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DYNAMICAL FORCES INSIDE FACET JOINTS, INTERVERTEBRAL DISCS AND LIGAMENTS OF HUMAN CERVICAL SPINE

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The paper presents a dynamic spatial mathematical model in which a head, seven cervical vertebrae, a group of 19 couples of neck muscles, 6 intervertebral discs, ligaments and facet joints are taken into consideration. The created model enables simulation of dynamical forces inside anatomical parts of human neck. The behaviour of the modelled body exposed to action of a force corresponding to the real enforcement, which occurs at a head-on collision during the road accident was simulated, and the model was verified on the basis of the data obtained from published experiments.

1. Introduction

Model researches concerning human body behaviour under dynamical loading are carried out mainly for motor industry needs. It is connected with injury consequences very often experienced by people involved in car accidents. In the time of treat rise carried by modern motorization, safety of road users has become very important and it is reflected in the latest car constructions. However, the applied safety increasing systems are still imperfect. It can be seen in the car accidents injury statistics, which prove the necessity of further research in the field of occupant protection [1, 4, 5].

During car crashes the head and human cervical spine are the two parts of human body mostly exposed to injuries, the result of which could be permanent disability or even death. Movement of head directly depends on internal forces between cervical vertebrae. Experimenting on people is usually impossible because of its dangerous character. As a result, crash tests are carried out on dummies and cadavers. Another way to study the behaviour of the human body during car crashes is the mathematical modelling, using modern numerical methods [1, 5, 6].

2. Model assumptions

The elaborated mathematical model of cervical spine consists of a detailed dynamical models of cervical vertebrae with intervertebral disks, ligaments, neck muscles and facet joints and of a simplified model of neighbouring members – head and trunk. Scheme of the modelled system is presented in Fig. 1.

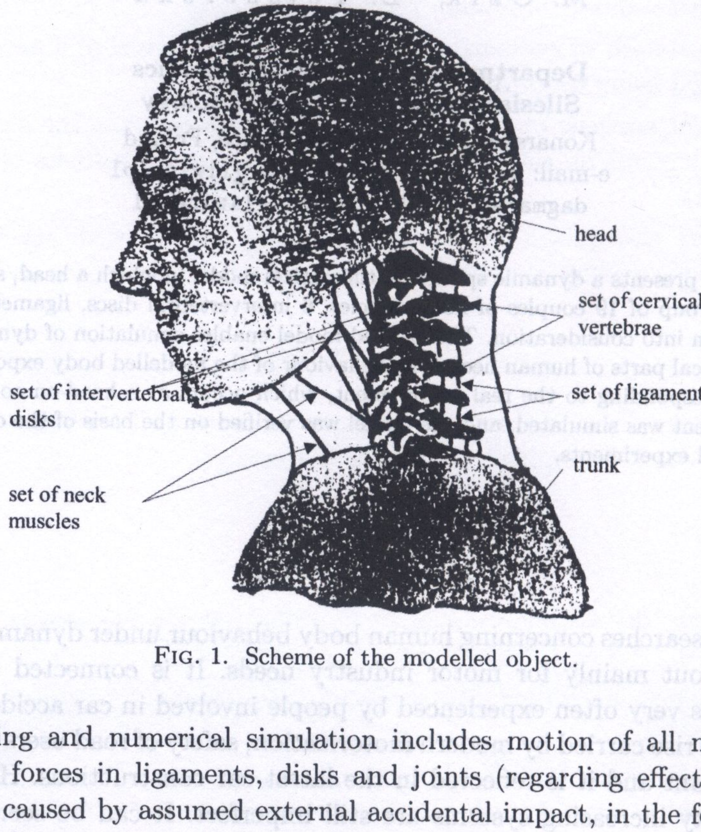

FIG. 1. Scheme of the modelled object.

Modelling and numerical simulation includes motion of all members and changes of forces in ligaments, disks and joints (regarding effects of possible fractures), caused by assumed external accidental impact, in the form of accelerations or forces, acting on certain points of body.

Basic detailed assumptions are listed below [2, 3, 5]:

- Vertebrae, head and trunk are treated as stiff bodies, with defined mass and inertial moments. All 6 degrees of freedom are considered, including possibility of some displacements in joints, in directions other than normal rotation, caused by elasticity or fracture of ligaments and cartilage, which may be very important when simulating short lasting shocks. The constraints, resulting from contact with neighbouring members, are determined as forces in intervertebral facet joints, ligaments and disks, depending on mutual position of members. Physical model of typical vertebrae is shown in Fig. 2, along with the used coordinate systems.

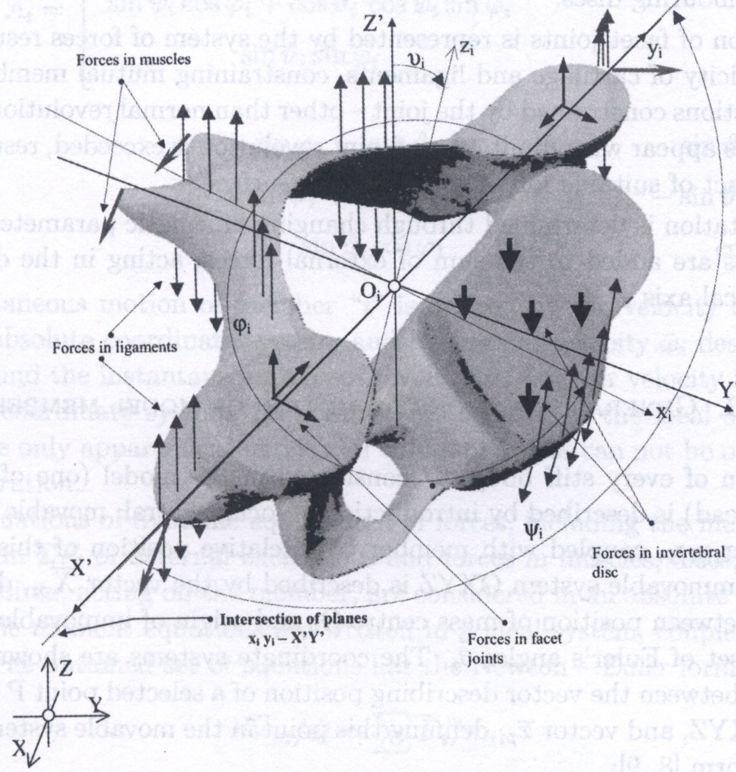

FIG. 2. Scheme of typical cervical vertebra with local and absolute coordinate system.

- Neck muscles are modelled as Hill's model, like elements with negligible mass, acting on connected members with compressive force and direction resulting from actual position of the conventional fixing points. Position of fixing points in local coordinate system is determined on the basis of Seireg's researches.
- Ligaments are modelled as weightless elements, acting on joined vertebrae with compressive forces, depending on actual elongation. Ligaments are divided in the model into several parallel strips, with independently calculated elongation. As a direction of forces were treated lines, connecting the conventional points of sticking of ligament strips to vertebral body. The dependence of force on elongation may be strongly nonlinear. The force is zero when shortening occurs.

- Intervertebral disks are treated as elastic elements, with mass added to neighbouring vertebrae. They are divided into segments, acting on vertebrae with forces depending on actual compression – change of distance between the centres of conventional points of sticking of segments to the neighbouring discs.
- Action of facet joints is represented by the system of forces resulting from elasticity of cartilage and ligaments, constraining mutual members, in the directions constrained by the joint – other than normal revolution. Additive forces appear when limitation of joint revolution is exceeded, resulting from contact of suitable forces with resisting surface.
- Excitation is determined through changing kinematic parameters. Gravity forces are added to the sum of external forces, acting in the direction of vertical axis Z.

3. General equations of motion of model member

Position of every stiff body "i" considered in the model (one of vertebrae, trunk or head) is described by introduction of local, central, movable coordinate system $O_i x_i y_i z_i$, coupled with member "i". Relative position of this system in absolute, immovable system OXYZ is described by the vector $\overline{X}_{oi}$, defining the distance between position of mass centre O_i and origin of immovable system O, and by a set of Euler's angles $\overline{\theta}_i$. The coordinate systems are shown in Fig. 2. Relations between the vector describing position of a selected point P in absolute system OXYZ, and vector $\overline{x}_{pi}$ defining this point in the movable system $O_i x_i y_i z_i$, have the form [8, 9]:

$$\overline{X}_p = \overline{\overline{A}}_i \cdot \overline{x}_{pi} + \overline{X}_{oi}, \qquad \overline{x}_{pi} = \overline{\overline{A}}_i^{\,-1} \cdot \left(\overline{X}_p - \overline{X}_{oi}\right), \tag{3.1}$$

where:

$$\overline{\overline{A}}_i^{\,-1} = \overline{\overline{A}}_i^{\,T}$$

and

$$\overline{X}_{oi} = [X_{oi},\, Y_{oi},\, Z_{oi}]^T ,$$

$$\overline{X}_p = [X_p,\, Y_p,\, Z_p]^T ,$$

$$\overline{x}_{pi} = [x_{pi},\, y_{pi},\, z_{pi}]^T .$$

Elements of the rotation matrix $\overline{\overline{A}}_i$ are equal to cosines of the angles formed by axes of movable and absolute system:

$$\overline{\overline{A}}_i = \begin{bmatrix} \cos\psi_i\cos\varphi_i - \cos\vartheta_i\sin\psi_i\sin\phi_1 & -\cos\psi_i\sin\varphi_i - \cos\vartheta_i\sin\psi_1\cos\varphi_i & \sin\vartheta_1\sin\psi_1 \\ \sin\psi_i\cos\varphi_i + \cos\vartheta_i\cos\psi_i\sin\varphi_i & -\sin\psi_i\sin\phi_1 + \cos\vartheta_1\cos\psi_1\cos\varphi_1 & -\sin\vartheta_1\cos\psi_1 \\ \sin\vartheta_i\sin\varphi_i & \sin\vartheta_i\cos\varphi_1 & \cos\vartheta_1 \end{bmatrix}. \tag{3.2}$$

Instantaneous motion of member "i" is defined by the velocity of its mass centre in absolute coordinate system, and by angular velocity $\bar{\omega}_i$ describing rotation around the instantaneous axis of revolution. Angular velocity is analysed in a local coordinate system. Its components on axes of the local system ω_{xi}, ω_{yi}, ω_{zi} are only apparent velocities and values of angles can not be obtained by their integration.

The equations of dynamic equilibrium of forces, including the inertial forces and the sum ΣF_i of external excitations and forces in muscles, discs, ligaments and facet joints, acting on the member, are considered in an absolute coordinate system. The moment equations are written in a local system, coupled with the member. The obtained set of equations has the Newton – Euler form [7, 8]:

$$\begin{aligned} d\overline{V}_{oi}/dt &= \sum \overline{F}_i/m_i, \\ d\overline{\omega}_i/dt &= \overline{\overline{T}}_i \cdot \Big\{ -\overline{\omega}_i \times \Big(\overline{\overline{T}}_i \cdot \overline{\omega}_i\Big) + \sum \overline{M}_i \Big\}, \end{aligned} \tag{3.3}$$

where:

$$\overline{V}_{oi} = [V_{Xoi}, V_{Yoi}, V_{Zoi}]^T,$$

$$\overline{\omega}_i = [\omega_{xi}, \omega_{yi}, \omega_{zi}],$$

$$\sum \overline{F}_i = \Big[\sum F_{Xi}, \sum F_{Yi}, \sum F_{Zi}\Big]^T,$$

$$\sum \overline{M}_i = \Big[\sum M_{xi}, \sum M_{yi}, \sum M_{zi}\Big]^T.$$

The system of equations, determining the time course of 12 parameters (V_{Xoi},... ω_{xi},... X_{oi},...υ_i, ψ_i, φ_i), defining linear and angular displacements and velocities of member "i", consists of six relations, describing equilibrium of forces and moments, and of six kinematic formulas (3). Complete system of nonlinear ordinary equations for the whole modelled system can be written in a general matrix form, assuming, that forces in muscles, discs, ligaments and facet joints depend on displacements of the connected members.

$$\frac{d\overline{X}}{dt} = \Phi\left(\overline{X}, \overline{F}_{\text{external}}\right), \qquad \overline{X} = \left[\overline{X}_1, ..., \overline{X}_i, ...\right]^T, \tag{3.4}$$

where

$$\overline{X}_i = \left[V_{Xoi}, V_{Yoi}, V_{Zoi}, \omega_{xi}, \omega_{yi}, \omega_{zi}, X_{oi}, Y_{oi}, Z_{oi}, \upsilon_i, \psi_i, \varphi_i\right]^T$$

4. Model of intervertebral disk

Intervertebral disc is represented by five forces (Fig. 3).

- 1 – force of nucleus, pulposus,
- 2 – 5 – forces of anulus fibrosus.

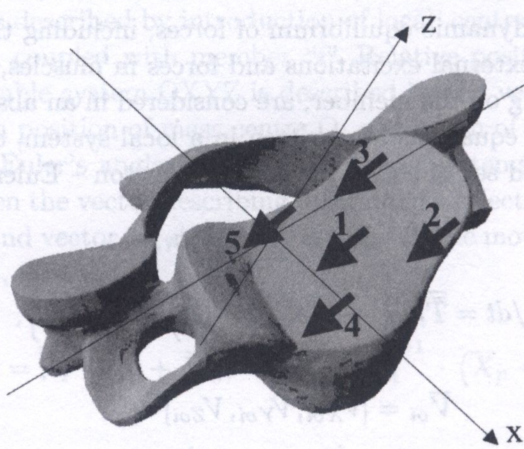

Fig. 3. Model of intervertebral disc.

Formulation of relations between the actual position of the members and forces acting on them is the most arduous part of elaboration of the set of model equations (3.4). It consists of determination of the positions of adequate points of cervical members in absolute coordinate system, calculation of their mutual displacement and components of the resulting elastic forces, and evaluation of moments in local coordinate system. The forces in disc depend on actual elasticity constant C_D, being assumed a function of elongation. The method of determination of the components of forces in absolute coordinate system are described by the relation:

$$\begin{pmatrix} X_{D(i+1)d} \\ Y_{D(i+1)d} \\ Z_{D(i+1)d} \end{pmatrix} = \overline{\overline{A}}_{i+1} \cdot \begin{pmatrix} X_{D(i+1)d} \\ Y_{D(i+1)d} \\ Z_{D(i+1)d} \end{pmatrix} + \begin{pmatrix} X_D(i+1) \\ Y_D(i+1) \\ Z_D(i+1) \end{pmatrix},$$

$$\begin{pmatrix} X_{Did} \\ Y_{Did} \\ Z_{Did} \end{pmatrix} = \overline{\overline{A}}_{i} \cdot \begin{pmatrix} X_{Did} \\ Y_{Did} \\ Z_{Did} \end{pmatrix} + \begin{pmatrix} X_{oi} \\ Y_{oi} \\ Z_{oi} \end{pmatrix},$$

$$L_{Did} = \sqrt{\left(X_{D(i+1)} - X_{Did}\right)^2 + \left(Y_{D(i+1)d} - Y_{Did}\right)^2 + \left(Z_{D(i+1)} - Z_{Did}\right)^2}, \tag{4.1}$$

$$\Delta L_{Did} = L_{Did} - L_{Did,0},$$

$$\begin{pmatrix} F_{Did,X} \\ F_{Did,Y} \\ F_{Did,Z} \end{pmatrix} = \begin{pmatrix} \left(X_{D(i+1)d} - X_{Did}\right) \cdot \Delta L_{Did} \cdot C_D d / L_{Did} \\ \left(Y_{D(i+1)d} - Y_{Did}\right) \cdot \Delta L_{Did} \cdot C_D d / L_{Did} \\ \left(Z_{D(i+1)d} - Z_{Did}\right) \cdot \Delta L_{Did} \cdot C_D d / L_{Did} \end{pmatrix},$$

$$\{F_{Did}\} = [C_D] \cdot \{\Delta X_{Did}\}.$$

where:

- A_i – rotation matrix,
- X_D, Y_D, Z_D – coordinates of characteristic disc points in the absolute system,
- x_D, y_D, z_D – coordinates of characteristic disc points in the local system,
- $\{F_{Did}\}$ – matrix of disc forces,
- $\{\Delta X_{Did}\}$ – matrix of unit elongation,
- $[C_D]$ – matrix of disc stiffness.

5. Matrix of equations described forces of facet joints

Facet joint is represented by the system of forces F_{Sis} (5.1), resulting from elasticity of the cartilage and ligaments, constraining mutual displacement of the joined members, nonlinearly dependent on mutual displacements of the members, in the directions constrained by the joint – other than normal revolution. Additive forces appear (M_{Siop}) when limitation of joint the revolution is exceeded, resulting from contact of the suitable forces with the resisting surface.

$$\begin{aligned}
&\{X_{S(i+1)s}\} = [A_{(i+1)}] \cdot \{x_{S(i+1)s}\} + \{X_{0(i+1)}\}, \\
&\{X_{Sis}\} = [A_i] \cdot \{X_{Sis}\} + \{X_{0i}\}, \\
&\{L_{Sis}\} = \{X_{S(i+1)s}\} - \{X_{Sis}\}, \\
&\Delta L = |\bar{L}_{Sis}| - |\bar{L}_{0Sis}|, \\
&\{F_{Sis}\} = [C_S] \cdot \{\Delta X_{Sis}\},
\end{aligned} \tag{5.1}$$

where

$\{F_{sis}\}$ – matrix of facet joint forces,

$\{\Delta X_{sis}\}$ – matrix of unit elongation,

$[C_S]$ – matrix of facet joint stiffness,

$$M_{Siop} = C(A + B^{\vartheta\mathrm{ext}}), \tag{5.2}$$

where:

$C = 1$ [Nm],

$A = 50$, $B = 5$ [dimensionless number],

M_{Siop} – resisting moment [Nm],

ϑext – nutation angle exceeded [dimensionless number].

6. Equations of intervertebral ligaments forces

Ligaments which connect the vertebrae (Fig.4.) are modelled as weightless elements, acting on joined vertebrae with forces depending on actual elongation. Wider ligaments are divided in the model into parallel strips, with independently calculated elongations.

- ligamentum longitudinale anterius – divided into five ($k = 5$) parallel strips,
- ligamenta intertransversaria, ligamentum nuchae, ligamenta flava – divided into three ($k = 3$) parallel strips.

$$\begin{bmatrix} x_{Wiw,k} \\ y_{Wiw,k} \\ z_{Wiw,k} \end{bmatrix} = \begin{bmatrix} x_{Wiw,k,p} + \Delta x_{Wiw} \cdot (2k - 1) / n_{Wiw} \\ y_{Wiw,k,p} + \Delta y_{Wiw} \cdot (2k - 1) / n_{Wiw} \\ z_{Wiw,k,p} + \Delta z_{Wiw} \cdot (2k - 1) / n_{Wiw} \end{bmatrix}. \tag{6.1}$$

As the directions of forces were treated the lines, connecting the conventional points of sticking of ligament strips to the vertebrae. The force is zero when shortening occurs.

$$
\begin{aligned}
\{X_{W(i+1)w}\} &= [A_{(i+1)}] \cdot \{x_{W(i+1)w}\} + \{X_{0(i+1)}\}, \\
\{X_{Wiw}\} &= [A_i] \cdot \{x_{Wiw}\} + \{X_{0i}\}, \\
\{L_{Wiw}\} &= \{X_{W(i+1)w}\} - \{X_{Wiw}\}, \\
\Delta L &= |\bar{L}_{Wiw}| - |\bar{L}_{0Wiw}|, \\
\{F_{Wiw}\} &= [C_W] \cdot \{\Delta X_{Wiw}\},
\end{aligned}
\tag{6.2}
$$

where

- $\{F_{wiw}\}$ – matrix of ligaments forces,
- $\{\Delta X_{wiw}\}$ – matrix unit elongation of ligaments,
- $[C_W]$ – matrix of ligament stiffness.

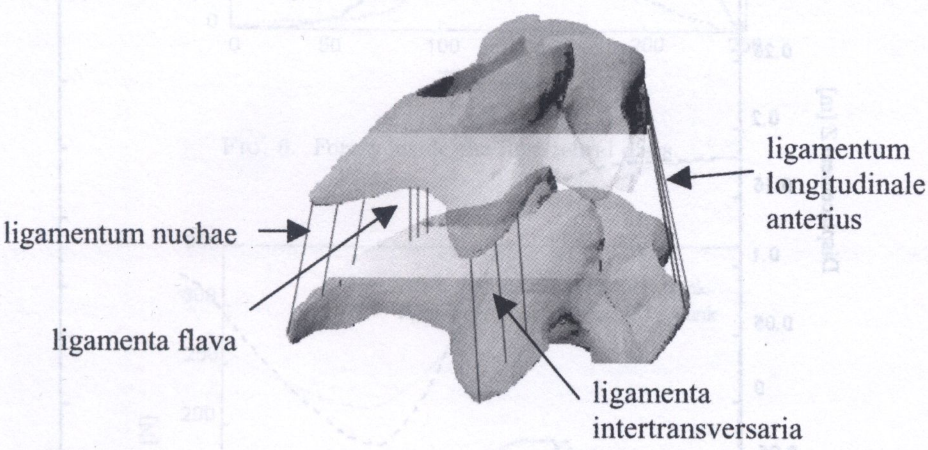

FIG. 4. Model of two vertebrae C4 and C5 connected by intervertebral ligaments.

7. Verification of model

The model was verified by comparing displacements of the gravity head centre obtained from numerical simulation and tests conducted on volunteers by the Naval Biodynamics Laboratory in New Orleans Fig. 5.

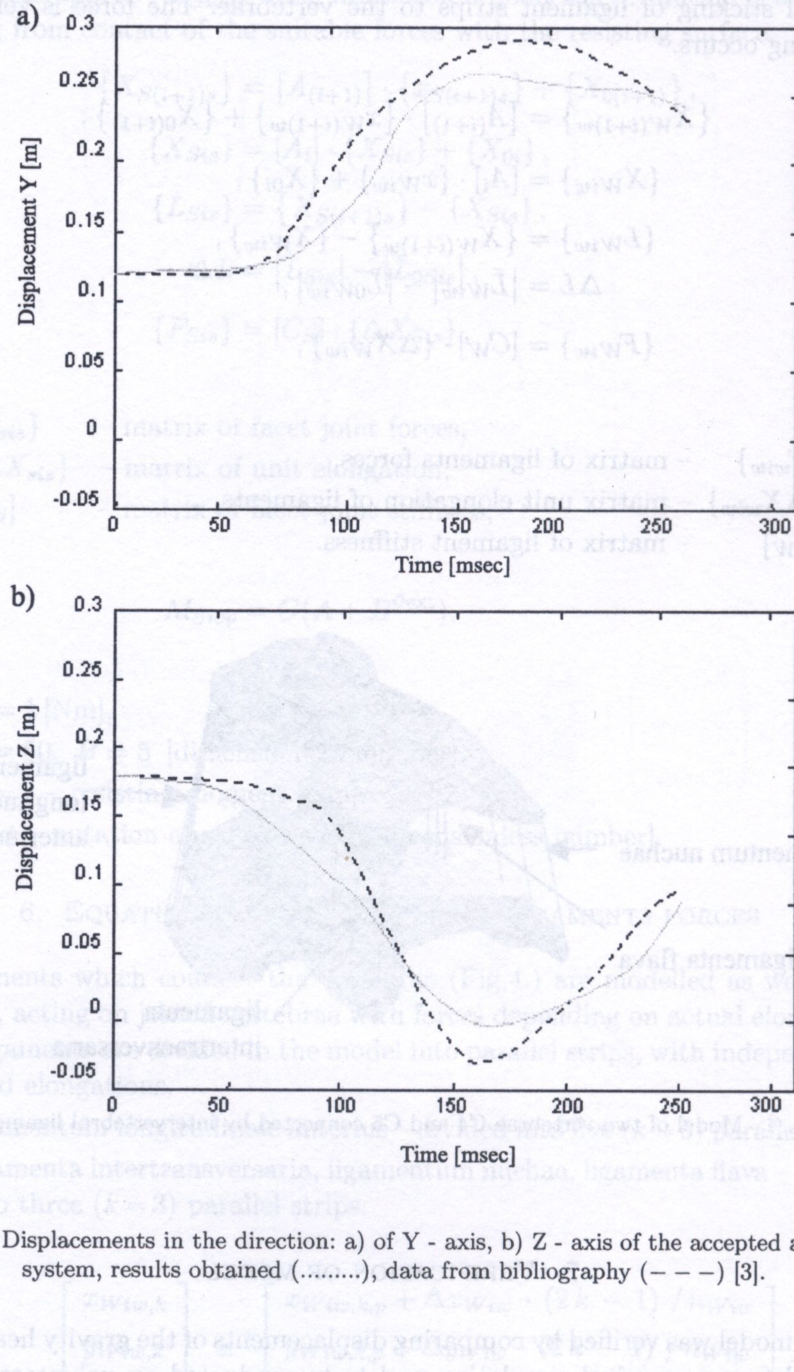

FIG. 5. Displacements in the direction: a) of Y - axis, b) Z - axis of the accepted absolute system, results obtained (.........), data from bibliography (– – –) [3].

8. Results of numerical simulation

The model presented can provide information about the forces inside the intervertebral facet joints, discs and ligaments, in situations corresponding to real head-on collision. Examples of dynamical forces inside the anatomical parts taken into consideration of human neck are presented below.

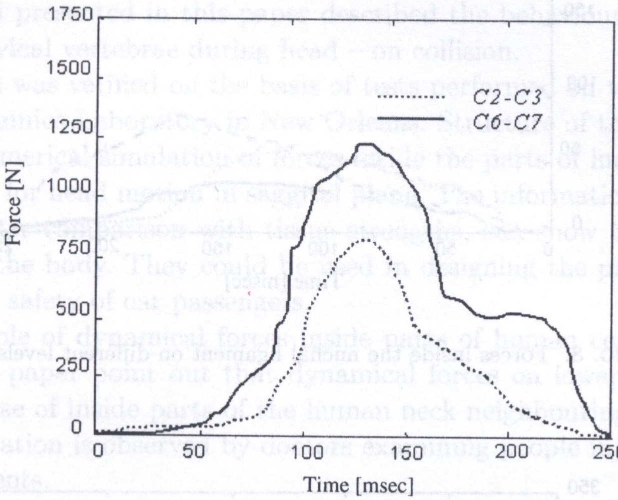

Fig. 6. Forces inside the invertebral discs.

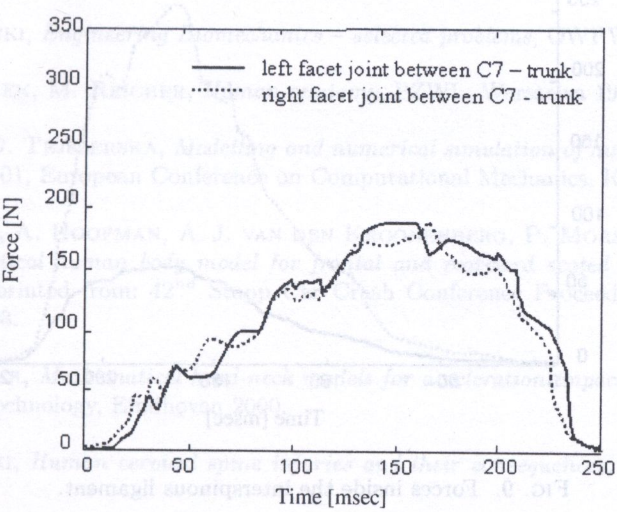

Fig. 7. Forces inside the invertebral facet joints.

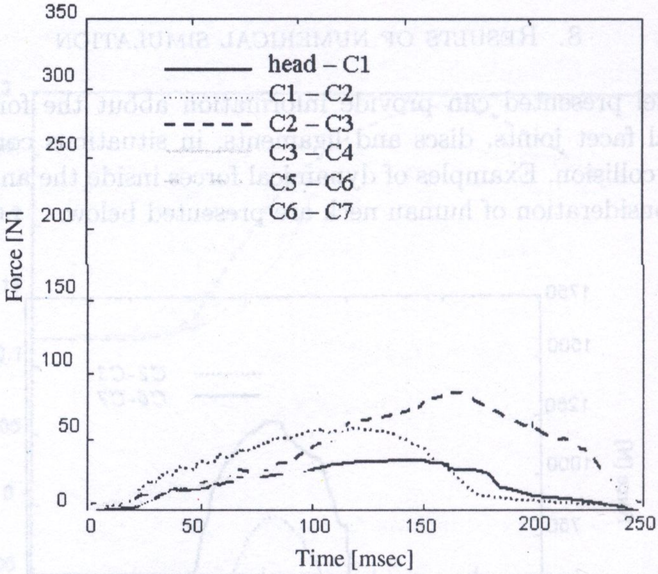

FIG. 8. Forces inside the nuchal ligament on different levels.

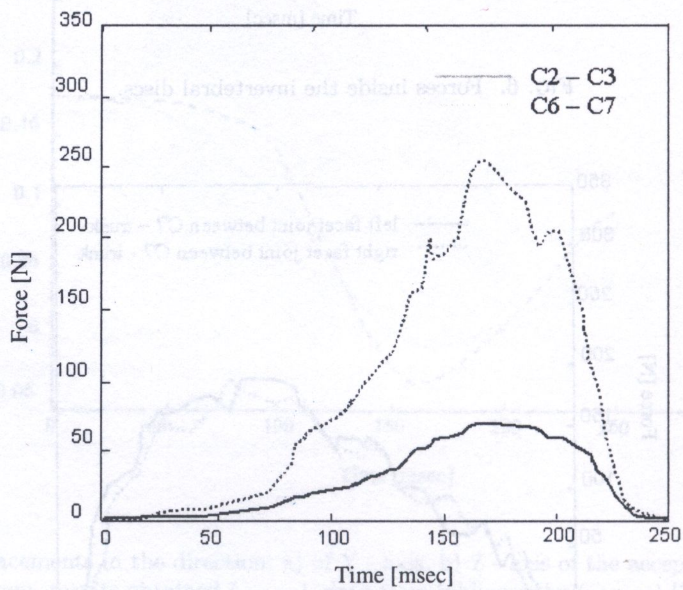

FIG. 9. Forces inside the interspinous ligament.

For the frontal impact, the forces on lower level are larger than in the case of internal parts of the human neck neighbouring with the head. The same situation is observed by doctors examining people injured during low velocity accidents.

9. Conclusion

The model presented in this paper described the behaviour of human head and seven cervical vertebrae during head – on collision.

The model was verified on the basis of tests performed on volunteers by the Naval Biodynamics Laboratory in New Orleans. Structure of the model enables to conduct numerical simulation of forces inside the parts of human neck which are important for head motion in saggital plane. The information obtained from simulation, after comparison with tissue strengths, can show the most vulnerable parts of the body. They could be used in designing the process of devices improving the safety of car passengers.

The example of dynamical forces inside parts of human cervical spine presented in this paper point out that dynamical forces on lower level are larger than in the case of inside parts of the human neck neighbouring with the head. The same situation is observed by doctors examining people injured during low velocity accidents.

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