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DYNAMIC EFFECTS IN THE RAIL VEHICLE-TRACK SYSTEM FOR ELASTIC-PLASTIC FOUNDATION (*)

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The paper is devoted to elaboration of a simulation method for an infinite straight Timoshenko beam on the elastic-plastic foundation with nonsymmetric characteristic under moving load. The inertialess distributed load moves along the beam with a constant velocity. This beam can be considered as a railway track and the loads as a train moving on the track. The stationary solution for the nonlinear equation was obtained by means of the approximation method applying the analytical solution for the case of the linear approach. Some results of the numerical calculations of dimensionless displacements are shown in the Figs. 3 – 7. The plastic deformation of the beam connected with the assumed elastic-plastic characteristic of foundation have been determined. The numerical program allows to sum up the plastic displacements computed in successive passing of the load. These dimensionless plastic deformations are shown in the Tables 2 – 5. The possibility of track destruction as a result of incorrect operation has also been considered.

NOTATIONS

- A beam cross-sectional area,
- b_p, b_m attenuation constants of linear characteristic component of the foundation, referred to beam length unit,
- c_p, c_m elasticity constants of linear characteristic component of the foundation, referred to beam length unit,
- E Young's modulus,
- G shear modulus,
- I moment of inertia of beam cross-section,
- r radius of inertia of beam cross-section,
- t time,
- v speed of travelling load,
- $\mathbb{I}(z)$ Heaviside's unit step function,
- κ Timoshenko shear coefficient,
- ϱ linear mass density of beam,
- ψ angle of rotation.

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1. Introduction

When high-speed railway transport is introduced, it is necessary to analyse the complete vehicle-track system and, in particular, to examine in detail the track characteristics. If in the analysis only linear dynamic systems are used assuming the nonlinear foundation characteristics, we cannot examine in detail the permanent deformation of the track. The track, especially a new one, can be damaged e.g. by incorrect selection of load parameters and the motion speed. This paper presents an analysis of a dynamic system composed of a track and a train running along it. The modelled track has been assumed to be a simple, infinite Timoshenko beam resting on an inertialess foundation of nonsymmetric elastic-plastic characteristics. The train is modelled by a distributed load of a constant intensity, acting on the beam in an inertialess track section, and moving along the beam at a constant speed.

Dynamical systems of beams resting on deformable elastic and viscoelastic foundation under rolling loads were analysed by many authors, e.g. [5, 12, 13]. The problem of linear steady vibrations of a beam resting on a viscoelastic half-space, and produced by a load moving at a constant speed was discussed in [3]. A similar analysis as a Timoshenko beam, taking into account the stability of its interaction with the foundation, was presented in [2]. Timoshenko beam was also the subject of investigation of Ref. [14]. The paper [4] presents an approach to track quality, considering the permanent ballast deformation in vertical direction. Author's contribution in this field are the papers [6] to [10]. In [9] were defined the permanent deflections of the beam modelling the track connected with the assumed elastic-plastic foundation. A numerical program was used which made it possible to determine steady-state nonlinear, close to real vibrations of the track. Using the program we can sum up the plastic deflections from subsequent load runs, and the analysis of the subsequent load run includes the change of foundation characteristics caused by plastic deformation due to the previous runs. Moreover, the subsequent runs can have different parameters such as load density, length of route segment, rolling speed. The program, therefore, makes it possible to analyse the event of track damage due to improper conditions of operation, negative plastic deflections behind the load, as well as to examine the conditions to eliminate it. The problems of track damage and the ways of preventing it are illustrated by the results of computations done for various traffic parameters presented in the paper.

The equations of motion are written under the assumption that the longitudinal d'Alembert inertial forces and the horizontal reaction of the foundation, resulting from the contact between the sleepers and the condensed ballast, are disregarded. Only the reaction of moment connected with that kind of interaction is taken into account. Moreover, the uniaxial (vertical) state of stresses for the elastic-plastic model of the foundation is assumed. Such an approach leads to a simplification of the analysis and enables the separation of the bending and longitudinal vibrations of the beam.

2. Dynamical model and its mathematical description

The model of a dynamical system has been presented in Fig. 1. The stationary coordinate system $\overline{O}\overline{x}_1\overline{y}$ is connected with the beam, Ox_1y – with the rolling load.

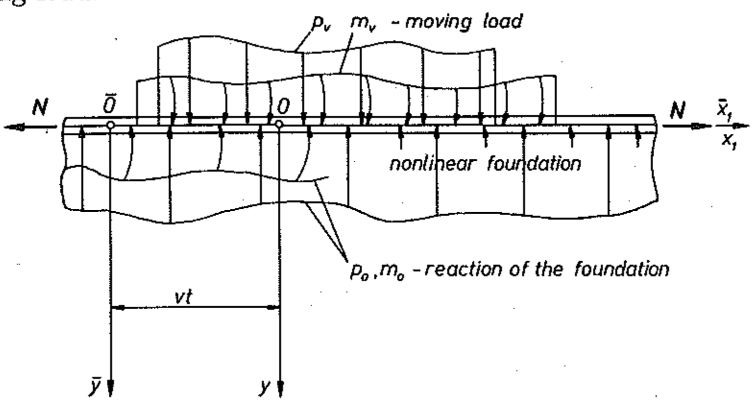

FIG. 1. Model of dynamical system.

It is assumed that the beam deflection $w = w(\overline{x}_1, t)$ is produced by the loads: $p = p(\overline{x}_1, t)$ – external distributed load, $m = m(\overline{x}_1, t)$ – external distributed moment, N – constant longitudinal force acting along the beam.

Partial differential equations for a Timoshenko beam are:

$$
(1)\qquad
\begin{aligned}
\kappa AG\left(\frac{\partial^2 w}{\partial \overline{x}_1^2} - \frac{\partial \psi}{\partial \overline{x}_1}\right) - \varrho\frac{\partial^2 w}{\partial t^2} + p = 0\,,\\
EI\frac{\partial^2 \psi}{\partial \overline{x}_1^2} + \kappa AG\left(\frac{\partial w}{\partial \overline{x}_1} - \psi\right) - \varrho r^2\frac{\partial^2 \psi}{\partial t^2} - N\frac{\partial w}{\partial \overline{x}_1} + m = 0\,.
\end{aligned}
$$

Let

$$p = p_v - p_0\,, \qquad m = m_v - m_0\,,$$

where

$$p_v = p_v(\overline{x}_1, t), \qquad m_v = m_v(\overline{x}_1, t),$$

are the given rolling loads. Assuming that the beam foundation has nonlinear characteristics, close to viscoelastic, we assume the following loads:

$$p_0 = p_0(\overline{x}_1, t), \qquad m_0 = m_0(\overline{x}_1, t),$$

according to the response of deformable foundation. They depend on the displacements caused by beam vibrations according to the relations

$$p_0 = c_p w + b_p \frac{\partial w}{\partial t} + p_0^*, \qquad m_0 = c_m \psi + b_m \frac{\partial \psi}{\partial t},$$

in which p_0^* denotes a nonlinear component. Searching for stationary solutions of equations (1), we introduce new dimensionless variables:

$$x = \frac{\overline{x}_1 - vt}{r}, \qquad u = \frac{w}{w_s},$$

where w_s denotes a known, constant and positive value of deflection w.

In a coordinate system Ox_1y connected with the moving load, equations (1) are transformed to ordinary differential equations which, after elimination of angle ψ, are reduced to the following differential equation of the 4th order:

$$F[u(x)] + f[u(x)] = g_m[\overline{m}_v(x)] + g_p[\overline{p}_v(x)]. \tag{2}$$

In Eq. (2) we have denoted:

$$\begin{aligned}
F[u(x)] = D(V^2)u^{\mathrm{IV}} - 2V\left[B(V^2 - V_1^2) + b(V^2 - V_2^2)\right]u''' \\
+ \left[V^2(V_1^2 + 1 + 4bB + C) - (S + C)V_1^2 - V_2^2\right]u'' \\
-2V\left[b(V_1^2 + C) + B\right]u' + (V_1^2 + C)u,
\end{aligned}$$

$$\begin{aligned}
f[u(x)] &= (V^2 - V_2^2)\overline{p}_0^{*\prime\prime} - 2BV\overline{p}_0^{*\prime} + (V_1^2 + C)\overline{p}_0^*, \\
g_m[\overline{m}_v(x)] &= -V_1^2\overline{m}_v'(x), \\
g_p[\overline{p}_v(x)] &= (V^2 - V_2^2)\overline{p}_v''(x) - 2BV\overline{p}_v'(x) + (V_1^2 + C)\overline{p}_v(x), \\
D(V^2) &= (V^2 - V_1^2)(V^2 - V_2^2), \qquad (') = \frac{d}{dx}.
\end{aligned}$$

The load and parameters of the beam and foundation used in Eq. (2) describe the following dimensionless values:

$$V = \frac{v}{r}\sqrt{\frac{\varrho}{c_p}}, \qquad V_1 = \frac{1}{r}\sqrt{\frac{\kappa AG}{c_p}}, \qquad V_2 = \frac{1}{r}\sqrt{\frac{EA}{c_p}},$$

$$b = \frac{b_p}{2\sqrt{c_p \varrho}}, \qquad B = \frac{b_m}{2r^2\sqrt{c_p \varrho}}, \qquad S = \frac{N}{r^2 c_p}, \qquad C = \frac{c_m}{r^2 c_p},$$

$$\overline{p}_0^* = \overline{p}_0^*[u(x)] = \frac{p_0^*}{p_s}, \qquad \overline{p}_v = \overline{p}_v(x) = \frac{p_v}{p_s}, \qquad \overline{m}_v = \overline{m}_v(x) = \frac{m_v}{p_s r},$$

where

$$p_s = c_p w_s .$$

Parameter V introduced above is a dimensionless speed of load motion.

Load p_0 is presented as a sum

$$p_0 = p_u + p_d ,$$

where p_u – component resulting from the elastic-plastic properties of the foundation, p_d – component resulting from the damping properties of the foundation.

The elastic-plastic characteristics assumed in the analysis are shown in Fig. 2, with the following notations: A_n, A_p – points determining the elastic limit when there is no permanent deflection of the foundation; A_{nz}, A_{pz} – points determining the elastic limit when permanent deflection occurs; w_t – permanent deflection of the foundation that remains when the load (of certain duration) has stopped, with foundation characteristics represented by straight lines l_e, l_n and l_p, and with elastic limits at points A_n and A_p; l_e, l_{ez} – straight lines referring to the elastic part of the characteristics before and after the deflection w_t, respectively; l_n, l_p – straight lines referring to the plastic part of the characteristics.

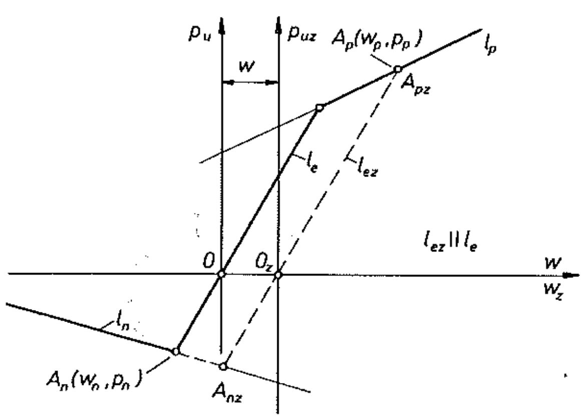

FIG. 2. Elastic-plastic characteristics of foundation and their change due to load's passage.

In system Owp_u, the equations of straight lines describing the characteristics are as follows:

$$\begin{aligned} l_e: &\qquad p_u(w) = c_p w\,, \\ l_n: &\qquad p_u(w) = p_n + c_p\xi_n(w - w_n), \\ l_p: &\qquad p_u(w) = p_p + c_p\xi_p(w - w_p), \end{aligned}$$

where ξ_n, ξ_p – coefficients determining the angle of inclination of straight lines referring to the plastic part of the foundation characteristics.

The assumed elastic-plastic characteristics has thus been described by five parameters: w_n, w_p, c_p, ξ_n and ξ_p. The following dimensionless parameters have been introduced:

$$u_n = \frac{w_n}{w_s}\,, \qquad u_p = \frac{w_p}{w_s}\,, \qquad u_t = \frac{w_t}{w_s}\,.$$

In a stationary case, the nonlinear load component (in dimensionless form) has the form

$$\overline{p}_0^* = \overline{p}_u - u\,,$$

where

$$\overline{p}_u = \frac{p_u}{p_s}\,.$$

After the load has passed along the beam with foundation characteristics corresponding to l_e, l_n and l_p, permanent deflection $w_t \neq 0$ may occur, at which a next load passage will take place at a new, changed characteristics (with different elastic limits – providing $\xi_n \neq 0$, or $\xi_p \neq 0$) determined by straight lines l_{ez}, l_n and l_p. Another load passage, not necessarily with the parameters of the previous one, may cause a new permanent deflection w_{t1}, which will be superposed on the previous deflection w_t.

3. Remarks on the method of analysis and a numerical example

Beam (track) deflection described by the nonlinear differential equation (2) is determined by the approximate method in which analytical solutions are used. In a linear case the nonlinear component appearing in (2) is $f \equiv 0$. Linear equation is solved using the transfer function, integral transformation and convolution theorem. A description of the method of analysis and applications of the elastic-plastic characteristics in a numerical program were given in [9] and a brief description – in [10].

In the present case the distributed travelling load of constant value q within the given section of dimensionless length

$$L = x_{bq} - x_{aq}\,,$$

has the dimensionless form

$$\overline{p}_v(x) = q_0 \{\mathbb{I}[-(x - x_{bq})] - \mathbb{I}[-(x - x_{aq})]\},$$

where

$$q_0 = \frac{q}{p_s}, \qquad q = \text{const}.$$

In the numerical analysis the results of which are shown below, $\overline{m}_v(x) \equiv 0$, $q_0 = 2$, $V_1 = 50$, $V_2 = 100$, $b = 0.2$, $L = 120$ and $C = S = B = x_{bq} = 0$ have been assumed. The assumed values V_1 and V_2 (dimensionless critical speeds) correspond to a railway track with UIC rails. The data of the track given in Table 1 were taken from [1] and [11]. Nonlinear elastic-plastic characteristics, representing qualitatively the railway track foundation properties are described by parameters

$$u_n = -0.4, \qquad \xi_n = -0.2, \qquad u_p = 2.1, \qquad \xi_p = 0.8.$$

Table 1.

A	I	r	k
$1.5372 \times 10^{-2}\text{m}^2$	$6.11 \times 10^{-5}\text{m}^4$	$6.3 \times 10^{-2}\text{m}$	0.667

(A, I – for two rails)

ϱ	E	G	c_p
250 kg/m	2.1×10^{11} Pa	8.1×10^{10} Pa	$1 \times 10^6 \div 1.5 \times 10^8$ N/m^2

The result of numerical analysis of five subsequent passages at $V = 12$ are dimensionless permanent deflections of the foundation given in Table 2. Their negative values mean that they lead to track damage, i.e. its rise upwards.

Table 2.

Passage number	1	2	3	4	5
u_t	−0.035	−0.090	−0.195	−0.325	−0.537

Comparison of displacements that occurred during these five passages of the load is given in Fig. 3.

Comparison of displacements after the fifth passage (linear case $V = 12$) is shown in Fig. 4.

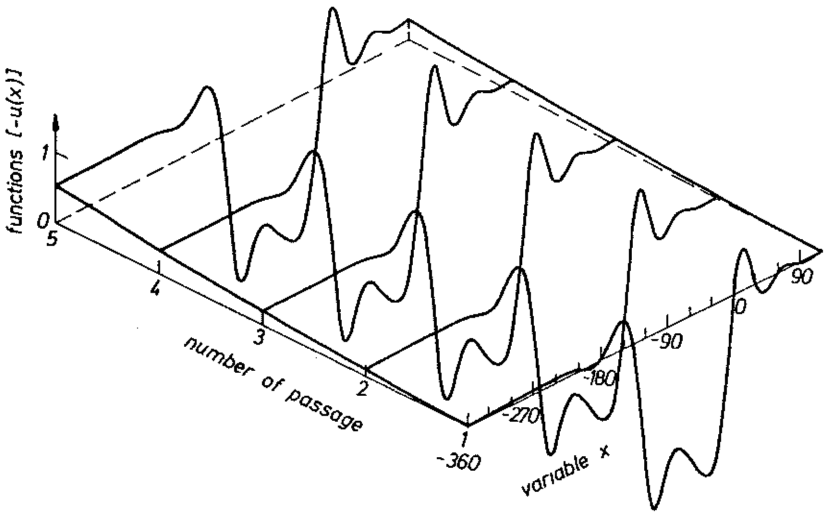

FIG. 3. Comparison of beam displacements during passages at $V = 12$.

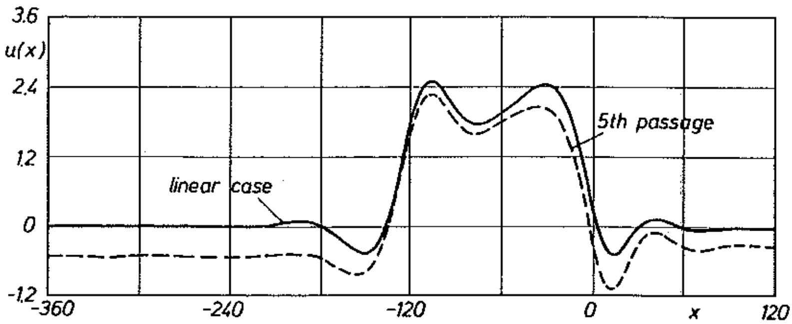

FIG. 4. Comparison of displacements after the fifth passage (linear case $V = 12$).

In case of passage of an unchanged load, at a lower speed, the permanent displacements of foundation have opposite signs, which is an evidence of stabilization of plastic deflections. The dimensionless permanent deflections of the foundation determined by $V = 10$, occuring during eleven subsequent passages along the beam on the foundation of the same initial parameters as before, have positive values (Table 3), which proves that the track level has become lower. Shake-down is also noticeable.

Table 3.

Passage number	1	2	3	4	5	6	7	8	9	10	11
u_t	0.048	0.086	0.116	0.141	0.161	0.177	0.190	0.200	0.209	0.215	0.220

After these eleven passages, parameters u_n and u_p of the characteristics have changed (at unchanged coefficients ξ_n and ξ_p) and are now equal to

$$u_n = -0.4441, \qquad u_p = 2.276.$$

During subsequent passages of the same load at a higher speed ($V = 12$), the permanent foundation deflections do not cause any track damage. Some values of u_t resulting from subsequent passages (including the thirty-fifth one) have been shown in Table 4.

Table 4.

Passage number	12	13	14	15	16	17	18	19	20	21	22
u_t	0.237	0.262	0.296	0.327	0.360	0.387	0.408	0.425	0.439	0.451	0.460

23	24	25	26	27	28	29	30	31	32	33	34	35
0.467	0.473	0.478	0.482	0.486	0.489	0.491	0.493	0.495	0.497	0.498	0.499	0.500

As it can be seen, beginning from the twelfth passage the plastic track deflection increases during subsequent passages, which means that the track continues to stabilize during the final passages (Fig. 5). Therefore, trains travelling at higher speeds are no longer destructive, since the track shake-down occur.

Beginning from the nineteenth passage, because of the changed characteristics of the foundation, the plastic limit (point A_{nz}) is no longer exceeded at negative deflections (upwards) of the beam.

The comparison of deflections that occurred after the eleventh and thirty-fifth passages of the load with linear case (determined at $V = 10$) has been shown in Fig. 6.

In case of characteristics of adopted parameters, there is a limit travel speed $v_{\rm lim}$ (in a dimensionless form $V_{\rm lim} \in < 10; 12 >$); above that limit, track destruction occurs. During passages at speeds slightly lower than $V_{\rm lim}$, shake-down of the foundation can be soon achieved. The possibility of determining such a speed is an important conclusion for practical purposes.

Dynamic effect of foundation stabilization can be also achieved by subsequently increasing values of the moving load. In the case presented below, the distributed travel load of constant value $k_q q$ within the given section of dimensionless length L has the dimensionless form

$$\overline{p}_v(x) = k_q q_0 \left\{ \mathbb{I}[-(x - x_{bq})] - \mathbb{I}[-(x - x_{aq})] \right\},$$

where k_q – coefficient determining the change of the load during subsequent passages.

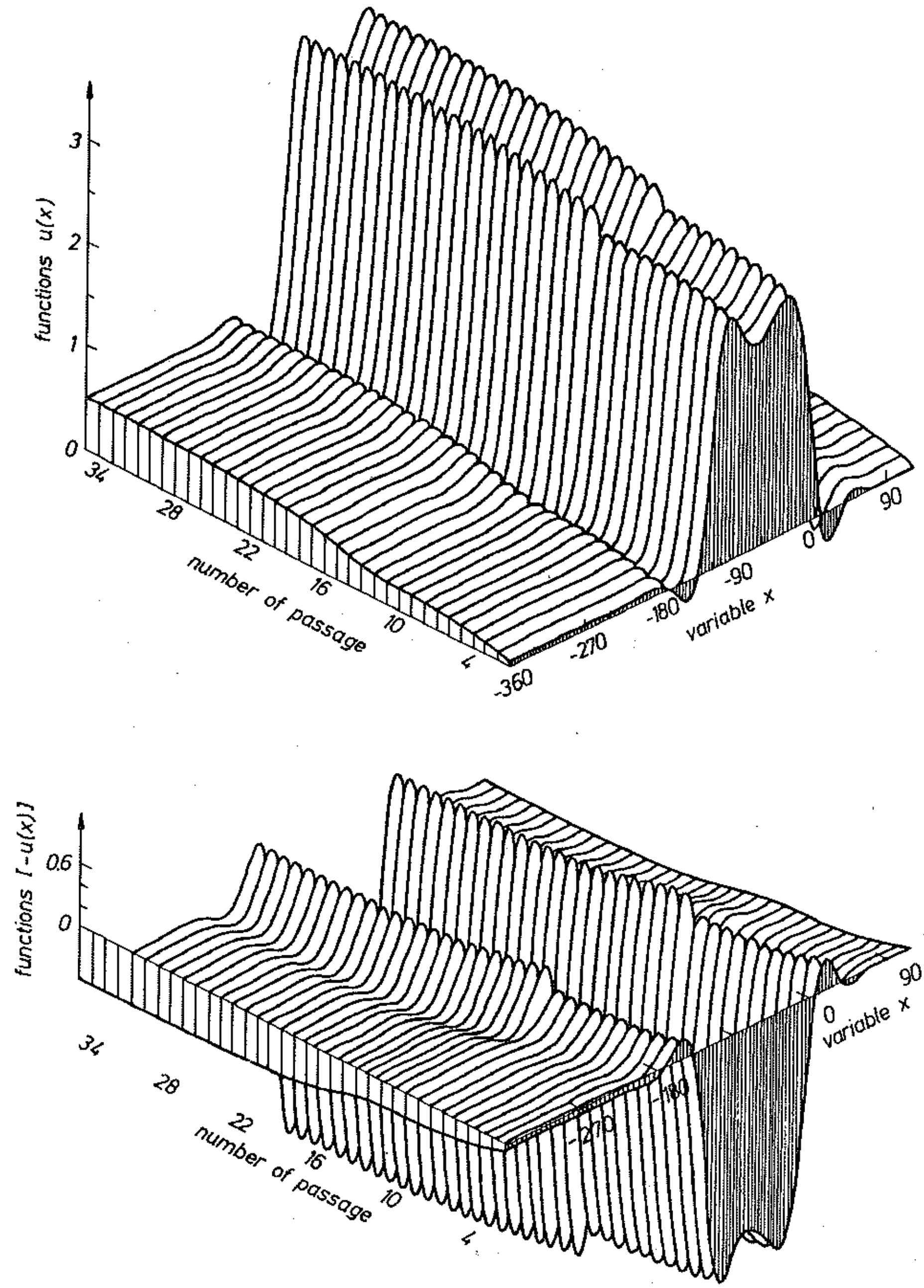

FIG. 5. Beam dislocations during passages at dimensionless speed $V = 10$ and $V = 12$.

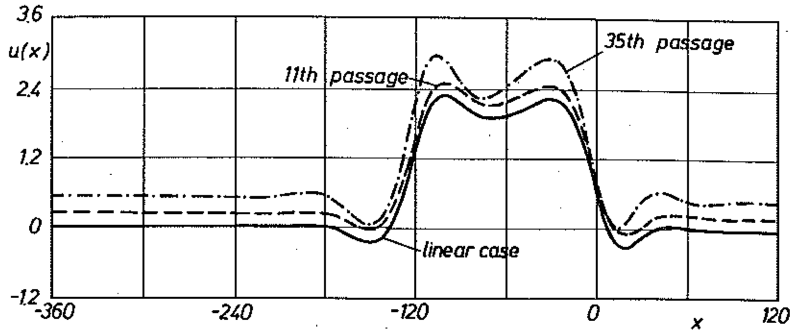

FIG. 6. The comparison of deflections after the eleventh and thirty-fifth passages of the load with the linear case (determined at $V = 10$).

The result of the numerical analysis of forty subsequent passages at $V = 12$ are dimensionless permanent deflections of the foundation given in Table 5.

Table 5.

Passage number	1	2	3	4	5	6	7	8	9	10	11	12
k_q	0.85		0.86		0.87		0.88		0.89		0.90	
u_t	0.006	0.010	0.019	0.027	0.039	0.049	0.063	0.075	0.090	0.102	0.118	0.131

13	14	15	16	17	18	19	20	21	22	23	24	25	26
0.91		0.92		0.93		0.94		0.95		0.96		0.97	
0.148	0.161	0.178	0.191	0.208	0.222	0.239	0.253	0.270	0.283	0.301	0.314	0.332	0.346

27	28	29	30	31	32	33	34	35	36	37	38	39	40
0.98		0.99		1.00									
0.363	0.377	0.394	0.408	0.425	0.439	0.451	0.460	0.467	0.473	0.478	0.482	0.486	0.489

Comparison of displacements that occurred during these forty passages of load is given in Fig. 7.

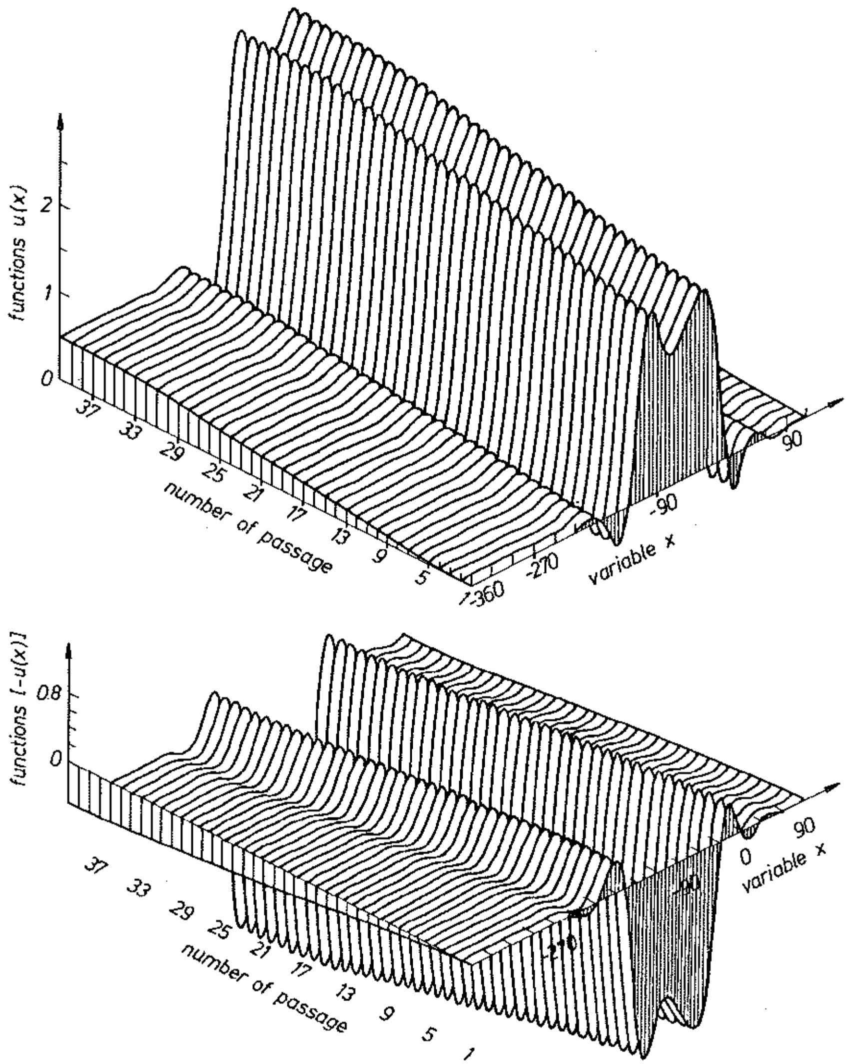

FIG. 7. Beam displacement during passages at dimensionless speed $V = 12$ and change k_q from 0.85 to 1.00.

4. Final remarks

The results obtained from the computer simulation of plastic deformation of the foundation prove the efficiency of the numerical program in predicting a proper way of adaptation of new railway subgrade for high-speed trains. However, a more complete (resulting from experimental investigations) knowledge of the parameters of track foundation characteristics would be desired in order to make the calculation results more reliable.

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