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DYNAMIC INTERACTION OF INELASTIC STRUCTURES AND FLUIDS (*)

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A semi-analytic algorithm for the analysis of dynamic interaction between an elasto-viscoplastic beam and the linear compressible fluid in a rectangular containment is presented. The inelastic parts of the strain in the beam are treated as unknown eigenstrains acting upon the linear elastic background structure. By means of this consistent eigenstrain analogy, the dynamic interaction problem is represented in a linear form. Using a substructure technique, dynamic influence functions for the eigenstrains are developed in the frequency domain and transformed back to the time domain, partly by analytic transformation, and partly by FFT. The eigenstrains are subsequently evaluated from the inelastic constitutive equations in a time stepping procedure, the influence functions being used, in connection with appropriate non-linear algorithms.

1. Introduction

It is well known that the dynamic fluid-solid interaction problem deserves a careful treatment, since simplified descriptions like Westergaard's added-mass formulation, [1], do not reflect the frequency content of the problem correctly.

Considerable amount of work has been devoted in the literature to the interaction between linear elastic structures and linear compressible fluids, especially in earthquake engineering. For instance, reference should be made to the paper by Chopra [2], Chakrabarti and Chopra [3], Nath [4], Porter and Chopra [5]. An analytic formulation for random vibrations considering fluid-solid interaction is due to Yang and Chiarito [6]. Valuable analytical and numerical, deterministic and random formulations have also been presented by Höllinger [7–10], who was the first to include random vibrations in a numerical BEM-FEM routine, and to use Green's function of the half-space for the BEM applied to the fluid domain; see also

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ZIEGLER, HÖLLINGER and ZHANG [11]. The influence of various complicating effects on the vibrations of linear elastic tanks filled with fluids has been discussed by FISCHER, RAMMERSTORFER and SCHARF [12].

In the present contribution, the dynamic fluid-solid interaction problem is extended to the case of elasto-viscoplastic structures. This practically important question occurs in the case of catastrophic loading of fluid containments, where structural elements are driven into the inelastic range by earthquakes or blasts. For an efficient structural analysis, both time and frequency domain formulations are used. The inelastic parts of strain are treated as additional eigenstrains (sources of selfstress) acting upon the linear elastic background structure, thereby following a classical procedure for static problems dating back to H. REISSNER [13]. This method of eigenstrain analysis has been extended to the dynamics of inelastic structures using various types of constitutive equations, see IRSCHIK and ZIEGLER [14], FOTIU, IRSCHIK and ZIEGLER [15, 16, 17], BRUNNER and IRSCHIK [18], and the literature cited there. The advantage of the eigenstrain analogy lies in the fact that linear elastic methods, like influence functions and modal analysis, can be used in the dynamics of inelastic and geometrically nonlinear structures in a thermodynamically consistent manner. No interaction of the inelastic structure with fluids has been considered in Refs. [14–18].

As an illustrative example for the extension of this eigenstrain analysis to the fluid-inelastic solid interaction problem, a rectangular fluid domain interacting with an elasto-viscoplastic beam is considered in the following. A frequency domain formulation for the linear elastic solid-fluid interaction due to unit time-harmonic eigenstrains is presented first. These dynamic influence solutions are transformed back to the time domain using analytical as well as numerical procedures. The time evolution and space distribution of the as yet unknown eigenstrains is calculated in a time-stepping procedure from the inelastic constitutive relations, using the dynamic influence functions. Any appropriate implicit non-linear algorithm may be used to solve the resulting set of non-linear equations for the eigenstrains. In the present paper, classical plasticity as well as PERZYNA'S elasto-viscoplastic formulation [19] is used to characterize the inelastic behaviour of the beam.

As the main result of our formulation, the dynamic fluid-inelastic solid interaction problem has to be solved only once, before the beginning of the time-stepping procedure, and it needs to be treated only as a linear problem in the frequency domain, despite the nonlinearity in the constitutive equations of the beam. The first account of this method considering the excitation by earthquakes has been given in Ref. [20]. In the present paper, the full set of equations is presented, and a blast-type loading is considered.

Due to the semi-analytic nature of the formulation, the presented numerical example may serve as a benchmark solution for the efficient computational treatement of the dynamic interaction between fluids and inelastic solids.

2. Linear dynamic fluid-solid interaction due to eigenstrains

Considering plane bending of linear elastic beams, the influence of imposed eigenstrains $\bar{\varepsilon}$ is manifested in the imposed curvature

$$\bar{\kappa} = \frac{1}{J} \int_A \bar{\varepsilon} z \, dA. \tag{2.1}$$

The cross-section of the beam is denoted by A, and its geometrical moment of inertia by J. The transverse coordinate is denoted by z. The imposed curvature $\bar{\kappa}$ enters the Bernoulli–Euler differential equation of bending, similarly to a thermal curvature, see Refs. [14–18]:

$$EJ \frac{\partial^4}{\partial x^4} w + \mu \, \dot{w} + \rho A \, \ddot{w} = q - p_F - EJ \frac{\partial^2}{\partial x^2} \bar{\kappa}, \tag{2.2}$$

compare e.g. Ziegler [21] for thermally loaded beams, w denotes the deflection, referred to the initial state. The dot stands for time derivative, ρ is the mass-density, and E is Young's modulus of the beam. The beam axis is denoted by x, $0 \leq x \leq L$, with appropriate boundary conditions at $x = 0$ and $x = L$. An overal linear viscous damping with damping parameter μ is considered without any loss of generality. More sophisticated linear damping models may be introduced likewise. An imposed force loading q of blast-type is considered. The dynamic interactive fluid pressure is denoted by p_F. The fluid pressure p_F is associated with both q and $\bar{\kappa}$. It is emphasized that Eq. (2.2) remains valid in the present case of elasto-viscoplastic beams. The eigenstrain $\bar{\varepsilon}$ in Eq. (2.1) then stands for the inelastic part of strain, see Refs. [14–18]. Geometrically linearized relations have been assumed in Eq. (2.2). Only physical non-linearity is considered in the present contribution. Note that, within a v. Kármán-type incremental formulation, geometrically non-linear terms also may be included in the imposed curvature $\bar{\kappa}$, compare Ref. [22]. The extension to geometrical non-linearity, however, is left for additional investigation.

Since Eq. (2.2) is formally linear, p_F as well as the beam deflection w can be split into a part (*) due to the force loading q and a part (**) due to $\bar{\kappa}$:

$$p_F = p_F^* + p_F^{**}, \qquad w = w^* + w^{**}. \tag{2.3}$$

Thus,

$$EJ\frac{\partial^4}{\partial x^4}w^* + \left(\mu i\omega - \rho A\omega^2\right) w^* = q - p_F^*, \tag{2.4}$$

$$EJ\frac{\partial^4}{\partial x^4}w^{**} + \left(\mu i\omega - \rho A\omega^2\right) w^{**} = -p_F^{**} - EJ\frac{\partial^2}{\partial x^2}\overline{\kappa}. \tag{2.5}$$

Equations (2.4) and (2.5) are formulated in the frequency domain. The factor $\exp(i\omega t)$ is used, ω denoting the frequency and $i = \sqrt{-1}$. In a slight abuse of notation, e.g. $\overline{\kappa}(x,\omega)$ in Eq. (2.5) denotes the Fourier transform of the imposed curvature defined in Eq. (2.1). Note that $w^*(x,\omega)$ as well as $w^{**}(x,\omega)$ refer to the linear elastic background beam. w^* is known in advance, since it is the solution of the linear interaction problem, compare HÖLLINGER [17].

A survey has shown that the dynamic coupling between solids and fluids due to eigenstrains has not been considered in literature so far. Therefore, the interaction problem associated with Eq. (2.5) will be formulated. The lines of this derivation at first follow the well-known procedure of force or earthquake loadings, compare Refs. [2–11]. As an illustrative example, a rectangular fluid domain $0 \le x \le L$, $0 \le z \le H$ with unit thickness perpendicular to the xz-plane is considered. The fluid in the containment is assumed to be inviscid, but linearly compressible. In the frequency domain, the fluid is then governed by the spatial form of the Helmholtz equation, compare Refs. [7–11]:

$$\frac{\partial^2}{\partial x^2}p^{**} + \frac{\partial^2}{\partial z^2}p^{**} + k^2 p^{**} = 0. \tag{2.6}$$

Here $k = \omega/c_F$, c_F denoting the speed of acoustical wave propagation in the fluid. $p^{**}(x,z,\omega)$ is the Fourier-transform of the dynamic fluid pressure in the containment associated with $\overline{\kappa}(x,\omega)$. Note that $p^{**}(x,z=0,\omega) = p_F^{**}(x,\omega)$ in Eq. (2.5). In the following, the frequency dependence is used again.

The fixed boundaries of the fluid containment in $z = H$ and $x = L$ are formulated as Neumann conditions:

$$\frac{\partial}{\partial z}p^{**}(x,z=H) = 0, \tag{2.7}$$

$$\frac{\partial}{\partial x}p^{**}(x=0,z) = 0. \tag{2.8}$$

The free boundary at $x = L$ is approximated using a Dirichlet-type formulation:

$$p^{**}(x=L,z) = 0. \tag{2.9}$$

In Eq. (2.9), the influence of the fluid surface waves is neglected. It has been shown within the context of earthquake engineering that this approximation does not lead to significant numerical errors in the hydrodynamic pressure in case of reservoirs with sufficiently large depth, see Ref. [23], and compare Refs. [1–11]. The Robin-type formulation of the free fluid surface, see [23], could be used likewise, but with more lengthy expressions. Geometric non-linearities, like fluid overtopping, are excluded. The interaction condition between the vibrating beam and the fluid in $z = 0$, is formulated as an inhomogeneous Neumann condition by means of Stoker's formulation, see Refs. [2–11] and [23]:

$$\frac{\partial}{\partial z}p^{**}(x, z=0) = \rho_F \omega^2 w^{**}(x), \tag{2.10}$$

where ρ_F denotes the density of the fluid.

The solution of the boundary value problem defined in Eqs. (2.6)–(2.10) is found using Bernoulli's method of separation of variables. The solution of the resulting ordinary differential equation for the x-direction gives

$$p^{**}(x,z) = \sum_n Z_n^{**}\cos(\lambda_n x), \qquad \lambda_n = \frac{2n-1}{2}\frac{\pi}{L}, \qquad n = 1,2,3,\ldots, \tag{2.11}$$

which satisfies the boundary conditions at $x = 0$ and $x = L$, Eqs. (2.8) and (2.9). The functions $Z_n^{**}(z)$ in Eq. (2.11) have to satisfy the remaining boundary conditions (2.7) and (2.10). Considering Eq. (2.7), one finds

$$Z_n^{**}(z) = \overline{Z}_n(z)A_n^{**} \tag{2.12}$$

with

$$\overline{Z}_n(z) = \left[e^{\mu_n(z-2H)} + e^{-\mu_n z}\right] \tag{2.13}$$

in case of $\mu_n^2 = \lambda_n^2 - k^2 > 0$. Analogous expressions apply for $\mu_n^2 < 0$, $\mu_n^2 = 0$. Following the sub-structure technique, compare Refs. [7–11], w^{**} in Eq. (2.10) is expanded into a series of the orthogonal set of the undamped beam eigenfunctions $\Phi_j(x)$:

$$w^{**}(x) = \sum_j Y_j^{**}\Phi_j(x). \tag{2.14}$$

$Y_j^{**} = Y_j^{**}(\omega)$ denotes the j-th modal coordinate corresponding to Eq. (2.5). In order to evaluate the parameter A_n^{**}, the eigenfunctions $\Phi_j(x)$ in Eq. (2.14) now are expanded into the set of cosine functions in Eq. (2.11). This leads to

$$A_n^{**} = \sum_m Y_m^{**}B_{nm}, \qquad B_{nm} = \frac{1}{\alpha_m}\frac{\rho_F\omega^2}{\mu_n(e^{-\mu_n 2H} - 1)}C_{nm}, \tag{2.15}$$

where

$$C_{nm} = \int_0^L \Phi_m(x)\cos(\lambda_n x)\,dx, \qquad \alpha_m = \int_0^L \Phi_m^2\,dx. \tag{2.16}$$

Proceeding with the substructure technique, the interactive fluid pressure itself is expanded into the set of beam eigenfunctions Φ_j. Using Eq. (2.11), it is found that

$$p_F^{**}(x) = \sum_j \sum_n \sum_m Y_m^{**} D_{nmj} \Phi_j(x), \tag{2.17}$$

with

$$D_{nmj} = \overline{Z}_n(z=0) C_{nj} B_{nm} \alpha_j^{-1}. \tag{2.18}$$

For the sake of convenience, Y_m^{**} in Eq. (2.17) is further subdivided into a part (0) due to $\overline{\kappa}$ and a part (+) due to the interactive pressure p_F^{**}

$$Y_m^{**} = Y_{m(0)}^{**} + Y_{m(+)}^{**}. \tag{2.19}$$

This, of course, is equivalent to a splitting of the deflection

$$w^{**} = w_{(0)}^{**} + w_{(+)}^{**} = \sum_j Y_{j(0)}^{**}\Phi_j + \sum_j Y_{j(+)}^{**}\Phi_j. \tag{2.20}$$

Equations (2.20) and (2.17) are inserted into the differential equation (2.5), and Galerkin's procedure is applied. The result is a set of coupled equations for $Y_{j(+)}^{**}$:

$$Y_{j(+)}^{**}\left(\omega_j^2 - \omega^2 + 2i\zeta_j\omega_j\omega\right) + \frac{1}{\rho A}\sum_m \sum_n D_{nmj} Y_{m(+)}^{**} = -\frac{1}{\rho A}\sum_m \sum_n D_{nmj} Y_{m(0)}^{**}. \tag{2.21}$$

Equation (2.21) is complex-valued, since light modal damping ζ_j of the linear elastic background beam has been added.

The partitions $Y_{j(0)}^{**}$ are the solution of problem (2.5) without fluid-structure interaction:

$$EJ\frac{\partial^4}{\partial x^4} w_{(0)}^{**} + \left(\mu i\omega - \rho A\omega^2\right) w_{(0)}^{**} = -EJ\frac{\partial^2}{\partial x^2}\overline{\kappa}, \tag{2.22}$$

with $w_{(0)}^{**}$ according to Eq. (2.20). Assuming $\overline{\kappa}$ to be known, the expansion

(2.23) $$EJ\frac{\partial^2}{\partial x^2}\overline{\kappa} = \sum_j k_j \Phi_j$$

is inserted into Eq. (2.22). Galerkin's procedure then gives

(2.24) $$Y^{**}_{j(0)} = -\frac{1}{\rho A}k_j\left[\omega_j^2 - \omega^2 + 2i\zeta_j\omega_j\omega\right]^{-1}.$$

Using a finite number of eigenfunctions, the system of linear equations (2.21) is solved numerically. Stepping the frequency gives the damped modal frequency response functions $Y^{**}_{j(+)}(\omega)$. The state variables of the beam follow by modal superposition. Especially, the curvature due to the interactive fluid pressure is

(2.25) $$\kappa^{**}_{(+)}(x,\omega) = -\frac{\partial^2}{\partial x^2}w^{**}_{(+)}(x,\omega) = -\sum_j Y^{**}_{j(+)}(\omega)\frac{\partial^2}{\partial x^2}\Phi_j(x).$$

The frequency response functions for deflection and curvature are transformed back into the time-domain by FFT, which results in time-domain representations $w^{**}_{(+)}(x,t)$ and $\kappa^{**}_{(+)}(x,t)$, respectively. For the sake of numerical efficiency, the (0)-parts are formulated directly in the time domain. At first, a partition into the quasi-static part $(0,s)$ and the dynamic part $(0,d)$ is performed,

(2.26) $$w^{**}_{(0)}(x,t) = w^{**}_{(0,s)}(x,t) + w^{**}_{(0,d)}(x,t),$$

where the quasi-static part is governed by

(2.27) $$EJ\frac{\partial^4}{\partial x^4}w^{**}_{(0,s)} = -EJ\frac{\partial^2}{\partial x^2}\overline{\kappa}.$$

The solution is given in the form of Maysel's integral

(2.28) $$w^{**}_{(0,s)}(x,t) = \int_0^L \overline{\kappa}(\xi,t)\widetilde{M}(\xi,x)\,d\xi,$$

where the kernel $\widetilde{M}(\xi,x)$ denotes the static bending moment in ξ due to a dummy single force applied in the point x of the beam, compare e.g. ZIEGLER [21] for thermally loaded beams. The dynamic part is governed by

(2.29) $$EJ\frac{\partial^4}{\partial x^4}w^{**}_{(0,d)} + \mu\,\dot{w}^{**}_{(0,d)} + \rho A\,\ddot{w}^{**}_{(0,d)} = -\mu\,\dot{w}^{**}_{(0,s)} - \rho A\,\ddot{w}^{**}_{(0,s)},$$

see Eqs. (2.2), (2.26) and (2.27). The solution of Eq. (2.29) again is formulated as a modal series expansion:

$$w^{**}_{(0,d)}(x,t) = \sum_j Y^{**}_{j(0,d)}(t)\Phi_j(x); \tag{2.30}$$

see Refs. [14–18] for details. Finally, the curvature follows from

$$\begin{aligned}\kappa^{**}_{(0)}(x,t) &= \kappa^{**}_{(0,s)}(x,t) + \kappa^{**}_{(0,d)}(x,t)\\ &= -\frac{\partial^2}{\partial x^2}\left(w^{**}_{(0,s)}(x,t) + w^{**}_{(0,d)}(x,t)\right).\end{aligned} \tag{2.31}$$

For beams with statically determinate support conditions, the quasi-static curvature is given by

$$\kappa^{**}_{(0,s)}(x,t) = \overline{\kappa}(x,t). \tag{2.32}$$

Note that the partition into a quasi-static and dynamic part is equivalent to a mode acceleration method. It allows us to consider a lower number of modes in the series expansion by formulating the quasi-static part in the closed form of Eqs. (2.28) and (2.32), and to avoid Gibb's phenomenon in the case of a discontinuous distribution of $\overline{\kappa}$.

3. Evaluation of eigenstrains from the constitutive equations

In Sec. 2, the solution for the dynamic fluid-solid interaction problem due to imposed eigenstrains has been solved formally. For a numerical evaluation of these solutions, the time evolution and the space distribution of $\overline{\kappa}$ have to be calculated from the inelastic constitutive law of the beam in an iterative time-stepping procedure. Therefore, time is subdivided into time intervals of length Δt, and the beam is subdivided into cells of length Δx. Step functions are used to approximate the spatial distribution of $\overline{\kappa}$,

$$\overline{\kappa}(x,t) = \sum_r \overline{\kappa}_r(t)\left[H\left(x - x_r + \frac{\Delta x}{2}\right) - H\left(x - x_r - \frac{\Delta x}{2}\right)\right], \tag{3.1}$$

while the eigenstrain in the r-th cell is assumed to vary linearly within the p-th time interval,

$$\overline{\kappa}_r(t) = \overline{\kappa}_{r,p-1}\frac{p\Delta t - t}{\Delta t} + \overline{\kappa}_{r,p}\frac{t-(p-1)\Delta t}{\Delta t}, \quad (p-1)\Delta t \le t \le p\Delta t. \tag{3.2}$$

The curvature in the s-th cell of the beam at the end of the n-th time interval is then calculated in the form

$$(3.3)\qquad \kappa^{**}(x = x_s, t = n\Delta t) = \kappa^{**I}(x = x_s, t = n\Delta t) + \sum_r \overline{\kappa}_{r,n}\widetilde{\kappa}_r^{**}(x_s, t = \Delta t),$$

with

$$(3.4)\qquad \kappa^{**I}(x = x_s, t = n\Delta t) = \sum_r \left[\sum_{p=0}^{n-1} \overline{\kappa}_{r,p}\widehat{\kappa}_r^{**}(x_s, t = (n-p)\Delta t) + \sum_{p=1}^{n-1} \overline{\kappa}_{r,p}\widetilde{\kappa}_r^{**}(x_s, t = (n-p)\Delta t)\right].$$

The functions $\widehat{\kappa}_r^{**}$ and $\widetilde{\kappa}_r$ denote dynamic influence functions, which are evaluated according to the formulas of Sec. 2, partly in the frequency domain at first, and partly in the time-domain. Especially, $\widehat{\kappa}_r^{**}(x, t)$ is the curvature of the beam due to an imposed curvature with triangular time-evolution:

$$(3.5)\qquad \begin{aligned} \overline{\kappa}(x,t) &= \left[H\left(x - x_r + \frac{\Delta x}{2}\right) - H\left(x - x_r - \frac{\Delta x}{2}\right)\right]\frac{\Delta t - t}{\Delta t}, \quad 0 \le t \le \Delta t,\\ &= 0, \qquad \Delta t \le t. \end{aligned}$$

Analogously, $\widetilde{\kappa}_r^{**}(x, t)$ is due to

$$(3.6)\qquad \begin{aligned} \overline{\kappa}(x,t) &= \left[H\left(x - x_r + \frac{\Delta x}{2}\right) - H\left(x - x_r - \frac{\Delta x}{2}\right)\right]\frac{t}{\Delta t}, \quad 0 \le t \le \Delta t,\\ &= 0, \qquad \Delta t \le t. \end{aligned}$$

Those triangular shape functions for the time-evolution of $\overline{\kappa}$ are chosen for the sake of convenience. The functions $\widehat{\kappa}_r^{**}$ and $\widetilde{\kappa}_r^{**}$ are evaluated before the beginning of the time-stepping procedure.

Assuming the state of the beam to be known at $t = (n-1)\Delta t$, the values of $\overline{\kappa}$ at time $t = n\Delta t$, namely the values $\overline{\kappa}_{r,n}$ in Eq. (3.3), have to be computed. From Eq. (2.1) we obtain

$$(3.7)\qquad \overline{\kappa}_{r,n} = \frac{1}{J}\int_A \overline{\varepsilon}(x = x_r, z, t = n\Delta t)z\,dA.$$

The inealastic constitutive equation is assumed in the form of an evolutionary non-linear differential equation:

$$(3.8)\qquad \dot{\overline{\varepsilon}} = F(\varepsilon, \overline{\varepsilon}).$$

The strain is connected with the total curvature by the Bernouilli – Euler assumption

$$\varepsilon = z(\kappa^* + \kappa^{**}), \tag{3.9}$$

where κ^{**} is given in Eq. (3.3) as a linear function of $\overline{\kappa}_{r,n}$.

Using the generalized mid-point rule, Eq. (3.8) is integrated:

$$\begin{aligned} \overline{\varepsilon}(t = n\Delta t) \\ = \overline{\varepsilon}(t = (n-1)\Delta t) + F[\varepsilon(t = (n-1+\theta)\Delta t), \overline{\varepsilon}(t = (n-1+\theta)\Delta t)]\Delta t, \end{aligned} \tag{3.10}$$

where

$$\varepsilon(t = (n-1+\theta)\Delta t) = \varepsilon(t = (n-1)\Delta t)(1-\theta) + \varepsilon(t = n\Delta t)\theta, \quad 0 \le \theta \le 1, \tag{3.11}$$

and analogously for $\overline{\varepsilon}$. In the following, $\theta = 1$ is assumed. CHRISFIELD's accelerated secant-Newton method [24] is applied: We start with a measure for the $\overline{\kappa}_{r,n}$, Eq. (3.3) is evaluated and inserted into Eq. (3.9). Afterwards, Eq. (3.10) is used to calculate a new value of the eigenstrain at $t = n\Delta t$. This is done across the beams' section in every cell, and new values of the $\overline{\kappa}_{r,n}$ are found by integration according to Eq. (3.7). The procedure is repeated according to Chrisfield's algorithm, until sufficient accuracy is reached.

4. NUMERICAL EXAMPLE

Figure 1 shows the geometry, loading and the dimensionless input parameters of a numerical example. An elastic-viscoplastic simply supported beam of length L and rectangular cross-section of height h and width b represents the carrying structure of a rectangular fluid containment. The beam is described by PERZYNA's constitutive law, see Ref. [19], where the inelastic parts of strain are calculated from the equation of evolution

$$\dot{\overline{\varepsilon}} = \frac{1}{2\eta}\left\langle \phi\left(1 - \frac{K\sqrt{3}}{|\sigma|}\right)\right\rangle \frac{2}{3}\sigma, \tag{4.1}$$

where

$$\phi\left(1 - \frac{K\sqrt{3}}{|\sigma|}\right) = \left(1 - \frac{K\sqrt{3}}{|\sigma|}\right)^m . \tag{4.2}$$

The fluid is assumed to be inviscid and linearly compressible, and it is governed by the differential equation, Eq. (2.6), and the boundary conditions, Eqs. (2.7)–(2.10).

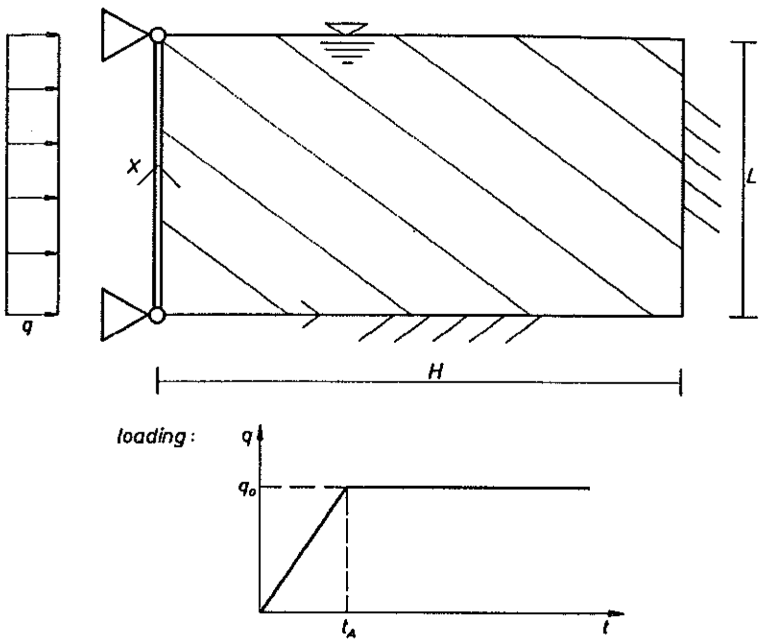

FIG. 1. Example: geometry and loading of the system.

The beam is subjected to a blast-type loading q which is approximated by a ramp-type time evolution. For the numerical work, the following parameters of the system and the loading are chosen:

$$\frac{h}{L} = \frac{1}{30}, \qquad \frac{b}{h} = \frac{1}{2}, \qquad \frac{H}{L} = 4, \qquad c_F \frac{T}{L} = 778, \qquad \frac{B}{L} = \frac{1}{12},$$

$$\rho \frac{L^2}{ET^2} = \frac{1}{4400}, \qquad \frac{\rho}{\rho_F} = 7.5, \qquad \frac{q_0 B}{EL^2} = \frac{1}{1600},$$

$$\zeta_j = \frac{1}{100}, \qquad \eta \frac{T}{E} = \frac{1}{6000}, \qquad \frac{K}{E} = \frac{1}{1100}, \qquad \frac{t_A}{T} = \frac{2}{3},$$

$$T = \frac{2L^2}{\pi}\sqrt{\rho A/EJ}, \qquad J = \frac{bh^3}{12},$$

where the fundamental period of the linear elastic background beam is denoted by T.

Figure 2 shows the time evolution of the midpoint deflection of the beam for two values of the exponent m in the constitutive law. The dashed lines show the quasi-static part w_s^{**}/h, representing the drift of the beam, that is the instantaneous value of permanent deformation.

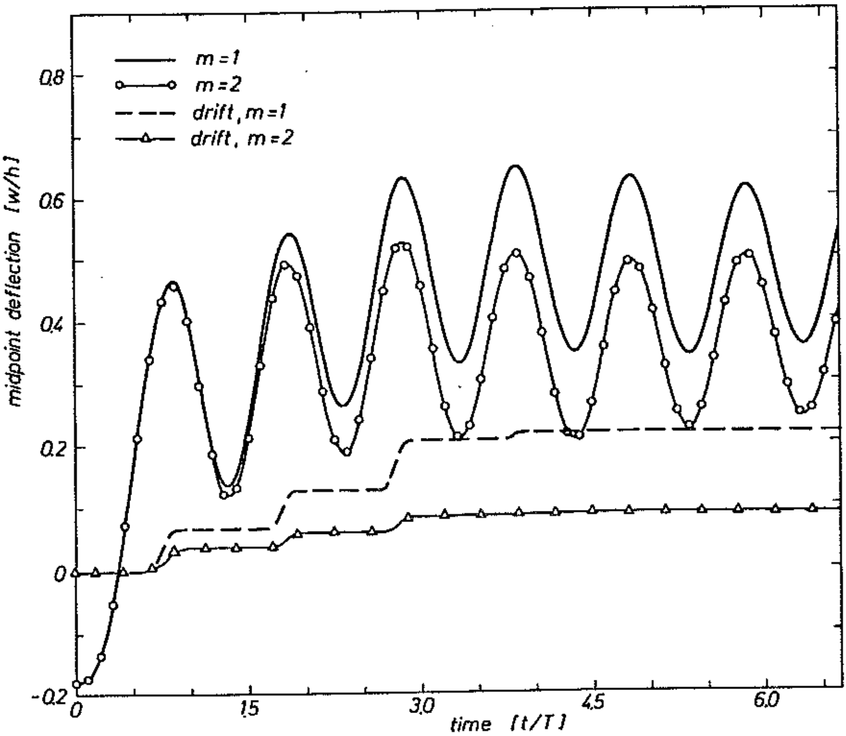

FIG. 2. Time evolution of midpoint deflection of beam.

5. Conclusion

The above partitioning method makes a convenient and appropriate use of well-known linear elastic solution strategies in the physically nonlinear problem of elastic-viscoplastic structures interacting with fluids. Special emphasis is given to catastrophic loadings. While compatibility as well as dynamic conditions are satisfied "exactly", the material law is approximately satisfied at discrete instants of time in order to determine the sources of selfstress acting in the linear background beam. Numerical advantages of the method, which allows us to use procedures for linear fluid-structure interaction, are due to the underlying powerful elastic-inelastic analogy.

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