ENGINEERING TRANSACTIONS • *Enging. Trans.* • **39, 3-4, 351-367, 1991**
Polish Academy of Sciences • Institute of Fundamental Technological Research

SIMULATION ANALYSIS OF THE INFLUENCE OF VARIATION IN SOME SELECTED DESIGN AND CONTROL PARAMETERS ON THE ACCELERATION TIME OF A TURBOJET ENGINE

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The equation governing the acceleration time of a turbojet engine is presented in an implicit form; expansion into power series contains first order partial derivatives. The derivatives represent the "sensitivity measure" of the acceleration time to the variation of a) – the design parameters of the compressor (stability margin, moment of inertia, efficiency, compression ratio), b) – technical conditions (deformation of blade profiles, due to erosion, turbine blade clearance), c) – service conditions due to flight speed and altitude. The relations derived enable the wide range simulation analysis of the problem.

1. Introduction

The acceleration time was shown in [4, 9 and 11] to be one of the most important operation parameters of a turbojet engine, its importance being still greater in the case of military aircraft. Although it is not the only quantity which can be used as a criterion for optimization of the design features of a turbojet engine, the sensitivity of the acceleration time to a variation in design and operation parameters with reference to their design values should be known in view of its importance in cases such as those described in [15], for instance. Single-flow, single-rotor engines being still used for military aircraft (although they are gradually superseded by turbofan engines [2, 10]), it is this particular class of engines which will be the subject of the present considerations.

To the group of design factors influencing the acceleration time of turbojet engines belong

a) *factors connected with the design parameters of the engine, namely* the theoretical compression ratio Π_s^*,

theoretical air flow intensity $\dot{m}$,
theoretical efficiency of the compressor η_s^*,
engine speed range from idling n_{bj} to maximum $n_{\max}$,
polar mass moment of inertia of the rotor J,

b) *factors resulting from the way in which the air passage through the engine is controlled*
air bleed,
adjustable nozzle blades of the inlet or outlet guide vanes
of the compressor or both,
propulsion nozzle with variable section,
active control of the turbine blade clearance.

To the group of operation factors belong
flying speed,
flight altitude,
variation of the surface condition of the air passage through the engine as a result of deformation of blade profiles of the compressor due to erosion [14], dust sedimentation [6] or mechanical damage to blades.

It is obvious that there is a relation between design and operation parameters, which will be taken into account in what follows.

The subject of the present considerations will be the model of acceleration of a single-flow turbojet engine as described in [11].

2. FUNDAMENTAL ASSUMPTIONS

The acceleration time can be described by an implicit function in the form

$$t = f(K_T;\ K_P;\ K_Z;\ Ma;\ H;\ n)\,, \tag{2.1}$$

where K_T – dynamic coefficient, K_Z – coefficient of relative stability margin of the compressor, K_P – power coefficient, n – engine speed, H – flight altitude, Ma – flying speed.

The parameters of the function (2.1) depend on those of the engine design, control and the flight conditions of the aeroplane. Hence, in agreement with [11], we can write

$$\begin{aligned} K_T &= f(J;\ \eta_s^*;\ \Pi_s^*;\ \dot{m})\,, \\ K_Z &= f(Z_u;\ Z_p)\,, \\ K_P &= f(\eta_s^*;\ \Pi_s^*)\,, \end{aligned} \tag{2.2}$$

where J – mass moment of inertia of the rotor, η_s^* – compressor efficiency, Π_s^* – compression ratio, $\dot{m}$ – air flow intensity, Z_u – coefficient of compressor stability range in steady operation, Z_p – coefficient of minimum stability margin of the compressor during acceleration.

The minimum admissible coefficient of stability of the compressor during acceleartion may vary with use of the engine. This variation results from erosion deformation of blade profiles and non-uniformity of the temperature, pressure and air speed in the inlet section of the compressor or a variation in the Reynolds number of flowing air in the case of climbing, for instance, and, possibly, a variation in blade clearance under varying flight conditions.

The coefficient of stability of the compressor in steady state is determined by the cooperation of engine subassemblies and may undergo a change as a result of air bleeding from the compressor, a change of the cross-sectional area of the propelling nozzle or the engine control by adjusting the guide vanes of the compressor. As a result, the relation for the coefficient of stability margin of the compressor under steady state conditions and the minimum coefficient of stability margin during an acceleration process can be expressed thus

$$\begin{aligned} Z_u &= f\left(\nu;\ \bar{l}_s;\ \varphi;\ \bar{A}_5\right)\ , \\ Z_p &= f\left(\mathrm{Re};\ \bar{\delta}\right)\ , \end{aligned} \tag{2.3}$$

where ν – parameter determining the quantity of air bled, $\bar{l}_s$ – distribution ratio of the work done by the compressor during the air bleeding operation, φ – angle of setting of inlet or outlet guide vanes or both, $\bar{A}_5$ - relative cross-sectional area of the propelling nozzle (as referred to the design value), Re – Reynolds number, $\bar{\delta}$ – relative compressor blade clearance (as referred to the blade length).

For small deviations of particular parameters we obtain, by expanding the relation (2.1) in Taylor's series, the following linear relation for the acceleration time of the engine

$$\begin{aligned} t = t_0 + \frac{\partial t}{\partial K_T} dK_T + \frac{\partial t}{\partial K_Z} dK_Z + \frac{\partial t}{\partial K_P} dK_P \\ + \frac{\partial t}{\partial \bar{n}} d\bar{n} + \frac{\partial t}{\partial H} dH + \frac{\partial t}{\partial Ma} dMa\ . \end{aligned} \tag{2.4}$$

On expanding the relation (2.2) in Taylor's series, we can represent them in the linear form

$$
\begin{aligned}
K_T &= K_{T0} + \frac{\partial K_T}{\partial J}\, dJ + \frac{\partial K_T}{\partial \eta_s^*}\, d\eta_s^* + \frac{\partial K_T}{\partial \Pi_s^*}\, d\Pi_s^* + \frac{\partial K_T}{\partial \dot{m}}\, d\dot{m}\,, \\
K_P &= K_{P0} + \frac{\partial K_P}{\partial \eta_s^*}\, d\eta_s^* + \frac{\partial K_P}{\partial \Pi_s^*}\, d\Pi_s^*\,, \qquad (2.5)\\
K_Z &= K_{Z0} + \frac{\partial K_Z}{\partial Z_u}\, dZ_u + \frac{\partial K_Z}{\partial Z_p}\, dZ_p\,.
\end{aligned}
$$

On expanding the relation (2.2) in Taylor's series, we obtain

$$
\begin{aligned}
Z_u &= Z_{u0} + \frac{\partial Z_u}{\partial \nu}\, d\nu + \frac{\partial Z_u}{\partial \bar{l}_s}\, d\bar{l}_s + \frac{\partial Z_u}{\partial \varphi}\, d\varphi + \frac{\partial Z_u}{\partial \bar{A}_5}\, d\bar{A}_5\,, \qquad (2.6)\\
Z_p &= Z_{p0} + \frac{\partial Z_p}{\partial \mathrm{Re}}\, d\mathrm{Re} + \frac{\partial Z_p}{\partial \bar{\delta}}\, d\bar{\delta}
\end{aligned}
$$

(parameters with subscript 0 concern the design values). On substituting Eqs.(2.5) and (2.6) into Eq.(2.4), the relation for the acceleration time of a turbojet engine takes the form

$$
\begin{aligned}
t = t_0 &+ \frac{\partial t}{\partial K_T}\left[\frac{\partial K_T}{\partial J}\, dJ + \frac{\partial K_T}{\partial \eta_s^*}\, d\eta_s^* + \frac{\partial K_T}{\partial \dot{m}}\, d\dot{m}\right] \qquad (2.7)\\
&+ \frac{\partial t}{\partial K_Z}\left[\frac{\partial K_Z}{\partial Z_u}\left(\frac{\partial Z_u}{\partial \nu}\, d\nu + \frac{\partial Z_u}{\partial \bar{l}_s}\, d\bar{l}_s + \frac{\partial Z_u}{\partial \varphi}\, d\varphi + \frac{\partial Z_u}{\partial \bar{A}_5}\, d\bar{A}_5\right)\right.\\
&\left. + \frac{\partial K_Z}{\partial Z_p}\left(\frac{\partial Z_p}{\partial\, \mathrm{Re}}\, d\mathrm{Re} + \frac{\partial Z_p}{\partial \bar{\delta}}\, d\bar{\delta}\right)\right] + \frac{\partial t}{\partial K_P}\left(\frac{\partial K_P}{\partial \eta_s^*}\, d\eta_s^* + \frac{\partial K_P}{\partial \Pi_s^*}\, d\Pi_s^*\right)\\
&+ \frac{\partial t}{\partial \bar{n}}\, d\bar{n} + \frac{\partial t}{\partial H}\, dH + \frac{\partial t}{\partial Ma}\, dMa\,.
\end{aligned}
$$

The partial derivatives in Eq.(2.7) are decisive for the sensitivity of the acceleration time to a variation of the independent variables [8, 16]. In [11] and [13] are mentioned relations describing the dependence of the dynamic and power coefficients and the coefficient of relative stability margin of the compressor on the design and operation parameters and in [17] – relations expressing the sensitivity of the acceleration time to a variaton in a particular independent variable. Those relations are illustrated graphically in Fig.1. By substituting them into (2.7) we obtain the acceleration time in a form conveninent for simulation studies:

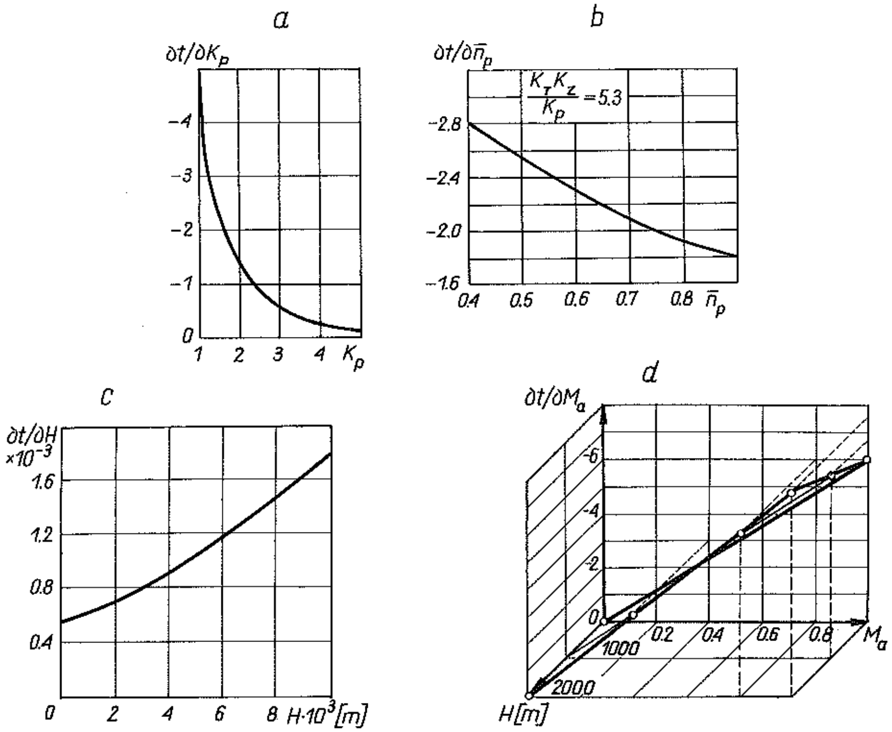

FIG. 1. Examples of variation of the sensitivity of the acceleration time and the coefficients of its components as functions of the independent variables.
a-d) The influence of the coefficient of power K_P , initial engine speed $\bar{n}_p$ and flight altitude and velocity (H, Ma) on the acceleration time.

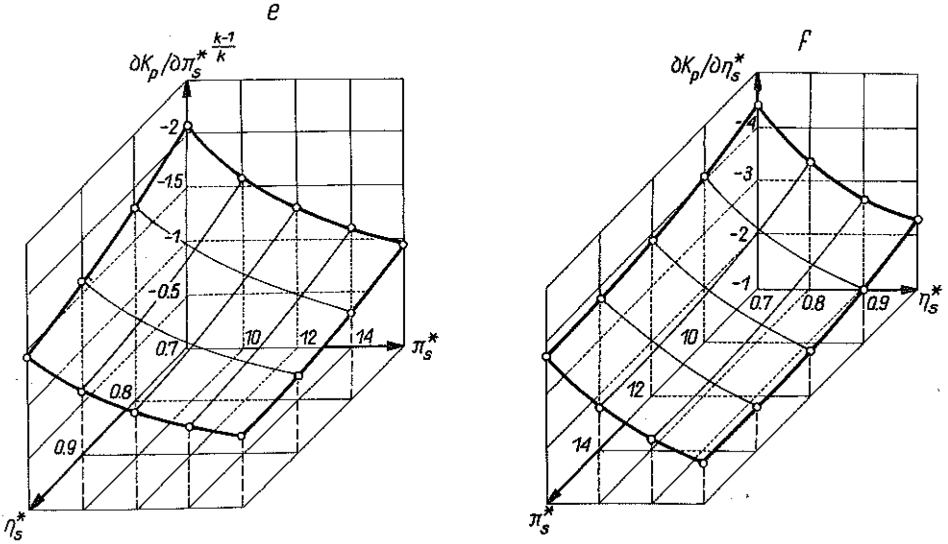

FIG. 1. e-f) The influence of the compression ratio and the efficiency of the compressor on the coefficient of power K_P.

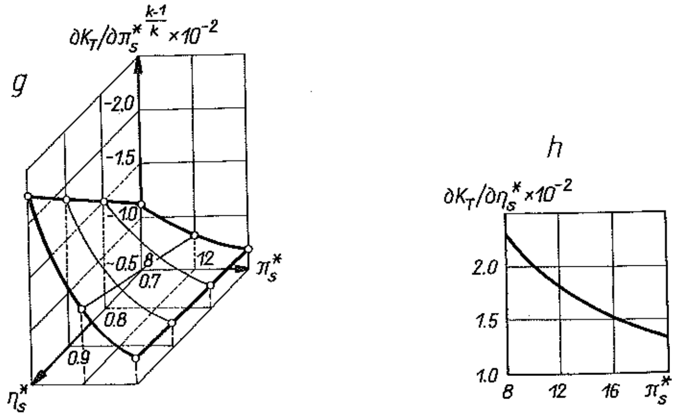

FIG. 1. g-h) The influence of the compression ratio and the efficiency of the compressor on the dynamic coefficient K_T.

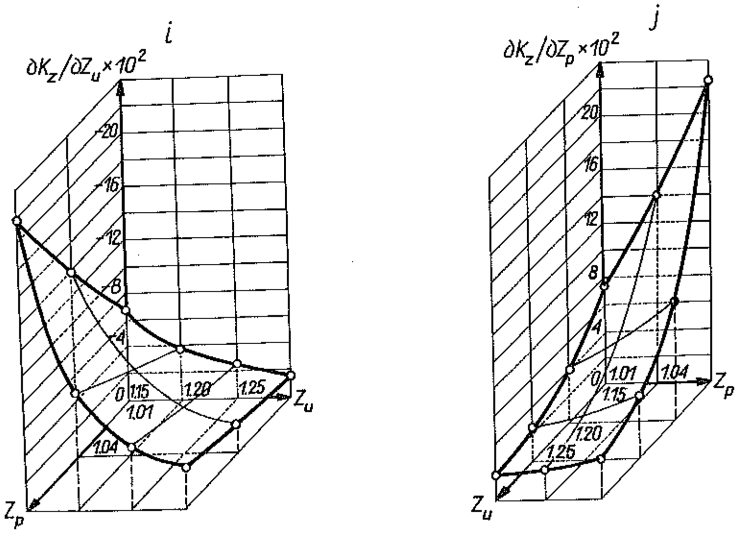

FIG. 1. i-j) The influence of the coefficient of stability margin of the compressor during steady operation Z_u and its minium value during the acceleration process Z_p on the relative coefficient of stability.

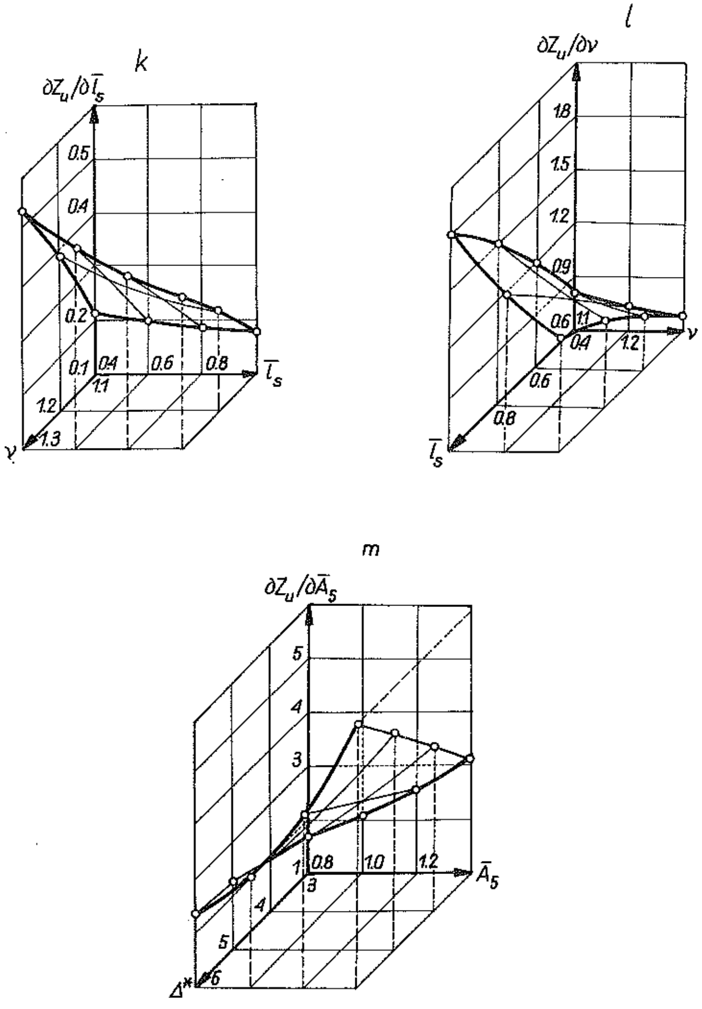

FIG. 1. k-m) The influence of the work distribution ratio of the compressor $\bar{l}_s$, the coefficient of air bleeding ν and the cross-sectional area of the propelling nozzle $\bar{A}_5$ on the coefficient of stability margin of the coefficient during steady operation.

$$t = t_0 + \Delta t\,, \qquad \Delta t = \bar{t}_K + \bar{t}_R + \bar{t}_E\,, \tag{2.8}$$

where $\bar{t}_K$ – the influence of a variation in design parameters of the compressor on the acceleration time, $\bar{t}_R$ – the influence of a variation in the control parameters on the acceleration time, $\bar{t}_E$ – the influence of a variation in flight conditions on the acceleration time, t_0 – the value of the acceleration time under the design conditions and

$$\bar{t}_K = \frac{dJ}{J} - \frac{(\Pi_s^* + d\Pi_s^*)^n - \Pi_s^{*n}}{(\Pi_s^{*n} - 1)} \left(1 + \frac{1}{\Pi_s^{*n}}\right) + 2\frac{dn_{\max}}{n_{\max}} ,$$

$$\text{(2.9)} \qquad \bar{t}_R = \frac{Z_u^n}{Z_p^n\,(Z_u^n - Z_p^n)} \left[(Z_p + dZ_p)^n - Z_p^n\right] - \frac{1}{Z_u^n - Z_p^n} \left[(Z_u + dZ_u)^n - Z_u^n\right] ,$$

$$\bar{t}_E = -\frac{2-k}{k} \frac{\bar{n}_p^{-2n}}{\bar{n}_k^{\frac{2-k}{k}} - \bar{n}_p^{\frac{2-k}{k}}} \left[(\bar{n}_p + d\bar{n}_p)^{\frac{2-k}{k}} - \bar{n}_p^{\frac{2-k}{k}}\right]$$

$$+\left[1.07E-4+1.4E-8H+1.08E-12H^2\right] dH - 1.2\left(1-\frac{H}{44300}\right)^{-4.756} Ma\,dMa,$$

respectively, where

$$n = \frac{k-1}{k} ,$$

$\bar{n}_k$ – relative engine speed at the end of the acceleration process, $\bar{n}_p$ – relative engine speed at the beginning of the acceleration process, k – isentropic exponent of air.

The total differential of the coefficient of stability margin in steady state for selected methods of the engine control is expressed by the relation

$$\text{(2.10)} \qquad dZ_u = Z_u^n \frac{\bar{c}_p\,\Delta^*\eta_s^*\eta_T^*(1 - \text{const}\,\bar{A}_5^m)\,[1 - \bar{c}_p\Delta^*\eta_s^*\eta_T^*(1 - \text{const})]}{[\bar{c}_p\Delta^*\eta_s^*\eta_T^*(1 - \text{const}\,\bar{A}_5^m) + \bar{l}_s\,(\nu - 1) + 1]^2} \times [\bar{l}_s d\nu + (\nu - 1)d\bar{l}_s]$$

$$+\frac{[\bar{c}_p\Delta^*\eta_s^*\eta_T^*\,(1 - \text{const}) + 1]\,\bar{c}_p\Delta^*\eta_s^*\eta_T^*\,\text{const}}{[\bar{c}_p\Delta^*\eta_s^*\eta_T^*(1 - \text{const}\,\bar{A}_5^m) + 1]^2} \left[(\bar{A}_5 + d\bar{A}_5)^m - \bar{A}_5^m\right] ,$$

where

$$m = \frac{k'-1}{k'} ,$$

k' – isentropic exponent of the combustion gases, $\bar{c}_p$ – ratio of specific heats of air, η_T^* – turbine efficiency, Δ^* – degree of heating of the flowing gas

$$\left(\Delta^* = \frac{T_3^*}{T_H^*}\right) ,$$

T_3^* – temperature before the turbine, T_H^* – temperature in the inlet section of the engine, const – a constant determined from the design conditions of the engine.

The method for evaluating $\bar{l}_s$ and ν is discussed in [12].

The influence of the Reynolds number and the setting angle of the guide vanes of the compressor are rejected in Eq.(2.9), no reliable literature data being available as regards the influence of those quantities on the stability margin of the compressor. In practice they can be taken into account by using regressive models based on a large number of measurements made during experimental investigations [1].

3. THE RESULTS OF SIMULATION STUDIES

The region of simulation studies is marked on the characteristic diagram in Fig.2. A variation in the blade clearance of the compressor or a decrease

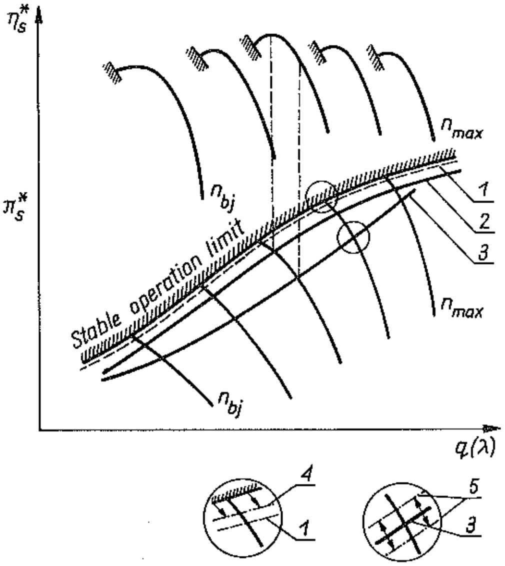

FIG. 2. The characteristic of an axial compressor:
1 – line of minimum stability margin of the compressor during the acceleration process, 2 – line of maximum efficiency of the compressor, 3 – line of steady operation of the engine, 4 – displacement of the stable operation limit, n_{bj} – idling speed, n_{max} – maximum speed, Π_s^* – compression ratio, η_s^* – compressor efficiency, $q(\lambda)$ – relative density of flow.

in Reynolds number of air flow through the engine result in a displacement of the limit of stable operation towards the line (4). In order to prevent the compressor from entering the instability region, the fuel supply system should be controlled so that the line of cooperation between engine subassemblies should move during the acceleration (1) process away from the line (4) by a distance resulting from the displacement of the stability limit to the line (4). This will ensure the minimum stability margin of the compression being preserved during the acceleration process of the engine. With reference to the design conditions this means an increase in the stability margin by a certain value dZ_p. Figure 4 shows the influence of an increase in the minimum coefficient of stability margin during the acceleration process as referred to the difference between the design values Z_u and Z_p on the increase in the acceleration time of the engine. From the diagram it follows that an increase in dZ_p by 50% results in a prolongation of the acceleration time by about 100%. An increase in the minimum coefficient of stability margin of the compressor may also result from incorrect control of the fuel supply system, so that the increase in the degree of preheating is too low as referred to the design conditions.

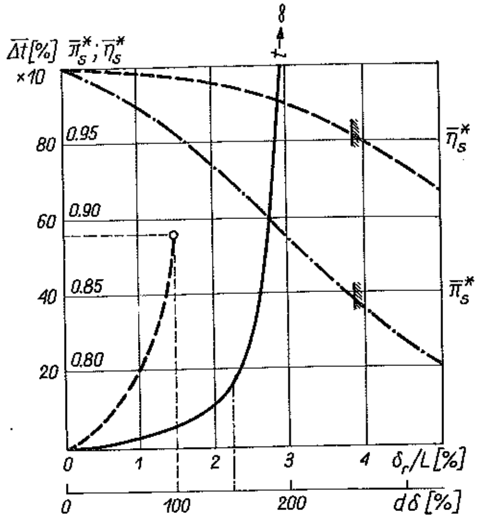

FIG. 3. The influence of the blade clearance of the compressor on the acceleration time of the engine, δ_r – blade clearance, L – blade length, $d\delta$ – increase in blade clearance.

An increase in blade clearance of the compressor results also in a decrease in the minimum coefficient of stability margin in agreement with the former description. According to [1] and [3], the average value of the blade clearance of a compressor is contained within the interval $\delta_r = (0.007...0.045)L$ (L – blade length), higher values being those for stages situated near the outlet. An initial state of $\delta_r/L = 1.5\%$ is assumed for the considerations. This means that an increase in blade clearance of the compressor by 1.5% means a 100% increase in blade clearance as referred to the initial state. From the form of the dashed line in Fig.3 it is inferred that an increase in blade clearance by 100% will result in the acceleration time of the engine being prolonged by about 55%. This time increases rapidly, if the increase in blade clearance exceeds 150% of its initial value. For small values of that clearance the increase in acceleration time does not exceed 20%.

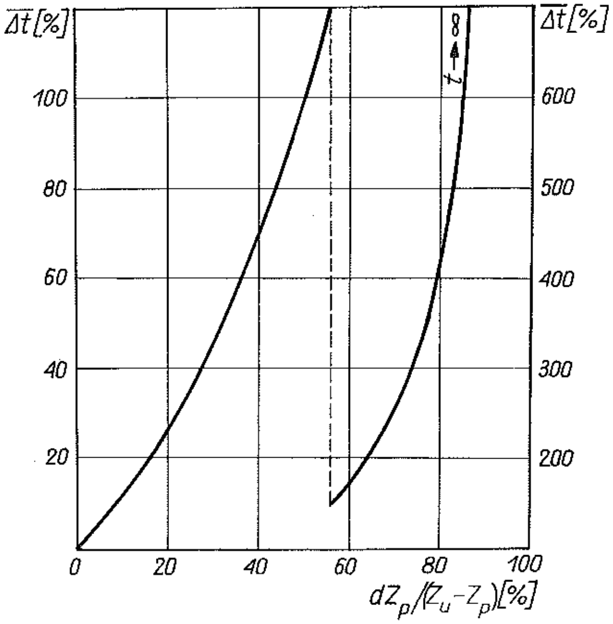

FIG. 4. The influence of an increase in the minimum stability margin of the compressor on the increase in the acceleration time.

Under the conditions of operation in a zone of high dustiness structural parts of the engine undergo damage by erosion. The compressor is an engine element which is exposed in a particular manner to the action of dust. Deformation of the air passage through the compressor results in a drop in the compression ratio efficiency, air flow rate and operation stability margin (Fig.5). According to [7], if the total amount of dust which has passed through the compressor is some 110 kg, the deformation of the passage through the compressor is so considerable that the stable operation limit is reached (for an engine of TW2 or TW3 class, with $\dot{m}$=8 kg/s).

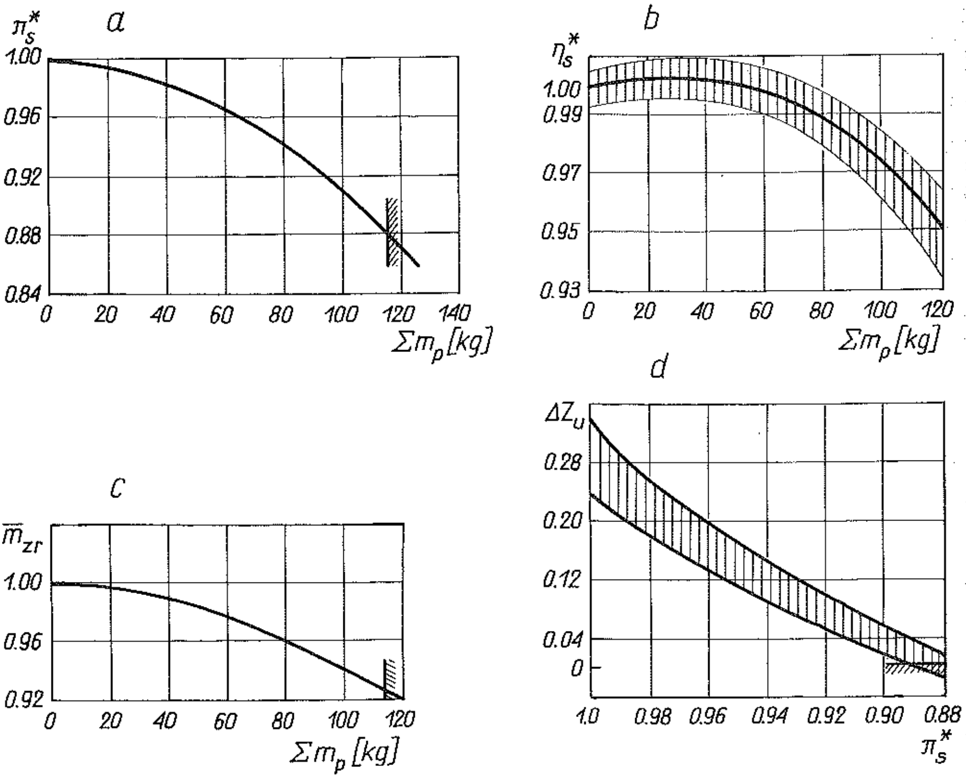

FIG. 5. The influence of the total mass of dust passing through the engine on: a) the compression ratio, b) compression efficiency, c) reduced flow rate through the compressor, d) the influence of a reduction in the compression ratio as a result of dust flowing through the compressor on the stable operation margin.

Figure 6 shows the dependence of the increase in the acceleration time of the engine on the total mass of dust which has passed through the engine. The rate of this increase becomes higher when the total mass of dust approaches the amount corresponding to the limit of stable operation. The character of the influence of the total mass of dust which has passed through the engine on the acceleration time of the latter is similar to that of the blade clearance.

The action of air bleeding from the compressor and that of varying the cross-sectional area of the propelling nozzle result in a change of position of

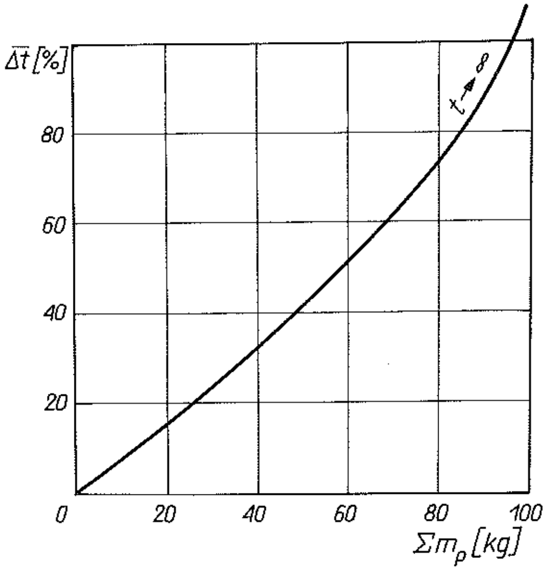

FIG. 6. The influence of the total mass of dust passing through the engine on the increase in the acceleration time.

the line of cooperation of engine subassemblies (Fig.3, line 2), thus causing a variation in the coefficient of stability margin during stationary operation. Figure 7 shows the influence of the increase in cross-sectional area of the propelling nozzle. An increase of about 20% of the design value results in the acceleration time being reduced by about 23%, which is in agreement with the data presented in [16], for instance.

As a result of some production reasons the real characteristics of the engine deviate from the design characteristics, which influences the acceleration time of the engine. Figure 8 shows the influence of deviation of the compression ratio from its design value on the acceleration time of the engine.

The influence is evaluated for an engine with an identical mass moment of inertia of the rotor.

Under such assumptions the acceleration time decreases with increasing compression ratio, because the dynamic coefficient decreases at a higher rate than that of power, and it has been shown in [11] that the acceleration time is proportional to the ratio of those two coefficients

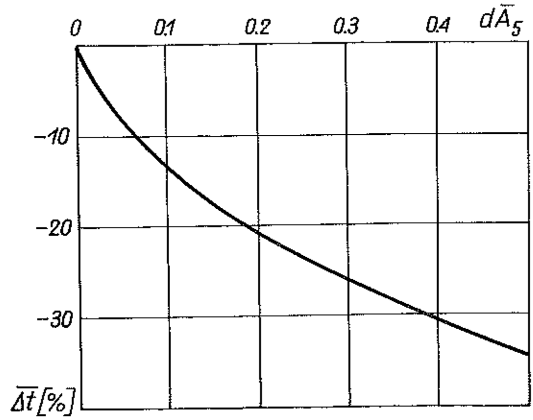

FIG. 7. The influence of an increase in the cross-sectional area of the propelling nozzle on the decrease in the acceleration time of the engine.

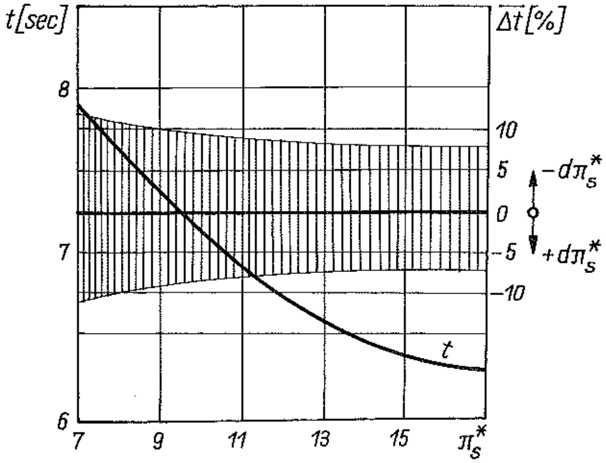

FIG. 8. The influence of the design compression ratio on the acceleration time of the engine and the influence of its 10% variation on the increase of that time $d\Pi_s^* = 0.1\Pi_s^*$.

$$t \sim \frac{K_T}{K_P}\,.$$

For high values of the compression ratio and the efficiency of the compressor, the coefficient of power is practically constant for high values of the compression ratio and the efficiency of the compressor, for $\Pi_s^* > 10$ in practice. A variation in the compression ratio of 10% with reference to the design value of the compression ratio results in a variation in the acceleration time for $\Pi_s^* > 10$ by about $\mp$ 8%.

4. INFERENCES

1. The linear relationship proposed here for expressing the acceleration time of a turbojet engine is found to be a tool which may be helpful for estimating the influence on that time of various design and operation factors.

2. The relation (2.8) completed with Eqs.(2.9) and (2.10) enable us to simulate the influence of the fuel supply system on the acceleration time of a turbojet engine.

3. The influence of the blade clearance of the compressor is particularly important if it exceeds its initial value by more than 50%.

4. If sufficient experimental data are available, the measurement of the acceleration time may be used in connection with the relation derived as a diagnostic signal.

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STRESZCZENIE

BADANIA SYMULACYJNE WPŁYWU ZMIAN WYBRANYCH PARAMETRÓW KONSTRUKCYJNYCH I REGULACYJNYCH SILNIKÓW TURBINOWYCH NA CZAS AKCELERACJI

Równanie opisujące czas akceleracji turbinowego silnika odrzutowego przedstawiono jako funkcję uwikłaną, która po rozłożeniu w szereg Taylora zawiera pierwsze pochodne cząstkowe. Pochodne te są miarą "czułości" czasu akceleracji na zmianę: a) parametrów konstrukcyjnych sprężarki (zapas statecznej pracy, moment bezwładności, sprawność i spręż); b) stanu technicznego sprężarki (erozyjne zniekształcenie profilu łopatek, luz wierzchołkowy łopatek); c) warunków eksploatacji związanych z prędkością i wysokością lotu. Otrzymane zależności umożliwiają badania symulacyjne w szerokim zakresie.

РЕЗЮМЕ

ИМИТАЦИОННЫЕ ИССЛЕДОВАНИЯ ВЛИЯНИЯ ИЗМЕНЕНИЙ ИЗБРАННЫХ КОНСТРУКЦИОННЫХ И РЕГУЛЯЦИОННЫХ ПАРАМЕТРОВ ТУРБИННЫХ ДВИГАТЕЛЕЙ НА ВРЕМЯ АКСЕЛЕРАЦИИ

Уравниение, описывающее время акселерации турбинного реактивного двигателя, представлено как неявную функцию, которая, после разложения в ряд Тейлора, содержит первые частные производные. Эти производные являются мерой "чувствительности" времени акселерации на изменение конструкционных параметров компрессора (запас устойчивой работы, момент инерции, эффективность и сжатие), технического состояния компрессора (эрозионное искажение профиля лопастей, верхний зазор лопастей), условий эксплуатации, связанных со скоростью и высотой полета. Полученные зависимости дают возможность исследовать имитационным образом явления в широком интервале.

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Received November 9, 1990.