ENGINEERING TRANSACTIONS • *Enging. Trans.* • 39, 2, 255-270, 1991
Polish Academy of Sciences • Institute of Fundamental Technological Research

INFLUENCE OF BOUNDARY CONDITIONS ON DAMPING PROPERTIES OF FLUID-SATURATED POROUS MATERIALS

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The object of this paper is the analysis of damping properties of elastic porous media filled with a viscous fluid and, in particular, examination of the influence of boundary conditions and material constants of the medium on the effect of damping. The problem is illustrated by an example of damped vibrations of a fluid-saturated porous cylinder. The attention is concentrated on the analysis of the damping coefficient and the dependence of this coefficient on the boundary conditions and the material constants. The examples analysed show that there exists a broad possibility of controlling this coefficient by a suitable choice of the material constants and the boundary conditions.

1. Introduction

Mechanical properties of fluid-saturated porous materials depend not only on physical properties of the components but also on the structure of porous material. It was ascertained in Ref. [1], where vibrations of a fluid-saturated porous cylinder were analysed, that the damping of vibrations depends not only on Darcy constant which for a given medium is a function of the fluid viscosity, permeability, and porosity, but also on the boundary conditions which stimulate the pore pressure and the amplitude of fluid displacement with respect to the skeleton.

The main aim of this paper is just a broader analysis of the influence of boundary conditions on damped vibrations of a cylinder consisting of porous elastic solid filled with a viscous fluid. We concentrate our attention on two kinds of boundary conditions: firstly, the stress in fluid on the boundary is assumed to be constant, and secondly, the value of this stress is varied proportionally to the value of the fluid displacement on the boundary.

The solution for vibrations of fluid-saturated porous cylinder is performed here, similarly as in Ref. [1], in the form of a series expanded with respect to two different sets of eigenfunctions. The duality of the eigenfunctions results from the biwave character of equations of motion.

Generally, each harmonic component of vibrations (mode shape) is characterized by its own damping coefficient. Only in the case of free vibrations with free boundary surfaces, the damping coefficient is common for all mode shapes of the first set of eigenvalues and common for all mode shapes of the second set of eigenvalues but, generally, different for these two sets.

The efficiency of damping of the first six mode shapes for each set of eigenvalues, being a function of pore pressure and the amplitude of fluid displacement that are directly influenced by the boundary condition, is examined. The analysis was carried out for various combinations of the material constants. The results obtained have shown that the mode shapes of slow vibrations, i.e. for the first set of eigenvalues, are generally more strongly damped than those for the fast vibrations, i.e. for the second set of eigenvalues, but there is a possibility of such a selection of the boundary values and the material constants that the relations become reversed.

2. Fundamental equations

The objective of the study are the damping properties of an elastic porous medium filled with viscous fluid and, in particular, some possibilities of control of these properties through adequate selection of the boundary conditions and constants of the medium.

It was ascertained in Ref. [1], where the vibrations of a porous cylinder filled with a viscous fluid were analysed, that the damping of vibrations depends not only on the parameter called the damping coefficient (Darcy constant) but it depends also on other parameters resulting from the adequate realization of boundary conditions and suitable choice of material constants. In this paper, we have focused our attention on these parameters.

The general solution for the vibrations of fluid-saturated porous cylinder has the following form [1]:

$$
\begin{aligned}
u_s(x,t) &= \sum_{n=1}^{\infty} \left[U_{sn}^{(1)}(x) T_n^{(1)}(t) + U_{sn}^{(2)}(x) T_n^{(2)}(t) \right], \\
u_f(x,t) &= \sum_{n=1}^{\infty} \left[U_{fn}^{(1)}(x) T_n^{(1)}(t) + U_{fn}^{(2)}(x) T_n^{(2)}(t) \right],
\end{aligned} \tag{2.1}
$$

where u_s and u_f are the displacement of the skeleton, and of the fluid in pores, respectively. Eigenfunctons for the skeleton $U_{sn}^{(1)}$ related to the slow vibrations (first set of eigenvalues) and $U_{sn}^{(2)}$ related to the fast vibrations (second set of eigenvalues), and the corresponding eigenfunctions for the fluid $U_{fn}^{(1)}$ and $U_{fn}^{(2)}$ have the forms

$$
\begin{aligned}
U_{sn}^{(k)}(x) &= A_{1n}^{(k)} \exp\left[\left(i\omega_n^{(k)}/c_w\right)x\right] + A_{2n}^{(k)} \exp\left[\left(-i\omega_n^{(k)}/c_w\right)x\right] \\
&\quad + A_{3n}^{(k)} \exp\left[\left(i\omega^{(k)}/c_s\right)x\right] + A_{4n}^{(k)} \exp\left[\left(-i\omega_n^{(k)}/c_s\right)x\right], \\
U_{fn}^{(k)}(x) &= \delta_1 \left\{A_{1n}^{(k)} \exp\left[\left(i\omega_n^{(k)}/c_w\right)x\right] + A_{2n}^{(k)} \exp\left[\left(-i\omega_n^{(k)}/c_w\right)x\right]\right\} \\
&\quad + \delta_2 \left\{A_{3n}^{(k)} \exp\left[\left(i\omega_n^{(k)}/c_s\right)x\right] + A_{4n}^{(k)} \exp\left[\left(-i\omega_n^{(k)}/c_s\right)x\right]\right\}.
\end{aligned}
\tag{2.2}
$$

for $k = 1, 2$, where

$$
\begin{aligned}
\delta_1 &= -(1 - c_w^2/a_s^2)/a_1, \qquad \delta_2 = -(1 - c_s^2/a_s^2)/a_1, \\
c_w^2 &= 0.5\{a_s^2 + a_f^2 - [(a_s^2 - a_f^2)^2 + 4a_1 a_2 a_s^2 a_f^2]^{0.5}\}, \\
c_s^2 &= 0.5\{a_s^2 + a_f^2 + [(a_s^2 - a_f^2)^2 + 4a_1 a_2 a_s^2 a_f^2]^{0.5}\}, \\
a_s^2 &= (2N + A)/\rho_s, \qquad a_f^2 = R/\rho_f, \\
a_1 &= Q/(2N + A), \qquad a_2 = Q/R.
\end{aligned}
\tag{2.3}
$$

Here N, A, R, Q denote the material constants of a fluid-saturated porous medium, introduced to the theory in Ref. [4], ρ_s and ρ_f stand for the partial mass densities for the skeleton and fluid, and quantities c_w and c_s are called the slow wave velocity and the fast wave velocity, respectively. Eigenvalues $\omega_n^{(1)}$ and $\omega_n^{(2)}$ are the n-th natural frequencies of vibrations of the slow wave (first set of eigenvalues) and of the fast wave (second set of eigenvalues), respectively. Letter i in Eq. (2.2) denotes the imaginary unit.

Eigenfunctions have to satisfy the orthogonality condition of the form [1, 2]

$$
\begin{aligned}
&\left[\omega_n^{(k)2} - \omega_m^{(l)2}\right] \int_0^h \left[\rho_s U_{sn}^{(k)}(x) U_{sm}^{(l)}(x) + \rho_f U_{fn}^{(k)}(x) U_{fm}^{(l)}(x)\right] dx \\
&\quad + \left\{\left[(2N + A) U_{sn}^{(k)\prime}(x) + Q U_{fn}^{(k)\prime}(x)\right] U_{sm}^{(l)}(x) + \left[Q U_{sn}^{(k)\prime}(x)\right.\right. \\
&\quad \left. + R U_{fn}^{(k)\prime}(x)\right] U_{fm}^{(l)}(x) - \left[(2N + A) U_{sm}^{(l)\prime}(x) + Q U_{fm}^{(l)\prime}(x)\right] U_{sn}^{(k)}(x) \\
&\quad \left.\left. - \left[Q U_{sm}^{(l)\prime}(x) + R U_{fm}^{(l)\prime}(x)\right] U_{fn}^{(k)}(x)\right\}\right|_0^h = 0,
\end{aligned}
\tag{2.4}
$$

where $k, l = 1, 2; n, m = 1, 2, 3\ldots,$ primes over the symbols denote ordinary derivatives, and h is the height of the porous cylinder.

A generalized coordinate $T_n^{(k)}(t)$ results from the solution of the following differential equation, [1]:

(2.5) $$\ddot{T}_n^{(k)}(t) + 2b_n^{(k)}\dot{T}_n^{(k)}(t) + \omega_n^{(k)2}T_n^{(k)}(t) = 0,$$

in which

(2.6) $$2b_n^{(k)} = bH_n^{(k)}/M_n^{(k)},$$

stands exactly for the damping coefficient of the n-th mode shape. This is the quantity which is examined here with great care. The constant b which occurs in Eq. (2.6), is sometimes called the Darcy constant. This constant depends on the fluid viscosity μ, on the porosity f, and on the permeability, k:

(2.7) $$b = \mu f^2/k.$$

However, we are interested first and foremost in the influence of the parameters $H_n^{(k)}$ and $M_n^{(k)}$ on the damping coefficient of the n-th mode shape. The first one is a function of square of the amplitude of the relative fluid displacement, [1]:

(2.8) $$H_n^{(k)} = \int\limits_0^h \left[U_{fn}^{(k)}(x) - U_{sn}^{(k)}(x)\right]^2 dx.$$

The second one expresses a generalized mass per unit area of the cylinder cross-section and it results from the orthogonality condition for the given boundary conditions.

Since the shape of eigenfunctions depends on the boundary conditions, we can say that both $H_n^{(k)}$ and $M_n^{(k)}$ depend indirectly also on the boundary conditions.

We have examined the parameter $H_n^{(k)}/M_n^{(k)}$ termed here "coefficient amplifying the damping effect" for the examples presented below.

3. Free vibrations of the fluid-saturated porous cylinder

First, we shall analyse the damping coefficient (2.6) for the simplest case, i.e. for the free vibration of the unloaded cylinder (Fig. 1). The boundary conditions in this case are as follows:

(3.1) $$\begin{aligned} \sigma_s(h,t) &= 0,\\ \sigma_f(h,t) &= 0,\\ u_s(0,t) &= 0,\\ u_f(0,t) &= 0, \end{aligned}$$

where σ_s and σ_f denote here the axial forces in the cylinder, which are transmitted by the skeleton and by the fluid, respectively, related to the total cross-sectional area A_0 of the cylinder.

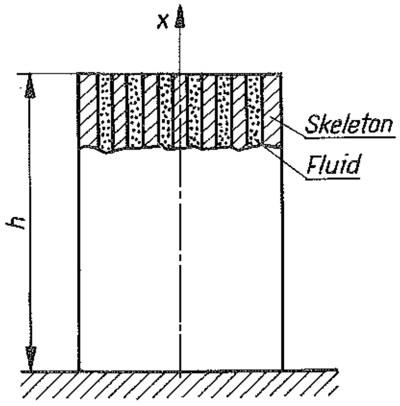

FIG. 1.

The relationships between stresses and derivatives of the displacements for the one-dimensional problem are as follows:

$$\begin{aligned} \sigma_s &= (2N + A)u_{s,x} + Qu_{f,x}, \\ \sigma_f &= Qu_{s,x} + Ru_{f,x}. \end{aligned} \tag{3.2}$$

Two sets of eigenvalues (natural frequencies) are obtained for the given boundary conditions, [3]:

$$\begin{aligned} \omega_n^{(1)} &= (2n-1)\Pi c_w/2h, \\ \omega_n^{(2)} &= (2n-1)\Pi c_s/2h. \end{aligned} \tag{3.3}$$

The eigenfunctions for these boundary conditions are, [3],

$$\begin{aligned} U_{sn}^{(1)}(x) &= C\sin(\omega_n^{(1)}x/c_w), & U_{sn}^{(2)}(x) &= C\sin(\omega_n^{(2)}x/c_s), \\ U_{fn}^{(1)}(x) &= C\delta_1\sin(\omega_n^{(1)}x/c_w), & U_{fn}^{(2)}(x) &= C\delta_2\sin(\omega_n^{(2)}x/c_s), \end{aligned} \tag{3.4}$$

where C is an arbitrary constant (here $C = 1$ was assumed). The orthogonality condition for the above boundary conditions is reduced to

$$\int\limits_0^h \left[\rho_s U_{sn}^{(k)}(x)U_{sm}^{(l)}(x) + \rho_f U_{fn}^{(k)}(x)U_{fm}^{(l)}(x)\right] dx \tag{3.5}$$

$$= \begin{cases} 0 & \text{for } k \neq l \text{ or } n \neq m, \\ M_n^{(k)} & \text{for } k = l \text{ and } n = m, \end{cases}$$

where

$$M_n^{(k)} = \int_0^h \left[\rho_s (U_{sn}^{(k)}(x))^2 + \rho_f (U_{fn}^{(k)}(x))^2\right] dx. \tag{3.6}$$

Substituting the above eigenfunctions to the formulae (2.8) and (3.6), one obtains the following coefficients amplifying the damping effect for the slow vibrations:

$$H_n^{(1)}/M_n^{(1)} = (1-\delta_1)^2/(\rho_s + \rho_f \delta_1^2), \tag{3.7}$$

and for the fast vibrations:

$$H_n^{(2)}/M_n^{(2)} = (1-\delta_2)^2/(\rho_s + \rho_f \delta_2^2) \tag{3.7'}$$

for the n-th mode shape.

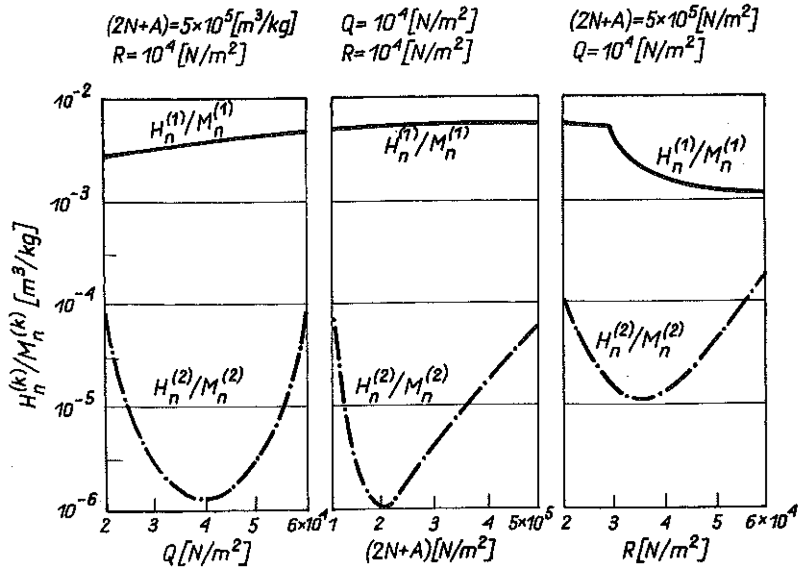

FIG. 2.

Figure 2 illustrates the dependence of $H_n^{(k)}/M_n^{(k)}$ on various values of material constants, $2N+A, R, Q$. Parameter $2N+A$ represents the elastic modulus of the porous solid, parameter Q expresses the dilatation coupling between the skeleton and the fluid, and parameter R characterizes the volume changes of the fluid. A detailed interpretation of these constants is given by BIOT and WILLIS [5]. The values of these constants were varied in a theoretically possible range in our numerical calculations.

It can be seen from Fig.2 that a suitable choice of the material constants changes the value of coefficient $H_n^{(k)}/M_n^{(k)}$ in a wide range. Particularly sensitivite to the changes of the constants is the coefficient $H_n^{(2)}/M_n^{(2)}$ connected with the fast vibrations.

4. Vibrations of the fluid-saturated porous cylinder, in which the pore pressure and the fluid displacement are stimulated by a spring

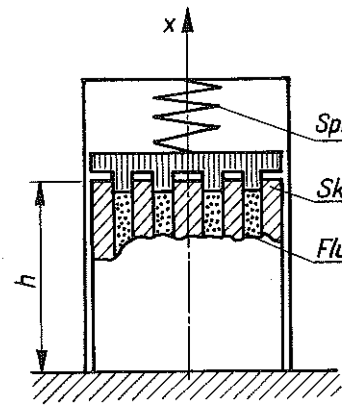

FIG. 3.

In order to illustrate how the boundary conditions influence the damping effect, we have considered a case in which the pressure as well as the amplitude of the fluid displacement on the boundary surface are stimulated by a spring characterized by constant k (Fig.3), called spring rate.

The boundary conditions for this case take the form

$$\begin{aligned} \sigma_s(h,t) &= 0, \\ A_0\sigma_f(h,t) &= ku_f(h,t), \\ u_s(0,t) &= 0, \\ u_f(0,t) &= 0. \end{aligned} \tag{4.1}$$

The eigenvalues are determined from the characteristic equation

$$\rho_s c_s c_w \omega_n^{(k)}(\delta_2-\delta_1)\cos\alpha_n^{(k)}\cos\beta_n^{(k)} + k(c_s\cos\alpha_n^{(k)}\sin\beta_n^{(k)} - c_w\cos\beta_n^{(k)}\sin\alpha_n^{(k)})/A_0 = 0. \tag{4.2}$$

The eigenfunctions corresponding to the particular mode shapes are expressed by trigonometric functions

$$
(4.3)\qquad
\begin{aligned}
U_{sn}^{(k)}(x) &= C_n^{(k)}[\sin(\omega_n^{(k)}x/c_w) + D_n^{(k)}\sin(\omega_n^{(k)}x/c_s)],\\
U_{fn}^{(k)}(x) &= C_n^{(k)}[\delta_1\sin(\omega_n^{(k)}x/c_w) + \delta_2 D_n^{(k)}\sin(\omega_n^{(k)}x/c_s)],
\end{aligned}
$$

where

$$
(4.4)\qquad
\begin{aligned}
D_n^{(k)} &= -(\rho_s+\rho_f\delta_1)c_w\cos\beta_n^{(k)}/(\rho_s+\rho_f\delta_2)c_s\cos\alpha_n^{(k)}),\\
\alpha_n^{(k)} &= \omega_n^{(k)}h/c_s, \qquad \beta_n^{(k)} = \omega_n^{(k)}h/c_w.
\end{aligned}
$$

$C_n^{(k)}$ is an arbitrary constant (here it is assumed that $C_n^{(k)} = 1$).

The orthogonality condition for the above eigenfunctions is the same as that in the previous section, i.e. it has the form Eq. (3.5).

The explicit form of expressions $H_n^{(k)}$ and $M_n^{(k)}$ is the following:

$$
(4.5)\qquad
\begin{aligned}
H_n^{(k)} = {}& (1-\delta_1)^2(h/2 - \sin\beta_n^{(k)}\cos\beta_n^{(k)})\\
&+(\sin\alpha_n^{(k)}\cos\beta_n^{(k)}/c_w - \sin\beta_n^{(k)}\cos\alpha_n^{(k)}/c_s)\\
&\times(1-\delta_1)(1-\delta_2)c_s^2c_w^2D_n^{(k)}/[\omega_n^{(k)}(c_w^2-c_s^2)]\\
&+(1-\delta_2)^2(h/2 - \sin\alpha_n^{(k)}\cos\alpha_n^{(k)})D_n^{(k)2},\\
M_n^{(k)} = {}& (\rho_s+\rho_f\delta_1^2)(h/2 - \sin\beta_n^{(k)}\cos\beta_n^{(k)})\\
&+(\sin\alpha_n^{(k)}\cos\beta_n^{(k)}/c_w - \sin\beta_n^{(k)}\cos\alpha_n^{(k)}/c_s)\\
&\times(\rho_s+\rho_f\delta_1\delta_2)c_s^2c_w^2D_n^{(k)}/[\omega_n^{(k)}(c_w^2-c_s^2)]\\
&+(\rho_s+\rho_f\delta_2^2)(h/2 - \sin\alpha_n^{(k)}\cos\alpha_n^{(k)})D_n^{(k)2}.
\end{aligned}
$$

The dependence of the coefficient amplifying the damping effect $2b_n^{(k)}/b = H_n^{(k)}/M_n^{(k)}$ for the n-th mode shape on the value of the spring rate k for different combinations of the material constants $2N+A, Q, R$ is presented in Fig.4. The coefficients amplifying the damping effect of the slow vibrations are represented by the solid line 1, and those amplifying the damping effect of the fast vibrations by the dashed line 2. It is seen from the graphs that all mode shapes belonging to various sets are damped with the same intensity. Only the mode shapes related to the fast vibrations are sensitive to the changes of the spring rate. In general, the modes of the slow vibrations are damped more strongly. However, certain combinations of the material constants may lead to a situation when the slow vibrations are damped less intensively than the fast ones.

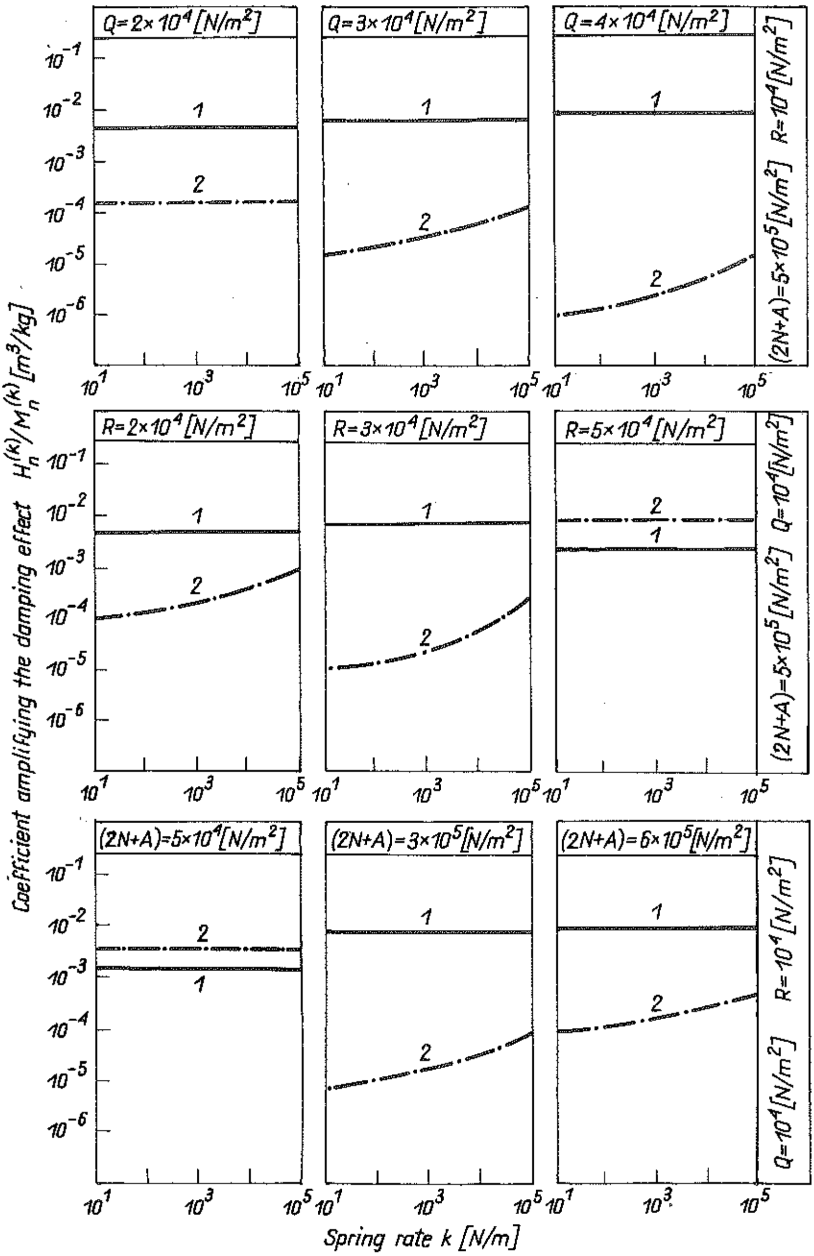
Q=2×10⁴ [N/m²]
Q=3×10⁴ [N/m²]
Q=4×10⁴ [N/m²]
R=2×10⁴ [N/m²]
R=3×10⁴ [N/m²]
R=5×10⁴ [N/m²]
(2N+A)=5×10⁴ [N/m²]
(2N+A)=3×10⁵ [N/m²]
(2N+A)=6×10⁵ [N/m²]
(2N+A)=5×10⁵ [N/m²] R=10⁴ [N/m²]
(2N+A)=5×10⁵ [N/m²] Q=10⁴ [N/m²]
Q=10⁴ [N/m²] R=10⁴ [N/m²]
Coefficient amplifying the damping effect H_n^(k)/M_n^(k) [m³/kg]
Spring rate k [N/m]

FIG. 4.

5. Vibrations of the porous cylinder with the rigid mass stimulating the pore pressure

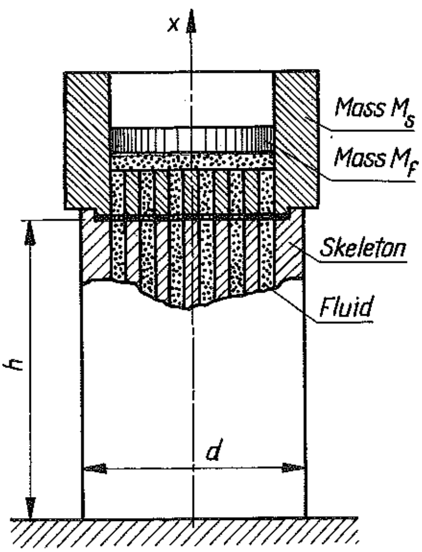

FIG. 5.

Let us now present an analysis of the coefficient amplifying the damping effect for the fluid-saturated porous cylinder, the base of which is kept fixed while the top is loaded by two concentrated masses: one mass in placed on the skeleton, and the second one on the fluid (Fig. 5). We assume that mass M_s is constant and it represents the mass of a technical device resting on the porous cylinder. The porous cylinder plays here the role of a vibroisolator. The mass M_f is to be changed here and its role is to stimulate the pore pressure.

The idea of stimulating the pore pressure may be realized also in another, quite different way. Our idea serves only as an example. The boundary conditions for the vibro-isolator presented above are the following:

$$
\begin{aligned}
A_0\sigma_s(h,t) &= -M_s\ddot{u}_s(h,t),\\
A_0\sigma_f(h,t) &= -M_f\ddot{u}_f(h,t),\\
u_s(0,t) &= 0,\\
u_f(0,t) &= 0.
\end{aligned}
\tag{5.1}
$$

For the above boundary conditions the eigenfunctions take the form

$$
\begin{aligned}
U_{sn}^{(k)} &= \rho_s[c_w\Omega_{1n}^{(k)}\sin\alpha_n^{(k)}x/h - c_s\Omega_{2n}^{(k)}\sin\beta_n^{(k)}x/h],\\
U_{fn}^{(k)} &= \rho_s[\delta_2 c_w\Omega_{1n}^{(k)} sin\alpha_n^{(k)}x/h - \delta_1 c_s\Omega_{2n}^{(k)}\sin\beta_n^{(k)}x/h].
\end{aligned}
\tag{5.2}
$$

where

$$
\begin{aligned}
\Omega_{1n}^{(k)} &= \cos\beta_n^{(k)} - m_s\beta_n^{(k)}\sin\beta_n^{(k)},\\
\Omega_{2n}^{(k)} &= \cos\alpha_n^{(k)} - m_s^{(k)}\alpha_n^{(k)}\sin\alpha_n^{(k)}.
\end{aligned}
\tag{5.3}
$$

The orthogonality condition for these eigenfunctions is expressed as follows:

$$
\begin{aligned}
&\int_0^h \left[\rho_s U_{sn}^{(k)}(x)U_{sm}^{(l)}(x) + \rho_f U_{fn}^{(k)}(x)U_{fm}^{(l)}(x)\right]dx\\
&\quad + h\left[\rho_s m_s U_{sn}^{(k)}(h)U_{sm}^{(l)}(h) + \rho_f m_f U^{(k)}(h)U^{(l)}(h)\right]\\
&\quad = \begin{cases} 0 & \text{for } n \neq m \text{ or } k \neq l,\\ M_n^{(k)} & \text{for } n = m \text{ and } k = l,\end{cases}
\end{aligned}
\tag{5.4}
$$

where

$$
\begin{aligned}
M_n^{(k)} = &\int_0^h \left[\rho_s(U_{sn}^{(k)}(x))^2 + \rho_f(U_{fn}^{(k)}(x))^2\right]dx\\
&+ h\left[\rho_s m_s(U_{sn}^{(k)}(h))^2 + \rho_f m_f(U_{fn}^{(k)}(h))^2\right],
\end{aligned}
\tag{5.5}
$$

and

$$
m_s = M_s/\rho_s hA_0, \qquad m_f = M_f/\rho_s hA_0.
\tag{5.6}
$$

The natural frequencies $\omega_n^{(k)}$ results from the characteristic equation of the form [1]

$$
\begin{aligned}
&(\delta_2 - \delta_1)(\cos\alpha\cos\beta + m_s m_f\alpha\beta\sin\alpha\sin\beta)\\
&\quad = (\delta_2 m_s - \delta_1 m_f)\alpha\sin\alpha\cos\beta + (\delta_2 m_f - \delta_1 m_s)\beta\sin\beta\cos\alpha,
\end{aligned}
\tag{5.7}
$$

where $\alpha = \omega h/c_s, \beta = \omega h/c_w$.

Two sets of natural frequencies (eigenvalues) follow from this equation, namely: $\omega_n^{(1)}$, which belong to the set of slow frequencies, and $\omega_n^{(2)}$, which belong to the set of fast frequencies. These frequencies depend, among others, on the ratio M_f/M_s, which can be easily controlled. For example, for $M_f/M_s = 0.1$, the first six frequencies from each group take the following values:

$$\{\omega_n^{(1)}\} = \{15.04, 229.03, 454.67, 681.06, 907.63, 1134.28\},$$
$$\{\omega_n^{(2)}\} = \{22.93, 483.71, 966.64, 1449.74, 1932.89, 2416.06\}.$$

After substituting the eigenfunctions (5.2) to formulae (2.8) and (5.5) determining $H_n^{(k)}$ and $M_n^{(k)}$, and integrating, we obtain the explicit forms of these quantities:

$$\begin{aligned}H_n^{(k)} &= 0.5(1-\delta_2)^2 c_w^2[(\cos\beta_n^{(k)} - m_s\beta_n^{(k)}\sin\beta_n^{(k)})^2\\ &\times(1-\sin\alpha_n^{(k)}\cos\alpha_n^{(k)}/\alpha_n^{(k)})] - 2(1-\delta_2)(1+\delta_1)c_s c_w\\ &\times[\cos\beta_n^{(k)} - m_s\beta_n^{(k)}\sin\beta_n^{(k)})(\cos\alpha_n^{(k)} - m_s\alpha_n^{(k)}\sin\alpha_n^{(k)})\\ &\times(\beta_n^{(k)}\sin\alpha_n^{(k)}\cos\beta_n^{(k)} - \alpha_n^{(k)}\sin\beta_n^{(k)}\cos\alpha_n^{(k)})]\\ &+0.5(1+\delta_1)^2 c_s^2[(\cos\alpha_n^{(k)} - m_s\alpha_n^{(k)}\sin\alpha_n^{(k)})\\ &\times(1-\sin\beta_n^{(k)}\cos\beta_n^{(k)}/\beta_n^{(k)})].\end{aligned}\tag{5.8}$$

$$\begin{aligned}M_n^{(k)} &= (\rho_s+\rho_f\delta_2^2)[(\cos\beta_n^{(k)} - m_s\beta_n^{(k)}\sin\beta_n^{(k)})c_w^2\\ &\times 0.5(1-\sin\alpha_n^{(k)}\cos\alpha_n^{(k)}/\alpha_n^{(k)})] + [(\cos\alpha_n^{(k)} - m_s\alpha_n^{(k)}\sin\alpha_n^{(k)})\\ &\times(\beta_n^{(k)}\sin\alpha_n^{(k)}\cos\beta_n^{(k)} - \alpha_n^{(k)}\sin\beta_n^{(k)}\cos\alpha_n^{(k)})\\ &\times 2(\cos\beta_n^{(k)} - m_s\sin\beta_n^{(k)})c_s c_w/(\alpha_n^{(k)2} - \beta_n^{(k)2})]\\ &\times(\rho_s+\rho_f\delta_1\delta_2) + (\rho_s+\rho_f\delta_1^2)\\ &\times[(\cos\alpha_n^{(k)} - m_s\alpha_n^{(k)}\sin\alpha_n^{(k)})c_s^2 0.5(1-\sin\beta_n^{(k)}\cos\beta_n^{k)}/\beta_n^{(k)})]\\ &+(\cos\beta_n^{(k)} - m_s\beta_n^{(k)}\sin\beta_n^{(k)})^2 c_w^2(\rho_s m_s + \rho_f m_f\delta_2^2)\sin^2\alpha^{(k)}\\ &-2(\cos\beta_n^{(k)} - m_s\beta_n^{(k)}\sin\beta_n^{(k)})(\cos\alpha_n^{(k)} - m_s\alpha_n^{(k)}\sin\alpha_n^{(k)})\\ &\times c_s c_w(\rho_s m_s + \rho_f m_f\delta_1\delta_2)\sin\alpha_n^{(k)}\cos\beta_n^{(k)}\\ &+(\cos\alpha_n^{(k)} - m_s\alpha_n^{(k)}\sin\alpha_n^{(k)})^2 c_s^2(\rho_s m_s + \rho_f m_f\delta_1^2)\sin^2\beta_n^{(k)}.\end{aligned}\tag{5.9}$$

Figure 6 illustrates the dependence of the coefficient $2b_n^{(k)}/b = H_n^{(k)}/M_n^{(k)}$ of the n-th mode shape on the ratio M_f/M_s for different combinations of the material constants. The solid lines are related to the coefficients $H_n^{(1)}/M_n^{(1)}$, and the dashed lines to $H_n^{(2)}/M_n^{(2)}$.

As it is seen in the above figure, practically only two first modes are influenced by the changes of parameter M_f/M_s, i.e. by the changes of the pore pressure. The damping coefficient of the higher order modes is insensitive

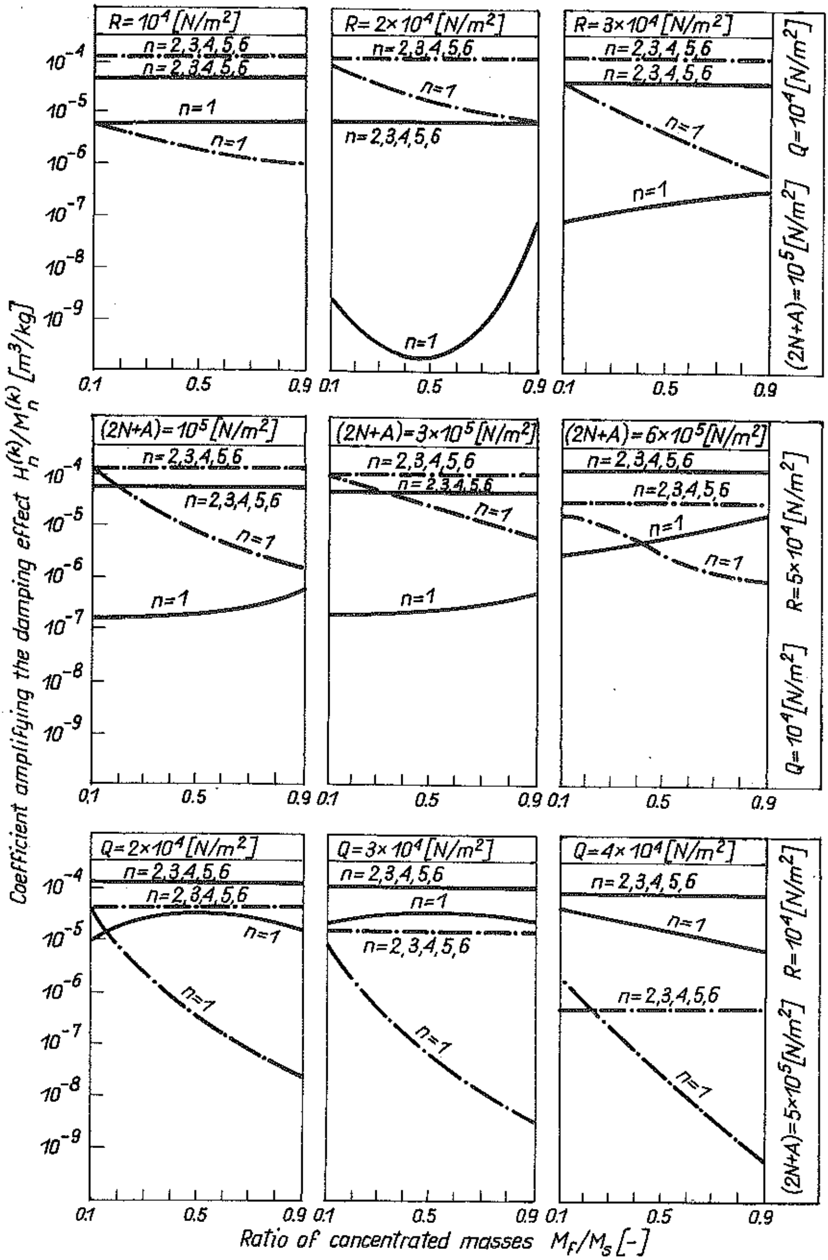
R= 10^4 [N/m^2]
R= 2×10^4 [N/m^2]
R= 3×10^4 [N/m^2]
Q=10^4 [N/m^2]
(2N+A)=10^5 [N/m^2]
(2N+A)=10^5 [N/m^2]
(2N+A)=3×10^5 [N/m^2]
(2N+A)=6×10^5 [N/m^2]
R=5×10^4 [N/m^2]
Q=10^4 [N/m^2]
Q=2×10^4 [N/m^2]
Q=3×10^4 [N/m^2]
Q=4×10^4 [N/m^2]
R=10^4 [N/m^2]
(2N+A)=5×10^5 [N/m^2]
n=2,3,4,5,6
n=1
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
0.1
0.5
0.9
Coefficient amplifying the damping effect H_n^(k)/M_n^(k) [m^3/kg]
Ratio of concentrated masses M_f/M_s [-]

FIG. 6.

to the changes of M_f/M_s. Its values are different for modes belonging to the first and the second sets. If we change the combinations of the material constants, we can influence the value of the coefficient amplifying the damping effect, similarly to the previous example. The situation when the slow vibrations are damped less than the fast vibrations is also possible but, in general, the opposite is true, what can be seen from the figures.

6. Remarks

The fundamental purpose of this paper was to study the influence of boundary conditions on the damping effect of fluid-saturated porous materials which can be used for construction of vibroisolators or dampers. Therefore we have concentrated our attention on nothing else but the analysis of the coefficient expressed by formula (2.6) only. We have not analysed the vibrations alone. The solutions for the damped vibrations were already presented in Ref. [1], but without such a detailed analysis of the damping coefficient as that done in this paper.

On the basis of our considerations carried out here, we know now that we change the damping effect not only trough changes of the constant b, i.e. trough a suitable choice of the porous material and the fluid filling the pores, but also through proper realization of the boundary conditions. The boundary conditions enable us to stimulate the pore pressure and the amplitude of the relative fluid displacement.

A stronger damping of slow vibrations can be explained by the fact that the amplitude of the relative fluid displacement for this kind of vibrations is greater than for the fast vibrations. This becomes obvious when the eigenfunctions are analysed in particular the simple eigenfunctions (3.4), which show us that the particles of the skeleton and the fluid performing slow vibrations move in opposite directions, i.e. $U_{sn}^{(1)} > 0$ and $U_{fn}^{(1)} < 0$, or inversely. This does not occur for fast vibrations, when $U_{sn}^{(2)}$ and $U_{fn}^{(2)}$ are always of the same sign, except maybe for practically unrealistic cases. This gives, in fact, a large amplitude of both the relative displacement and the velocity of the fluid for the first sets of eigenfunctions. Let us observe now that when we multiply the numerator and the denominator of the parameter $H_n^{(k)}/M_n^{(k)}$ by the square velocity of the generalized coordinate $(\dot{T}_n^{(k)})^2$, then this parameter expresses the square of the relative velocity for the n-th mode shape integrated along the length of the cylinder and related to the average kinetic energy of the cylinder.

The parameter $H_n^{(k)}(\dot{T}_n^{(k)})^2/M_n^{(k)}(\dot{T}_n^{(k)}) \equiv H_n^{(k)}/M_n^{(k)}$ can be then considered as a measure of the dissipation energy for the n-th mode of vibrations, since the dissipation energy is proportional to the square of the relative velocity (see e.g. [4]). So, an increase of the amplitude of the relative velocity causes an increase of the dissipated energy, and thus the damping is stronger. The effect of such a strong damping does not occur in the case of fast vibrations, where the particles move always in the same directions, what was ascertained on the basis of the eigenfunctions. In this case the velocity amplitudes are smaller, and, of course, the damping is also smaller.

The analysis of vibrations is difficult due to the fact that the solution of equation (2.5) in generalized coordinates takes three different forms, depending on the relation between the damping coefficient $b_n^{(k)}$ and the frequency $\omega_n^{(k)}$. For $b_n^{(k)} > \omega_n^{(k)}$ the damping is overcritical, for $b_n^{(k)} = \omega_n^{(k)}$ critical, and for $b^{(k)} < \omega_n^{(k)}$ subcritical.

It may happen, for example, that the first two modes are damped subcritically, when the next ones – overcritically. We have to verify the relation between $b_n^{(k)}$ and $\omega_n^{(k)}$ for each mode. Of course, application of the computer techniques makes this verification simple. This technique has been already used in Ref. [1].

This paper has been aimed at presenting only a few examples illustrating the influence of boundary condition on the damping effect of fluid-saturated porous materials. It has been shown here that the influence is different for different material constants and boundary values. It follows that in real dampers or vibro-isolators one should seek for optimal values of both the material constants and the boundary values under the given boundary conditions.

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STRESZCZENIE

NIEKONWENCJONALNE STEROWANIE WŁASNOŚCIAMI TŁUMIĄCYMI MATERIAŁÓW POROWATYCH WYPEŁNIONYCH CIECZĄ

Przedmiotem pracy są własności tłumiące sprężystego ośrodka porowatego wypełnionego cieczą lepką, a w szczególności niekonwencjonalne możliwości sterowania tymi własnościami przez odpowiedni dobór warunków brzegowych i stałych ośrodka. Zagadnienie to zilustrowano na przykładzie drgań porowatego walca wypełnionego cieczą, przy czym uwagę skupiono na analizie tylko współczynnika tłumienia i jego zależności od warunków brzegowych i stałych materiałowych bez analizowania samych drgań. Na przykładach pokazano, że istnieją duże możliwości sterowania tym współczynnikiem zarówno przez odpowiedni dobór stałych materiałowych, jak i warunków brzegowych.

Резюме

НЕКОНВЕНЦИОНАЛЬНОЕ УПРАВЛЕНИЕ ЗАТУХАЮЩИМИ СВОЙСТВАМИ ПОРИСТЫХ МАТЕРИАЛОВ ЗАПОЛНЕННЫХ ЖИДКОСТЬЮ

Предметом работы являются затухающие свойства упругой пористой среды, заполненной вязкой жидкостью, а в частности неконвенциональные возможности управления этими свойствами путем соответствующего подбора граничных условий и постоянных среды. Этот вопрос илюстрируется на примере колебаний пористого цилиндра, заполненного жидкостью, причем внимание сосредоточено на анализе только коэффициента затухания и его зависимости от граничных условий и материальных постоянных, без анализа самих колебаний. На примерах показано, что существуют больше возможности управления этим коэффициентом так путем соответствующего подбора материальных постоянных, как и граничных условий.

POLISH ACADEMY OF SCIENCES
INSTITUTE OF FUNDAMENTAL TECHNOLOGICAL RESEARCH.

Received November 15, 1990.