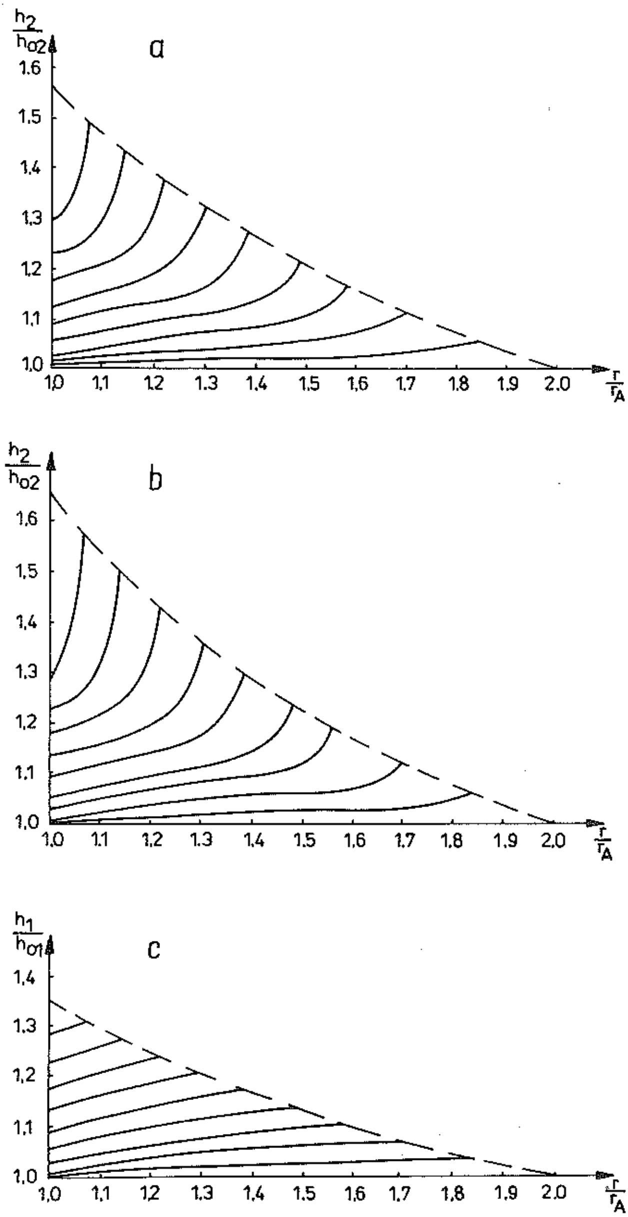

FIG. 4. a – zinc coat, b – tin coat, c – steel layer.

of 0.05 mm. If the initial external diameter $D_0 = 100$ mm, and the shell diameter $d = 50$ mm, then $d/D = 0.5$.

Figures 3a,b present the change of draw stresses at the inner rim of the middle steel layer and the coats during the deformation process. In spite of the fact that the coats are very thin, the results of calculations indicate a great influence of the coats on the stresses. The coats reduce the stresses occurring in the steel layer. Figures 4a,b,c illustrate the changes of the relative thicknesses of the steel layer and coats during the process. The increase of thickness of the zinc or tin coats is greater than that of the steel layer. However, the difference between thicknesses of the middle steel layer and the coatless steel is not important. The strain intensity value in zinc and tin coats is always larger than that in the steel layer, and reaches a critical value earlier than in the steel sheet.

4. Deep drawing on a conical die

Deep drawing without a blankholder is possible if the sheet metal is relatively thick. The application of a conical or curvilinear die profiles is useful in this case due to the reduction of the surface contact between the metal and the die. This contact is then effected on the external metal edge only and the friction force is small enough to be negligible. The deformation of the side wall of the shell is unconstrained and the curvature of the wall varies in time. The diameter of the initially flat material disk is D_0 and a punch of diameter d moves with vertical velocity v_0 (Fig. 5a). As previously, the sheet metal will be composed of a principal middle layer h_1 and two equal coats $h_2' = h_2''$. In spite of variable geometry of the side wall, the flow velocity is sufficiently determined at each time instant by a single radial component $v_r(r,t)$ which is the same for all layers. Previous considerations and assumptions concerning the stress state are then valid, and the relations (2.1), (2.2), (2.3), (2.4) established for plane stress conditions can also be applied.

The geometry of an axially symmetrical shell is described by the equation of the curve determined by cross-section of the shell along its meridian. If this equation has the form $r = f(z)$ and its derivatives $r' = (\partial f(z))/\partial z$ and $r'' = (\partial^2 f(z))/dz^2$, the radii of curvature are equal

$$\frac{1}{\rho_\alpha} = \frac{-r''}{(1+r'^2)^{\frac{3}{2}}} \qquad \text{and} \qquad \frac{1}{\rho_\beta} = \frac{1}{r\sqrt{1+r'^2}}. \tag{4.1}$$

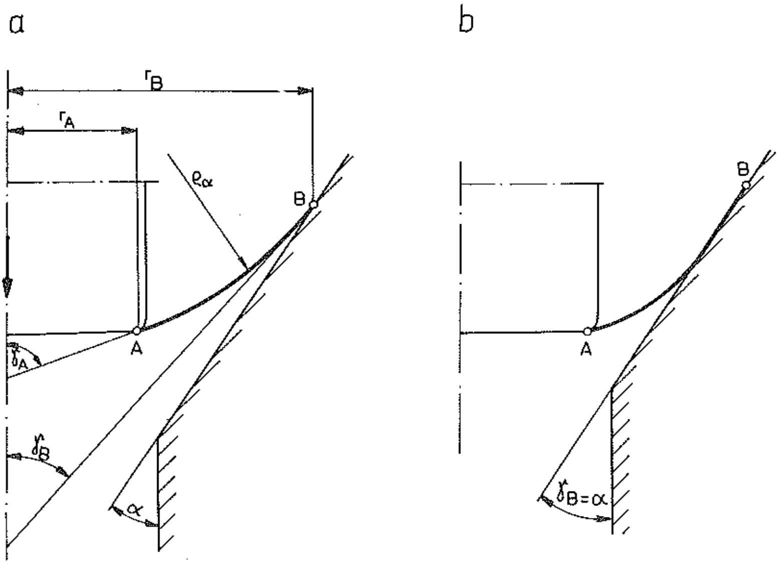

FIG. 5.

The strain-rates in directions $\{r, \theta\}$ are expressed for each layer by the following relations:

$$\dot{\varepsilon}_{r_1} = \dot{\varepsilon}_{r_2} = \frac{1}{A}\frac{\partial v_r}{\partial r}, \qquad \dot{\varepsilon}_{\theta_1} = \dot{\varepsilon}_{\theta_2} = \frac{v_r}{Ar}, \tag{4.2}$$

where

$$A = \sqrt{1 + r'^2}.$$

The incompressibility condition

$$\frac{1}{h_{1,2}}\left(\frac{\partial h_{1,2}}{\partial t} + v_r \frac{\partial h_{1,2}}{\partial r}\right) + \frac{1}{A}\frac{\partial v_r}{\partial r} + \frac{v_r}{Ar} = 0. \tag{4.3}$$

possesses a line of characteristics with the following relations satisfied along it:

$$dr - v_r dt = 0,$$

(4.4)

$$\frac{dh_{1,2}}{dr} = \frac{h_{1,2}}{Ar}\,\frac{2\cos\omega_{1,2}}{\sqrt{1 + 2R_{1,2}}\sin\omega_{1,2} - \cos\omega_{1,2}}.$$

The latter relation was finally obtained as previously, by introducing the functions (2.8).

Along the second line of characteristics (for t=const) we have to fulfil the relations (2.14) and (2.15).

The problem of deep drawing on a conical die has initial and boundary conditions expressed in coordinates $\{r, t\}$, but it is not accurate in coordinates $\{r, z\}$.

For each moment of time t the condition of vanishing of radial stress at the external edge is $\sigma_r(r_B, t) = 0$. At the inner edge the condition for radial velocity can be formulated as $v_r(r_A, t) = 0$. The initial conditions for thicknesses are as follows:

$$h_1(r, 0) = h_{01}, \qquad h_2'(r, 0) = h_2''(r, 0) = h_{02} \qquad \text{and} \qquad h_2 = h_2' + h_2''.$$

For $t = 0$, the values of $\omega_{1,2}(r)$ can be derived by numerical integration of Eq. (2.15), and the initial velocity distribution $v_r(r, 0)$ — by integration of Eq. (2.14).

The relation $(4.4)_1$ make possible the calculation of the new r-positions of material particles after the time increment dt. New thicknesses of layers at the considered r-points are found from Eq. $(4.4)_2$ under the assumption that $t = 0$, $r' = 0$ and $A = 1$. Unfortunately, we miss a relation which would enable us to determine the variation of z-positions of the material during the time interval dt. This can be evaluated simultaneously with the r-displacements. For this purpose, we can attempt to calculate the z-displacement of the external edge knowing its r-displacement and the geometric form of the die (e.g. the conical die with vertex angle α). Nevertheless, the vertical displacement of the external edge calculated as $dz_B = dr_B \mathrm{tg}\alpha$ is not correct because this approach does not satisfy the condition of conservation of the material volume. In fact, the external material edge is not joined with the die, and it can freely slip and rotate about the die surface. The assumption of linear form of the side wall after deformation is also not correct [7].

To obtain the first approximation of the $z(r)$ function assume that the radius of curvature ρ_α is constant at this moment and compare the side wall areas before and after this deformation rate. A new r-position of the external material edge is $r_B'' = r_B' - v_{rB}' dt$, and the shell area can be expressed in terms of angles of tangents to the side wall at the inner and external edges γ_A, γ_B, respectively. The condition of area equality may then be written as

$$\rho_\alpha'' \left(\sqrt{\rho_\alpha''^2 - r_A^2} - \sqrt{\rho_\alpha''^2 - r_B^2} \right) = \rho_\alpha' \left(\sqrt{\rho_\alpha'^2 - r_A^2} - \sqrt{\rho_\alpha'^2 - r_B^2} \right), \tag{4.5}$$

where

$$\rho'_\alpha = \frac{r_A}{\sin\gamma'_A} \qquad \text{and} \qquad \rho''_\alpha = \frac{r_A}{\sin\gamma''_A}.$$

From this condition, the value of γ''_A can be calculated numerically. For each point i of the considered n points of the side wall, we have $\gamma_i = \gamma_{i-1} + \frac{\gamma_B - \gamma_A}{n}$ and $dz_i = dr_i \operatorname{tg}\gamma_i$.

In the second approximation, the value of r' is calculated numerically for each point on the basis of the position of three adjacent points. After determining the value of r', A is introduced to the relation $(4.4)_2$. Next, the conservation of material volume can be verified for each wall element and its z-position is corrected. The final r- and z-positions determine the form of the side wall after time dt.

Now we can calculate $dh_{1,2}/dr$, $d\sigma_{p1,2}/dr$, stresses are found from (2.15), and velocity v_r — from Eq. (2.14). In this calculation, the strain-hardening law is described, as previously, by the Eq. (3.3).

The numerical examples will also be found for a carbon steel sheet coated with zinc or tin, characterized by the previously evaluated constants C, φ_0, n.

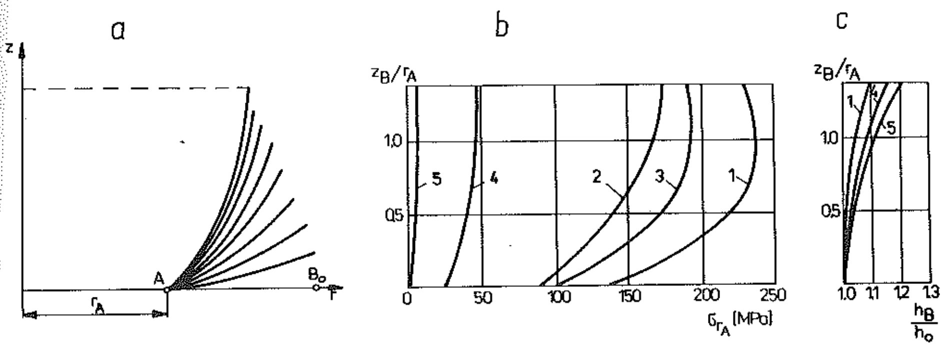

FIG. 6. 1 – coatless steel, 2 – middle steel layer of zinc coated steel, 3 – middle steel layer of tin coated steel, 4 – zinc coat, 5 – tin coat.

Figure 6 presents the results concerning the third approximation. The numerical calculations show that the deformation of the shell's side wall is almost independent of the material properties, and it is practically the same for material with or without coats. The side wall profile variation in time (Fig. 6a) can be then considered as valid for all materials with a given d/D ratio. The draw stress variation depends mainly on the properties of the sheet metal and its coats (Fig. 6b). The action of coats not only reduces the draw stress in the middle steel layer, but also displaces the maximum of draw stress in time. This fact may be used to determine the optimum die angle.

During the initial stage of the process, the material remains in contact with the die at a very limited surface of the external edge only (Fig. 5a). If the slope of the side wall γ_B attains the value of the die angle α, the surface of contact increases and a considerable friction force appears (Fig. 5b). This contact should take place after the deformation force reaches its maximum and starts to decrease. Therefore, the die angle can not be greater than the slope of the side wall at the external ege γ_B at the moment of maximum draw stress. If this maximum of force takes place just after the beginning of the deformation process, the admissible die angle is relatively great and the profile of the die may be short.

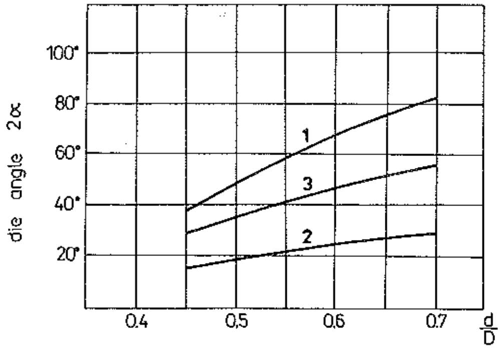

FIG. 7. 1 – coatless steel, 2 – zinc coated steel, 3 – tin coated steel.

Performing the calculations for different d/D ratios, we have determined the maximum die angle values for steel both with and without coats (Fig. 7). From this point of view, the presence of coats is not advantageous for the deep drawing process because it requires longer dies.

Figure 6c shows the change of the maximum relative thickness of the middle steel layer and zinc or tin coats, this maximum appearing on the external edge of the shell. The increments of relative thickness and strain intensity are greater in the zinc and tin coats than in the middle steeel layer.

5. FINAL REMARKS

Solution of the plane stress deep drawing problem obtained by the method of characteristics with regard to the influence of superficial coats on sheet metal may prove to be useful. To this end, a very simple model of the stress state acting on the layers, including the kinematic condition of equal radial velocity of all layers was proposed. With these assumptions, the equations of characteristics take into account the stresses in the layers and their physical properties such as the normal anisotropy factor and the strain hardening law. In this way, the stresses in the layers and the behaviour of layers (e.g. growth of thickness, strain intensity) can be estimated theoretically; this problem is difficult to solve by other methods.

The numerical examples show that the zinc or tin coats placed on the steel sheet reduce the draw stress in the middle steel layer, and that the thickness and strain intensity of superficial coats increase during the deformation process.

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Streszczenie

ZAGADNIENIE PLASTYCZNEGO KSZTAŁTOWANIA, W WARUNKACH PŁASKIEGO STANU NAPRĘŻENIA, NACZYŃ CYLINDRYCZNYCH Z BLACH WARSTOWYCH

Metodę charakterystyk zastosowano do rozwiązania osiowo-symetrycznego zagadnienia plastycznego płynięcia blachy warstwowej przy utrzymaniu warunków płaskiego stanu naprężenia, pomimo istnienia różnych warstw. Własności materiałów warstw zostały zmodelowane przez wprowadzenie różnych wskaźników anizotropii Lankforda i różnych krzywych wzmocnienia dla poszczególnych warstw. Wyprowadzone związki wzdłuż charakterystyk zastosowano do rozwiązania procesu tłoczenia naczyń cylindrycznych w matrycy płaskiej i stożkowej bez dociskacza. Przykłady liczbowe ilustrują wpływ powłok z cynku i cyny na naprężenia i odkształcenia przy tłoczeniu blach stalowych powlekanych.

Резюме

ЗАДАЧА ПЛАСТИЧЕСКОГО ФОРМИРОВАНИЯ, В УСЛОВИЯХ ПЛОСКОГО НАПРЯЖЕННОГО СОСТОЯНИЯ, ЦИЛИНДРИЧЕСКИХ СОСУДОВ ИЗ СЛОИСТЫХ ЖЕСТЕЙ

Метод характеристик применен для решения осесимметричной задачи пластического течения слоистой жести, при поддержании условий плоского напряженного состояния, несмотря на существование разных слоев. Свойства материалов слоев моделированы путем введения разных показателей анизотропии Ланкфорда и разных кривых упрочнения для отдельных слоев. Выведенные соотношения вдоль характеристик применены для решения процесса штамповки цилиндрических сосудов в плоских и конических матрицах без прижима. Числовые примеры иллюстрируют влияние оболочек из цинка и олова на напряжения и деформации, при штамповке стальных жестей с покрытиями.

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