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# ON THE PROBLEM OF BENDING OF TRANSVERSALLY ISOTROPIC PLATES

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The possibility of analysis of transversally isotropic plates of medium thickness by means of representing the displacements in the form of finite series along the lateral coordinate is examined. Differential equations of the twelfth order are derived. Conditions on the plate boundaries are determined. Examples of numerical calculations are presented. The article is a sequel to papers [11 – 13].

## 1. Introduction

Many papers devoted to the construction of a generalized theory of thin plates do not employ the main hypothesis of the Kirchhoff's classical theory of thin plates bending – the hypothesis of undeformed normals. Such approaches are based on the theories of Timoshenko [14], Reissner [10] and Ambartsumyan [1]. Almost all of them are based on the assumption of the parabolic law of distribution of shearing stresses $\sigma_{rz}$, $\sigma_{\theta z}$ across the thickness of the plate. Such theories seldom take into account the influence of normal stresses $\sigma_{zz}$; the possibility of determining the lateral deformations $\varepsilon_{zz}$ is also rarely taken into consideration. Most of the specified theories of plates of medium and large thickness which include shear and normal deformations [1 – 6, 9] do not differ much, and their results give a small increase of the accuracy of the solution, in spite of substantial increase of the complexity of the governing equations.

In devolopment of the author's earlier inverstigations [11], in this paper the differencial equations of bending are reduced to the equations similar to the equations of Timoshenko's type with simultaneous addition of terms which take into account the effect of transversal contraction and tangential loads applied upon the faces of the plate.

## 2. Problem formulation and solution

Let us examine the stress-deformed state of the circular plate made of transversally isotropic material with radius $a$ and thickness $2h$, referred to the cylindrical system of coordinates $r$, $\theta$, $z$. The axis $Oz$ is directed vertically downwards. Let us accept that the median plane of the plate, which is the plane of elastic isotropy, coincides with the coordinate plane $z = 0$, outer surfaces being described by equations $z = \pm h$, $\leq a =$ const and $r = a$, $-h \leq z \leq h$ (Fig. 1). Lower and upper surfaces of the plates $S^+$ and $S^-$ are under the influence of external forces $\mathbf{q}^{\pm} = \tau_r^{\pm}\mathbf{r}^0 + \tau_\theta^{\pm}\boldsymbol{\theta}^0 + q_z^{\pm}\mathbf{z}^0$ $(z = \pm h)$, where $\mathbf{r}^0$, $\boldsymbol{\theta}^0$, $\mathbf{z}^0$ are unit vectors of the cylindrical system of coordinates $r$, $\theta$, $z$ in the examined point. Then, the boundary conditions on $S^+$ and $S^-$ may be written down as follows:

$$\sigma_{zz}\big|_{z=\pm h} = \pm q_z^{\pm}, \quad \sigma_{rz}\big|_{z=\pm h} = \pm\tau_r^{\pm}, \quad \sigma_{\theta z}\big|_{z=\pm h} = \pm\tau_\theta^{\pm}. \tag{2.1}$$

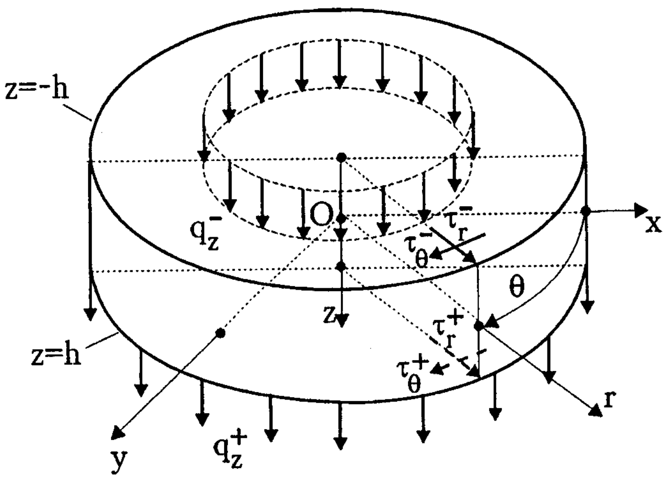


Fig. 1.

Having denoted the components of elastic displacement vector as $U_r$, $U_\theta$, $W$, we shall look for the solutions of the problem in the form of truncated series [12]

$$\{U_r(r,\theta,z), U_\theta(r,\theta,z)\} = \{u_r(r,\theta), u_\theta(r,\theta)\} + \sum_{n=1,3} \{u_{rn}(r,\theta), u_{\theta 1}(r,\theta)\} z^n, \tag{2.2}$$

$$W(r,\theta,z) = w(r,\theta) + \sum_{n=1}^{4} f_n(r,\theta) z^n,$$

where

$$u_{r2} = \frac{\tau_{r1}}{2G'h}, \; u_{\theta 2} = \frac{\tau_{\theta 1}}{2G'h}, \; \tau_{r1} = \frac{1}{2}\left(\tau_r^+ - \tau_r^-\right), \; \tau_{\theta 1} = \frac{1}{2}\left(\tau_\theta^+ - \tau_\theta^-\right),$$

$u_r(r,\theta)$, $u_\theta(r,\theta)$, $u_{r1}(r,\theta)$, $u_{\theta 1}(r,\theta)$, $u_{r3}(r,\theta)$, $u_{\theta 3}(r,\theta)$, $f_n(r,\theta)$ – are as yet unknown functions.

As in the papers [11 – 13], we shall use the following expressions for resultant forces in the isotropy plane $S_{rr}$, $S_{\theta\theta}$, $T_{r\theta}$, shearing forces $N_r$, $N_\theta$ and bending and twisting moments $M_r$, $M_\theta$, $H_{r\theta}$ in the plate,

(2.3)
$$\begin{aligned}
\{S_{rr}, S_{\theta\theta}, T_{r\theta}, N_r, N_\theta\} &= \int_{-h}^{h} \{\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{r\theta}, \sigma_{rz}, \sigma_{\theta z}\}\, dz, \\
\{M_r, M_\theta, H_{r\theta}\} &= \int_{-h}^{h} \{\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{r\theta}\}\, zdz,
\end{aligned}$$

which satisfy the equilibrium equations

(2.4)
$$\begin{aligned}
\frac{\partial S_{rr}}{\partial r} + \frac{1}{r}\frac{\partial T_{r\theta}}{\partial \theta} + \frac{S_{rr} - S_{\theta\theta}}{r} &= -\tau_{r2}, \\
\frac{\partial T_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial S_{\theta\theta}}{\partial \theta} + 2\frac{T_{r\theta}}{r} &= -\tau_{\theta 2}, \\
\frac{\partial N_r}{\partial r} + \frac{1}{r}\frac{\partial N_\theta}{\partial \theta} + \frac{N_r}{r} &= -q_2, \\
\frac{\partial M_r}{\partial r} + \frac{1}{r}\frac{\partial H_{r\theta}}{\partial \theta} + \frac{M_r - M_\theta}{r} &= N_r - h\tau_{r1}, \\
\frac{\partial H_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial M_\theta}{\partial \theta} + 2\frac{H_{r\theta}}{r} &= N_\theta - h\tau_{\theta 1},
\end{aligned}$$

where

$$q_1 = \frac{1}{2}\left(q_z^+ - q_z^-\right), \quad q_2 = q_z^+ + q_z^-,$$

$$\tau_{r2} = \tau_r^+ + \tau_r^-, \quad \tau_{\theta 2} = \tau_\theta^+ + \tau_\theta^-.$$

If we find the components of deformations Eq. (2.2) and, by using Hooke's law and boundary conditions (2.1), the expressions for resultant forces and moments (2.3) and equilibrium Eq. (2.4), we shall obtain the system of differential equations

$$\Delta^2\varphi = 2\nu'' h\Delta\overline{q}_1 - \frac{2(1+\nu)m_1}{h}, \tag{2.5}$$

$$D\Delta^2\tilde{w} = q_2 + m_2 - \varepsilon_1\Delta q_2 + \varepsilon_1'\Delta m_2, \tag{2.6}$$

$$D\Delta\tilde{w}_\tau = \varepsilon_\tau(q_2 + m_2), \quad \Delta\Omega - k_0^2\Omega = 0. \tag{2.7}$$

Here $\Omega(r,\theta) = \dfrac{1}{\Lambda' k_0^2}\left(\dfrac{\partial N_\theta}{\partial r} - \dfrac{1}{r}\dfrac{\partial N_r}{\partial \theta}\right)$ is the function of shear angles; $\varphi(r,\theta)$ is the function of the forces in the median surface of the plate which determines the resultant forces in the isotropy plane,

$$S_{rr} = \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r}, \quad S_{\theta\theta} = \frac{\partial^2\varphi}{\partial r^2},$$

$$T_{r\theta} = \frac{1}{r^2}\frac{\partial\varphi}{\partial\theta} - \frac{1}{r}\frac{\partial^2\varphi}{\partial r\partial\theta},$$

$$\tilde{w} = w + \frac{\overline{q}_2\varepsilon_2}{D}, \quad \tilde{w}_\tau = w_\tau - 4q_\tau, \quad m_i = h\left(\frac{\partial\tau_{ri}}{\partial r} + \frac{1}{r}\frac{\partial\tau_{\theta i}}{\partial\theta}\right),$$

$$\overline{q}_i = q_i + \frac{1}{3}m_i, \quad q_\tau = \int\frac{\tau_{r2}}{10G'}dr = \int\frac{\tau_{\theta 2}}{10G'}d\theta,$$

$$\varepsilon_1 = \frac{h^2}{10(1-\nu)}\left(8\frac{G}{G'} - 3\nu''\right), \quad \varepsilon_1' = \frac{h^2}{30(1-\nu)}\left(4\frac{G}{G'} - \nu''\right),$$

$$\varepsilon_2 = \frac{h^4}{20(1-\nu^2)}\left(1 - \nu'(1+\nu)\frac{G'}{E'}\right)\frac{E}{E'},$$

$$\varepsilon_\tau = \frac{4h^2}{5(1-\nu)}\frac{G}{G'}, \quad \Lambda' = \frac{5}{3}G'h, \quad \nu'' = \frac{E}{E'},$$

$$\alpha = \frac{\nu''}{2}\frac{G'}{G}, \quad k_0^2 = \frac{5G'}{2Gh^2}, \quad D = \frac{2Eh^3}{3(1-\nu^2)},$$

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} \quad (i = 1, 2),$$

$\Delta$ – two-dimensional Laplacian operator in a cylindrical system of coordinates; $E \sim E_r = E_\theta$, $E' \sim E_z$ – elasticity moduli for the directions parallel and perpendicular to the isotropy plane $z = 0$; $\nu \sim \nu_{r\theta} = \nu_{\theta r}$, $\nu' \sim \nu_{zr}$, $\nu'' \sim \nu_{rz}$ –

Poisson's ratios; $G' \sim G_{r\theta} = G_{\theta r}$ – shear modulus; $w_\tau$ – component of vertical displacement caused by lateral shear.

The constructed system of differential equations is of the tenth order. It is divided into two independent parts. One equation of the fourth order (2.5) refers to the generalized plane problem which includes the load applied to the surface of the plate. The system of Eqs. (2.6), (2.7) of the sixth order describes the problem of lateral bending of transversally isotropic plates.

Besides Eqs. (2.5), (2.6), the functions $\varphi$, $\Omega$, $w$, $w_\tau$ must also satisfy five boundary conditions on the face (cylindrical surface) $r = a =$ const of the plate. In particular, the conditions of a simply supported plate at $r = a$, $z = z_0(-h \le z_0 \le h)$ are written down in the form:

$$S_{rr} = M_r = H_{r\theta} = 0, \quad U_\theta\,|_{z=z_0} \cong U_\theta|_{z=0} = u_\theta = 0, \tag{2.8}$$
$$W\,|_{z=z_0} \cong W|_{z=0} = w = 0(r = a);$$

the conditions of rigid fixing of the surface in the form:

$$U_r\,|_{z=z_0} \cong U_r|_{z=0} = u_r = 0,\ U_\theta\,|_{z=z_0} \cong U_\theta|_{z=0}$$
$$= u_\theta = 0,\ W\,|_{z=z_0} \cong W|_{z=0} = w = 0, \tag{2.9}$$
$$\left.\frac{\partial U_r}{\partial z}\right|_{z=z_0} \cong (u_{r1} + 3u_{r3}z^2)|_{z=z_0} = 0,\ \left.\frac{\partial U_\theta}{\partial z}\right|_{z=z_0}$$
$$\cong (u_{\theta 1} + 3u_{\theta 3}z^2)|_{z=z_0} = 0(r = a).$$

The last two conditions meant that the rotation of elements at points $z = \pm z_0$, symmetrical with respect to the median plane, are equal to zero.

In view of possible approximation, the two last boundary conditions are written as

$$\left.\frac{\partial U_r}{\partial z}\right|_{z=z_0} \cong \left.\frac{\partial U_r}{\partial z}\right|_{z=0} = u_{r1} = 0,$$
$$\left.\frac{\partial U_\theta}{\partial z}\right|_{z=z_0} \cong \left.\frac{\partial U_\theta}{\partial z}\right|_{z=0} = u_{\theta 1} = 0(r = a).$$

The condition of fixed support clamping can be simplified to the form:

$$u_r = u_\theta = w = u_{r1} = u_{\theta 1} = 0(r = a). \tag{2.10}$$

In the particular case of loading the plate by normal forces ($\tau_r^\pm = \tau_\theta^\pm = 0$), the average inner force factors look as follows:

$$
\begin{aligned}
S_{rr} &= B\left(\frac{\partial u_r}{\partial r} + \frac{\nu}{r}\frac{\partial u_\theta}{\partial \theta} + \nu\frac{u_r}{r}\right) + 2\lambda h q_1,\\
S_{\theta\theta} &= B\left(\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} + \nu\frac{\partial u_r}{\partial r}\right) + 2\lambda h q_1,\\
T_{r\theta} &= 2Gh\left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} + \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right),\\
N_r &= \Lambda'\left(\frac{\partial w_r}{\partial r} + \gamma_r\right), \quad N_\theta = \Lambda'\left(\frac{1}{r}\frac{\partial w_r}{\partial \theta} + \gamma_\theta\right),\\
M_{rr} &= D\left(\frac{\partial \gamma_r}{\partial r} + \frac{\nu}{r}\frac{\partial \gamma_\theta}{\partial \theta} + \nu\frac{\gamma_r}{r}\right) + \frac{2}{5}\lambda h^2 q_2,\\
\gamma_r &= \frac{\partial}{\partial r}(w_\tau - w_r) - \frac{4}{5r}\frac{\partial \Omega}{\partial \theta},\\
M_\theta &= D\left(\frac{1}{r}\frac{\partial \gamma_\theta}{\partial \theta} + \frac{\gamma_r}{r} + \nu\frac{\partial \gamma_r}{\partial r}\right) + \frac{2}{5}\lambda h^2 q_2,\\
\gamma_\theta &= \frac{1}{r}\frac{\partial}{\partial \theta}(w_\tau - w_r) + \frac{4}{5}\frac{\partial \Omega}{\partial r},\\
H_{r\theta} &= \frac{(1-\nu)D}{2}\left(\frac{\partial \gamma_\theta}{\partial r} - \frac{\gamma_\theta}{r} + \frac{1}{r}\frac{\partial \gamma_r}{\partial \theta}\right),\\
w_r &= w - \frac{\kappa_0}{4(3+\kappa_0)}w_\tau + \frac{hq_2}{E_0},\\
\kappa_0 &= \frac{3\nu''}{2G/G' - \nu''}, \quad B = \frac{2hE}{1-\mu^2},\\
\lambda &= \frac{\nu''}{1-\nu}, \quad E_0 = \frac{40(3+\kappa_0)E'}{9}.
\end{aligned}
\tag{2.11}
$$

It is noticeable that functions $\gamma_r$, $\gamma_\theta$ for the moments play the role as functions $u_r$, $u_\theta$ for the resultant forces in the plane of the plate.

## 3. Example and final remarks

As an example, let us examine the plate deflections under the action of uniformly distributed load on the upper surface of the plate ($q_z^- = q$, $q_z^+ = 0$, $q_1 =$

$-q/2$, $q_2 = q$). This load leads to the axi-symmetric character of the problem ($U_\theta \equiv 0$) and ensures the independence of the solution of the angle $\theta$. Integrating the system of Eqs. (2.5), (2.6) and taking account of the condition of simply supported edge of plate (2.8), we shall find, in particular, expressions for vertical displacement $W$, displacement $u = u_r$ and normal stress $\sigma_{rr}$:

$$W(r,z) = \frac{q(a^2-r^2)}{64D}\left[\frac{5+\nu}{1+\nu}a^2 - r^2 + \frac{8nh^2}{5(1-\nu^2)}\right] + \frac{q}{2E'}(z_0 - z) \tag{3.1}$$

$$-\frac{q\nu'' z^2(a^2-r^2)}{8(1-\nu)D} + \frac{\nu'' q(z_0^2 - z^2)}{16(1-\nu)D}\left[\frac{1-\nu}{1+\nu}a^2 + \frac{4nh^2}{5(1-\nu^2)}\right]$$

$$-\frac{\alpha_0 q}{8hE'}\left[6A_2(z_0^2 - z^2) - \frac{A_3}{h^2}(z_0^4 - z^4)\right],$$

$$u(r) = \frac{\nu' q}{2E'}r, \quad \sigma_{rr}(r,z) = \frac{3qz}{32h^3}\left[(3+\nu)(a^2-r^2) + m\left(\frac{z^2}{3} - \frac{h^2}{5}\right)\right],$$

$$\alpha_0 = \frac{1}{2} - \nu'\lambda, \quad n = 4\frac{E}{G'} - \nu''(7-\nu), \quad m = \frac{4}{1-\nu}\left[\frac{E}{G'} - \nu''(3+\nu)\right],$$

$$A_2 = 1 + \frac{\lambda E'}{2\alpha_0 G'}, \quad A_3 = 1 + \frac{\lambda E'}{2\alpha_0 G'}\left[1 - \nu'(1+\nu)\frac{G'}{E'}\right].$$

We find it necessary to note that at $z_0 = 0$, the obtained expressions concide with the formulas of the thick plates theory [6]. At the same time, the offered modification of the plate theory enables the realization of a more natural way of simple supporting, when the plate is supported at the edge of its lower face $z = h$, that is, when $z_0 = h$ (Fig. 2 a).

Columns 4, 6 of Table 1 contain tabulated values of dimensionless displacement $\overline{W}(z) = W(0,z)E/(qa)$ on the outer and median surfaces ($z = \pm h; 0$), calculated by formula (2.12) for material $E/E' = G/G' = 1$; $\nu = \nu'' = 0.25$ and transversally isotropic material which, by its thickness, has properties of composite $E/E' = 25$; $G/G' = 2.5$; $\nu = 0.25$; $\nu'' = 0$. These results are compared with the data obtained for the same isotropic material by DS method [7, 8] in spatial problem statement of elasticity theory [8] and by means of the Reissner and Ambarcumyan theories. Calculations have been done for two different values of the relative thickness of the plate. The absolute relative error of the obtained value is given in brackets.

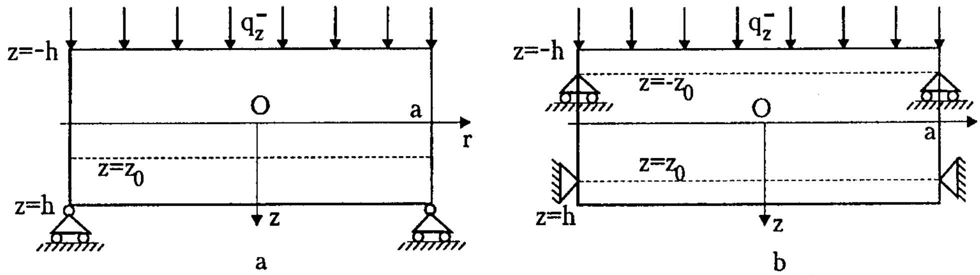


FIG. 2.

**Table 1. Tabulated displacement $\overline{W}(z)$ for simply supported plate.**

| | | Isotropic material | | | Transversally isotropic | |
|---|---|---|---|---|---|---|
| $\frac{h}{a}$ | $\frac{z}{h}$ | DS method [8] | By Eq. (3.1) | Reissner's theory | By Eq. (3.1) | Ambartsumyan's theory |
| | −1 | 1.821 | 1.744 (4.2%) | 1.336 (26.6%) | 15.12 | 2.612 |
| 0.5 | 0 | 1.549 | 1.489 (3.9%) | 1.336 (13.8%) | 4.956 | 2.612 |
| | 1 | 1.317 | 1.245 (5.5%) | 1.336 (1.4%) | 2.612 | 2.612 |
| | −1 | 12.92 | 12.99 (0.5%) | 13.03 (0.3%) | 21.22 | 16.22 |
| 0.2 | 0 | 13.33 | 13.21 (0.9%) | 13.03 (1.4%) | 17.15 | 16.22 |
| | 1 | 12.72 | 12.79 (0.5%) | 13.03 (1.9%) | 16.22 | 16.22 |

It is seen that the values of the isotropic plate deflection, found by formula (3.1), agree with the exact solution (see [8]) even for the plate of large thickness ($h = 0.5a$). The maximal absolute relative error on the lower and upper face of the thick plate amounts to 5.5%; on the median surface it is the smallest – 3.9%. For 2.5 times thinner plates the difference does not exceed 1% and amounts, respectively, to 0.5% and 0.9%. As compared with formula (3.1), the Reissner's theory gives larger error which amounts even to 27% for thick plates; however, the error reduces fast for smaller thickness and at $h = 0.2a$ it does not exceed 2%.

At the same time, it is seen from Table 1 that displacements of the plates made of anisotropic material differ greatly from the corresponding displacements for isotropic plates. Furthermore, great differences between the displacements of the points located at different depths of the same thickness plates are also observed. Maximal displacement of the loaded thick plate surface center ($h = 0.5a$), for example, is 5.8 times greater than the displacement of the loaded surface. The difference for the thin plate $h = 0.2a$ equals 30.8%. The main reason for the above mentioned differences is the influence of transversal contraction.

It should be noted that if $\nu'' = 0$, the obtained displacement of the lower surface center ($r = 0, z = h$) will correspond to maximal deflection of transversally isotropic plates, found by the theories of Reissner and Ambartsumyan. However, the plate deflections are considered to be similar all over the thickness.

When the clamping conditions (2.10) are satisfied on the plate boundary (Fig. 2b), the expressions for vertical displacement and normal stresses at the center of the plate equal

$$W(0,z) = \frac{qa^4}{64D}\left[1 + \frac{8(8G/G' + \nu'')}{5(1-\nu)}\frac{h^2}{a^2}\right] + \frac{\alpha_0 q}{E'}(z_0 - z) - \frac{q\nu'' a^2}{16(1-\nu)D}\left[z_0^2 + z^2 - 4(z_0^2 - z^2)\frac{8G/G' + \nu''}{5(1-\nu)}\frac{h^2}{a^2}\right] - \frac{\alpha_0 q}{8hE'}\left[6A_2(z_0^2 - z^2) - \frac{A_3}{h^2}(z_0^4 - z^4)\right]; \tag{3.2}$$

$$\sigma_{rr}(0,z) = \sigma_{\theta\theta}(0,z) = \frac{3qa^a z(1+\nu)}{32h^3}\left[1 + \frac{4h^2}{(1-\nu)a^2}\left(m_1 + m_2\frac{z^2}{h^2}\right)\right] - \frac{q\nu''}{2(1-\nu)}, \tag{3.3}$$

$$m_1 = -\frac{2G}{5G'} + \frac{\nu''}{5}\cdot\frac{11+\nu}{1+\nu}, \quad m_2 = \frac{2G}{3G'} - \nu''\frac{3+\nu}{3(1+\nu)}.$$

The results obtained by formulas (3.2), (3.3) at $\nu = \nu'' = 0.25$ for isotropic material (column 4) and transversally isotropic $\left(\frac{E}{E'} = 2, \frac{G}{G'} = 5\right)$ material (column 7) as well as by DS method [7] and by formulas of the Kirchoff's theory of thin plates for isotropic material and the Reissner's theory are compared in Table 2. Tabulated dimensionless normal stresses $\overline{\sigma}_{rr}(z) = \sigma_{rr}(0,z)/q$ are calculated at the upper face center ($r = 0, z = -h$), where they are maximal; it was accepted that parameter $z_0 = h$.

Data analysis shows that for the examined thickness $h/a = 0.2, 0.1$, the errors of the formula (3.2) for median surface of isotropic plate (column 4) do not exceed 1% in comparison with the exact values of DS method.

The errors of the Reissner's theory, which takes into account lateral shear only (column 5), are also negligible in the case. However, it is accepted in shear theories that vertical displacements of median surface are similar to displacements of other parallel surfaces, though, as the analysis of the results of Table 2 shows, they may greatly differ.

**Table 2. Tabulated displacement $\overline{W}(0)$ and stresses $-\sigma_{rr}(-h)$ for a clamped plate.**

| | $\frac{h}{a}$ | Isotropic material | | | | Transversally isotropic material | |
|---|---|---|---|---|---|---|---|
| | | DS method [8] | By Eqs. (3.2), (3.3) | Reissner's theory | Kirchoff's theory | By Eqs. (3.2) (3.3) | Reissner's theory |
| $\overline{W}(0)$ | 0.2 | 4.543 | 4.553 (0.22%) | 4.622 (1.74%) | 2.747 (39.5%) | 11.94 | 12.12 |
| | 0.1 | 25.61 | 25.56 (0.20%) | 25.72 (0.43%) | 21.97 (14.2%) | 40.47 | 40.72 |
| $-\sigma_{rr}(-h)$ | 0.2 | 3.631 | 3.410 (6.1%) | 3.130 (13.8%) | 2.930 (19.3%) | 4.076 | 3.130 |
| | 0.1 | 13.15 | 12.20 (7.2%) | 11.92 (9.3%) | 11.72 (10.9%) | 12.87 | 11.92 |

The errors of formula (3.3) for stress $\sigma_{rr}$ in isotropic plates equal 6.1% and 7.2%, correspondingly. The above mentioned formula gives underestimated values in comparison with the exact ones. Still more underestimated are the results of the Reissner's theory which do not take account of lateral normal deformation. As it is seen from Table 2 (columns 5, 8), both the Reissner's and Kirchhoff's theories do not pay sufficient attention to the effects of lateral anisotropy of the plate.

Thus, the generalized theory of plates indicates that transversally isotropic plates analysis should take into account not only the corrections for lateral shear deformation but also the corrections for lateral normal stresses and deformations. The latter ones may in some cases considerably exceed the shear corrections.

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