ENGINEERING TRANSACTIONS • Engng. Trans. • 41, 2, 167–175, 1993
Polish Academy of Sciences • Institute of Fundamental Technological Research

MODELLING OF LOADING RATE EFFECT ON FATIGUE CRACK GROWTH

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A mechanical model of fatigue crack growth at low and intermediate ΔK values is presented. In the model the effect of the loading rate on the fatigue crack growth is considered. The local stress and strain are calculated based on the Hutchinson, Rice and Rosengren (HRR) solution. The required data for predicting fatigue crack growth rate can be found in standard material handbooks where cyclic and fatigue properties of the materials are presented.

1. Introduction

Experimental data indicate that the fatigue crack growth rate (FCGR) may be described by a sigmoidal curve in a $\log(da/dN)$ versus $\log(\Delta K)$ coordinate scale. The behaviour of the FCGR is frequently described as having three ranges. Range I is associated with the very slow FCGR behaviour in the vicinity of the threshold stress-intensity factor (ΔK_{th}). Range II describes stable, subcritical FCGR which is frequently analytically expressed by Paris–Erdogan [1] law in the form

$$da/dN = A(\Delta K)^m, \tag{1.1}$$

where A and m are empirically determined constants. Range III describes the behaviour exhibited at very high FCGR which is characterized by the critical stress intensity ΔK_c. The FCGR may be influenced by many factors associated with load history and environment [2]. In the present paper special attention is paid to the effect of loading rate on FCGR. Experimental data suggest that fatigue crack propagation rate decreases with increase of frequency. On the other hand, Yokobori and Sato [3] have shown that exponent m of ΔK is a constant and independent of frequency. Based on the conducted experiments they showed that da/dN is inversely proportional to $f^{\lambda}(\lambda > 0)$. In this case they proposed the modification of Eq. (1.1) to the following form

$$da/dN = A(\Delta K)^m f^{-\lambda} \tag{1.2}$$

where f is a frequency and λ is a material constant.

In contrast to the monotonic loading, in the cyclic loading due to repeated loading and unloading of the plastic zone in front of the crack, a crack-tip blunting occurs. Furthermore, microcracks are produced in the plastic zone, or more particularly in front of the crack in the generally termed damage process zone.

Thus, special care has to be taken in analyzing this region when continuum theories are used to determine the stress and strain distribution ahead of the crack.

A desirable feature of crack propagation model under cyclic loading would be the incorporation of the fatigue properties in nonlinear fracture mechanics. It is the subject of this paper to present such a model with loading rate effect to determine FCGR.

2. A model for fatigue crack growth

Analysis of the fatigue crack growth during cyclic loading requires a fatigue failure criterion and specification of the zone where such a criterion can be applied. The finite element calculations of stress and strain ahead of the crack tip have shown that there exists a small region, the damage process zone, denoted herein by δ, where the stress and strain have a finite magnitude. This zone, as shown in experiments, is a few times smaller than the plastic zone. It would be pertinent at this stage to mention that a process zone, δ, may be associated with the microstructure and/or micro-failure mechanism. The damage process zone can be defined as the set of cells that have reached the state of fatigue damage, which is proportional to the plastic strain energy density.

Since the fatigue damage is generally caused by the cyclic plastic strain, the plastic strain energy plays an important role in damage process. Therefore, to describe the damage process in the front of the crack we should apply the fatigue criterion based on plastic strain or plastic strain energy density.

2.1. *Fatigue criterion*

In the present paper the criterion based on total strain energy density is adopted [4, 5]. Since the process of damage in the process zone is controlled by plastic strain range, the fatigue criterion can be expressed in the form

$$\Delta W^p = \zeta (2N_f)^{\beta} f^{\gamma}, \tag{2.1}$$

where ζ and β are material parameters. In the cases where the values of ζ and β are not available, the approximate relationships to compute them can be obtained throught the Manson–Coffin law, i.e.

$$\begin{aligned} \Delta\varepsilon^p &= 2\varepsilon'_f (2N_f)^c f^d, \\ \Delta\sigma &= 2\sigma'_f (2N_f)^b f^e. \end{aligned} \tag{2.2}$$

Therefore

$$\beta \simeq b + c, \qquad \zeta \simeq 4\sigma'_f \varepsilon'_f \frac{1-n'}{1+n'}, \qquad \gamma = d + e, \tag{2.3}$$

where n' is the cyclic strain-hardening exponent, (σ'_f/E) and ε'_f are the strain amplitude corresponding to the elastic and plastic intercept for one cycle, b is the fatigue strength exponent and c is the fatigue ductility exponent.

2.2. *Stress and strain distribution ahead of the crack tip*

For a strain hardening material, stress-strain relation for most metals can be expressed as

$$\varepsilon^* = \sigma^* + \alpha (\sigma^*)^N, \tag{2.4}$$

where ε^*, σ^*, are nondimensional strain and stress; α, N are material parameters.

Generalizing this uniaxial relationship by J_2 deformation theory of plasticity, Hutchinson [6], and Rice and Rosengren [7] obtained similar solutions for the elastic-plastic stress and strain distribution ahead of a crack tip (frequently referred to as the HRR singularity fields) in antiplane shear (Mode III) under small scale yielding. According to Hutchinson's plane stress solution [6], the stress and strain components normal to the plane of crack growth can be calculated from following equations:

$$(2.5)\qquad \varepsilon = \frac{\sigma_y}{E}\left[\frac{K_{\mathrm{III}}^2}{\alpha I \sigma_y x}\right]^{1/1+N}(\sigma_0 - \nu\sigma_r) + \alpha\frac{\sigma_y}{E}\left[\frac{K_{\mathrm{III}}^2}{\alpha I \sigma_y x}\right]^{N/1+N}(\sigma_0 - 0.5\sigma_r),$$

$$(2.6)\qquad \sigma = \sigma_y\left[\frac{K_{\mathrm{III}}^2}{\alpha I \sigma_y x}\right]^{1/1+N}\sigma_0\,.$$

Parameters σ_0 and σ_r are nondimensional functions of the strain hardening exponent N (and the polar coordinate 0 in general case). Parameter I is a nondimensional function of N. The values of the parameters σ_0, σ_r and I for certain values of N are given by HUTCHINSON [6].

A similar analytical solutions for tensile loading (Mode I), which generally is the most critical case from the engineering application viewpoint, is not yet available. MCCLINTOCK [8] has shown that there is an analogy between Mode III and I for the case of small scale yielding, where displacements parallel to the crack are small compared to those normal to the crack surface. Thus an analogous equations to (2.5) and (2.6) can be written for Mode I, in the following form:

$$(2.7)\qquad \varepsilon = \frac{\sigma_y}{E}\left[\frac{K^2}{\alpha I \sigma_y x}\right]^{1/1+N}(\sigma_0 - \nu\sigma_r) + \alpha\frac{\sigma_y}{E}\left[\frac{K^2}{\alpha I \sigma_y x}\right]^{N/1+N}(\sigma_0 - 0.5\sigma_r),$$

$$(2.8)\qquad \sigma = \sigma_y\left[\frac{K^2}{\alpha I \sigma_y x}\right]^{1/1+N}\sigma_0\,.$$

In the case of cyclic loading the stress-strain relation of metals can be represented by

$$(2.9)\qquad \frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon^e}{2} + \frac{\Delta\varepsilon^p}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{1/n'},$$

where $\Delta\varepsilon$ is the cyclic strain range, $\Delta\sigma$ – the stress range, n' – the cyclic strain hardening exponent and K' is a coefficient with dimensions of stress. The relations between the constants in Eqs. (2.4 and (2.9) is

$$(2.10)\qquad N = 1/n', \qquad \alpha = \frac{2E}{(2K')^N \sigma_y^{(1-N)}}\,.$$

As it was mentioned above, Eqs. (2.7) and (2.8) were obtained from Hutchinson's plane stress solution for monotonic loading. For cyclic loading, one should use $\Delta K/2$, instead of K; replace N by $1/n'$ and σ_y by σ'_y.

Therefore we get

$$(2.11)\qquad \Delta\varepsilon = \frac{2\sigma'_y}{E}\left[\frac{\Delta K^2}{4\alpha I\sigma'_y x}\right]^{n'/1+n'}(\sigma_0-\nu\sigma_r) + \alpha\frac{2\sigma'_y}{E}\left[\frac{\Delta K^2}{4\alpha I\sigma'_y x}\right]^{1/1+n'}(\sigma_0-0.5\sigma_r),$$

$$(2.12)\qquad \Delta\sigma = 2\sigma'_y\left[\frac{\Delta K^2}{4\alpha I\sigma'_y x}\right]^{n'/1+n'}\sigma_0\,.$$

Equations (2.11) and (2.12) exhibit a singularity as $x \to 0$. It has been widely recognized [9–14] that a small region, denoted herein by δ, exists immediately ahead of the crack tip where, due to non-proportional plasticity and crack blunting, the strains and stresses have finite magnitudes. In this region the stress and strain gradients are much smaller than those predicted by the HRR fields. Good agreement was demonstrated between the finite-element solution and HRR prediction for a region adjacent to the crack tip, of diameter equal to several times the crack opening displacement. Ahead of the crack tip, the HRR stress and strain fields showed a trend whereby stresses were higher and strains were lower than those predicted by the finite-element solutions.

However, prediction based on Eqs. (2.11), (2.12) is not satisfactory when $x \to 0$ resulting in strain energy density $\Delta W^p \to \infty$. This is obviously unreasonable since the stress range $\Delta\sigma$, and the plastic strain range $\Delta\varepsilon$ cannot exceed $2\sigma'_f f^e$ and $2\varepsilon'_f f^d$, as predicted by Eq. (2.1).

Let us now introduce a critical blunting radius, r_c, associated with the threshold stress intensity factor $\Delta K_{\rm th}$, below which cracks will not propagate. Thus, replacing x by $x+r_c$ and leaving for the time being r_c undefined, Eqs. (2.11), (2.12) can be rewritten as

$$(2.13)\qquad \Delta W^p = \frac{1-n'}{1+n'}\Delta\sigma\,\Delta\varepsilon^p = \frac{1-n'}{1+n'}L\frac{\Delta K^2}{IE(x+r_c)}\,,$$

where

$$L = \sigma_0(\sigma_0 - 0.5\sigma_r).$$

2.3. Fatigue crack growth expression

The plastic strain energy density within the process zone may be calculated from Eq. (2.13) by setting $x = \delta$. The corresponding number of cycles ΔN, required for the crack to penetrate through δ, can be determined from Eq. (2.1). Substituting Eq. (2.1) into (2.13) the crack growth rate per cycle da/dN can therefore be estimated as follows

$$\frac{da}{dN} = \frac{\delta}{\Delta N} = \frac{L(1-n')\Delta K^2}{(1+n')IE\zeta f^{\gamma}} \frac{(2\Delta N)^{-\beta}}{\Delta N} - \frac{r_c}{\Delta N} . \tag{2.14}$$

The r_c can be calculated by assuming that for $\Delta K = \Delta K_{\text{th}}$, $da/dN = 0$. The experiments show unstable crack growth when the stress intensity range approaches the critical value, i.e. $\Delta K - \Delta K_c$. Then putting $2\Delta N = 1$ in Eq. (2.14) we can calculate the value of δ as follows:

$$\delta = L\frac{1-n'}{1+n'} \frac{\Delta K_c^2 - \Delta K_{\text{th}}^2}{\zeta IEf^{\gamma}} . \tag{2.15}$$

On the other hand, threshold stress intensity range is strongly dependent on the stress ratio. In literature many models describing this effect have been proposed. In the present approach the formula proposed by KLESNIL and LUCAS [15] is used

$$\Delta K_{\text{th}}^{*}(R) = \Delta K_{\text{th}}^{0}(1-R)^{\eta} , \tag{2.16}$$

where R is the stress ratio, η is the material constant and ΔK_{th}^{0} is the threshold stress intensity range at $R = 0$.

Rearranging Eq. (2.14), the FCGR can be described as:

$$\frac{da}{dN} = 2\delta \left[L\frac{\Delta K^2 - (\Delta K_{\text{th}}^{*})^2}{\zeta\dfrac{(1+n')}{(1-n')}IE\delta f^{\gamma}} \right]^{-1/\beta} = D\left(\Delta K^2 - \Delta K_{\text{th}}^{*2}\right)^{-1/\beta} , \tag{2.17}$$

where

$$D = 2\delta L^{-1/\beta} / \left[\zeta\frac{(1+n')}{(1-n')} I\, E\, \delta\, f^{\gamma}\right]^{-1/\beta} . \tag{2.18}$$

In the case when $\Delta K_{\text{th}}^{*} \ll \Delta K$ we obtain

$$\frac{da}{dN} = 2\delta \left[L\frac{(\Delta K)^2}{\zeta\dfrac{(1+n')}{1-n'} I\, E\, \delta\, f^{\gamma}} \right]^{-1/\beta} . \tag{2.19}$$

In this case, putting

$$
\begin{aligned}
m &= -2/\beta, \\
A &= 2\delta L^{-1/\beta} / \left[\zeta \frac{(1+n')}{(1-n')} I\, E\, \delta\right]^{-1/\beta}, \\
\lambda &= -\gamma/\beta,
\end{aligned}
\tag{2.20}
$$

we obtain the Yokobari–Sato relationship, Eq. (1.2).

It is worth to mention that a formula of the same type as Eq. (1.2), including the frequency effect, was obtained by Yokobori who used the dislocation theory of fatigue crack growth.

3. Comparison with experiment and discussion

In the present study the experimental data for SM-50 steel [3], and aluminium alloy 2024-T3 [3] were used. In the analysis the values of ζ, β and γ were calculated from Eq. (2.3). Therefore, substituting relationship (2.3) into (2.17) we obtain the following relationship for predicting the FCGR of the materials

$$
\frac{da}{dN} = 2\delta \left[L \frac{\Delta K^2 - (\Delta K^*_{\mathrm{th}})^2}{4\sigma'_f \varepsilon'_f I E \delta f^{d+e}} \right]^{-1/b+c} . \tag{3.1}
$$

The experimental and theoretical results for these materials are shown in Figs. 1 and 2 by solid lines. Predictions of the proposed model of the materials are in good agreement with the FCGR in the analyzed loading rate range.

In this model the HRR plane stress solution for the calculation of the plastic strain energy density distribution ahead of the crack tip is used. A fatigue failure criterion based on the plastic strain energy density is adopted to define a region which can be described in macroscopic terms. The effect of loading rate on the FCGR is considered in this approach.

The model developed herein indicates that constants A, m and λ in the Yokobori–Sato empirical equations are mutually dependent. The required data can be found in the material handbooks where fatigue properties of materials are listed.

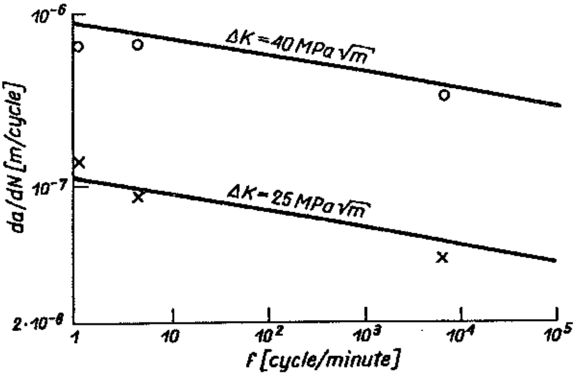

FIG. 1. Theoretical predictions and experimental data of fatigue crack growth rate for SM–50 steel [3].

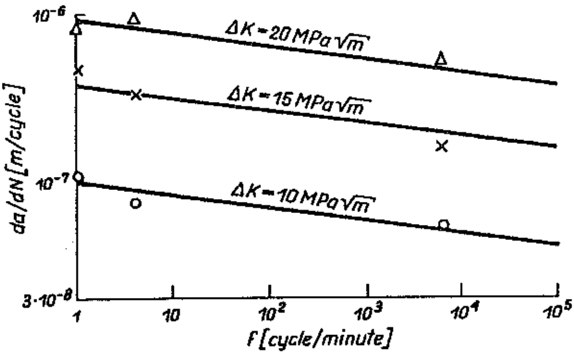

FIG. 2. Theoretical predictions and experimental data of fatigue crack growth rate for 2024-T3 aluminium alloy [3].

REFERENCES

1. P.C.PARIS and F.ERDOGAN, *A critical analysis of crack propagation*, Trans. Am. Soc. Mech. Engrs., J. Basic Engng. Series D, **85**, 528–534, 1963.
2. S.KOCAŃDA and J.SZALA, *Foundations of fatigue analysis* [in Polish], PWN, Warszawa 1985.
3. T.YOKOBORI and K.SATO, *The effect of frequency on fatigue crack propagation rate and striation spacing in 2024-T3 aluminium alloy and SM-50 steel*, Engng. Fracture Mechanics, **8**, 81–88, 1976.
4. K.GOŁOŚ, *Energy based fatigue criterion*, Arch. Bud. Maszyn, **25**, 1–2, 5–16, 1988.
5. K.GOŁOŚ and F.ELLYIN, Trans. ASME, *A total strain energy theory for cumulative fatigue damage*, J. Press. Vessel Tech., **110**, 35–41, 1988.

6. J.W.HUTCHINSON, *Singular behaviour at the and of a tensile crack in a hardening material*, J. Mech. Phys. Solids, **16**, 30-31, 1968.
7. J.R.RICE and G.F.ROSENGREN, *Plane strain deformation near a crack tip in a power-law hardening material*, J. Mech. Phys. Solids, **16**, 1–12, 1968.
8. F.A.MCCLINTOCK, [in:] Fracture of Solids, D.DRUCKER [Ed.], 65–102, Wiley, New York 1963.
9. S.D.ANTOLOVICH, A.SAXENA and G.R.CHANANI, *A model for fatigue crack propagation*, Engng. Fract. Mech., **7**, 649–652, 1975.
10. S.MAJUMDAR and J.MORROW, *Correlation between fatigue crack propagation and low cycle fatigue properties*, ASTM STP, **559**, 159–182, 1974.
11. G.GLINKA, *Cumulative model of fatigue crack growth*, Int. J. Fatigue, **4**, 161–166, 1982.
12. L.C.RADON, *A model for fatigue crack growth in a threshold region*, Int. J. Fatigue, **4**, 161–166, 1982.
13. D.KUJAWSKI and F.ELLYIN, *A fatigue crack propagation model*, Engng. Fracture Mech., **20**, 695–704, 1984.
14. K.GOŁOŚ, *Crack propagation model for cyclic loading*, Proc. 5th Int. Conf. on Mechanical Behaviour of Metals (ICM 5), Pergamon Press, 567–573, 1987.
15. M.KLESNIL and P.LUKAS, *Influence of strength and stress history on growth and stabilisation of fatigue cracks*, Engng. Fracture Mech., **4**, 77–92, 1972.
16. T.YOKOBORI, *A critical evaluation of mathematical equations for fatigue crack growth*, ASTM STP, **675**, 683–706, 1979.

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Received February 26, 1992, new version January 22, 1993.