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Research Paper

Pressure Drop Reduction in Fluid Flows with a Polymer Solution

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This work is targeted at making available some data to enhance the development of better predictive models for drag reduction (DR) in two-phase flows. Oil-water flows studies were carried out by using a horizontal acrylic pipe (14 mm ID) with tap water and a middle distillate oil the flowing liquids. A hydrolysed polyacrylamide served as the polymer in the water phase. Polyethylene oxide (PEO) with two different molecular weights was equally investigated. At an optimal polymer concentration of 20 ppm, drag was lowered as Reynolds number increased. While drag reduction was effectively described by models, it increased with polymer weight.

Key words: polymer; oil-water; model; concentration; pressure drop; molecular weight.

1. Introduction

There is an increasing need to optimize energy requirements for movement of fluids across vast distances in the chemical, and oil and gas industries. The use of drag reducing agents (natural and synthetic) have been known to reduce frictional pressure drop in flow systems, thereby reducing the energy requirements [1–4]. DR is the deliberate reduction of the drag that occurs as fluids flow by the use of certain materials and pipeline modifications [5]. Some of these materials are heavy molecular weight (MW) polymers such as polyethylene oxide (PEO), polyisobutylene (PIB), polyacrylamide, okra mucilage, aloe vera, guar gum, among others [4, 6–10]. The molecular weight of these materials is usually in excess of a million grams per mole [11].

Over 80% of reduction in energy losses has been reported by the use of very minute quantities (in parts per million, ppm) of the materials called drag reducing agents (DRA). These materials are widely used in industries including long distance transportation of crude oil and some of its derivatives, oilfield operations, management of oil-sand tailings, public heating and cooling, biomedical uses in the reduction of hemorrhagic shock, treatment of wastewater and in firefighting [11–13]. Their use in the movement of drinking water from water for public consumption with no negative side effects has also been reported [14].

The ability of the DRAs to lower pressure drop (drag) is usually given as DR [%]:

$$\mathrm{DR} = \frac{\Delta P - \Delta P_p}{\Delta P} \cdot 100\ [\%]. \tag{1.1}$$

Here ΔP_p and ΔP are pressure drop of flow with and without polymer addition respectively. Although the friction factor of the flow can also be used, the ease of using pressure drop arises from the fact that the measuring device is configured to read pressure drop values.

The knowledge of drag reduction has been available since the epic report by Toms [15]. The first major commercial application of a polymeric drag-reducing additive is reported in the Trans-Alaskan Pipeline System. For drag-reduced flows, very scanty mathematical models are available, particularly for flows of two or more phases. This limitation hampers the ability to effectively and accurately predict the effect of polymer addition in such systems. An explicit description has been reported by Virk [16], with several equations for drag-reduced single-phase flows. Virk showed the limit of DR possible for a system, a term now referred to as the maximum drag reduction asymptote (MDRA). His report was, however, limited to single-phase flows. The scanty nature of data for DR modelling is one of the major challenges to the development of a robust predictive model.

The work presented in this paper aims to provide some data on both water-only flows and multiphase flows as a contribution to the development of models for more accurate and reliable prediction of DR phenomenon. In addition, this work will provide more insight into the effects of polymers on fluid (single and two-phase) flows. The predictive accuracy of the developed model obtained from the modification of the two-fluid model will be tested using the experimental data of drag-reduced flows.

2. Materials and methods

The study was carried out in a pipe made of acrylic with a length of 4 m and an internal diameter of 14 mm with tap water (density, 1000 kg/m; viscosity, 1.0 mPa · s), and Exxsol D140 distilled oil (density, 828 kg/m; viscosity, 5.5 mPa · s at 20°C) as test fluids. Figure 1 is the sketch that shows the flow facility used in the investigation. Two 160 litre tanks store the water and oil, which enter the flow section with the help of two separate centrifugal pumps. Two variable area flowmeters (7.5 l/min ± 0.20%) are used to control the amount

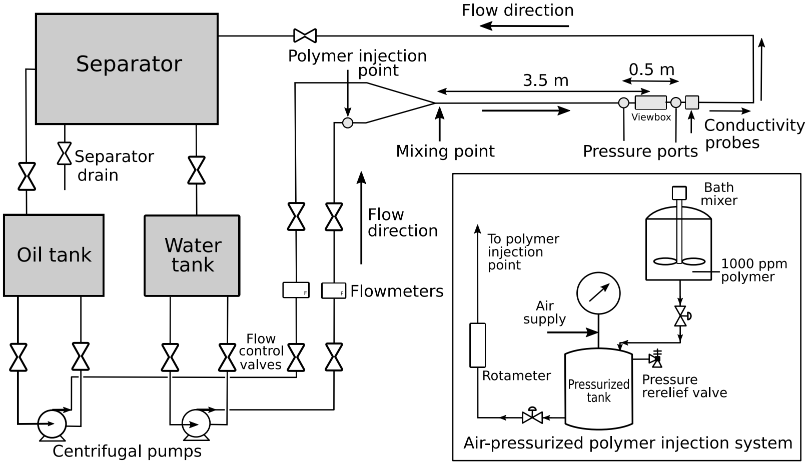

FIG. 1. Schematic diagram of the oil-water flow facility with the polymer injection system.

of fluids that enter the flow section. A spare 35 l/min capacity flowmeter is used for flows greater than 8 l/min particularly for water flows. A Y-junction that reduces mixing is used to bring the two liquids together in the flow section and they end up in the separator tank that has a capacity of 250 litres with the oil been returned to its tank after gravity separation. Freshwater was always used for every flow cycle to avoid the degradation of the polymer by the pumps.

Superficial oil (U_{so}) and water velocities (U_{sw}) that cover a wide spectrum of the flow pattern map [17] were investigated for the oil-water flows. Twenty ppm of Magnafloc 1011 (a co-polymer of polyacrylamide and sodium acrylate; HPAM, mol wt. = $10 \cdot 10^6$ g/mol) added to the water phase only was used for the study. The concentration was selected because of the very high-pressure drop reduction reported for similar systems [3, 18, 19]. Some of the experiments were conducted using PEO as the active polymer. The MWs of the PEO are $5 \cdot 10^6$ g/mol (5MPEO) and $8 \cdot 10^6$ g/mol (8MPEO) and were equally added to the water phase in the same concentration. Before commencement of the tests, a polymer solution of 1000 ppm (master solution) was prepared by gently adding 10 g of the solid polymer into a stirred water container that has 10 litres. The stirring was achieved with the help of a three-bladed mechanical stirrer (Heidolph, D-91126) for about four hours. To allow for proper water absorption and release of trapped gas molecules, the solution was left for over 12 hours (mostly overnight). In order to avoid the introduction of disturbances into the flow, the master solution was introduced into the water line of the flow section with the help of an air-pressurized system (see inset in Fig. 1). Pressurized air at 2 bars was used to push the polymer solution (1000 ppm) into the flow section. A 1.50 mm diameter hole at the bottom of the inlet pipe was used as the inlet of the polymer. It was situated 0.50 m upstream the point where the two liquids met. To improve the mixing of the polymer solution and water, an acrylic mesh was inserted in the water flow line of the Y-junction. Drag as pressure drop readings were taken by using a differential pressure transmitter (ABB 266MST 0.04% BA) that was connected to two pressure ports that are positioned at 3.25 m and 3.75 m (0.5 m apart) respectively from the Y-junction. Pressure drop readings were taken at least three times (SD $<$ 2%) for the separate values investigated. The mean position of the interface was determined with the use of conductance probes [17, 20, 21].

3. Results and discussion

3.1. DR in water flows

A substantial amount of the drag in the water flows was lowered when the polymer solution was injected. Figure 2 typifies water flow with 20 ppm of polymer added at different Reynolds numbers (Re) in the 14 mm pipe. Drag (pressure drop) is observed to be reduced when 20 ppm of the polymer solution was injected into the flow and this increases with the Reynolds number in contrast to flows of the pure fluid in the pipe. In addition, the observation at low Reynolds number, corresponding to laminar flows, confirms some earlier reports where results show insignificant DR while it was completely unnoticeable in other reports [16, 22, 23]. At Re of around 1500, the DR of 5% was recorded and can be identified as the beginning/starting point of DR for this flow system.

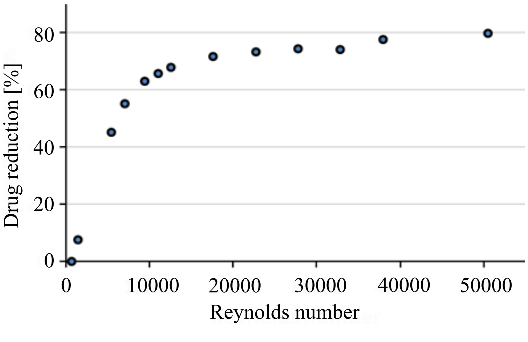

FIG. 2. Pressure gradient reduction with 20 ppm polymer in water flows at different Re.

Equation (3.1) gives the mathematical expression for determining the friction factor f in flow systems:

$$f = \frac{\Delta P D}{2\rho l U^2}. \tag{3.1}$$

Here f is the Fanning friction factor, ΔP is a pressure drop, ρ is the density of test fluid, U is the velocity, l is the pipe length, and D is the diameter of test pipe.

A closer look showed that at first DR increased with the concentration of polymer solution up to 20 ppm and then reached its zenith and remained somewhat constant with no further significant increase as polymer concentration increased beyond 20 ppm as can be seen in Fig. 3.

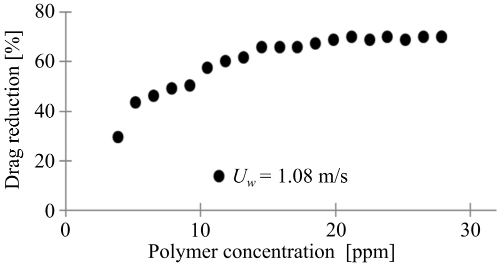

Fig. 3. DR *vs* polymer concentration in water-only flows.

It is interesting to note that with as little as 2.5 ppm of the polymer solution, about 30% DR was recorded as was also reported earlier in [24] where they recorded significant DR after applying just 2 ppm of polymer in their oil-water flow experiments. The Trans-Alaskan pipeline system reported a 33% reduction in drag by simply adding 1 ppm of an oil-soluble polymer without providing its name for a proprietary purpose.

Equations (3.2) and (3.3) give the derived mathematical link between Re of flows and the corresponding friction factor for water flows with drag reducing agents (HPAM) for both normal and von Karman points. The reported errors are less than 1%, and hence this relationship can be conveniently inserted into the already established two-fluid model for accurate determination of pressure drop lowering with polymer solution for oil-water flows, and particularly horizontal flows [17, 25]

$$f = 1.2022\mathrm{Re}^{-0.651}, \tag{3.2}$$

$$f^{-1/2} = 0.858\left(\mathrm{Re}\sqrt{f}\right)^{0.4833}. \tag{3.3}$$

Here,

$$\mathrm{Re} = \frac{\rho V D}{\mu}, \tag{3.4}$$

where $\rho$is the density of test fluid, V is the velocity, D is the diameter of the test pipe, and μ is the fluid viscosity.

Figure 4 shows the Prandtl-Karman (P-K) coordinates and the comparison of the data obtained from Magnafloc 1011 (HPAM) and those from the PEOs alongside the Virk's [16] maximum drag reduction asymptote (MDRA).

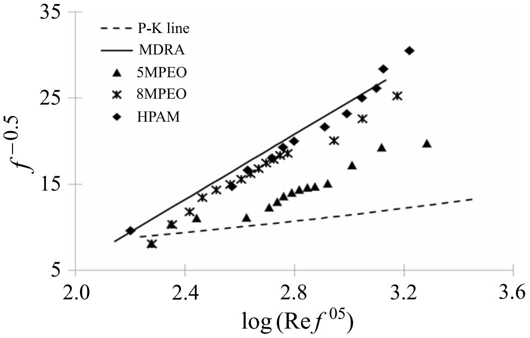

FIG. 4. DR with HPAM, 5MPEO & 8MPEO *vs* Virk's MDRA.

The Fanning friction factor (f) for flows with and without polymer is obtained from the pressure drop. Equation (3.1) gives the P-K line while Eq. (3.5) gives the MDRA as shown:

$$f = 0.58\,\mathrm{Re}^{-0.58}. \tag{3.5}$$

As can be seen, similar effects were noticed for both HPAM and 8MPEO in the water phase compared to 5MPEO, which showed far less drag reducing effect (Fig. 4). Again, for Re above 2000, which are within the turbulent flow regime, there is a significant lowering of the friction factor with polymer addition. For Re over 45 000, the reported maximum DR are 80%, 72%, and 55% for HPAM (see Fig. 2 also), 8MPEO and 5MPEO, respectively.

Evidently, polymer molecular weight (MW) significantly influences DR as shown in Fig. 4, where higher DR is achieved with higher MW. The ionic structure has been identified as a possible cause for this observation [9, 11, 12] and can provide an explanation for the difference in the measured friction factors of HPAM and 8MPEO solutions especially at Re > 15 000. In this region, the gradient of the friction factors of PEO is less steep when they are compared to those of HPAM. This observation is understandable since PEO polymers are known to be easily degraded when subjected to mechanical shear associated with high Re when compared to HPAM that has reportedly displayed better resistance [26–29]. It has been reported that polymers with higher MWs form better entanglements and aggregates, which are known to increase DR effectiveness [11]. This may explain the difference in the DR observed for 5MPEO and 8MPEO.

3.2. *Addition of polymer to oil-water flow*

The reduction in pressure drop for different polymer concentrations investigated in oil-water flows is depicted in Fig. 5. There is a noticeable huge lowering of pressure drop in the oil-water flows after the polymer solution ranging from 10 ppm to 50 ppm was added to the water phase. The superficial velocities of the liquids (water, U_{sw}; oil, U_{so}) displayed in the figure are within the limits of the stratified/stratified wavy and early stage of dual continuous flows [3] and are a reflection of the performance of the polymer over the range of velocities investigated.

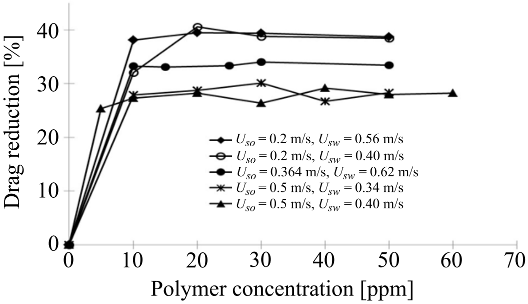

Fig. 5. Effect of polymer concentration on DR in oil-water flows at different superficial oil and water velocities.

It can be seen in Fig. 5 that the DR increased until 20 ppm concentration and then generally flattened out after that. This agrees with the observation in water-only flows earlier discussed and as reported by other investigators [30–32]. Some other researchers [18] also noticed that beyond 20 ppm, there were no substantial pressure drop reductions in the oil-water system. It has been opined that the region of strong turbulence activities within the flow may be filled with sufficient polymer aggregates so that equilibrium is attained such that further addition of the polymer solution in aggregate form does not yield increased destruction/reduction of the turbulence activities. Some have reported that further increase in polymer concentration with the raise of the fluid viscosity results in the unwanted effect of increased drag [29, 33]. As a result, the selected optimal concentration was used for further experiments in the oil-water flows where up to 55% reduction in pressure drop was recorded after polymer solution was added to dispersed flows and it changed to stratified flow.

The derived friction factor relationship for flows with a polymer solution (Eq. (3.2)) was applied as a replacement for the water phase wall friction factor in the modified two-fluid model (2FMpol). The model (2FMpol) also included a new correction for interface curvature as discussed in [17] while the roughness factor was not included because of the dampening effect of the polymer on the interfacial waves as earlier reported in [3, 30]. It is important to note that the interface curvature for the oil-water flows was retained despite polymer addition to the flow and a novel equation that relates the height at the wall and that at the pipe's centre was derived for insertion and application in the 2FMpol model. The new correlation is given as:

$$h_b = 0.7619 h_w + 0.0004151, \tag{3.6}$$

where h_b is the height of the water phase at the middle of the pipe while h_w is the height at the pipe's centre.

The data used in these comparisons are the same as those reported by Edomwonyi-Otu and Angeli [17]. Comparison of the results obtained from the modified two- fluid model and the predictions from the homogeneous model proposed by Al-Sarkhi and co-workers [31] for oil-water flows with polymer solutions was carried out. The experimentally derived friction factor for the Al-Sarkhi's model in flows with polymer solutions is shown in Eq. (3.7), while the corresponding pressure drop is given by Eq. (3.8)

$$f_m = 0.614 \left(\mathrm{Re}_m \left(\frac{U_{sw}}{U_{so}} \right)^{0.5} \right)^{-0.5}, \tag{3.7}$$

$$\left(\frac{\mathrm{d}p}{\mathrm{d}z} \right)_{\mathrm{pol}} = \frac{2 f_m \rho_m U_m^2}{D}, \tag{3.8}$$

where $\left(\frac{\mathrm{d}p}{\mathrm{d}z} \right)_{\mathrm{pol}}$ is the pressure drop measured after polymer addition, U_m (mixture velocity) is the sum of the superficial velocities of the oil (U_{so}) and water (U_{sw}) phases, f_m is the friction factor (Fanning type) of the oil and water mixture and the mixture density ρ_m, is as given in Eq. (3.9):

$$\rho_m = H_w \rho_w + H_o \rho_o, \tag{3.9}$$

where ρ_o and ρ_w are densities while H_o and H_w are the hold-ups for the oil and water phases, respectively.

An improvement of about 6 % in prediction accuracy was observed when Eqs (3.2) and (3.3) were used in comparison to that obtained when friction factor and interface heights for flows without polymer solution were used [17]. As can be seen from Fig. 6, the DR prediction accuracy using the modified two-fluid model (2FMpol) was well within the ± 15% boundary of the experimental data, while in some cases about 42% under-prediction of the experimental data was noted for the model proposed by Al-Sarkhi and co-workers.

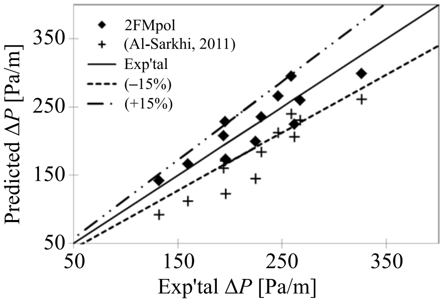

Fig. 6. Comparison of predictive accuracies of DR models.

Of note is the improvement in predictive accuracy as the superficial oil (U_{so}) and water (U_{sw}) velocities increased. The unsuitability of the homogeneous model as applied to pressure drop in the stratified flow region may be responsible for the measured under-prediction of the Al-Sarkhi's model. The peculiarity of the curved interface that has been reported for oil-water flows may not have been accounted for in the model. Similarly, a 32% under-prediction of data by the Al-Sarkhi model has also been reported by Al-Wahaibi and coworkers [34].

Figure 7 shows the results of the comparison of the prediction of oil-water flows with a polymeric solution by the developed model (2FMpol) and those from the different interfacial shear stress correlations discussed in [17]. While the Brauner [35] and Brauner *et al.* [36] model over-predicted the experimental data in some cases, the models by Hall [37] and Taitel *et al.* [38] have similar prediction with the derived model (2FMpol) with appreciable closeness to the experimental data. Nevertheless, the prediction trends of all the models were similar.

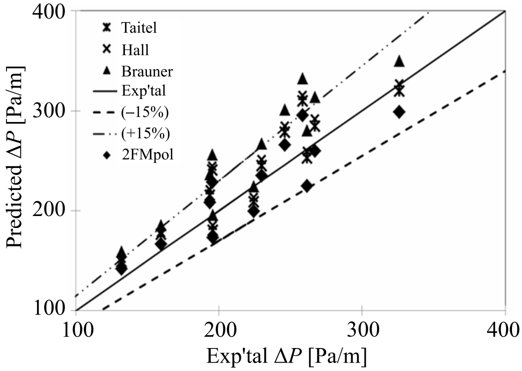

Fig. 7. Pressure drop prediction by the modified model and different interfacial shear stress correlations in the two-fluid model for flows with the polymer solution.

4. Conclusion

From the results obtained from this investigation, it can be concluded that:

1) Pressure drop/drag reduction increases with increasing Re, particularly for single-phase water flows.
2) Drag reduction increases with polymer concentration and reaches a maximum beyond which no further increases are observed.
3) Optimal polymer concentration for this system is 20 ppm.
4) Drag reduction is sensitive to polymer type and increases with polymer MW.
5) Drag reduction in both single-phase and two-phase flows can be accurately described by models.

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