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KINEMATIC APPROACH TO DYNAMIC CONTACT PROBLEMS – THE GEOMETRICAL SOFT WAY METHOD (*)

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A method of taking into account the geometrical constraints in evolution problems of solid systems that limit the possibility of motion by the history of variation of the velocity field is developed in the paper. The presented formulation can be adapted to numerous problems of solid systems subjected to dynamic effects with large deformations, large displacements, large rotations. The computational cost of one iteration in the method proposed is the same as in other classical methods because of the frontal approach. However, the iterative process converges faster. If an adaptive space and time meshing were chosen, it could become a less expensive method. A numerical example of the contact analysis, in which both the spatial and temporal partition was adapted according to the evolution of the geometry, proved the approach to be more efficient.

1. Introduction

Several possibilities of taking into account the unilateral contact conditions are described in the literature [1, 2]. However, numerical experiments show that the time of computation is not the same for each approach chosen for a particular problem. The choice of geometrical constraints as restrictions imposed on the variation of the velocity field enables us to reduce the computational time.

First the velocity formulation is presented. It can be adapted particularly to a wide range of problems such as rigid mechanisms [3] or deformable solid systems subjected to dynamic effects [4]. It must be emphasized that the choice of the velocity formulation is not incompatible with the description of large deformations, large displacements and large rotations. Moreover, the expressions are less intricate and the formulation of the contact evolution between deformable solids is easier than in a tangent space formulation. The fundamental general formulation (instantaneous updated Lagrangian formulation) is convenient for discretization by the space-time finite element

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method [5–10] which avoids the separation of the spatial and time discretization. The interpolation formulas depend on both the spatial and time terms. We can recall here that in the classical approach used in solutions of dynamic problems, the space is discretized by one discrete method (finite element method, finite difference method) applied to a structure in certain moments, while time derivatives are integrated by another method (central difference, Runge-Kutta, etc.). In our approach the virtual work principle is formulated by the integration in time of the virtual power principle. The velocity field is considered as the principal variable. Space-time elements used in the formulation have linear time-dependence of the real interpolation functions. The appropriate choice of the virtual velocity field and virtual interpolation functions are discussed.

The problem of points entering into contact is formulated by means of the introduced special elements. These elements have additional nodal points in time that are eliminated by the normal contact conditions. The interpolation functions are piece-wise continuous in time. The tangential conditions are formulated by friction laws with the same type of formulation as the rheological laws. The tensor theory in dynamics of surface allows us to join the space behaviour and the surface behaviour. Certain conditions of compatibility have to be fulfilled, but a phenomenological way permits to use a more suitable law to a specific problem with dry or lubricated friction. The viscoplastic friction law is used as a more realistic behaviour, commonly applied, particularly to the forming problems.

The solution algorithm uses a fixed point method that leads to a gain of computation time in comparison with the Newton–Raphson method. In the former paper [4] the Newton–Raphson method was used to a similar mechanical problem. In the present paper the solving by a fixed point method gives good results because of the low number of arithmetical operations and the well-conditioning of the system of equations. The algorithm uses a frontal method of the solution of the system of equations. It reduces the size of the problem to the same size as we have in the classical finite element method. Therefore, the usual classical software can be simply implemented.

Numerical examples related to different application fields are presented.

2. Velocity formulation

The velocity is considered as the main variable. Generally, the virtual power principle can be formulated by

$$\mathcal{P}_i^* + \mathcal{P}_e^* - \mathcal{P}_a^* = 0, \tag{2.1}$$

where for a deformable solid

$$
\mathcal{P}_i^* = -\int_{\Omega} \boldsymbol{\sigma} : \mathbf{D}^* \, d\Omega \,,
$$

$$
\mathcal{P}_e^* = \int_{\Omega} \rho \, \mathbf{f} \cdot \mathbf{v}^* \, d\Omega + \int_{\partial\Omega} \mathbf{F} \cdot \mathbf{v}^* \, d(\partial\Omega) \,, \tag{2.2}
$$

$$
\mathcal{P}_a^* = -\int_{\Omega} \rho \frac{d\mathbf{v}}{dt} \, \mathbf{v} \, d\Omega \,.
$$

$\mathbf{D}$ – the Euler strain rate tensor, $\boldsymbol{\sigma}$ – the Cauchy stress tensor, ρ – the mass density, $\mathbf{f}$ – external body forces, $\mathbf{F}$ – external surface forces.

The virtual work $\mathcal{W}^*$ within the interval $[t_0; t_1]$ is defined by the integral

$$
\mathcal{W}^* = \int_{t_0}^{t_1} \mathcal{P}^* \, dt \,. \tag{2.3}
$$

Therefore it is calculated by the integration over the space and time domain $\{\Omega \times [t_0, t_1]\}$:

$$
\mathcal{W}^* = \int_{t_0}^{t_1} \int_{\Omega} \mathcal{F} \, v^* \, d\Omega \, dt \,, \tag{2.4}
$$

where $\mathcal{F}$ is a generalized force and v^* is a virtual velocity. The chosen virtual velocity field $\mathbf{v}^*$ is formulated as a product of a space and time functions

$$
\mathbf{v}^* = \mathbf{f}(\mathbf{x}(t)) \, \mathbf{g}(t) \,, \tag{2.5}
$$

where $\mathbf{f}(\mathbf{x}(t))$ is a spatial distribution which in a general case can vary in time, since the geometry depends on time and $\mathbf{g}(t)$ can be either a constant distribution within the step of time or composed of one or several Dirac peaks. In the first case we recognize the momentum theorem and in the second case we recognize the equation of motion.

After the integration (2.3), the equations are implicitly described by the velocity field

$$
\phi(\mathbf{v}, \overline{\boldsymbol{\Omega}}) = 0 \,. \tag{2.6}
$$

Boundary conditions include contact dissipations, and the variation of the domain and its boundary $\overline{\boldsymbol{\Omega}}$ is the function of variation of the velocity field. Points $\mathbf{x}$ of $\overline{\boldsymbol{\Omega}}$ are estimated by

$$
\mathbf{x} = \mathbf{x}_0 + \int_{t_0}^{t} \mathbf{v} \, dt \,. \tag{2.7}
$$

In the case of large rotations the objectivity of the rheological law is guaranteed by an objective derivative used in the formulation of the law. The commonly used derivatives are either the Jauman derivative or the Truesdell derivative.

However, the respective expressions in a fixed coordinate system are not easy to evaluate. We determine here the variation of stress tensor in the relative coordinates of Jauman. For instance, the elastic behaviour can be well formulated by hypoelasticity, where the relation between strain rate tensor and the time derivative of the Cauchy stress tensor is linear when the components are considered in the corotational axis [11]. Truesdell derivative is well adapted when the Piola stress tensor is considered. The components can also be considered in corotational axes.

3. Dynamic contact conditions

The solid S_j is considered as a reference one. The external normal (effective or predicted) $\mathbf{n}_{ji}$ is determined at the effective or predicted contact point. Afterwards the base of the tangent plane defines two remaining reference axes.

Below we will use indices i and j to denote (number) the body. n is the normal direction and T is the tangential one.

The relative normal displacement of a point of the solid S_i with respect to a solid S_j is

$$u_{n_{ij}} = \mathbf{u}_{ij} \cdot \mathbf{n}_{ji}, \qquad \mathbf{u}_{ij}(t) = \int_{t_0}^{t} \mathbf{v}_{ij}\, dt. \tag{3.1}$$

Let us notice that $u_{ij}(t_0) = 0$ and $\mathbf{v}_{ij}$ is the relative velocity of the solid S_i with respect to the reference axis associated with S_j. $\mathbf{v}_{ij}$ has a normal and a tangential component

$$\mathbf{v}_{ij} = \mathbf{v}_{n_{ij}} + \mathbf{v}_{T_{ij}}. \tag{3.2}$$

The normal force from the solid S_j to the solid S_i is defined by

$$F_{n_{ij}} = \mathbf{F}_{ji} \cdot \mathbf{n}_{ji}, \tag{3.3}$$

and the tangential contact force $\mathbf{F}_{T_{ji}}$ by

$$\mathbf{F}_{T_{ji}} = \mathbf{F}_{ji} - F_{n_{ij}} \mathbf{n}_{ji}. \tag{3.4}$$

A friction law takes here the form

$$\mathbf{F}_{T_{ji}} = \mathbf{H}(\mathbf{v}_{T_{ij}}) . \tag{3.5}$$

As an example, the Norton–Hoff law can be formulated by

$$\mathbf{F}_{T_{ji}} = -\alpha \|\mathbf{v}_{T_{ij}}\|^{p-1} \mathbf{v}_{T_{ij}} . \tag{3.6}$$

The Signorini conditions are satisfied at time t_0 and t_1:

$$\begin{aligned} u_{n_{ij}} - d_0 &\le 0 , \\ F_{n_{ji}} &\le 0 , \\ F_{n_{ji}}(u_{n_{ij}} - d_0) &= 0 , \end{aligned} \tag{3.7}$$

where d_0 is the distance between i and j. Furthermore, at any time t we have the power of the normal force within the time step

$$F_{n_{ji}} v_{n_{ij}} = 0 , \tag{3.8}$$

where $v_{n_{ij}}$ is the normal velocity. Therefore the work is equal to zero,

$$\int_{t_0}^{t_1} F_{n_{ji}} v_{n_{ij}} \, dt = 0 . \tag{3.9}$$

This expression is associated to the duality of the normal force and the normal relative velocity. Therefore the virtual velocity field is chosen to be compatible with these contact conditions.

We can integrate (3.9) by parts to obtain

$$\Big[F_{n_{ij}}(u_{n_{ij}} - d_0) \Big]_{t_0}^{t_1} - \int_{t_0}^{t_1} \frac{dF_{n_{ij}}}{dt} \Big(u_{n_{ij}} - d_0 \Big) \, dt = 0 . \tag{3.10}$$

Therefore, with the Signorini conditions for the interval $[t_0;\ t_1]$ we have

$$\int_{t_0}^{t_1} \frac{dF_{n_{ij}}}{dt} \Big(u_{n_{ij}} - d_0 \Big) \, dt = 0 . \tag{3.11}$$

4. Space and time discretization

The instantaneous updated Lagrangian description is used in our formulation. The new velocity field is the main variable which is iteratively calculated, based on the initial velocity field and the description of evolution. In each iteration the improved final geometry in the time interval (time step) is calculated [4]. The final geometry is then succesively improved. The inegrations of the elemental matrices are performed over space and time. Let us notice that the spatial domain is altered by the iterative process. This technique is contributed as a new element.

When the motion of any point of the spatial domain is regular during the time-step, we can interpolate the velocity field by the linear interpolation in time. The interpolation must be chosen so as to take into account the fact that, when a point comes into a contact, the velocity becomes a discontinuous function and the acceleration is not defined. For most of the problems it suffices to assume such a simple kind of interpolation. A higher degree of interpolation could be chosen if another strategy of contact analysis was used, for example the mesh would be refined in time.

We must emphasize that the test examples presented in the paper allow to check the ability of the proposed modeling. However, the method similar to the one applied here has already been used for more complex problem, i.e. the viscoplastic deformation problem [12].

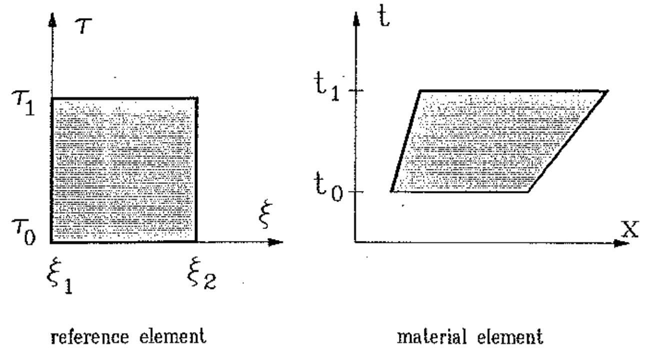

FIG. 1. Space-time element.

The choice of the interpolation in space is determined mainly by the type of the material behaviour. When the motion of a boundary point becomes non-smooth since it comes into a contact during the time-steps, it is necessary to divide the time interval into several steps of time. In the domain of a reference space-time element (Fig. 1) we consider a space-time interpolation function defined by a product of a spatial interpolation function and time interpolation function (Fig. 3):

$$\begin{aligned} P_{ij}(x,\tau) &= N_i(x)g_j(\tau), \\ \mathbf{v}(x,\tau) &= \sum P_{ij}(x,\tau)\mathbf{V}(x_i,\tau_{ij}), \end{aligned} \tag{4.1}$$

$\tau = (t - t_0)/h$, where $h = t_1 - t_0$.

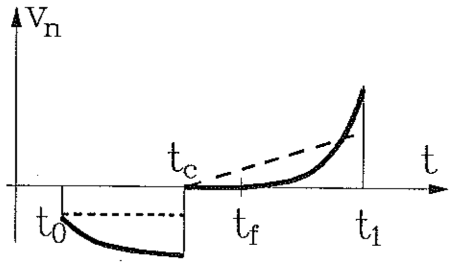

FIG. 2. Variation of the normal relative velocity within a step of time and piece-wise linear interpolation.

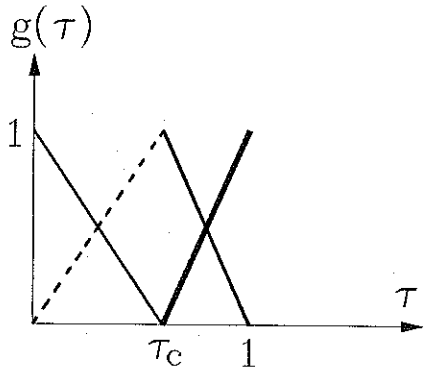

FIG. 3. Time interpolation functions.

If the solid S_i comes to a contact at time t_c within the time step h, two space and time elements are considered instead of one (Fig. 4). The step of time is assumed to be sufficiently short. In the first element the initial value of the velocity v_0 is known. At time t_c^- it is possible to assume the same velocity or to apply an explicit linear interpolation function from the previous step of time.

In the second time step the initial value of the velocity at time t_c^+ is zero because of the contact. The velocity is discontinuous at time t_c (Fig. 2). The velocity at time t_1 is unknown. For instance, in Fig. 4 we present a rod subjected to a dynamic elongation. In the second time step the right-hand end of the rod comes into contact with a wall. A space-time remeshing is performed for the elements neighboring with the contact area. The Fig. 4 shows only the principle. The depth of the space and time remeshing in practice is not large. The range of the remeshing, both in space and time, is limited to the nearest space-time elements in the mesh.

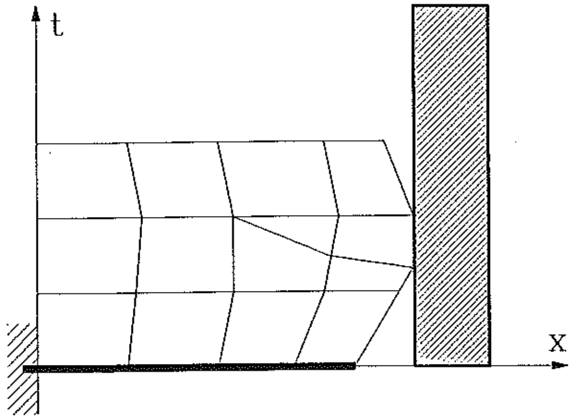

FIG. 4. Space-time mesh evolution.

5. NUMERICAL RESULTS

Two examples were solved to verify the efficiency of the presented approach. In the first example the elastic bar of length $L = 1$, Young modulus $E = 1$ and mass density $\rho = 1$, hits the rigid wall with a speed $v = 1$. The spatial mesh is composed of two elements. The elements were divided in the contact zone into trapezoidal and triangular element as depicted in Fig. 4. Figure 5 presents the time-dependent positions of both ends on the bar.

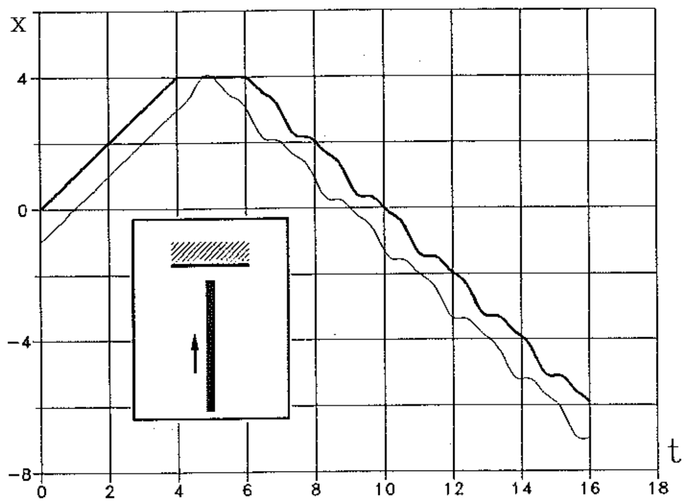

FIG. 5. Collision of an elastic bar with rigid wall.

The second example was calculated with the same mesh as in the previous case. However, here the end of the rod was fixed while for the frontal nodal point, the initial velocity v=1 was assumed. In Fig. 6 displacements of two

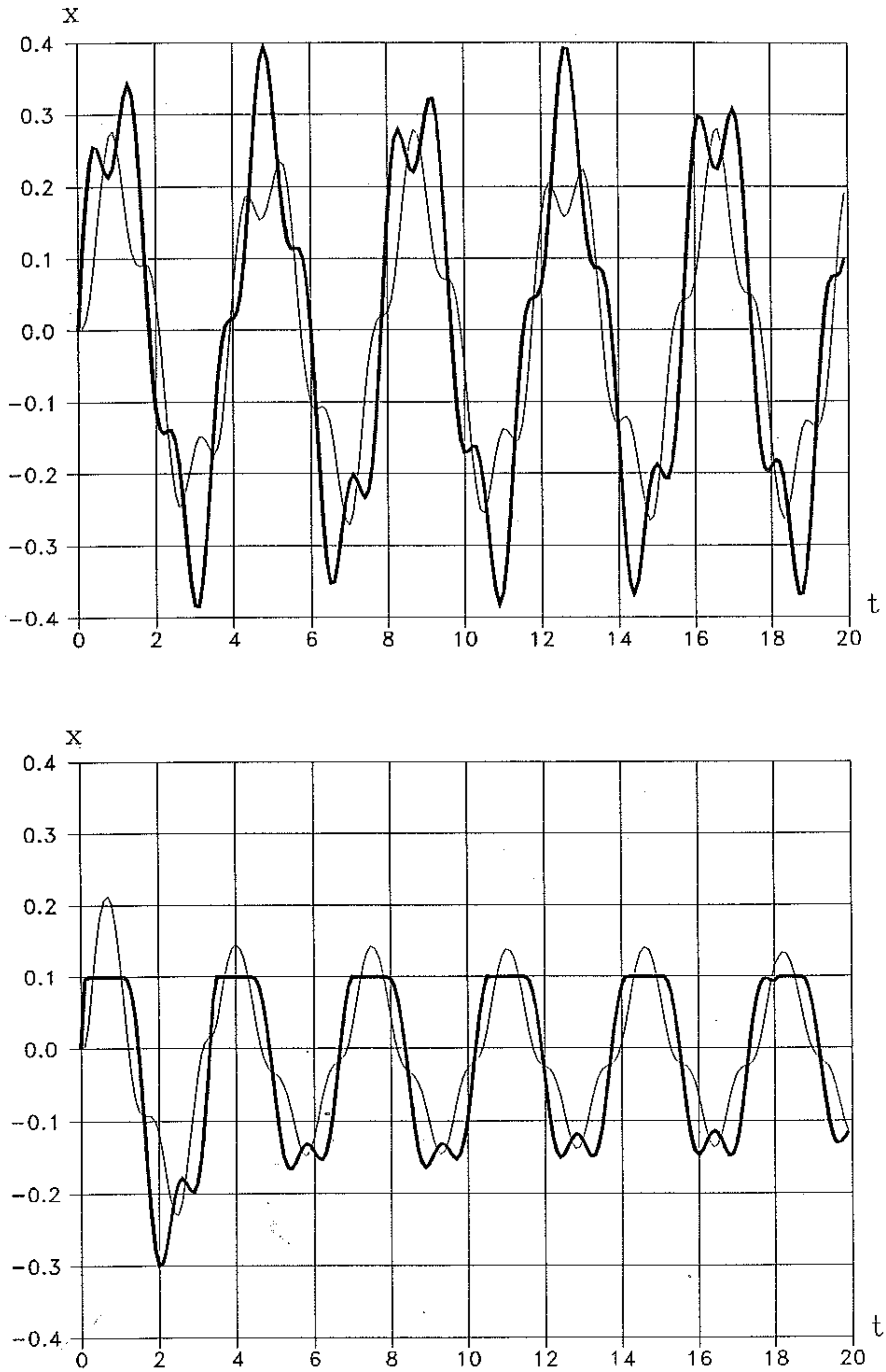

FIG. 6. Vibration of a bar without contact (upper figure) and with contact (lower figure).

remaining points are presented as functions of time (point of the contact – thick line, inner point – thin line). A case with the unilateral constraint additionally imposed upon a nodal point was compared with the case with only one point fixed.

6. Conclusions

The presented formulation and analysis of contact conditions has proved to be very efficient in view of the small computational effort and the simplicity of application. It is the result of the kinematic approach to the formulation and the space-time interpolation, which takes into account the change of the status of a boundary point during the time-step. The presented method of analysis is also convenient in the case involving thermo-mechanical problems and thermal processes during which the change of phases take place.

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