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OPTIMIZATION OF THIN-WALLED GIRDERS IN PROBABILISTIC FORMULATION

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This paper presents, in a probabilistic formulation, the optimization method of box and I-beams subjected to bending. Dimensions of the girder, material constants and the bending moment are random variables. The minimum area of the cross-section is assumed as the objective function. The conditions of strength and of local and global stability represent the constraints. The examples of calculations serve as a basis for comparative analysis of the results obtained and the results of deterministic optimization.

1. Introduction

Analysis of a structure and its elements are usually carried out under assumption that the load and strength of the material as well as its geometrical features are deterministic quantities. However, consideration of real structures indicates that these quantities are of a random character. The main reason for simplifications enabling us to treat these quantities as deterministic ones lies in the difficulty of determining the probability distribution of random variables. Moreover, probabilistic methods are much more complex. That is why the methods of optimization in deterministic formulation are mainly used in publications and are more frequently applied than the methods of probabilistic optimization.

In probabilistic optimization, the cost or, similarly to the deterministic optimization, the volume [4] constitutes the objective function. In the case of a structure composed of elements of constant cross-section and, moreover, regarding mass density as a deterministic quantity, the optimization procedure can be limited to the cross-sectional area.

Regarding the design variables, as well as the parameters connected with material properties, as random quantities, the problem is confined to stochastic programming [2, 5].

It consists in minimizing the objective function

$$F(x) \tag{1.1}$$

under the constraints

$$P_j(x) = P[g_j(x) > G_j(x)] \leq p_j, \qquad j = 1, \ldots M, \tag{1.2}$$

where $x = [x_1, x_2 \ldots x_n \ldots x_N]$ is a vector of random variables (the first n ones are design variables), $g_j(x)$ is the response of the structure, i.e. stress or displacement, whereas $G_j(x)$ are its constraints. Inequality (1.2) indicates that the probability of occurrence $P_j(x)$ exceeding the permissible limit $G_j(x)$ must be smaller or equal to the permissible probability of destruction p_j.

With regard to the minimum of mass, the strength optimization in probabilistic formulation was reduced in papers [2,5] by S. JENDO and W. MARKS to an equivalent problem of nonlinear deterministic programming. This method consists in writing an objective function in the form

$$F(x) = k'\bar{f} + k''\sigma_f, \tag{1.3}$$

where k' and k'' are treated as non-negative weights indicating the coefficients of importance, $\bar{f}$ is the mean value, and σ_f – standard deviation of the variable subjected to optimization (minimization).

Constraints, however, take the following form:

$$\bar{g}_j + \Phi(p_j)\sigma_{gj} \leq 0, \tag{1.4}$$

where, as in formula (1.3), $\bar{g}_j$ is the mean value, σ_{gj} – standard deviation related to the constraints, and $\Phi(p_j)$ – the value of the standarized random variable corresponding to probability p_j.

Application of the given method for the optimization of box and I-beams and an analysis of the results are presented in this paper.

2. FORMULATION OF THE PROBLEM

The paper is aimed at solving the problem which consists in making the optimal choice of parameters of box and I-section beams with respect to the minimum surface area of the cross-section (minimum girder weight). Considerations were carried out for simply supported beams of constant cross-sections, subjected to pure bending (Fig. 1).

FIG. 1.

The non-deterministic character of the load, the material constants as well as of the cross-section parameters determined were taken into consideration. It was assumed that these variables are subject to normal probability distribution, whose function of probability density is expressed by the formula

$$f(y) = \frac{1}{\sigma_y\sqrt{2\pi}} \exp\left(-\frac{1}{2}\frac{(y-\bar{y})^2}{\sigma_y^2}\right), \tag{2.1}$$

where y – random variable, $\bar{y}$ – expected value, σ_y^2 – variance.

In the solution of the problem, conditions of strength and those of local and integral stability were considered. It was assumed that particular conditions should be fulfilled at the probability determined for these conditions.

3. FORMULATION OF THE OBJECTIVE FUNCTION AND CONSTRAINTS

In our considerations, models of the cross-sections presented in Fig. 2 (2a – box section, 2b – I section) were adopted. They may be applied to many constructions such as cranes, diggers etc. [1]. Parameters concerning the box-section were denoted by subscripts *1*, whereas parameters concerning I-section – by *2*; for parameters concerning both types of sections no indices were used.

The section is subjected to the bending moment M. The section is described by four parameters: height – h, width – b, flange thickness – g, and web thickness – s. These dimensions, beam length l, as well as the bending moment M, and the material constants (E – Young's modulus and R_y – permissible stress due to bending) were treated as random variables subjected to normal probability distribution (Gauss).

The random variables form the vector

$$\mathbf{Y} = [b, g, h, s, l, R_g, E, M]^T$$

or, more generally,

$$\mathbf{Y} = [y_i]^T, \qquad i = 1, 2, \ldots 8.$$

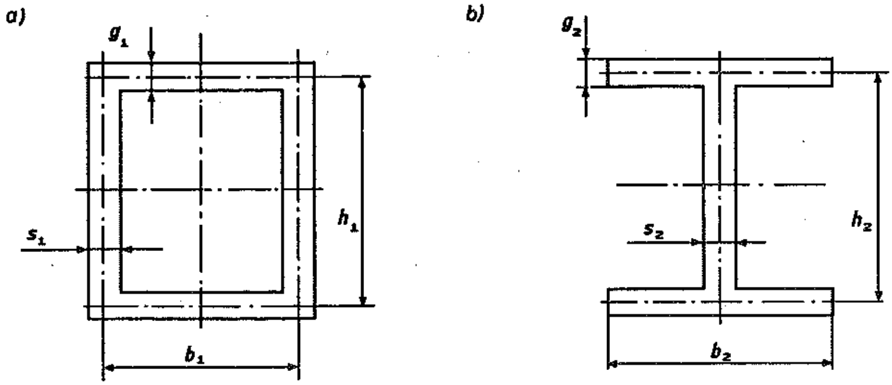

FIG. 2.

The normal distribution of the variable y_i can be described by its mean value $\bar{y}_i$ and the standard deviation σ_{yi}. For the description, the coefficient of variability α, defined by formula

$$\alpha_{yi} = \sigma_{yi}/\bar{y}_i\,, \tag{3.1}$$

can be used instead of the variance.

As it was mentioned above, the problem consists in minimization of the cross-section area. The area of the cross-section f is determined by the formulae

$$\begin{aligned} f_1 &= f_1(b_1, g_1, h_1, s_1) = 2(b_1 g_1 + h_1 s_1), \\ f_2 &= f_2(b_2, g_2, h_2, s_2) = 2b_2 g_2 + h_2 s_2. \end{aligned} \tag{3.2}$$

Due to the fact that quantities b, g, h, s are variables of Gauss distribution, the surface area f will also be a quantity characterized by this distribution. Its parameters can be determined, within a good approximation, to yield

$$\begin{aligned} \bar{f} &= f(\bar{y}), \\ \sigma_f^2 &= \sum_i \left(\frac{\partial f}{\partial y_i}\right)^2 \bigg|_{\bar{y}} \sigma_{yi}^2\,, \qquad i = 1, 2, 3, 4. \end{aligned} \tag{3.3}$$

The problem of optimization can thus be defined as minimization of the mean value of the surface area $\bar{f}$ and its standard deviation σ_f. The objective function F is adopted in agreement with formula (1.3).

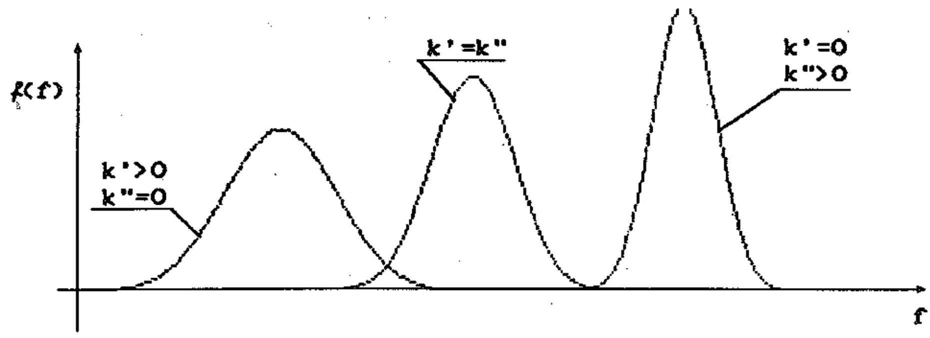

FIG. 3.

Influence of coefficients k' and k'' on the shape of the function of probability distribution density of the area f, obtained as a result of optimization of the objective function F, is presented in Fig. 3.

The objective function takes the final form

$$
\begin{aligned}
F_1 &= k_1' \left(\bar{b}_1\bar{g}_1 + \bar{h}_1\bar{s}_1\right) + k_1'' \left[\left(\bar{b}_1\bar{g}_1\right)^2 \left(\alpha_{b1}^2 + \alpha_{g1}^2\right)\right. \\
&\qquad \left. + \left(\bar{h}_1\bar{s}_1\right)^2 \left(\alpha_{h1}^2 + \alpha_{s1}^2\right)\right]^{1/2}, \\
F_2 &= k_2' \left(2\bar{b}_2\bar{g}_2 + \bar{h}_2\bar{s}_2\right) + k_2'' \left[\left(2\bar{b}_2\bar{g}_2\right)^2 \left(\alpha_{b2}^2 + \alpha_{g2}^2\right)\right. \\
&\qquad \left. + \left(\bar{h}_2\bar{s}_2\right)^2 \left(\alpha_{h2}^2 + \alpha_{s2}^2\right)\right]^{1/2}.
\end{aligned} \tag{3.4}
$$

The section should fulfil the strength condition and, moreover, the conditions of stability should be satisfied. They all will constitute the constraints in the optimization problem of the objective function F. They are as follows:

$$g_{1k} = S_{gk} - R_{gk} \le 0, \tag{3.5}$$

$$g_{2k} = S_{bk} - R_{bk} \le 0, \tag{3.6}$$

$$g_{3k} = S_{hk} - R_{hk} \le 0, \tag{3.7}$$

$$g_{42} = M - M_{\mathrm{cr}} \le 0, \qquad k = 1,\ 2. \tag{3.8}$$

Inequality (3.5) represents the condition of strength, where S_g is the maximum normal stress in the section. Inequalities (3.6) and (3.7) represent the conditions of stability of the flange and the web, and S_b and S_h denote maximal stress in the upper flange and the web, respectively, whereas R_b and R_h are the critical values of the stresses. Moreover, inequality (3.8) is the condition of stability of the whole beam, where M_{cr} is the critical moment at which torsional deviation of the beam occurs. This phenomenon was observed only in case of the I-beam because the box-beam, due to great torsional rigidity, does not exhibit such deformations within the range of the lengths [1] applied in practice.

Due to a small difference between stresses S_g, S_b and S_h, approximate values of those stresses are

$$S_k = \frac{2M}{h_k J_{\eta k}} = \frac{M}{w_{\eta k}}, \qquad k = 1, 2, \tag{3.9}$$

where the formulae

$$\begin{aligned} w_{\eta 1} &= b_1 g_1 h_1 + \frac{1}{3} s_1 h_1^2, \\ w_{\eta 2} &= b_2 g_2 h_2 + \frac{1}{6} s_2 h_2^2 \end{aligned} \tag{3.10}$$

will be used in further calculations.

Critical stress R_b and R_h can be determined from formulae [3],

$$\begin{aligned} R_{b1} &= k_{b1} D \left(\frac{g_1}{b_1}\right)^2, \\ R_{b2} &= k_{b2} D \left(\frac{2g_2}{b_2}\right)^2, \\ R_{hk} &= k_{hk} D \left(\frac{s_k}{h_k}\right)^2, \qquad k = 1, 2, \end{aligned} \tag{3.11}$$

where:

$$D = \frac{\pi^2 E}{12(1-\nu^2)}, \tag{3.12}$$

and k_{bk} and k_{hk} denote the numerical coefficients dependent on the ratio of the length of the plates to their width, and on the way the plate edges are supported. Values of these coefficients were adopted [3] such as for the plates of the length-to-width ratio equal to infinity. Hence, the values of these coefficients are $k_{b1} = 4.0$; $k_{b2} = 0.456$; $k_{hl} = k_{h2} = 24.0$.

The critical moment $M_{\rm cr}$ can be determined on the basis of the formula [3]

$$M_{\rm cr} = \frac{\pi}{l_2}\left[EJ_{\xi 2} GJ_{s2}\left(1 + \frac{\pi^2}{l_2^2}\frac{EC_{\omega 2}}{GJ_{s2}}\right)\right)^{1/2}, \tag{3.13}$$

where J_{s2} – polar moment of inertia, $J_{\xi 2}$ – moment of inertia with respect to axis ξ, $C_{\omega 2}$ – constant of deplanation.

These quantities are given by the relations

$$
(3.14)\qquad
\begin{aligned}
J_{s2} &= \frac{2}{3}b_2g_2^3 + \frac{1}{3}h_2s_2^3\,,\\
J_{\xi 2} &= \frac{1}{6}b_2g_2^3 + \frac{1}{12}h_2s_2^3\,,\\
J_{\omega 2} &= \frac{1}{24}g_2h_2^2b_2^3\,.
\end{aligned}
$$

Taking into consideration relations (3.14), constraints (3.11) are written in the following form:

for a box section

$$
(3.15)\qquad
\begin{aligned}
g_{11} &= M/(b_1g_1h_1 + s_1h_1^2/3) - R_{g1} \le 0,\\
g_{21} &= M/(b_1g_1h_1 + s_1h_1^2/3) - A_1(g_1/b_1)^2 \le 0,\\
g_{31} &= M/(b_1g_1h_1 + s_1h_1^2/3) - B_1(s_1/h_1)^2 \le 0,\\
g_{41} &= 0;
\end{aligned}
$$

for a I-section

$$
(3.16)\qquad
\begin{aligned}
g_{12} &= M/(b_2g_2h_2 + s_2h_2^2/6) - R_{g2} \le 0,\\
g_{22} &= M/(b_2g_2h_2 + s_2h_2^2/6) - A_2(g_2/b_2)^2 \le 0,\\
g_{32} &= M/(b_2g_2h_2 + s_2h_2^2/6) - B_2(s_2/h_2)^2 \le 0,\\
g_{42} &= M - \frac{\pi}{l_2}\left[EJ_{\omega 2}GJ_{s2}\left(1 + \frac{\pi^2}{l_2^2}\frac{EC_{\omega 2}}{GJ_{s2}}\right)\right]^{1/2} \le 0,
\end{aligned}
$$

where

$$
(3.17)\qquad A_1 = k_{b1}D, \qquad B_1 = k_{h1}D, \qquad a_2 = 4k_{b2}D, \qquad B_2 = k_{h2}D.
$$

Since the variables occurring in the constraints are random ones, probability P, arbitrarily great (but smaller than 100%) of their fulfilment p can be expected.

$$
(3.18)\qquad P[g_{jk} \le 0] \ge p_{jk}\,, \qquad P_{jk} \in (0-1), \quad j = 1,2,3,4, \quad k = 1,2.
$$

Equation (3.18) will be fulfilled if constraints (3.15) and (3.16) are presented in a new form (1.4), [2]

$$
(3.19)\qquad G_{jk} = \bar{g}_{jk} + \phi(p_{jk})\sigma_{gjk} \le 0, \qquad j = 1,\,2,\,3,\,4, \quad k = 1,\,2.
$$

Due to their complex final form, let us present the constraints in the version concerning the box section only:

$$G_{11} = \frac{\overline{M}}{\delta} - \overline{R}g + \phi(p_{11})\left[\frac{\beta^2 - \overline{M}^2}{\delta^4}\left(\alpha_{b1}^2 + \alpha_{g1}^2\right) + \left(\frac{\beta\overline{M} + 2\gamma\overline{M}}{\delta^2}\right)^2 \alpha_{h1}^2 + \frac{\gamma^2\overline{M}^2}{\delta^2}\alpha_{s1}^2 + \frac{\overline{M}^2}{\delta^2}\alpha_M^2 + \overline{R}_{g1}^2\alpha_{Rg1}^2\right]^{1/2} \le 0,$$

$$(3.20) \qquad G_{21} = \frac{\overline{M}}{\delta} - \frac{\overline{A}_1\bar{g}_1^2}{\bar{b}_1^2} + \phi(p_{21})\left[\left(\frac{2\overline{A}_1\bar{g}_1^2}{\bar{b}_1^2} + \frac{\beta^2\overline{M}}{\delta^2}\right)^2 \alpha_{b1}^2 + \left(\frac{2\overline{A}_1\bar{g}_1^2}{\bar{b}_1^2} - \frac{\beta^2\overline{M}}{\delta^2}\right)^2 \alpha_{g1}^2 + \left(\frac{\beta\overline{M} + 2\gamma\overline{M}}{\delta^2}\right)^2 \alpha_{h1}^2 + \frac{\gamma^2\overline{M}^2}{\delta^4}\alpha_{s1}^2 + \frac{\overline{M}^2}{\delta^2}\alpha_M^2 + \frac{\overline{A}_1^2\bar{g}_1^4}{\bar{b}_1^4}\alpha_{A1}^2\right]^{1/2} \le 0,$$

$$G_{31} = \frac{\overline{M}}{\delta} - \frac{\overline{B}_1\bar{s}_1^2}{\bar{h}_1^2} + \phi(p_{31})\left[\frac{\beta^2\overline{M}^2}{\delta^4}\left(\alpha_{b1}^2 + \alpha_{g1}^2\right) + \left(\frac{2\overline{B}_1\bar{s}_1^2}{\bar{h}_1^2} + \frac{\beta\overline{M} + 2\gamma\overline{M}}{\delta^2}\right)^2 \alpha_{h1}^2 + \left(\frac{2\overline{B}_1\bar{s}_1^2}{\bar{h}_1^2} - \frac{\gamma\overline{M}}{\delta^2}\right)^2 \alpha_{s1}^2 + \frac{\overline{M}^2}{\delta^2}\alpha_M^2 + \frac{\overline{B}_1^2\bar{s}_1^4}{\bar{h}_1^4}\alpha_{B1}^2\right]^{1/2} \le 0.$$

Here

$$\begin{aligned} \beta &= \bar{b}_1\bar{g}_1\bar{h}_1, \\ \gamma &= \bar{s}_1\bar{h}_1^2/3, \\ \delta &= \bar{b}_1\bar{g}_1\bar{h}_1 + \bar{s}_1\bar{h}_1^2/3. \end{aligned}$$

In constraints (3.20), as well as in objective function (3.4), coefficients of variability α_i (3.1) occur. Coefficients α_i, $(i = 1, 2, 3, 4, 5)$ for sectional dimensions should be determined due to the possibility of ensuring the accuracy of the given dimension. The variability coefficients of material constants α_6 and α_7 can be adopted on the basis of metallurgical standards, whereas the coefficient of variability of the bending moment $\alpha_M = \alpha_8$ is proposed to be calculated from the formula

$$(3.21) \qquad \alpha_M = \Delta M / \left(2\overline{M}\phi\frac{1+p}{2}\right),$$

in which p denotes the probability that the random variable M lies within the interval $\overline{M} \pm \frac{1}{2}\Delta M$.

It is worth noticing that the factors multiplying the expression $\phi(P_{jk})$ in constraints (3.20) include the dispersion of nondeterministic variables.

The larger are the dispersion and the probability p_{jk}, the more difficult it is to fulfil the constraints. The problem formulated is of a general character since the variables of design and parameters are treated as random variables. If the variables of design were treated in a deterministic manner, the objective function would also be a deterministic quantity.

4. Solution of the problem

A solution of the problem consisting in minimization of the objective function described by formula (3.4) at constraints (3.20) in a precise, analytical way is practically impossible due to the complicated form of the formulae. Hence an algorithm of numerical analysis was prepared, on the basis of which a computer program was written to determine the values of optimal dimensions, i.e. such ones which minimize the section at required probabilities of constraint fulfilment.

The algorithm is as follows.

1. Assume the quantities: $\overline{R}_g$, $\overline{M}$, $\overline{\alpha}_i$, $\overline{I}$, $\overline{E}$, ν, ϕ_{jk} and assume the initial value of the objective function F (greater than the anticipated optimal value), and the required accuracy of calculations and steps s_{bg}, s_h, s_s, s_b.
2. Assume the initial value of the quotient $(\bar{b}\bar{g})$ (smaller than the anticipated optimal value).
3. Increase the quotient $(\bar{b}\bar{g})$ by step s_{pg}.
4. Assume of the initial value $\bar{h}$ (smaller than the anticipated optimal value).
5. Increase $\bar{h}$ by step s_h.
6. Assume the initial value $\bar{s}$ (smaller than the anticipated optimal value).
7. Increase $\bar{s}$ by step s_s.
8. Verify the web stability condition.
9. If the condition is fulfilled, go to point 7.
10. Assume the initial value $\bar{b}$ (greater than the anticipated optimal value).
11. Reduce $\bar{b}$ by step s_b.
12. Calculate the value $\bar{g}$ as $(\bar{b}\bar{g})/\bar{b}$.

13. Verify the flange stability condition.

14. If the condition is not fulfilled, go to point 11.

15. Verify the strength condition and the integral stability.

16. If the conditions are not fulfilled, go to point 5.

17. Calculate of the value of the objective function.

18. If the value of the objective function is smaller than the ones calculated so far, go to point 3; if it is greater, return to the sectional dimensions for which the objective function has reached the minimum.

19. If the accuracy of calculations is satisfying (in agreement with the assumption made), print the values $\bar{h}$, $\bar{s}$, $\bar{b}$, $\bar{g}$, and the programme is completed.

20. Apply tenfold reduction of the steps s and go to point 3.

It should be noticed that in determining the starting point, the formulae defining the optimal dimensions at the deterministic approach to the problem [1] are helpful. Initial values can also be adopted with a great margin, without preliminary calculations. In such a case, applications of relatively large steps s will quickly shift the starting point towards the optimum point. This will not significantly increase the calculation time (one or two more calculation cycles will be performed, but the number of calculation points in the cycles will be small).

5. Analysis of the obtained results and conclusions

Basing upon the described algorithm, a computer programme was prepared by means of which a number of calculations were performed. They were aimed at:

a) establishing the influence of constraint fulfilment probability on the cross-sectional dimensions,

b) comparison of the cross-sectional area of box and I-beams,

c) comparison of the results obtained in a deterministic and probabilistic case.

a

The influence of the probability of satisfaction of the limitations imposed on the cross-sectional dimensions is shown in Fig. 4. For the sake of clarity of the diagram of probability, p_{jk}, the values $j = 1, 2, 3, 4$ and $k = 1, 2$ were assumed. This is a situation when fulfilment of the condition of local stability and strength is equally probable. Condition for g_{42} (3.16) was ignored to introduce no disturbances into the diagram of the I-beam. In other words, if the condition g_{42} is disregarded, the accurate solution will be obtained for short beams ($1 \le l_g$, l_g – the length limit above which the constraint g_{42} becomes active).

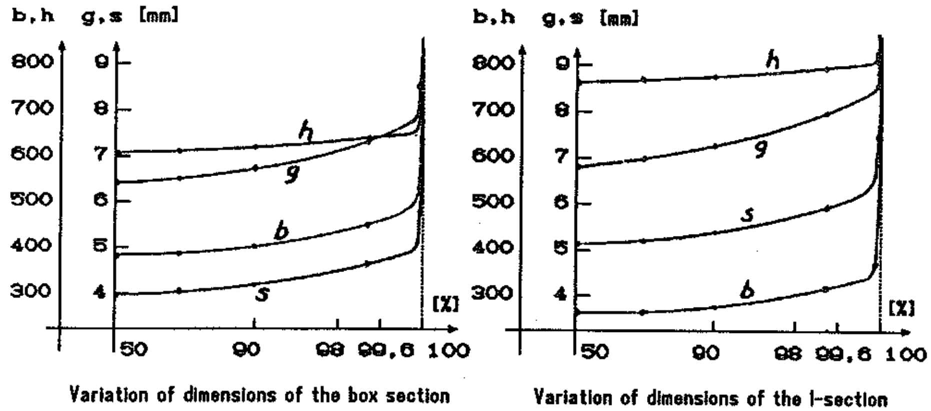

FIG. 4.

Calculations were carried out for a beam made of a material of R_g = 200 MPa. The moment $M = 400$ kNm acts in the considered section. The material constants are $E = 2.1 \cdot 10^5$ MPa, $\nu = 0.3$. The values of coefficients α were adopted as follows:

$$\begin{aligned} \alpha_b^2 &= \alpha_h^2 = 10^{-7}, & \alpha_M^2 &= 0.007, \\ \alpha_g^2 &= 0.0013, & \alpha_{A1}^2 &= 1.6 \cdot 10^{-5}, \\ \alpha_s^2 &= 0.0026, & \alpha_{A2}^2 &= 3.24 \cdot 10^{-6}, \\ \alpha_{Rg}^2 &= 0.0004, & \alpha_{B1}^2 &= \alpha_{B2}^2 = 5.76 \cdot 10^{-4}. \end{aligned}$$

It follows from the diagrams that the increase in reliability of the constraint fulfilment causes an increase in cross-sectional dimensions of the beam, whereas for probabilities approaching 100% the surface of the cross-section increases very fast.

b

The influence of the length of the I-beam on the cross-sectional area is shown in Fig. 5. As it was mentioned above, for short beams ($1 \le l_g$) the condition for g_{42} is not active; it means that the critical moment $M_{\rm cr} > M$. For long beams the constraint g_{42} becomes active and influences the cross-sectional dimensions.

The probabilities of constraint fulfilment adopted in our considerations are: line *1* – $p_{j2} = 50\%$, line (curve) *2* – $p_{j2} = 90\%$, and line (curve) *3* – $p_{j2} =$

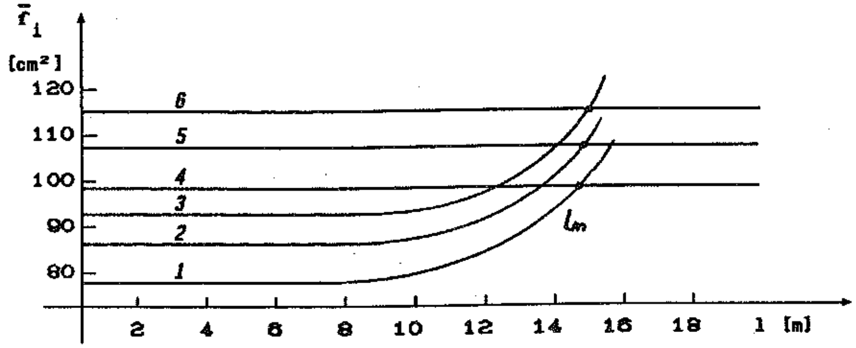

FIG. 5.

99% (j = 1, 2, 3, 4). Curves of the values of cross-sectional area of a box beam – lines *4*, *5*, and *6* were also plotted for comparison (for p_{j1} the same as for a I-beam: j = 1, 2, 3). It can be seen that, within the range $l < l_m$, the cross-sectional area of the I-beam is smaller; within the range $l > l_m$, the cross-sectional area of box beams is smaller. This results from the fact that, for the lengths considered, the condition of twisting in box-beams is not active [1], so it was not taken into consideration. However, it follows from the diagrams that this condition influences the change of I-beam cross-section. Hence, within the range $1 < 1m$, application of I-beams is more advisable due to their small weight and, consequently, lower cost of the material and labour as compared to box-beams.

c

Carrying out optimization in a conventional manner, the nondeterministic character of variables is not considered, which results in the adoption of $\alpha_{yi} = 0$ Eq. (3.1). In that case the objective function can be described by formulae (3.4) for $k' = 1$ and $k'' = 0$. If, in constraints (3.20), $\alpha_{yi} = 0$ is adopted (for $i = 1, 2, \ldots 8$), then the whole expression under the root sign will vanish. The same effect will be obtained by adopting the probabilities of fulfilment of all constraints $p_j = 50\%$ (at $\alpha_{yi} \neq 0$; $i = 1, 2, \ldots 8$).

Thus the dimensions calculated by the classical method would correspond to the dimensions which would fulfil the constraints with a 50% probability. An increase in probability can be attained by applying sufficiently high safety coefficients in the classical method. For a clear illustration of the problem, an example is presented in which only one coefficient of safety $x > 1$ increasing the moment M is considered. Its influence on the value of the cross-sectional surface area calculated in a classical manner for a box-beam is shown in the lower part of the diagram (Fig. 6). Upper part of the diagram

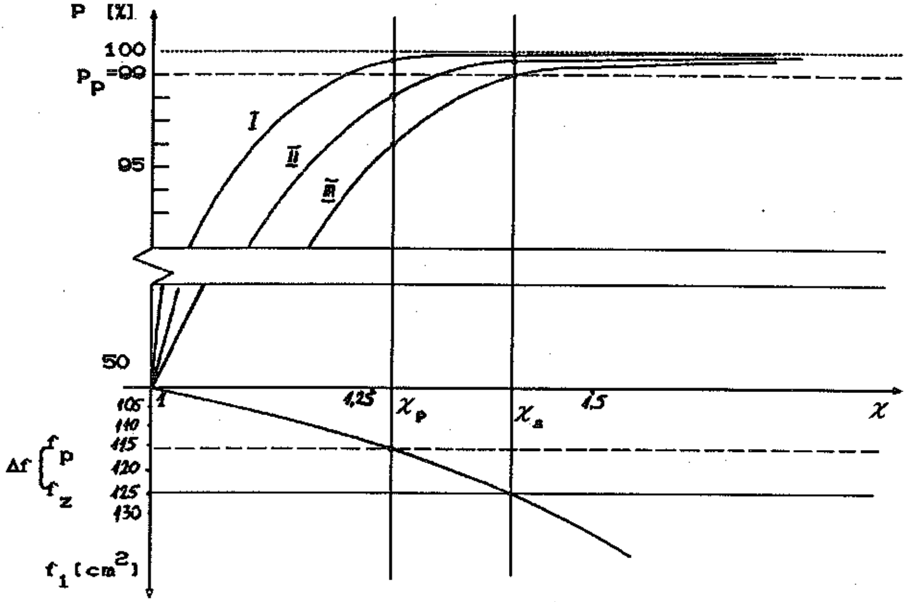

FIG. 6.

shows the probabilities of the fulfilment of particular constraints for beam dimensions calculated in this manner. Particular lines refer to the respective conditions: line I – to the strength condition, II – to the condition of flange stability, III – to the condition of web stability. Dashed lines denote the assumed probability $p_p = 99\%$ and the corresponding cross-sectional surface area of beam f_p obtained from the probabilistic analysis for the previously adopted input data. It follows from the diagram that the area obtained from the classical analysis for $\chi = \chi_p$ is equal to the area obtained from the probabilistic analysis, whereas the dimensions obtained from classical calculations fulfil particular constraints for various probabilities, i.e.

- strength with probability $p > p_p$ – line I,
- local stability of the flange with probability $p < p_p$ – line II,
- local stability of the web with probability $p < p_p$ – line III.

It follows that the classical analysis and assumption of a coefficient of safety $\chi > 1$ generally ensures an increase in the probability of constraint fulfilment, but not to the same degree. In order to ensure the satisfaction of all constraints at a probability greater or equal p_p, $\chi = \chi_2$ should be assumed and then the obtained area f_z will be greater by Δf than the area f_p.

Classical calculations do not always give a possibility of increasing the probability of not violating the individual constraints to the same degree. They do not enable us to determine the real degree of increase in the reliability of constraint fulfilment with the increase in the parameter χ. As a result, the coefficient χ determined in this manner is too high; consequently, an unjustified increase in the cross-sectional area (hence also in the beam weight) follows.

References

1. T.Gibczyńska, *Optimum design of thin-walled box beams* [in Polish], Zesz. Nauk. Pol. Krak., Mech., 4, 63, 1980.
2. S.Jendo and W.Marks, *Structural optimization by stochastic programming methods* [in Polish], Ossolineum, Warszawa 1983.
3. S.P.Timoshenko and J.M.Gere, *Theory of elastic stability* [Polish translation], Arkady, Warszawa 1963.
4. D.Frankpol and J.Rondal, *Le dimensionnement probabiliste optimal des structures*, Het Ingenierusblad, 46, 7, 189–195, 1977.
5. S.Jendo, W.Marks and A.M.Brandt, *Probabilistic approach to reliability-based optimum structural design*, Rozpr. Inż., **32**, 1, 57–74, 1984.

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