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AN ESTIMATE OF THE INFLUENCE OF THE COMPRESSOR STABILITY MARGIN ON THE THRUST OF A TURBOJET ENGINE

M. ORKISZ (DĘBLIN)

The relations derived in the paper make it possible to analyze the influence of the compressor stability margin upon the thrust of a turbojet engine. The engines operating within the economy range (minimum fuel consumption) and optimum range (maximum thrust) are considered.

1. Introduction

One of the parameters of a modern aircraft engine which are subject to control during flight is the stability margin of the compressor. As examples let us mention the Rolls-Royce RB-211-535C engine propelling the Boeing 757 aeroplane, RB-199 (Tornado) and AL-21F-3 (SU-22M4). Depending on the structure of the automatic control system, the control of the stability margin may be continuous, according to the flight conditions and the degree of fluctuation of inlet air stream, or periodic, according to the task to be performed at a given moment by the pilot, such as, for instance, firing of rocket missiles. The stability margin of a compressor varies in the course of operation of a turbojet engine in a steady or a transitory state as well. In a transitory state it is decisive for the time of attaining a definite speed after rapid displacement of the control lever by the pilot.

2. The computation model

The stability margin of a compressor is described by the relation [7]

$$\Delta Z = \frac{\pi^*_{gr}\dot{m}}{\pi^*\dot{m}_{gr}} - 1, \tag{2.1}$$

where π^*_{gr}, π^* is the compression ratio at the limit of stable operation of the compressor and on the line of cooperation of the compressor with a turbine under steady conditions (Fig.1), respectively, $\dot{m}_{gr}$, $\dot{m}$ – air flow intensity at

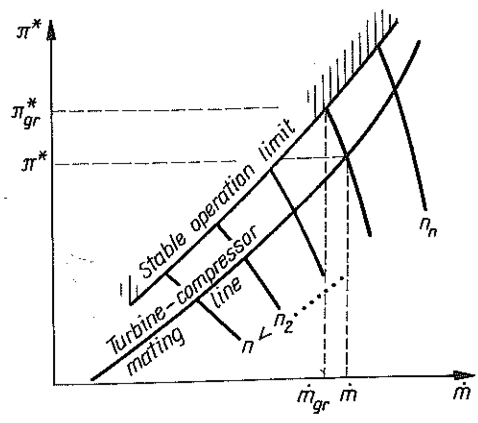

FIG. 1. Characteristic lines of an axial compressor, π^* – compresion ratio, $\dot{m}$ – air flow rate, n – r.p.m.

the limit of stable operation and on the line of co-operation between the compressor and the turbine, respectively.

On the basis of [2] and [4], the relation determining the thrust of a turbojet engine operating on the ground ($H = 0,\ Ma = 0$) may be expressed in the form

$$K = A_5 p_H [f(\lambda_5)\pi_D^* - 1], \tag{2.2}$$

where A_5 is the cross-sectional area of the propelling nozzle π_D^* – total pressure drop in the propelling nozzle, $f(\lambda_5)$ – relative outflow velocity from the propelling nozzle, p_H – ambient pressure. The function expressing the relative outflow velocity of the combustion products has the form [8]

$$f(\lambda_5) = (1 + \lambda_5^2)\left(1 - \frac{k_s - 1}{k_s + 1}\lambda_5^2\right)^{\frac{1}{k_s - 1}}, \tag{2.3}$$

where λ_5 is the relative outflow velocity of combustion products from the propelling nozzle

$$\lambda_5 = \frac{c_5}{a_{cr}},$$

c_5 – outflow speed of combustion products from the propelling nozzle, a_{cr} – critical speed

$$a_{cr} = \sqrt{\frac{2}{k_s + 1} R_s T_5^*},$$

k_s – isentropic exponent of the combustion products ($k_s = 1.33$ or as stated in [2, 9]), T_5^* – temperature of the combustion products, R_s – gas constant of the combustion products ($R_s = 289$ J/kgK or as stated in [2, 9]).

The pressure drop in the propulsion nozzle may be described in an approximate manner as follows

$$\pi_D^* \cong \frac{\pi^*}{\pi_T^* \sigma_{ks}^*}. \tag{2.4}$$

where π_T^* is the pressure drop in the turbine, σ_{ks}^* – the coefficient of pressure loss in the combustion chamber.

For model conditions it is assumed that the coefficient of pressure loss in the combution chamber is constant over the entire variation range of the engine speed. In the neighbourhood of the points of co-operation between the compressor and the turbine under fixed conditions for a given speed of the rotor assembly, a change in the compression ratio caused small changes in the relative speed function of the combustion products at the outlet, therefore it will be assumed for further considerations that $f(\lambda_5)_{n=\mathrm{const}} =$ const. It follows that the greatest influence on the variation in engine thrust is that of the variation in the propelling nozzle, which is connected with the compression ratio and the pressure drop in the turbine. For a modern turbojet engine it may be assumed, for the entire range of variation of engine speed essential from operational point of view, that $\pi_T^* =$ const [3, 5, 10, 11] and its value lies within the interval $\pi_T^* = 2,\ldots,4$ [5].

Thus, to find a relation enabling us to estimate the influence of the stability margin of the compressor on the thrust of a turbojet engine we must determine the relation $\pi^* = f(\Delta Z)$. This is a consequence of the relation (2.4) quoted above. On transforming (2.1) we find

$$\pi^* = \frac{\pi_{\mathrm{gr}}^* \dot{m}}{\dot{m}_{\mathrm{gr}}(\Delta Z + 1)}. \tag{2.5}$$

As a result of the stability margin of the compressor being increased by δZ, the compression ratio varies to become

$$\pi_{\delta Z}^* = \frac{\pi_{\mathrm{gr}}^* \dot{m}_{\delta Z}}{\dot{m}_{\mathrm{gr}}(\Delta Z + \delta Z + 1)}. \tag{2.6}$$

Hence the relative variation in the compression ratio becomes

$$\bar{\pi}^* = \frac{\pi^*}{\pi_{\delta Z}^*}$$

and, on substituting Eq. (2.5) and (2.6),

$$\bar{\pi}^* = \frac{\dot{m}(\Delta Z + \delta Z + 1)}{\dot{m}_{\mathrm{gr}}(\Delta Z + 1)}. \tag{2.7}$$

For axial compressors characterized by step characteristics branches for constant speeds it may be assumed, on the basis of statistical data and the characteristics presented in [12] and [13], for instance, with an error not exceeding 3.2%, that $\dot{m} = \dot{m}_{\delta Z}$ (which is justified, in particular, if the compressor has a theoretical compression ratio $\pi^* > 5$). Finally, the relation (2.7) is simplified to become

$$\bar{\pi}^* = 1 + \frac{\delta Z}{\Delta Z + 1}.$$

On introducing a proportionality coefficient $k' = \frac{\delta Z}{\Delta Z}$ (if $k' > 0$ the stability margin increases), we obtain

$$\bar{\pi}^* = 1 + k' \frac{\Delta Z}{\Delta Z + 1}. \tag{2.8}$$

In modern engines the stability margin of a compressor lies within the interval $\Delta Z = 0.15, \ldots, 0.25$ [5, 7].

On substituting Eq.(2.7) into Eq.(2.2), the relation (2.4) being borne in mind, we obtain

$$K = A_5 p_H \left[f(\lambda_5) \frac{\pi^*}{\pi_T^* \sigma_{ks}^* \bar{\pi}^*} - 1 \right]. \tag{2.9}$$

Assuming, for prescibed conditions of co-operation between the subassemblies of the engine, the simplifying expression

$$F = f(\lambda_5) \frac{\pi^*}{\pi_T^* \sigma_{ks}^*},$$

we obtain

$$K = A_5 p_H \left[F \frac{1}{\bar{\pi}^*} - 1 \right]. \tag{2.10}$$

If the stability margin of the compressor increases ($\bar{\pi}^* > 1$), the thrust of a turbojet engine decreases, and if the former decreases ($\bar{\pi}^* < 1$), the thrust increases.

Figure 2 represents the dependence of the relative thrust of a turbojet engine on the proportionality coefficient k' for various original values of the stability margin. From the form of the diagram it follows that a variation in the stability margin of the compressor by some 10% produces a variation in the engine thrust by some 2,...,4%, depending on the original value of the stability margin. This entails a proportional variation in the specific fuel consumption.

From the theory of turbojet engines it is known that the maximum specific thrust k_j is attained for values of the compression ratio other than the minimum specific fuel consumption. This statement is illustrated in Fig. 3.

As regards real structures, it is known that the following inequalities are satisfied [1]

$$\pi^*_{\mathrm{opt}} < \pi^*_{\mathrm{obl}} < \pi^*_{\mathrm{ek}},$$

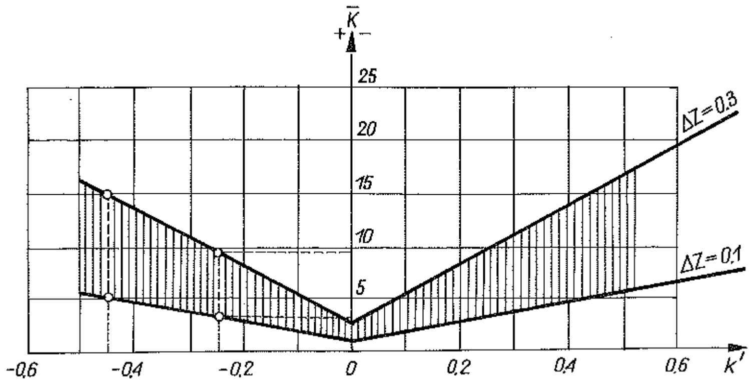

FIG. 2. Dependence of the relative increase in thrust on the coefficient of proportionality of stability margin.

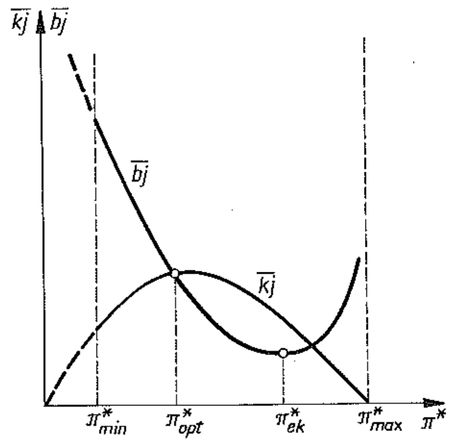

FIG. 3. Dependence of the specific thrust k_j and specific fuel consumption b_j of a single flow engine on the compression ratio.

where π^*_{obl} is the compression ratio selected for the computation conditions.

If the automatic control system of a turbojet engine enables continuous regulation of the stability margin of the compressor to be effected, there exists a possibility of adjusting the parameters of co-operation between the compressor and the turbine to suit the task to be performed by the pilot (cruising, climbing or interception of an object). The operation of a power plant has also an optimum as regards the combat mission to be performed by the power plant, determined by the combat effectiveness of the aircraft [6]. The criteria of combat effectiveness depend very much or the duration of the manoeuvre and the quantity of fuel necessary. At the same time there exists, for any altitude, a flight velocity for which the fuel consumption is minimum, which is determined also by the selection of an appropriate operating range of the power plant. To make possible the determination of the maximum effectiveness of the power plant during the combat mission to be performed, the stability margin of the compressor should be determined within the range of optimum and economic compression ratio.

3. Variation of the stability range of the compressor within the range of optimum compression ratio

The effective operation of the thermodynamic cycle of a turbojet engine is described by the relation [9]

$$l_e = c_p T_3^* \left[1 - \left(\frac{1}{\pi_T^*}\right)^{\frac{k-1}{k}}\right] \eta_r a - c_p T_H^* \left[\pi^{*\frac{k-1}{k}} - 1\right] \frac{1}{\eta_s},$$

where a is a coefficient for determining the specific heat of air and combustion products (for $T_3^* = 1000,\ldots,1400$ K and $\pi = 5,\ldots,20$ we have $a = 1.02,\ldots,1.04$), c_p – specific heat of air, T_3^* – temperature of the combustion gases before the turbine, η_s – efficiency of the compression process, η_r – efficiency of the expansion process, k – isentropic exponent. The operating range of the engine is optimum when

$$\frac{\partial l_e}{\partial \pi^{*\frac{k-1}{k}}} = 0.$$

By performing appropriate operations we obtain the following relation for the optimum compression ratio:

$$\pi_{opt}^* = \frac{(\Delta^* \eta_r \eta_s a)^{\frac{k}{2(k-1)}}}{\left(1 + \frac{k-1}{2} Ma^2\right)^{\frac{k-1}{k}} \sigma_{WL}^*}, \tag{3.1}$$

where Δ^* is the degree of preheating of the stream, $\Delta^* = T_3^*/T_H^*$, σ_{WL}^* coefficient of pressure losses at the inlet to the engine.

By rewriting the relation (2.1) for the case of optimum conditions of engine operation we obtain

$$\Delta Z_{\text{opt}} = \frac{\pi_{\text{gr}}^* \dot{m}_{\text{opt}}}{\pi_{\text{opt}}^* \dot{m}_{\text{gr}}} - 1. \tag{3.2}$$

By representing the relation for the flow intensity as a gas-dynamic function in the form [3]

$$\dot{m} = jAq(\lambda)\frac{p_2^*}{\sqrt{T_2^*}},$$

where j is the gas-dynamic parameter:

$$j = \sqrt{\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}} \frac{k}{R}},$$

A – cross-sectional area of the compressor, $q(\lambda)$ – relative density of the stream:

$$q(\lambda) = \sqrt{\frac{2}{k-1}\left(\frac{k+1}{2}\right)^{\frac{k+1}{k-1}}} \sqrt{\left(\frac{1}{\pi^*}\right)^{\frac{2}{k}} - \left(\frac{1}{\pi^*}\right)^{\frac{k+1}{k}}}$$

and expressing the pressure and the temperature at the outlet of the compressor as functions of the compression ratio, we find, after transformation, the relation (3.2) in the form

(3.3) $$\Delta Z_{\text{opt}} = \frac{q_{\text{opt}}(\lambda)}{q_{\text{gr}}(\lambda)} \frac{\pi_{\text{gr}}^{*\frac{k-1}{2k}}}{\pi_{\text{opt}}^{*\frac{k-1}{2k}}} - 1$$

and, on substituting Eq. (3.1),

(3.3′) $$\Delta Z_{\text{opt}} = \frac{q_{\text{opt}}(\lambda)}{q_{\text{gr}}(\lambda)} \frac{\left(1+\frac{k-1}{2} M\overset{2}{a}\right)^{0.5} \pi_{\text{gr}}^{*\frac{k-1}{2k}}}{(\Delta^* \eta_r \eta_s a)^{0.25}} - 1.$$

From statistical data and the characteristics of compressors of present engines it follows that, for optimum and limiting conditions, the ratio of relative stream densities is

$$\frac{q_{\text{opt}}(\lambda)}{q_{\text{gr}}(\lambda)} = 1.28$$

with a standard deviation $s_{\alpha,t} = 0.07$ for various branches of the characteristic of the compressor. In agreement with [1] it may be assumed that $\eta_r \eta_s a = 0.8$, therefore (3.3′) becomes

(3.3″) $$\Delta Z_{\text{opt}} = 1.28 \frac{\left(1 + 0.2 M\overset{2}{a}\right)^{0.5} \pi_{\text{gr}}^{*0.14}}{\Delta^{*0.25}} - 1.$$

Figure 4 shows the variation of the stability margin of the compressor, under optimum conditions, as a function of the flying speed and the degree of preheating of the air stream. From the form of the curve in Fig. 4a it follows, for the SO-3W engine, that an increase in the degree of preheating of the air stream by 50% with reference to the theoretical condition results in a drop in the stability margin by about 40%. Figure 4b shows that it is

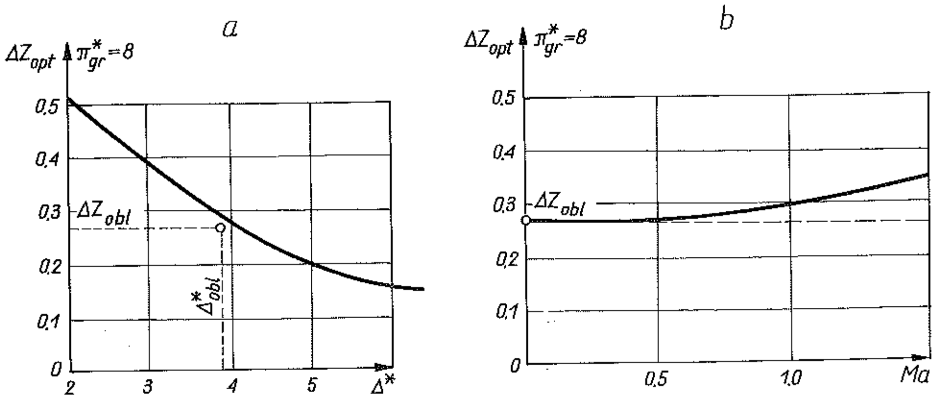

FIG. 4. Dependence of the stability margin of the compressor on the degree of preheating of the stream and the flying speed under optimum conditions.

only for a flight speed of above $Ma = 0.5$ that the stability margin of the compressor increases, this increase being insignificant, however, as referred to the variation due to Δ^*. From the form of the curve $\Delta Z = f(Ma)$ it is seen that an increase in the flying speed causes a drop in optimum compression ratio. Thus, for instance, for $\Delta^* = 4$ an increase in Ma of 0 to 1.0 results in a drop in optimum compression ratio of about 54%, therefore the increase of the stability margin is as shown in Fig. 4b.

4. Variation of the stability margin of a compressor within the economic range of the compression ratio

The value of the economic compression ratio depends on the temperature before the turbine and for $T_3^* = 1100,\ldots,1300$ K it is $\pi_{ek} = 19,\ldots,35$, respectively [9].

This is in considerable excess of the usual value for most present single-flow turbojet engines. The value of the compression ratio becomes economic at the point of maximum overall efficiency of the engine, which is illustrated in Fig. 5. In view of the necessity of (among other conditions to be satisfied) reducing the mass of the engine, we give up making the compression ratio economic, in order to increase the specific thrust of the engine instead. As a result, the economic value of the temperature of the combustion gases before the turbine is exceeded. Owing to this fact, the number of stages of the compressor can be reduced, it being borne in mind that the compression ratio of a single stage is, nowadays, approximately, $\pi^*_{st} = 1.4,\ldots,1.6$.

The economic value of the compression ratio can be found from the relation [1]

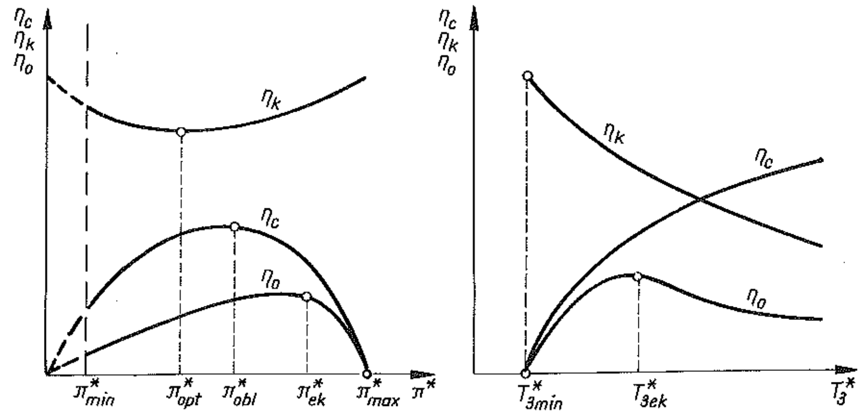

FIG. 5. Dependence of the efficiency of a turbojet engine on the compression ratio and the temperature of the combustion gases before the turbine: η_k – propulsion efficiency, η_o – overall efficiency, η_c – thermal efficiency, obl – assumed value of the compression ratio.

$$\pi^*_{\mathrm{ek}} = \left[\frac{\left[\Delta^* - \left(\frac{1}{\eta_s} - 1\right)\right]\eta_s}{\frac{2}{a\eta_r} - 1}\right]^{\frac{k}{k-1}} . \tag{4.1}$$

On substituting this, with appropriate symbols, into Eq. (3.3), we obtain

$$\Delta Z_{\mathrm{ek}} = \frac{q_{\mathrm{ek}}(\lambda)}{q_{\mathrm{gr}}(\lambda)} \frac{\pi^{*\frac{k-1}{2k}}_{\mathrm{gr}}}{\left\{\left[\Delta^* - \left(\frac{1}{\eta_s} - 1\right)\right]\eta_s\right\}^{0.5}} \left(\frac{2}{a\eta_r} - 1\right)^{0.5} - 1 \tag{4.2}$$

and, making use of the statistical data available,

$$\Delta Z_{\mathrm{ek}} = 1.31 \frac{\pi^{*0.14}_{\mathrm{gr}}}{0.92\Delta^{*0.5} - 0.39} - 1. \tag{4.2'}$$

Figure 6 shows the dependence of the stability margin of the compressor on the degree of preheating of the air stream for various limit values of the compression ratio. By confronting it with Fig. 4 it is seen that the influence of the degree of preheating of the stream is, in the case of economic operation conditions of the engine, much stronger than in the case of optimum conditions. It follows that if the engine is controlled within the economic range, the adjustment of the temperature of the combustion gases before the turbine must be accurate.

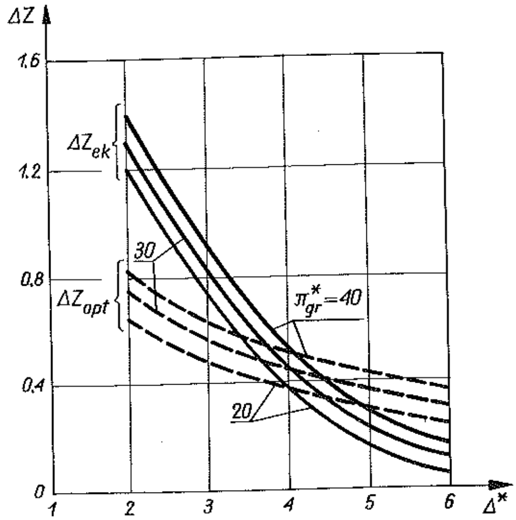

FIG. 6. Dependence of the stability margin of the compressor on the degree of preheating for various values of the limit compression ratio.

5. INTERDEPENDENCE BETWEEN THE STABILITY MARGIN OF THE COMPRESSOR FOR THE ECONOMIC AND THE OPTIMUM COMPRESSION RATIO

Taking into consideration the coefficient of stability margin, we obtain from Eq. (2.1)

$$\Delta Z = Z - 1,$$

where Z is the coefficient of stability margin.

By expressing the variation in the stability coefficient in the form

$$\bar{Z} = \frac{Z_{\mathrm{ek}}}{Z_{\mathrm{opt}}}$$

and making use of the relations (3.3) and (4.2), we obtain

$$\bar{Z} = \frac{q_{\mathrm{ek}}(\lambda)}{q_{\mathrm{opt}}(\lambda)} \frac{\left(\frac{2}{a\eta_r} - 1\right)^{0.5} (\Delta^* \eta_r \eta_s a)^{0.25}}{\left\{\left[\Delta^* - \left(\frac{1}{\eta_s} - 1\right)\right] \eta_s\right\}^{0.5} \left(1 + \frac{k-1}{2} M\mathring{a}^2\right)^{0.5}}. \tag{5.1}$$

Making use of the statistical data available and assuming that, for a prescribed engine speed, the difference between the relative values of the density of the stream for the optimum range and the economic range is small $q_{\mathrm{ek}} = q_{\mathrm{opt}}$, we obtain

$$\bar{Z} = \frac{\Delta_*^{0.25}}{0.9\Delta^{*0.5} - 0.38}. \tag{5.1'}$$

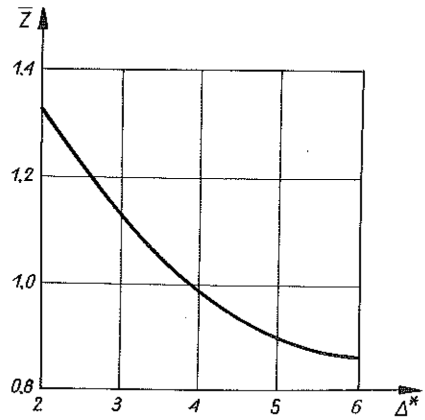

FIG. 7. Dependence of the relative coefficient of stability on the degree of preheating of the stream.

From the relation (5.1) it follows that ratio of coefficients of stability margin for economic and optimum conditions depends only on the degree of preheating. Figure 7 illustrates the dependence of the ratio $\bar{Z}$ of the stability range on the degree of preheating. The form of the curve may suggest that increased degree of preheating of the air stream reduces the relative value of the coefficient of stability margin and, within the range of $\Delta^* = 4$ it has a value $\bar{Z} = 1$. This means a possibility of control of the engine in such a manner that it is possible, in the case of this value of the degree of preheating, that the specific thrust is high and the specific fuel consumption is low.

6. Final remarks

Depending on the structural form of a modern turbojet engine, its compression ratio may reach considerable values (Fig. 8). It follows, for bypass engines, in particular, with and without afterbourning, control of the stability margin of the compressor is possible depending on the combat mission of the aeroplane. The present analysis concerns the influence of definite operating conditions of a turbojet engine on the variation of the stability margin of the compressor in steady states only. As regards transient states, a separate analysis must be made.

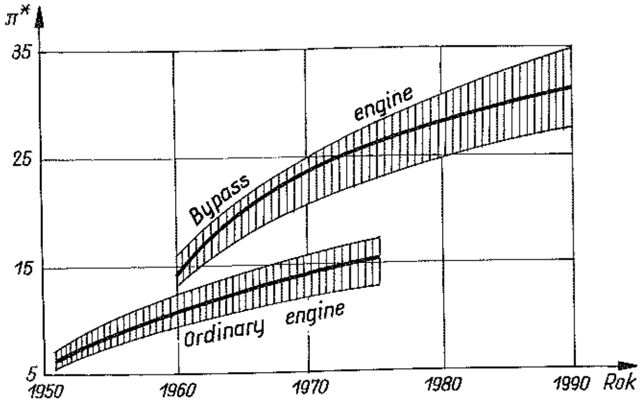

FIG. 8. Statistical dependence of the compression ratio of turbojet engines on the time of beginning of the production [12, 13].

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STRESZCZENIE

OCENA WPLYWU ZAPASU STATECZNOŚCI SPRĘŻARKI NA CIĄG TURBINOWEGO SILNIKA ODRZUTOWEGO

Przedstawiono zależności pozwalające na przeprowadzenie analizy wpływu zapasu stateczności sprężarki na ciąg turbinowego silnika odrzutowego. Wyprowadzono związki pozwalające na obliczenie wartości zapasu stateczności sprężarki dla silnika pracującego według kryterium ekonomicznego (minimalnego zużycia paliwa) i kryterium optymalnego (maksymalnego ciągu jednostkowego).

РЕЗЮМЕ

ОЦЕНКА ВЛИЯНИЯ ЗАПАСА УСТОЙЧИВОСТИ КОМПРЕССОРА НА ТЯГУ ТУРБИННОГО РЕАКТИВНОГО ДВИГАТЕЛЯ

Представлены зависимости, позволяющие провести анализ влияния запаса устойчивости компрессора на тягу турбинного реактивного двигателя. Выведены соотношения, позволяющие рассчитать значения запаса устойчивости компрессора для двигателя, работающего согласно экономическому критерию (минимальный расход топлива) и оптимальному критерию (максымальной единичной тяги).

AIR FORCE ACADEMY, DĘBLIN.

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