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THERMOELASTIC WAVES IN A TRANSVERSELY ISOTROPIC PLATE WITH THERMAL RELAXATIONS

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The generalized form of heat conduction equation is used to study the thermoelastic waves in a transversely isotropic, thermally conducting and infinitely extended, stretched elastic plate due to a cylindrical projectile, in the context of the Green and Lindsay theory of thermoelasticity. The Laplace transform technique is employed to obtain small time solutions. The discontinuities in stresses and temperature have also been discussed at their wavefronts. The jumps obtained have been computed numerically and are illustrated graphically for a single crystal of zinc.

1. INTRODUCTION

The thermoelasticity theory which includes the temperature rate in constitutive equations, developed by LORD and SHULMAN [2] and GREEN and LINDSAY [3] is a generalization of the conventional coupled thermoelasticity theory [1] and predicts the finite speed for the propagation of thermal signals. The generalized theory of thermoelasticity have been extended to anisotropic media by DHALIWAL and SHERIEF [4]. BANERJEE and PAO [5] also discussed the propagation of thermoelastic waves in anisotropic solids based on the theory of thermoelasticity which includes the effect of thermal phonon relaxation, and illustrated their results numerically and graphically for NaF and solid helium crystals. SHARMA [6] studied the transient generalized thermoelastic waves in transversely isotropic medium with a cylindrical hole. SHARMA *et al.* [7] investigated the distribution of displacements, temperature, and stresses due to a thermal shock in a homogeneous transversely isotropic elastic solid with cylindrical hole, in the context of generalized theories [2, 3] of thermoelasticity. KUMAR [8] studied the coupled thermoelastic waves in an infinitely extended plate resulting from a suddenly punched cylindrical hole. SHARMA and CHAND [9, 10] studied the thermoelastic waves in a homogeneous isotropic elastic plate due to suddenly punched hole in the context of generalized theories of thermoelasticity [1, 2].

In this article, the distributions of displacements, temperature, and stresses in a homogeneous transversely isotropic stretched elastic plate due to sudden punching of a cylindrical hole have been studied in the context of the generalized theory of thermoelasticity [3] by employing the Laplace transform technique.

2. Formulation of the problem

We consider a homogeneous transversely isotropic thermally conducting, infinitely extended stretched plate of thickness d, initially at temperature T_0 in the undeformed state. We take the origin of the coordinate system on the plane surface, and the z-axis pointing normally into the plate which is thus represented by $z \geq 0$. The z-axis is assumed to coincide with the axis of elastic and thermal symmetry of the material, and the planes of isotropy are perpendicular to z-axis. Let a flat nose [10] cylindrical projectile of radius a, moving with velocity v, strike the plate and begin to punch a hole of radius equal to its own. The following assumptions are taken into considerations:

- The plastic flow due to punching is localized in the neighbourhood of punching section and the punching starts instantaneously at $t = 0$ over the whole punched section, based on small thickness d of the plate and large value of impact speed v.
- The punching action takes place at an average speed $v/2$, which is the projectile's velocity in the compressional wave that develops in both the projectile and the plate, on a large portion of the diameter of the projectile and the plate. Thus the punching time $2d/v = l'$ is based on a large ratio of the diameter of projectile to the plate thickness.

We choose the origin of the cylindrical coordinate system (r, θ, z) at the axis of the cylindrical hole. Assuming the radial symmetry, the non-zero displacement component $u = u(r,t)$ is obtained. Then the governing field equations of motion and heat conduction, in the absence of body forces and heat sources, are [4]

$$
\begin{aligned}
&c_{11}\left[u_{,RR} + (R^{-1}u)_{,R}\right] - \beta_1(T + \tau_1 \dot{T})_{,R} = \rho\, \ddot{u}, \\
&K(T_{,RR} + R^{-1}T_{,R}) - \rho C_e(\dot{T} + \tau_0 \ddot{T}) = T_0\beta_1(\dot{u}_{,R} + R^{-1}\dot{u}),
\end{aligned} \tag{2.1}
$$

where

$\beta_1 = (c_{11} + c_{12})\alpha_1 + c_{13}\alpha_3$ is the coupling between elastic and thermal fields,
c_{ij} are isothermal elastic parameters,
α_1, α_3 coefficients of linear thermal expansions,
K thermal conductivity,
ρ material density,
C_e specific heat at constant strain,
τ_1, τ_0 thermal relaxation times satisfying the inequalities

$$\tau_1 \geq \tau_0 \geq 0. \tag{2.2}$$

The comma is used to denote spatial derivatives, and a superposed dot represents the time derivatives.

Define the following physical quantities:

$$\begin{aligned} r &= \omega^* R/v_p, & \tau &= \omega^* t, & U &= \rho\omega^* v_p u/T_0\beta_1, \\ Z &= T/T_0, & \tau_1' &= \omega^* \tau, & \tau_0' &= \omega^* \tau_0, \\ \varepsilon &= T_0\beta_1^2/\rho c_{11} C_e, & \omega^* &= c_{11} C_e/K, & v_p^2 &= c_{11}/\rho, \end{aligned} \tag{2.3}$$

where ω^* is the characteristic frequency, ε is the coupling constant, and v_p be the velocity of the longitudinal wave.

Introduce the physical quantities (2.3) into Eqs. (2.1), we get

$$\begin{aligned} U_{,rr} + r^{-1}U_{,r} - r^{-2}U - \ddot{U} &= Z_{,r} + \tau_1' \dot{Z}_{,r}, \\ Z_{,rr} + r^{-1}Z_{,r} - (\dot{Z} + \tau_0' \ddot{Z}) &= \varepsilon(\dot{U}_{,r} + r^{-1} \dot{U}), \end{aligned} \tag{2.4}$$

where we have supressed the dashes in τ_1 and τ_0.

The boundary of the hole $R = a$, is given by

$$r = \omega^* a/v_p = \eta \qquad \text{(say)}. \tag{2.5}$$

The initial and regularity conditions are given by

$$\begin{aligned} &U = 0 = Z \quad \text{at} \quad \tau = 0, \quad r = \eta, \\ &\frac{\partial U}{\partial \tau} = 0 \quad \text{at} \quad r = 0, \end{aligned} \tag{2.6}$$

$$U = 0 = Z \quad \text{for} \quad \tau = 0, \quad \text{when} \quad r \to \infty. \tag{2.7}$$

The boundary conditions are

$$S_{rr} = \left\{ \begin{array}{ll} 0, & \tau < 0 \\ -\sigma\tau/l, & 0 < \tau < l \\ -\sigma, & \tau > l \end{array} \right\} \quad \text{at} \quad r = \eta \quad \text{and} \quad l = 2d\omega^*/v, \tag{2.8}$$

$$Z(\eta, \tau) = 0, \tag{2.9}$$

where

$$\begin{aligned} S_{rr} &= U_{,r} + br^{-1}U - (Z + \tau_1 \dot{Z}), \\ b &= c_{12}/c_{11} \qquad \text{and} \qquad \tau_1^* = \tau_1 + s^{-1}. \end{aligned} \tag{2.10}$$

S_{rr} is the dimensionless form of the stress in the radial direction.

3. Solution of the problem

Applying the Laplace transform defined by

$$\overline{\psi}(r, s) = \int_0^\infty \psi(r, t)e^{-st}dt, \tag{3.1}$$

to Eqs. (2.4) we obtain

$$\begin{aligned} \left[D(D + r^{-1}) - s^2\right]\overline{U} &= \tau_1^* sD\overline{Z}, \\ \left[(D + r^{-1})D - s^2\tau_0^*\right]\overline{Z} &= \varepsilon s(D + r^{-1})\overline{U}, \end{aligned} \tag{3.2}$$

where $D = d/dr$, $\tau_1^* = (\tau_1 + s^{-1})$, and $\tau_0^* = (\tau_0 + s^{-1})$.

Simplifying Eqs. (3.2) we get

$$\begin{aligned} \left[\{D(D + r^{-1})\}^2 - (m_1^2 + m_2^2)D(D + r^{-1}) + m_1^2m_2^2\right]\overline{U} &= 0, \\ \left[\{(D + r^{-1})D\}^2 - (m_1^2 + m_2^2)D(D + r^{-1}) + m_1^2m_2^2\right]\overline{Z} &= 0, \end{aligned} \tag{3.3}$$

where m_i^2, $(i = 1, 2)$ are the roots of the equation

$$m^4 - s(\lambda_1 + s\lambda_2)m^2 + \tau_0^* s^4 = 0, \tag{3.4}$$

where

$$\lambda_1 = 1 + \varepsilon, \qquad \lambda_2 = 1 + \varepsilon\tau_1 + \tau_0. \tag{3.5}$$

Solving Eqs. (3.3) we get

$$\begin{aligned} \overline{U} &= G_1K_1(m_1r) + G_2K_1(m_2r), \\ \overline{Z} &= H_1K_0(m_1r) + H_2K_0(m_2r), \end{aligned} \tag{3.6}$$

where $K_1(m_ir)$ and $K_0(m_ir)$ are the modified Bessel functions of the first and zeroth order, respectively.

From Eqs. $(2.10)_2$, $(3.2)_1$ and (3.6), we get

$$H_i = (s^2 - m_i^2)G_i/m_i, \qquad i = 1, 2. \tag{3.7}$$

Therefore from Eqs.$(2.10)_1$, (3.6) and (3.7), we obtain

$$\begin{aligned} \overline{S}_{rr} &= G_1 A_1(m_1 r) + G_2 A_2(m_2 r), \\ \overline{Z} &= G_1 B_1(m_1 r) + G_2 B_2(m_2 r), \end{aligned} \tag{3.8}$$

where

$$\begin{aligned} A_1(m_1 r) &= [m_1 \beta_2^2 K_1(m_1 r) + r s^2 K_0(m_1 r) \\ &\qquad + r\tau_1 s(s^2 - m_1^2) K_0(m_1 r)]/r m_1, \\ A_2(m_2 r) &= [m_2 \beta_2^2 K_1(m_2 r) + r s^2 K_0(m_2 r) \\ &\qquad + r\tau_1 s(s^2 - m_2^2) K_0(m_2 r)]/r m_2, \\ B_1(m_1 r) &= (s^2 - m_1^2) K_0(m_1 r)/m_1, \\ B_2(m_2 r) &= (s^2 - m_2^2) K_0(m_2 r)/m_2, \\ \beta_2^2 &= (c_{11} - c_{12})/c_{11}. \end{aligned} \tag{3.9}$$

Applying the boundary conditions (2.8) and (2.9) to Eq.$(3.8)_1$, we get

$$\begin{aligned} G_1 &= -\sigma(1 - e^{-ls}) B_2(m_2\eta)/ls^2\Delta, \\ G_2 &= \sigma(1 - e^{-ls}) B_1(m_1\eta)/ls^2\Delta, \end{aligned} \tag{3.10}$$

where

$$\Delta = A_1(m_1\eta)B_2(m_2\eta) - A_2(m_2\eta)B_1(m_1\eta). \tag{3.11}$$

Substituting formulae (3.10) in Eqs. (3.6) and (3.8) we get

$$\begin{aligned} \overline{U}(r,s) &= -\sigma(1 - e^{-ls})[K_1(m_1 r)B_2(m_2\eta) \\ &\qquad - K_1(m_2 r)B_1(m_1\eta)]/ls^2\Delta, \\ \overline{Z}(r,s) &= \sigma(1 - e^{-ls}) \Big[m_1(s^2 - m_2^2)K_0(m_2 r)B_1(m_1\eta) \\ &\qquad - m_2(s^2 - m_1^2)K_0(m_1 r)B_2(m_2\eta)\Big] / m_1 m_2 l s^2\Delta, \\ \overline{S}_{rr}(r,s) &= \sigma(1 - e^{-ls})[A_2(m_2 r)B_1(m_1\eta) \\ &\qquad - A_1(m_1 r)B_2(m_2\eta)]/ls^2\Delta. \end{aligned} \tag{3.12}$$

4. Small time approximations

Since the thermal relaxation effects, i.e. "the second sound" effects, are of short duration [11], the discussion is confined to small time approximations, i.e. s is assumed to be large. The roots of the Eq. (3.4) are given by

$$m_i = sv_1^{-1} + \phi_i + o(s^{-1}), \qquad i = 1, 2, \tag{4.1}$$

where

$$\begin{aligned} v_{1,2}^{-1} &= \left[\lambda_2 \pm (\lambda_2^2 - 4\tau_0)^{1/2}\right]^{1/2} / \sqrt{2}, \\ \phi_{1,2} &= \left[\lambda_1 \pm (\lambda_1\lambda_2 - 2)/(\lambda_2^2 - 4\tau_0)^{1/2}\right] / 2\sqrt{2}\left[\lambda_2 \pm (\lambda_2^2 - 4\tau_0)^{1/2}\right]^{1/2}. \end{aligned} \tag{4.2}$$

The above analysis (4.1) shows that there exist two types of waves, namely, an elastic wave and a thermal wave. The former follows the latter one. The modified Bessel function $K_n(z)$ has the asymptotic expansion [12]

$$K_n(z) = \left(\frac{\pi}{2z}\right)^{1/2} e^{-Z}\left[1 + \frac{(4n^2 - 1^2)}{(8z)\ \underline{|1}} + \frac{(4n^2 - 1^2)(4n^2 - 3^2)}{(8z)^2\ \underline{|2}} + \ldots\right]. \tag{4.3}$$

Using Eqs. (3.9), (4.3) in Eqs. (3.12) and solving them, we obtain

$$\begin{aligned} \overline{U}(r, s) &= \sigma(\eta/r)^{1/2}\left[\left\{v_1(v_2^2 - 1)(s^{-2} + E_1 s^{-3} + \ldots)e^{-m_1 R'}\right\}\right. \\ &\quad \left. - \left\{v_2(v_1^2 - 1)(s^{-2} + E_1' s^{-3} + \ldots)e^{-m_2 R'}\right\}\right] / (v_1^2 - v_2^2), \\ \overline{Z}(r, s) &= \sigma(\eta/r)^{1/2}(v_1^2 - 1)(v_2^2 - 1)\left[(s^{-1} + E_2 s^{-2} + \ldots)e^{-m_1 R'}\right. \\ &\quad \left. - (s^{-1} + E_2' s^{-2} + \ldots)e^{-m_2 R'}\right] / (v_1^2 - v_2^2), \\ \overline{S}_{rr}(r, s) &= \sigma(\eta/r)^{1/2}\left[(E_3 + s^{-1}E_4 + \ldots)e^{-m_1 R'}\right. \\ &\quad \left. - (E_3' + s^{-1}E_4' + \ldots)e^{-m_2 R'}\right] / (v_1^2 - v_2^2), \end{aligned} \tag{4.4}$$

where

$$\begin{aligned} E_1 &= \left[3v_1\eta(v_2^2 - 1) - rv_2(v_2^2 - 1) - 8r\eta\left\{(\phi_2 v_2 + \phi_2 v_2^3)\right.\right. \\ &\quad \left.\left. + (v_2^2 - 1)\Gamma\right\}\right] / 8r\eta(v_2^2 - 1), \\ E_1' &= \left[(v^2 - 1)\{3v_2\eta - rv_1\} - 8r\eta\left\{(\phi_1 v_1 + \phi_1 v_1^3)\right.\right. \\ &\quad \left.\left. + (v_1^2 - 1)\Gamma\right\}\right] / 8r\eta(v_1^2 - 1), \end{aligned} \tag{4.5}$$

(4.5) [cont.]

$$E_2 = \Big[(v_1^2-1)(v_2^2-1)\{8r\eta\Gamma-(v_1\eta+v_2r)\}-8r\eta\Big\{(v_2^2-1)(\phi_1v_1+\phi_1v_1^3)+(v_1^2-1)(\phi_2v_2+\phi_2v_2^3)\Big\}\Big]\Big/8r\eta(v_1^2-1)(v_2^2-1),$$

$$E_2' = \Big[(v_1^2-1)(v_2^2-1)\{8r\eta\Gamma-(v_2\eta+v_1r)\}-8r\eta\Big\{(v_1^2-1)(\phi_2v_2+\phi_2v_2^3)+(v_2^2-1)(\phi_1v_1+\phi_1v_1^3)\Big\}\Big]\Big/8r\eta(v_1^2-1)(v_2^2-1),$$

$$E_3 = (v_1^2-1)(v_2^2-1)\tau_1 = E_3',$$

$$E_4 = \tau_1(v_1^2-1)(v_2^2-1)E_2 + v_1(v_2^2-1),$$

$$E_4' = \tau_1(v_1^2-1)(v_2^2-1)E_2' + v_2(v_1^2-1),$$

$$\Gamma = \Big[v_2^2\Big\{(v_1+v_2)+8\eta(\phi_1v_1+\phi_1v_1^3)\Big\} + 8\beta_2^2\Big\{v_2(v_1^2-1)-v_1(v_2^2-1)\Big\} + v_1^2\Big\{(v_1+v_2)(v_2^2-1) + 8\eta(\phi_2v_2+\phi_2v_2^3)\Big\}\Big]\Big/8\eta(v_2^2-v_1^2),$$

and

$$R' = (r-\eta).$$

Now, inverting the Laplace transforms of Eqs. (4.4) we get

$$U(r,\tau) = \sigma(\eta/r)^{1/2}\Big[v_1(v_2^2-1)\{1+E_1(\tau-R'/v_1)\}(\tau-R'/v_1) \times H(\tau-R'/v_1)e^{-\phi_1R'} - v_2(v_1^2-1)\{1+E_1'(\tau-R'/v_2)\} \times(\tau-R'/v_2)H(\tau-R'/v_2)e^{-\phi_2R'}\Big]\Big/(v_1^2-v_2^2),$$

(4.6)

$$Z(r,\tau) = \sigma(\eta/r)^{1/2}(v_1^2-1)(v_2^2-1)\left[\{1+E_2(\tau-R'/v_1)\} \times H(\tau-R'/v_1)e^{-\phi_1R'} - \{1+E_2'(\tau-R'/v_2)\} \times H(\tau-R'/v_2)e^{-\phi_2R'}\right]\Big/(v_1^2-v_2^2),$$

$$S_{rr}(r,\tau) = \sigma(\eta/r)^{1/2}\Big[\{E_3\delta(\tau-R'/v_1)+E_4H(\tau-R'/v_1)\}e^{-\phi_1R'} - \{E_3'\delta(\tau-R'/v_2)+E_4'H(\tau-R'/v_2)\}e^{-\phi_2R'}\Big]\Big/(v_1^2-v_2^2).$$

5. Long time solutions

The long time solutions can be obtained by expanding the values of m_i^2, $(i=1,2)$ of equation (3.4) for small values of s into Taylor's series. The roots m_1, m_2 can be obtained as

$$m_1 = (1+\varepsilon)^{1/2}\sqrt{s} + O(s^{3/2}),$$
$$m_2 = (1+\varepsilon)^{-1/2}s + O(s^2).$$

The roots m_1, m_2 do not contain thermal relaxation times up to the first order, which indicates that the "second sound" effects are of short duration. Therefore, the small time solutions are more useful than long time solutions. However, the expressions for displacement, temperature and stress can easily be obtained by using these roots in different relevant equations.

6. Discussions of the results

The detailed analysis sketched above shows that there are two kinds of waves i.e., the dilatational wave and the thermal wave travelling with velocities v_1 and v_2, respectively. The expressions containing $H(\tau - R'/v_1)$ and $H(\tau - R'/v_2)$ represent the contributions of the dilatational wave and the thermal wave in the vicinity of their wave fronts $R' = v_1\tau$ and $R' = v_2\tau$, respectively.

The displacement is found to be continuous, but the temperature and stress are found to be discontinuous and are given by the formulae

$$
\begin{aligned}
(Z^+ - Z^-)_{R'=v_1\tau} &= \sigma(\eta/r)^{1/2}\Big[(v_1^2-1)(v_2^2-1) \\
&\qquad \times \exp(-\phi_1 v_1 \tau)\Big]/(v_1^2 - v_2^2), \\
(Z^+ - Z^-)_{R'=v_2\tau} &= -\sigma(\eta/r)^{1/2}\Big[(v_1^2-1)(v_2^2-1) \\
&\qquad \times \exp(-\phi_2 v_2 \tau)\Big]/(v_1^2 - v_2^2), \\
(S_{rr}^+ - S_{rr}^-)_{R'=v_1\tau} &= \sigma(\eta/r)^{1/2}\Big[v_1^2(v_2^2-1) \\
&\quad +\tau_1\Big\{(v_1^2-1)(v_2^2-1)(8\eta r\Gamma - (v_1\eta + v_2 r)) \\
&\quad -8\eta r((v_2^2-1)(\phi_1 v_1 + \phi_1 v_1^3) + (v_1^2-1)(\phi_2 v_2 + \phi_2 v_2^3))\Big\}/8\eta r\Big] \\
&\quad \times \{\exp(-\phi_1 v_1\tau)\}/(v_1^2 - v_2^2), \\
(S_{rr}^+ - S_{rr}^-)_{R'=v_2\tau} &= -\sigma(\eta/r)^{1/2}\Big[v_2^2(v_1^2-1) + \tau_1\Big\{(v_1^2-1)(v_2^2-1) \\
&\quad \times(8\eta r\Gamma - (v_2\eta + v_1 r)) - 8\eta r\Big((v_1^2-1)(\phi_2 v_2 + \phi_2 v_2^3) \\
&\quad +(v_2^2-1)(\phi_1 v_1 + \phi_1 v_1^3)\Big)\Big\}/8\eta r\Big]\{\exp(-\phi_2 v_2\tau)\}/(v_1^2 - v_2^2).
\end{aligned} \tag{6.1}
$$

From the above expressions, it is clear that the discontinuities decay exponentially with radial distance.

7. Particular cases

i) If $\tau_1 = \tau_0 = 0$, i.e. in the case of conventional coupled thermoelasticity, we have

$$\lambda_1 = 1 + \varepsilon, \quad \lambda_2 = 1, \quad v_1 = 1, \quad v_2 \to \infty, \quad \phi_1 = \varepsilon/2, \quad \phi_2 \to \infty.$$

From Eqs. $(4.6)_{2,3}$, it is seen that the temperature at both the wavefronts and stress at the thermal wavefront become continuous. The stress suffers a finite jump at the elastic wavefront, given by

$$(S_{rr}^{+} - S_{rr}^{-})_{R'=v_1\tau} = -\sigma(\eta/r)^{1/2}\exp[-\varepsilon\tau/2].$$

ii) If the strain field and thermal field are not coupled to each other, i.e., $\varepsilon = 0$, then

$$\lambda_1 = 1, \quad \lambda_2 = (1 + \tau_0), \quad v_1 = 1, \quad v_2 = (\tau_0)^{-1/2}, \quad \phi_1 = 0, \quad \phi_2 = v_2/2.$$

Here again the stress is found to be discontinuous at the elastic wavefront and the jump is given by

$$(S_{rr}^{+} - S_{rr}^{-})_{R'=v_1\tau} = -\sigma(\eta/r)^{1/2}.$$

iii) When $\varepsilon = 0$, $\tau_1 = \tau_0 = 0$, i.e. the coupling and relaxation effects are ignored, then

$$\lambda_1 = 1, \quad \lambda_2 = 1, \quad v_1 = 1, \quad v_2 \to \infty, \quad \phi_1 = 0, \quad \phi_2 \to \infty.$$

The results obtained agree with case ii).

8. Numerical results and discussion

Various jumps obtained theoretically for temperature and stress at their respective wavefronts are computed numerically for a single crystal of zinc [13] for which the physical data are

$$\begin{aligned}
\rho &= 7.14 \times 10^3\,\mathrm{kg\,m^{-3}}, & \varepsilon &= 0.022,\\
c_{11} &= 1.628 \times 10^{11}\,\mathrm{Nm^{-2}}, & c_{12} &= 0.362 \times 10^{11}\,\mathrm{Nm^{-2}},\\
c_{13} &= 0.508 \times 10^{11}\,\mathrm{Nm^{-2}}, & \beta_1 &= 5.75 \times 10^{6}\,\mathrm{Nm^{-2}deg^{-1}},\\
C_e &= 3.9 \times 10^{2}\,\mathrm{Jkg^{-1}deg^{-1}}, & T_0 &= 296^\circ\mathrm{K}.
\end{aligned}$$

The variations of jumps with respect to time for different relaxation times $\tau_1, \tau_0 = 0.0,\ 0.1,\ 0.5$, are plotted as shown in the Figs. 1 and 2. It is observed that these jumps decay exponentially with time. It is also observed that the jumps at the thermal wavefront decay exponentially at a higher rate than those at the elastic wavefronts.

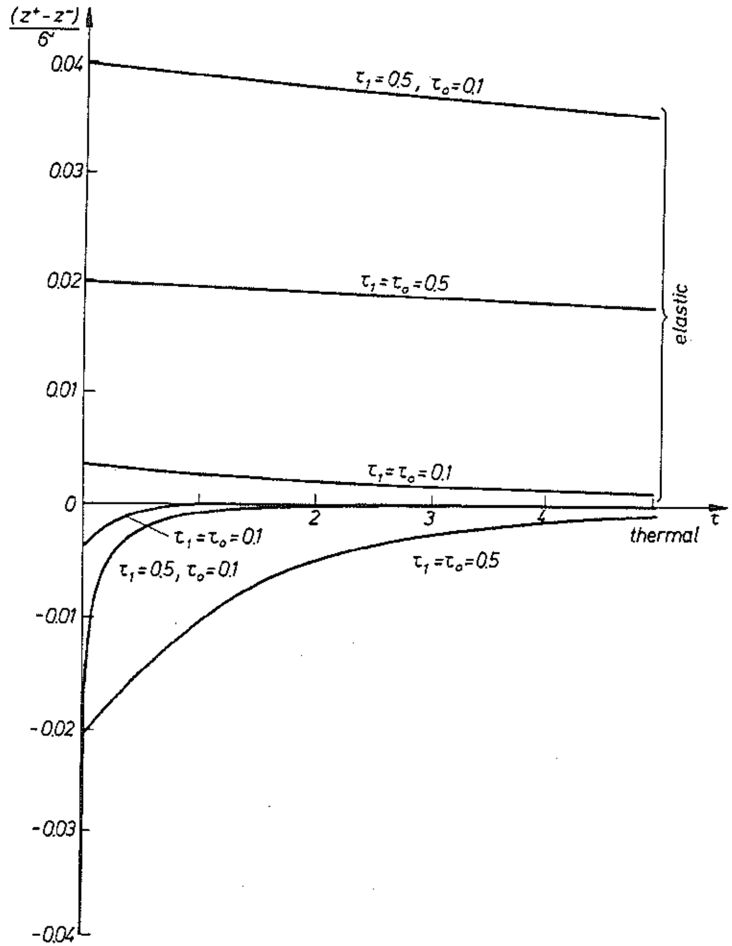

FIG. 1. The variation of jumps in temperature with respect to time, at the wavefronts.

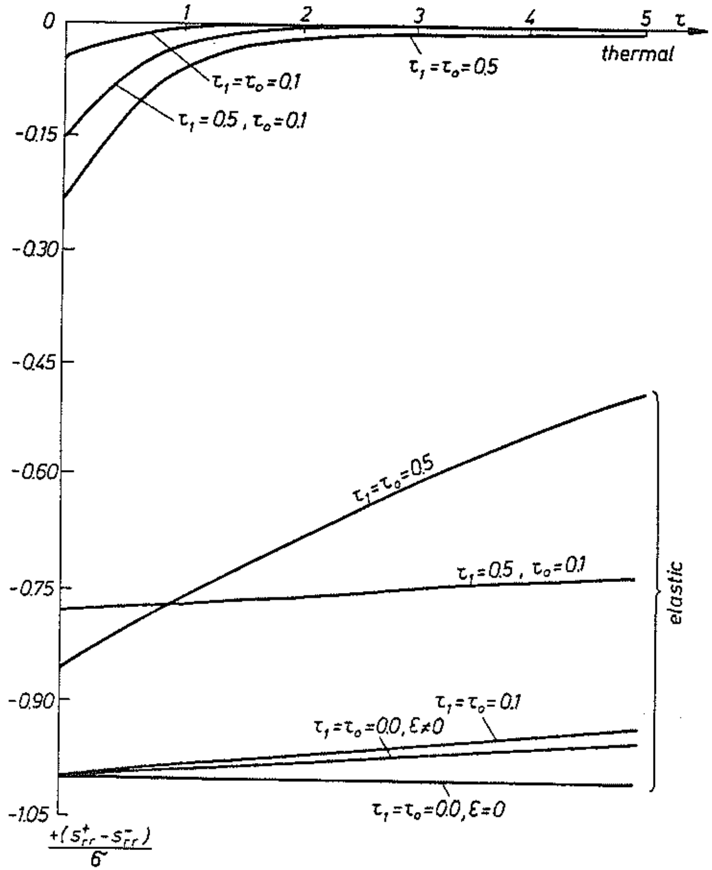

FIG. 2. The variation of jumps in stress with respect to time, at the wavefronts.

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